Submission Date: 01st November 2023

1 Linear Regression

The table below gives information about individual medical costs billed by a public health insurance company. The two deciding parameters to calculate the costs are the Body Mass Index 'BMI' and 'Age' of the person.
BMI is the ratio of the weight of the person to body height expressed as kg/m^2.

Age	**BMI**	**Charges**
18	53.13	1163.43
58	49.06	11381.33
23	17.38	2775
45	21	7222
63	21.66	14349
36	28.59	6548

Use Linear Regression to solve for the parameters. Do not include the bias term. Use the closed form solution. Further calculate the changes for a person with **Age** = 40, **BMI** = 32.5. Use Python.

2 Logistic Regression

1. Starting from negative loglikelihood (binary cross entropy loss), derive the update rule for w for gradient descent. Ignore the bias term for the moment.

   Negative loglikelihood:

   $$J = -\sum_{n=1}^{N} y_n \log p_n + (1 - y_n) \log(1 - p_n) \tag{1}$$

   with $p_n = h_{\boldsymbol{w}}(\boldsymbol{x}_n) = P(y = 1|X = \boldsymbol{x}_n; \boldsymbol{w}) = \sigma(\boldsymbol{x}_n)$

   For sigmoid use

   $$\sigma(\boldsymbol{x}_n) = \frac{e^{\boldsymbol{x}_n \boldsymbol{w}}}{1 + e^{\boldsymbol{x}_n \boldsymbol{w}}} \tag{2}$$

   which is equivalent to $\frac{1}{1+e^{-\boldsymbol{x}_n \boldsymbol{w}}}$ (try it)

2. Perform one gradient descent update step for using the initial parameters as $\boldsymbol{w}^{(0)} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T$ and learning rate $\alpha = 0.5$ using the following data:

x_1	x_2	y
2	4	1
3	3	1
-4	-2	0
-2	-6	0

Note: Think about a good way to include the bias in the data instead of deriving its update rule on its own.

Predict $P(y = 1|X = \begin{bmatrix}-1 & 1\end{bmatrix})$