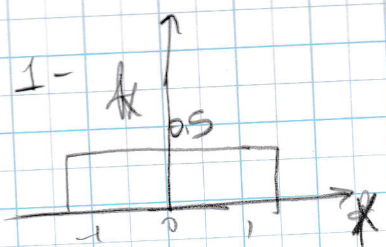
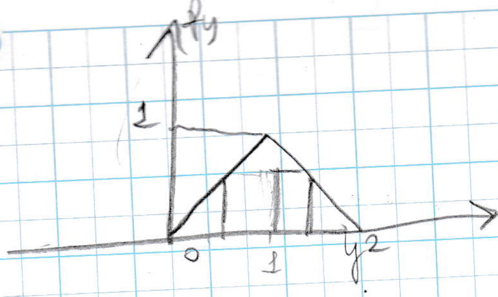
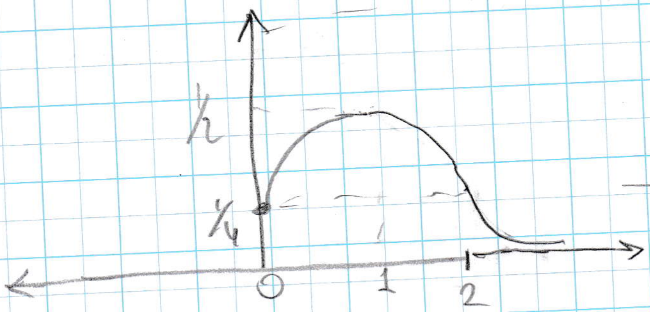
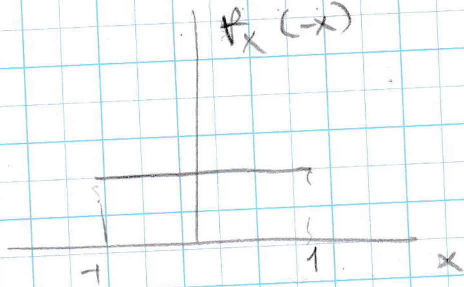
1-





f_z :





$$(f*g)(0) = \int_0^1 0.5 \cdot x \, dx = \frac{x^2}{4}\Big|_0^1 = \frac{1}{4}$$

$$(f*g)(2) = \frac{1}{4}$$

$$(f*g)(3) = 0$$

$$(f*g)(1) = \int_0^1 0.5 \cdot x \, dx + \int_1^2 0.5 \cdot 2 - x \, dx$$

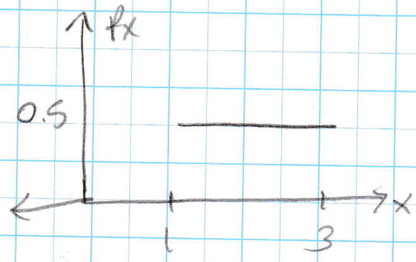
$$1 + \left(-\frac{x^2}{4}\Big|_1^2\right)$$

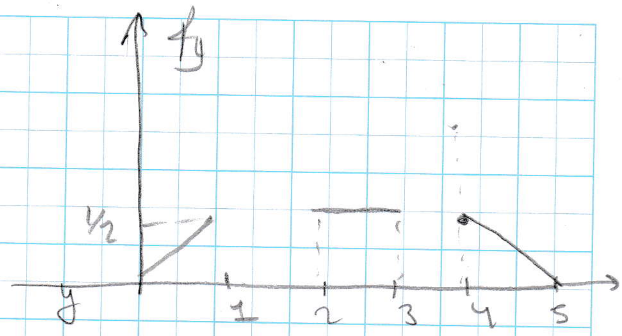
$$= 1 - 1 + \frac{1}{4} = \frac{1}{4}$$

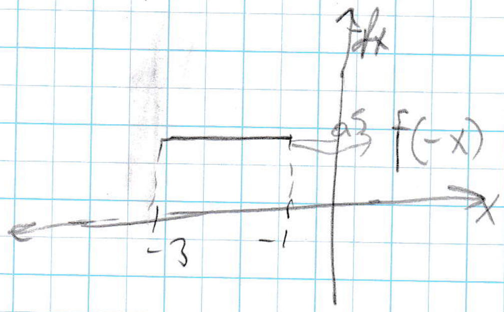
$$= \frac{1}{4} + \frac{1}{4}$$

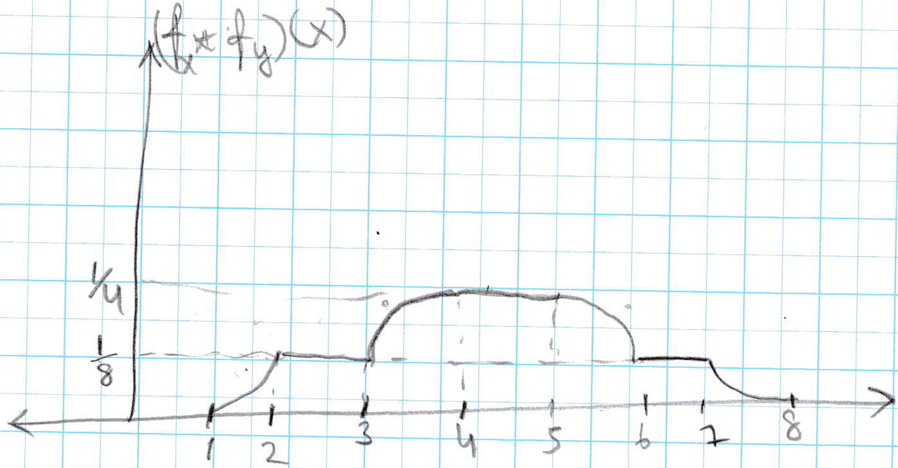
$$= \frac{1}{2}$$

2-









$(f_x * f_y)(2) = \int_0^1 0{,}5 \cdot x/2 \, dx = \frac{x^2}{8}\Big|_0^1 = \frac{1}{8}$

$7 \cdot \frac{1}{4} - \frac{6}{8} = \frac{14-6}{8} = 1$ ✓

$(f_x * f_y)(4) = \int_2^3 0{,}5 \cdot 0{,}5 \, dx = \frac{1}{4}$

Altında kalan alan 1

3-

$$P(X \geq a) \leq \frac{E[X]}{a}$$

let $T = 100 - X$

$$P(T \geq 90) \leq \frac{E[T]}{90}$$

$$E[T] = 100 - E[X]$$

$$E[T] = 100 - 40 = 60$$

$$\cancel{*} \quad P(100 - X \geq 90) = \frac{6}{9} = \frac{2}{3}$$

$$P(100 - X \geq 90) = P(X \leq 10)$$

$$P(-X \geq -10)$$

$$P(X \leq 10)$$

4-

$$P(|X-\mu|^2 \geq c^2) \leq \frac{E[(X-\mu)^2]}{c^2}$$

$$P(|X-\mu| \geq c) \leq \frac{\sigma^2}{c^2}$$

$$E[X] = \sum_{i=1}^{100} E[m] = E\left(\sum_{i=1}^{100} m\right) = 100 \cdot 3{,}5 = 350$$

iid olduklarından

m: Bir zarın beklenen değeri (sonucunun)

$$\text{var}(X) = \sum_{i=1}^{100} \text{var}(m)$$

iid oldukları için

$$\text{var}(m) = E[X^2] - E[X]^2$$

$$= \frac{91}{6} - \frac{49}{4}$$

$$P(|X - 350| \geq 50) \leq \frac{100(2{,}91)}{50 \cdot 50}$$

$$P(|X - 350| \geq 50) \leq 0{,}11$$

13:59 16 23 36

1-

$$\underset{p}{\text{argmax}} \; P(x_1, x_2, \dots x_n ; p)$$

$$\underset{p}{\text{argmax}} \prod_{i=1}^{n} f(x_i ; p) = \underset{p}{\text{argmax}} \prod_{i=1}^{n} p(1-p)^{k-1}$$

$$\underset{p}{\text{argmax}} \; p^n (1-p)^{\Sigma k - n}$$

$$0 = n p^{n-1} (1-p)^{\Sigma k - n} + p^n \left[(\Sigma k - n)(1-p)^{\Sigma k - n - 1} (-1) \right)$$

$$0 = p^{n-1} (1-p)^{\Sigma k - n - 1} \left(n(1-p) + p(n - \Sigma k) \right)$$

$$p : 1, 0, \frac{n}{\Sigma k}$$

$$n - \cancel{np} + \cancel{n}p - p \Sigma k = 0$$

$$p = \frac{n}{\Sigma k}$$

2-

Sample mean $= \frac{1}{n} \Sigma X_i = 168,83$

$S_n^2 = \frac{1}{n-1} \left(\Sigma (x - E[x])^2 \right) \rightarrow m_n$ kullaracağız

Dağılımını bilmiyoruz Yeterli

n için $m_n \cong E[x]$

$$= \frac{1}{6} \left(4 + 6,6 + 0,6 + 100 + 0 + 108 + 3 \right) = \frac{222}{6} = 37$$

std $= \sqrt{S_n^2} = 6,9$

3-

a) $Z_n = \dfrac{34,5 - \mu}{\sqrt{\dfrac{55,21}{144}}} = \dfrac{34,5 - \mu}{0,6} \sim N(0,1)$

$\Phi(z) = 0,05 \longrightarrow 1,65$

$z \in [-1,65 \, , \, 1,65]$

$\mu \in [\, 0,6(-1,65) + 34,5 \, , \, 0,6(1,65) + 34,5]$

$\mu \in [\, 33,5 \, , \, 35,5]$

b)

$Z_n = \dfrac{16,7 - \mu}{\sqrt{\dfrac{7,5}{16}}} \sim N(0,1) = \dfrac{16,7 - \mu}{0,68}$

Burada $\Psi_{15}(Z) = 0,025$ kullanıyorum normal değilim olduğu için

$z \in [-2,131 \, , \, 2,131]$

$\mu \in [\, 0,68 \cdot (-2,131) + 16,7 \, , \, 0,68(-2,131) + 16,7]$

4-

a) $P(X > \varepsilon \mid \lambda = 23) = 0,1$

$P(\lambda e^{-\lambda x} > \varepsilon \mid \lambda = 23) = 0,1$

$$0,1 = \int_{\varepsilon}^{\infty} 23 e^{-23x} \, dx$$

$$-e^{-23x} \Big|_{\varepsilon}^{\infty} = e^{-23\varepsilon} = 0,1$$

$\varepsilon = \ln(0,1) \, (-23) \cong 53$

b) $P(X < \varepsilon^{53} \mid \lambda = 1) = ?$

$P(e^{-1x} (53)) = ?$

$$\int_{-\infty}^{53} e^{-x} \, dx = -e^{-x} \Big|_{53}^{-\infty}$$

$$= e^{-53} = 9,6 \cdot 10^{-24}$$

0'a çok yakın