This document is prepared to provide the necessary descriptions and mathematical model of the reaction wheel controller that is designed in Simulink.

1. Background

Without delving into much detail, reaction wheels can be defined as actuators that are in use for the attitude control of satellites. This control is in terms of the angular position of the satellite with respect to its inertial reference frame. Most satellites require a certain pointing requirement for the mission's success. Reaction wheels basically maintain that accuracy.

Typically, there are two main components involved in the design of a reaction wheel: a DC motor and a flywheel. While the DC motor provides the angular rotation, the attached flywheel generates the required control torque. This torque generation process should be regulated according to the mission requirements and space environment of the satellite's operational orbit.

Normally, mission requirements and external (and internal) disturbance sources dictate the hardware design of the reaction wheel. Then, a control model is prepared according to the hardware.

This model was prepared for the prototype of StrathCUBE, the CubeSat project of the StrathAIS from the University of Strathclyde. Since this is a starting point for the development of the AOCS, only one axis control model was considered here.

2. Mathematical Model of the System

Mathematical modelling starts with an understanding of the expected input and output signals from the system and the involved components in the system. Since the reaction wheel operates to generate torque, naturally model would be about the control of the generated torque by the reaction wheel.

As mentioned, our prototype consists of a BLDC motor and a flywheel, however model was prepared by using a DC motor for simplicity. Equations involved in the mathematical model can be examined in three parts: controller, DC motor, and satellite dynamics.

- **Controller**

$$\boldsymbol{\theta_r - \theta_a = \theta_e} \quad (1)$$

$$\boldsymbol{T_c = K_p * \theta_e + K_d * \dot{\theta}_e + K_i * \int \theta_e dt} \quad (2)$$

Where,

$\boldsymbol{\theta_r}$ is the reference angular position, $\boldsymbol{\theta_a}$ is the actual position, $\boldsymbol{\theta_e}$ is the error signal of the system.

PID controller is selected in this model which is represented by the equation 2. This equation takes the error signal and generates the control torque to compensate this error.

- **DC Motor**

  To generate torque we need inertia, which comes from the flywheel, and rotational velocity which comes from the electric motor. Ergo proper modelling for the motor is crucial.

  Since the PID produces a control torque signal, we should start from here for this section:

$$\boldsymbol{T_c = K_m * i_c} \quad (3)$$

  Where K_m is the torque constant and i_c is the control current.

$$\boldsymbol{V - V_b = R_m * i_m + L * \frac{di_m}{dt}} \quad (4)$$

$$\boldsymbol{V_b = K_b * w = K_b * (w_{rw} - w_{sc})} \quad (5)$$

  Where V is the voltage, V_b is the back emf, Rm is the terminal resistance, i_m is the motor current, L is the inductance, K_b is the back emf constant, w is the relative rotational velocity, w_{rw} is the rotational velocity of the reaction wheel and w_{sc} is the rotational velocity of the spacecraft.

$$\boldsymbol{T_r = i_m * K_m} \quad (6)$$

  Here T_r is the generated reaction torque.

- **Satellite Dynamics**

$$\boldsymbol{T = I * \ddot{\theta}} \quad (7)$$

  Torque is the product of the inertia and rotational acceleration values. Based on this, since we get the reaction torque earlier, we can calculate how fast the spacecraft and flywheel rotate. The integration of rotation speed would give us the actual position.

It should be noted, this is only the basic modelling for the reaction wheel case.

3. Simulink Design

Figure 1 demonstrates the basic model for the one-axis controller. The flow of the model progresses as explained in the mathematical model section. Figure 2 shows the structure of the subsystem called the RW Model. This is the modelling for the DC motor and torque generation.

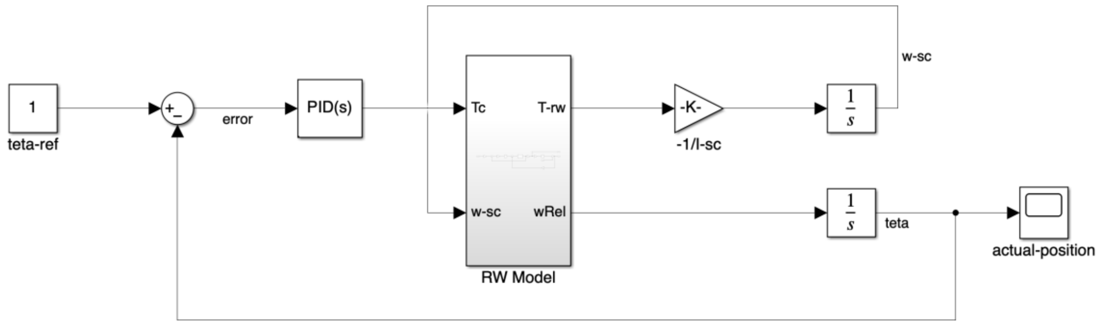

Figure 1-Reaction Wheel Simulink Model

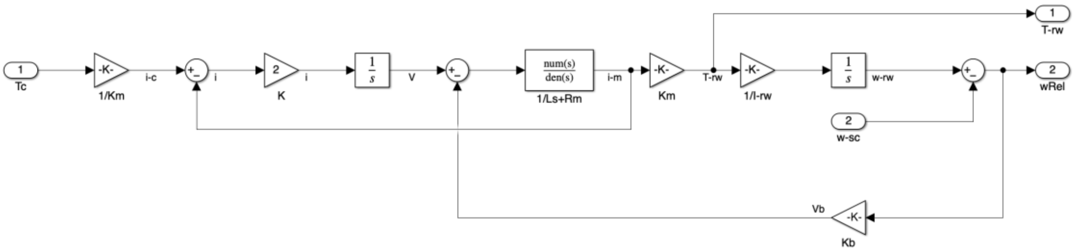

Figure 2-DC Motor Model

It is worth noting that the provided angle values in the model are in radians, not degrees.

The inertia values for the simulation are directly taken from the prototype models of the StrathCUBE team. However, the parameters of the DC motor were selected based on a literature review.

Terminal Resistance (ohm)	0.199
Terminal Inductance (H)	0.000113
Torque Constant (Nm/A)	0.0217
Back emf constant (Vs/rad)	0.0219

4. Results

The simulation run for the following PID gains:

- P=0.01
- I=0.01
- D=0.0025

Literature research showed that these are some typical gain values for reaction wheel applications except the integration gain. It was observed that bigger integration values disrupt the simulation results. This could be investigated in future studies.

Figure 3 demonstrates the simulation result which clearly shows that the model achieves its given reference angle in a short time. Figure 4 shows the result of the PD controller. It is possible to say that there is almost no change between these two models.

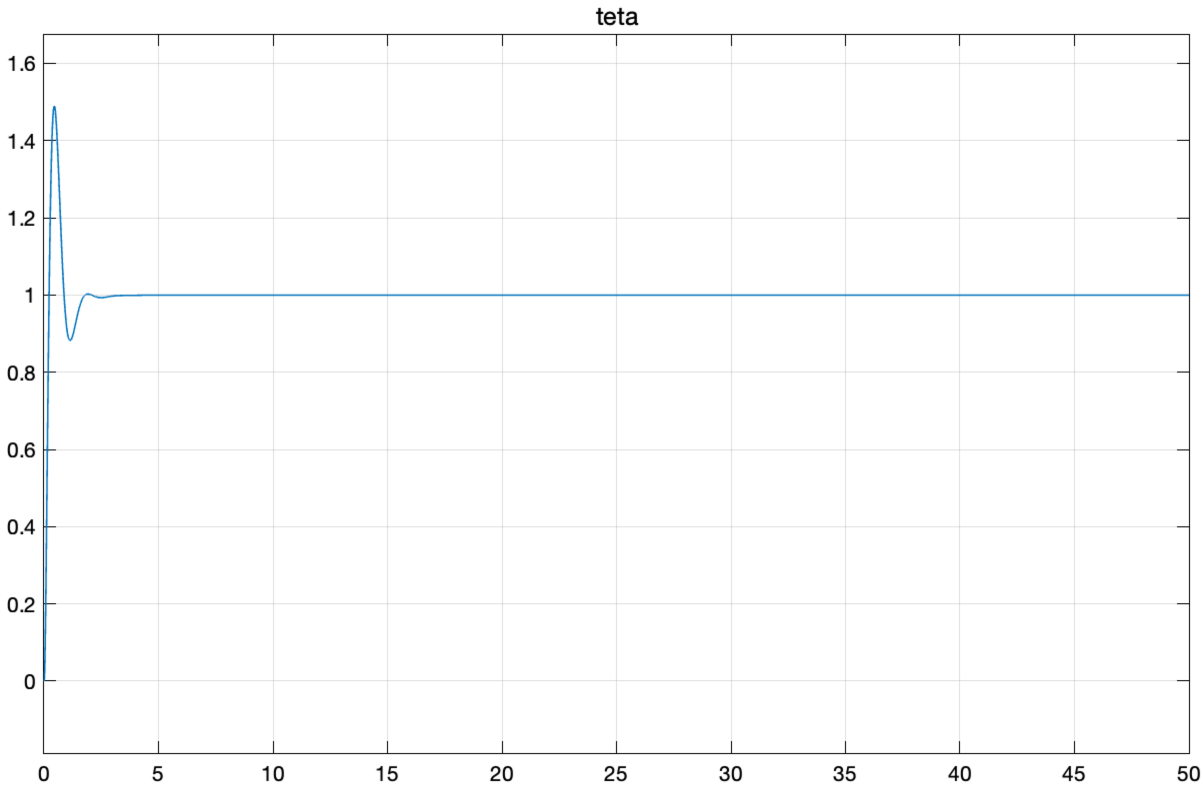

Figure 3-PID Simulation Result

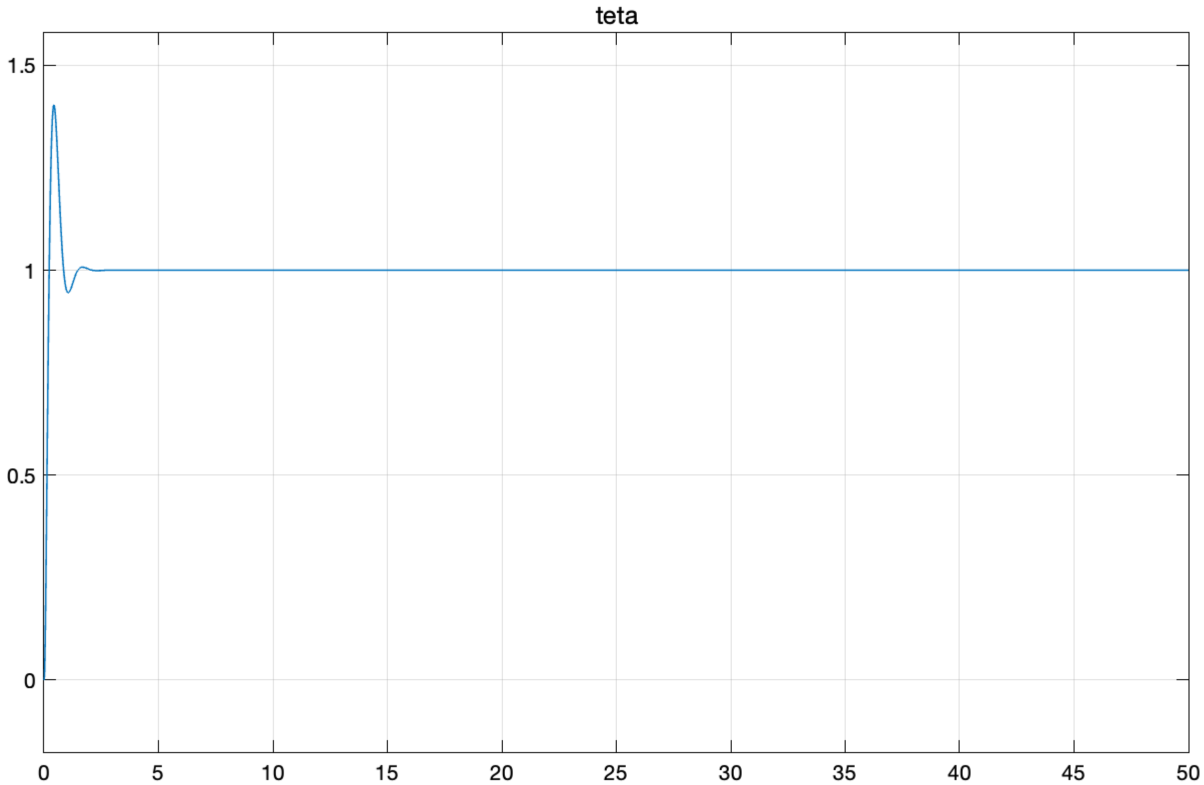

Figure 4-PD Simulation