



**ELECTRONIC ENGINEERING
DEPARTMENT**

MATH 214 NUMERICAL METHODS

2020 – 2021 FALL

PROJECT 2

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I. The Goal of The Project

The goal of this project is calculating the impressed voltage in a RL circuit.

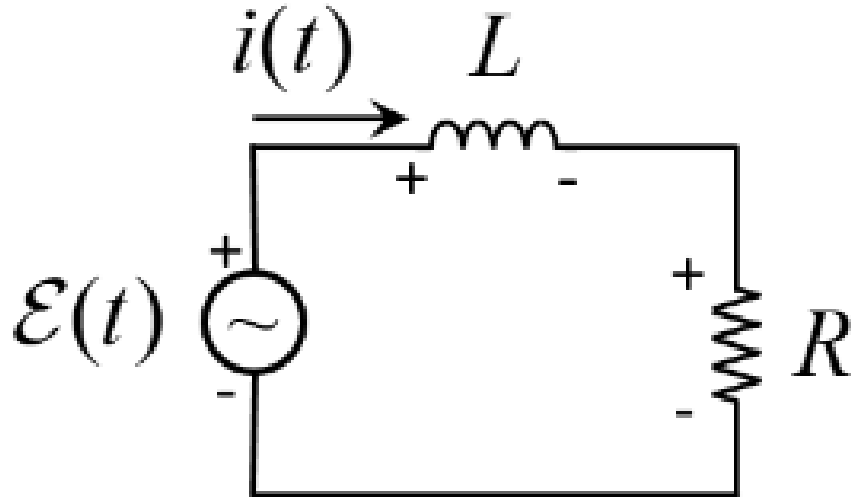


Figure 1: RL Circuit

The schematic representation of RL circuit is given at Figure 1. in our example, $L=0.98$ H and $R=14.2$ Ω values are given. Since the source is an alternating voltage source, the current is changing relative to time, therefore the ϵ is changing too.

The formula of ϵ is:

$$\mathcal{E}(t) = L \frac{d}{dt} i(t) + i(t)R$$

Current values were measured at different data sets as shown in Table 1.

Table 1: Data Sets

Data File	Time Step Size Δt
current1.dat	75 ms
current2.dat	50 ms
current3.dat	25 ms
current4.dat	10 ms

For each data set, time step is different so that the accuracy of these data sets can be assumed to be different.

II. Formulation

For calculating $\mathcal{E}(t)$, first the derivative of the current must be found. In this case, we going to use three different method for finding the numerical differentiation.

a) Forward Difference Method

Let the function is f, the derivative of f at x_0 point is:

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h}$$

Where h is a small positive number.

Example:

Let the g(x) function is $g(x) = x^2$

Where h=0.1 the derivative of g at 1 is

$$g'(1) = \frac{g(1 + 0.1) - g(1)}{0.1} = 2.1$$

b) Backward Difference Method

Let the function is f, the derivative of f at x_0 point is:

$$f'(x_0) = \frac{f(x_0) - f(x_0 - h)}{h}$$

Example:

Let the g(x) function is $g(x) = x^2$

Where h=0.1 the derivative of g at 1 is

$$g'(1) = \frac{g(1) - g(1 - 0.1)}{0.1} = 1.9$$

c) Three Point Midpoint Formula

Let the function is f, the derivative of f at x_0 point is:

$$f'(x_0) = \frac{f(x_0 + h) - f(x_0 - h)}{2h}$$

Example:

Let the g(x) function is $g(x) = x^2$

Where h=0.1 the derivative of g at 1 is

$$g'(1) = \frac{g(1 + 0.1) - g(1 - 0.1)}{0.2} = 2$$

The actual value of $g'(1) = 2$ so all of the methods can be assumed as relatively correct. Since the three point midpoint method uses three points unlike forward and backward difference methods which use only two, we can expect the three point method to be most accurate method between three of these methods. And since forward difference and backward difference methods has relatively same formula, we can expect that their error rates to be close each other.

III. Explanation of Code

We can write a matlab code for solving this solution.

Taking data sets

```
clear all;  
clc;  
format long;  
load current1.dat  
load current2.dat  
load current3.dat  
load current4.dat
```

First of all, data sets must be taken from computer by using load command.

forward-difference formula

```
for i=1:1:8
    forward1(i)=(current1(i+1,2)-current1(i,2))/0.0750;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.0750=%f\n",current1(i,1),current1(i+1,2),current1(i,2),forward1(i));
    E1_forward(i)= 0.98*forward1(i)+14.2*current1(i,2);
    fprintf("impressed voltage is %f\n",E1_forward(i));
end
fprintf("\n current2\n");
for i=1:1:12
    forward2(i)=(current2(i+1,2)-current2(i,2))/0.050;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.050=%f\n",current2(i,1),current2(i+1,2),current2(i,2),forward2(i));
    E2_forward(i)= 0.98*forward2(i)+14.2*current2(i,2);
    fprintf("impressed voltage is %f\n",E2_forward(i));
end
fprintf("\ncurrent3\n");
for i=1:1:24
    forward3(i)=(current3(i+1,2)-current3(i,2))/0.025;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.025=%f\n",current3(i,1),current3(i+1,2),current3(i,2),forward3(i));
    E3_forward(i)= 0.98*forward3(i)+14.2*current3(i,2);
    fprintf("impressed voltage is %f\n",E3_forward(i));
end
fprintf("\ncurrent4\n");
for i=1:1:60
    forward4(i)=(current4(i+1,2)-current4(i,2))/0.010;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.010=%f\n",current4(i,1),current4(i+1,2),current4(i,2),forward4(i));
    E4_forward(i)= 0.98*forward4(i)+14.2*current4(i,2);
    fprintf("impressed voltage is %f\n",E4_forward(i));
end
```

The formula of forward difference method is

$$f'(x_a) = \frac{f(x_a + \Delta t) - f(x_a)}{\Delta t}$$

For the current1, the $\Delta t=0.075$, current2 $\Delta t=0.05$, current3 $\Delta t=0.025$, current4 $\Delta t=0.01$

For applying forward difference method, the derivative of current1 must be determined and stored inside of an array. Then, with using derivative of current1, impressed voltage can be calculated by applying according to the given formula.

In forward difference method, for calculating the derivative of a point, another point which is bigger than concerned point must be known. Since there are no point which bigger than last point, the derivative at the last step of the data set can not be determined.

$$\mathcal{E}(t) = L \frac{d}{dt} i(t) + i(t)R$$

Applying this algorithm for current2, current3 and current4 too, gives us the following results.

Current1

derivative of current at $t=0.000000$ is: $((0.593746)-0.100000)/0.0750=6.583282$
impressed voltage is 7.871616
derivative of current at $t=0.075000$ is: $((0.760295)-0.593746)/0.0750=2.220647$
impressed voltage is 10.607430
derivative of current at $t=0.150000$ is: $((0.816474)-0.760295)/0.0750=0.749060$
impressed voltage is 11.530264
derivative of current at $t=0.225000$ is: $((0.835424)-0.816474)/0.0750=0.252670$
impressed voltage is 11.841550
derivative of current at $t=0.300000$ is: $((0.841817)-0.835424)/0.0750=0.085230$
impressed voltage is 11.946552
derivative of current at $t=0.375000$ is: $((0.843973)-0.841817)/0.0750=0.028749$
impressed voltage is 11.981971
derivative of current at $t=0.450000$ is: $((0.844700)-0.843973)/0.0750=0.009698$
impressed voltage is 11.993919
derivative of current at $t=0.525000$ is: $((0.844946)-0.844700)/0.0750=0.003271$
impressed voltage is 11.997949

current2

derivative of current at $t=0.000000$ is: $((0.484030)-0.100000)/0.050=7.680607$
impressed voltage is 8.946995
derivative of current at $t=0.050000$ is: $((0.670121)-0.484030)/0.050=3.721805$
impressed voltage is 10.520600
derivative of current at $t=0.100000$ is: $((0.760295)-0.670121)/0.050=1.803482$
impressed voltage is 11.283125
derivative of current at $t=0.150000$ is: $((0.803990)-0.760295)/0.050=0.873916$
impressed voltage is 11.652622
derivative of current at $t=0.200000$ is: $((0.825164)-0.803990)/0.050=0.423475$
impressed voltage is 11.831671
derivative of current at $t=0.250000$ is: $((0.835424)-0.825164)/0.050=0.205204$
impressed voltage is 11.918432
derivative of current at $t=0.300000$ is: $((0.840396)-0.835424)/0.050=0.099436$
impressed voltage is 11.960475
derivative of current at $t=0.350000$ is: $((0.842805)-0.840396)/0.050=0.048184$
impressed voltage is 11.980847
derivative of current at $t=0.400000$ is: $((0.843973)-0.842805)/0.050=0.023349$
impressed voltage is 11.990719
derivative of current at $t=0.450000$ is: $((0.844539)-0.843973)/0.050=0.011314$
impressed voltage is 11.995503
derivative of current at $t=0.500000$ is: $((0.844813)-0.844539)/0.050=0.005482$
impressed voltage is 11.997821
derivative of current at $t=0.550000$ is: $((0.844946)-0.844813)/0.050=0.002657$
impressed voltage is 11.998944

current3

derivative of current at $t=0.000000$ is: $((0.326418)-0.100000)/0.025=9.056722$
impressed voltage is 10.295588
derivative of current at $t=0.025000$ is: $((0.484030)-0.326418)/0.025=6.304492$
impressed voltage is 10.813538
derivative of current at $t=0.050000$ is: $((0.593746)-0.484030)/0.025=4.388632$
impressed voltage is 11.174090
derivative of current at $t=0.075000$ is: $((0.670121)-0.593746)/0.025=3.054979$
impressed voltage is 11.425074
derivative of current at $t=0.100000$ is: $((0.723286)-0.670121)/0.025=2.126607$
impressed voltage is 11.599787
derivative of current at $t=0.125000$ is: $((0.760295)-0.723286)/0.025=1.480356$
impressed voltage is 11.721407
derivative of current at $t=0.150000$ is: $((0.786057)-0.760295)/0.025=1.030494$
impressed voltage is 11.806068

derivative of current at $t=0.175000$ is: $((0.803990)-0.786057)/0.025=0.717339$
 impressed voltage is 11.865002
 derivative of current at $t=0.200000$ is: $((0.816474)-0.803990)/0.025=0.499348$
 impressed voltage is 11.906026
 derivative of current at $t=0.225000$ is: $((0.825164)-0.816474)/0.025=0.347602$
 impressed voltage is 11.934584
 derivative of current at $t=0.250000$ is: $((0.831214)-0.825164)/0.025=0.241970$
 impressed voltage is 11.954463
 derivative of current at $t=0.275000$ is: $((0.835424)-0.831214)/0.025=0.168438$
 impressed voltage is 11.968301
 derivative of current at $t=0.300000$ is: $((0.838356)-0.835424)/0.025=0.117252$
 impressed voltage is 11.977934
 derivative of current at $t=0.325000$ is: $((0.840396)-0.838356)/0.025=0.081620$
 impressed voltage is 11.984640
 derivative of current at $t=0.350000$ is: $((0.841817)-0.840396)/0.025=0.056817$
 impressed voltage is 11.989307
 derivative of current at $t=0.375000$ is: $((0.842805)-0.841817)/0.025=0.039551$
 impressed voltage is 11.992557
 derivative of current at $t=0.400000$ is: $((0.843494)-0.842805)/0.025=0.027532$
 impressed voltage is 11.994819
 derivative of current at $t=0.425000$ is: $((0.843973)-0.843494)/0.025=0.019165$
 impressed voltage is 11.996393
 derivative of current at $t=0.450000$ is: $((0.844306)-0.843973)/0.025=0.013341$
 impressed voltage is 11.997489
 derivative of current at $t=0.475000$ is: $((0.844539)-0.844306)/0.025=0.009287$
 impressed voltage is 11.998252
 derivative of current at $t=0.500000$ is: $((0.844700)-0.844539)/0.025=0.006465$
 impressed voltage is 11.998783
 derivative of current at $t=0.525000$ is: $((0.844813)-0.844700)/0.025=0.004500$
 impressed voltage is 11.999153
 derivative of current at $t=0.550000$ is: $((0.844891)-0.844813)/0.025=0.003133$
 impressed voltage is 11.999410
 derivative of current at $t=0.575000$ is: $((0.844946)-0.844891)/0.025=0.002181$
 impressed voltage is 11.999590

current4

derivative of current at $t=0.000000$ is: $((0.200502)-0.100000)/0.010=10.050213$
 impressed voltage is 11.269209
 derivative of current at $t=0.010000$ is: $((0.287448)-0.200502)/0.010=8.694545$
 impressed voltage is 11.367785
 derivative of current at $t=0.020000$ is: $((0.362665)-0.287448)/0.010=7.521743$
 impressed voltage is 11.453064
 derivative of current at $t=0.030000$ is: $((0.427736)-0.362665)/0.010=6.507139$
 impressed voltage is 11.526840
 derivative of current at $t=0.040000$ is: $((0.484030)-0.427736)/0.010=5.629395$
 impressed voltage is 11.590664
 derivative of current at $t=0.050000$ is: $((0.532731)-0.484030)/0.010=4.870049$
 impressed voltage is 11.645879
 derivative of current at $t=0.060000$ is: $((0.574862)-0.532731)/0.010=4.213131$
 impressed voltage is 11.693646
 derivative of current at $t=0.070000$ is: $((0.611310)-0.574862)/0.010=3.644824$
 impressed voltage is 11.734970
 derivative of current at $t=0.080000$ is: $((0.642842)-0.611310)/0.010=3.153176$
 impressed voltage is 11.770720
 derivative of current at $t=0.090000$ is: $((0.670121)-0.642842)/0.010=2.727846$
 impressed voltage is 11.801647
 derivative of current at $t=0.100000$ is: $((0.693719)-0.670121)/0.010=2.359888$
 impressed voltage is 11.828403

derivative of current at t=0.110000 is: $((0.714135)-0.693719)/0.010=2.041564$
impressed voltage is 11.851550
derivative of current at t=0.120000 is: $((0.731797)-0.714135)/0.010=1.766179$
impressed voltage is 11.871574
derivative of current at t=0.130000 is: $((0.747076)-0.731797)/0.010=1.527940$
impressed voltage is 11.888897
derivative of current at t=0.140000 is: $((0.760295)-0.747076)/0.010=1.321837$
impressed voltage is 11.903884
derivative of current at t=0.150000 is: $((0.771730)-0.760295)/0.010=1.143535$
impressed voltage is 11.916849
derivative of current at t=0.160000 is: $((0.781623)-0.771730)/0.010=0.989284$
impressed voltage is 11.928065
derivative of current at t=0.170000 is: $((0.790181)-0.781623)/0.010=0.855840$
impressed voltage is 11.937768
derivative of current at t=0.180000 is: $((0.797585)-0.790181)/0.010=0.740396$
impressed voltage is 11.946163
derivative of current at t=0.190000 is: $((0.803990)-0.797585)/0.010=0.640525$
impressed voltage is 11.953425
derivative of current at t=0.200000 is: $((0.809532)-0.803990)/0.010=0.554125$
impressed voltage is 11.959707
derivative of current at t=0.210000 is: $((0.814326)-0.809532)/0.010=0.479379$
impressed voltage is 11.965142
derivative of current at t=0.220000 is: $((0.818473)-0.814326)/0.010=0.414716$
impressed voltage is 11.969844
derivative of current at t=0.230000 is: $((0.822060)-0.818473)/0.010=0.358775$
impressed voltage is 11.973912
derivative of current at t=0.240000 is: $((0.825164)-0.822060)/0.010=0.310380$
impressed voltage is 11.977431
derivative of current at t=0.250000 is: $((0.827849)-0.825164)/0.010=0.268513$
impressed voltage is 11.980475
derivative of current at t=0.260000 is: $((0.830172)-0.827849)/0.010=0.232294$
impressed voltage is 11.983109
derivative of current at t=0.270000 is: $((0.832182)-0.830172)/0.010=0.200960$
impressed voltage is 11.985387
derivative of current at t=0.280000 is: $((0.833920)-0.832182)/0.010=0.173852$
impressed voltage is 11.987358
derivative of current at t=0.290000 is: $((0.835424)-0.833920)/0.010=0.150401$
impressed voltage is 11.989064
derivative of current at t=0.300000 is: $((0.836726)-0.835424)/0.010=0.130114$
impressed voltage is 11.990539
derivative of current at t=0.310000 is: $((0.837851)-0.836726)/0.010=0.112563$
impressed voltage is 11.991815
derivative of current at t=0.320000 is: $((0.838825)-0.837851)/0.010=0.097379$
impressed voltage is 11.992919
derivative of current at t=0.330000 is: $((0.839667)-0.838825)/0.010=0.084244$
impressed voltage is 11.993874
derivative of current at t=0.340000 is: $((0.840396)-0.839667)/0.010=0.072880$
impressed voltage is 11.994701
derivative of current at t=0.350000 is: $((0.841027)-0.840396)/0.010=0.063050$
impressed voltage is 11.995415
derivative of current at t=0.360000 is: $((0.841572)-0.841027)/0.010=0.054545$
impressed voltage is 11.996034
derivative of current at t=0.370000 is: $((0.842044)-0.841572)/0.010=0.047187$
impressed voltage is 11.996569
derivative of current at t=0.380000 is: $((0.842452)-0.842044)/0.010=0.040822$
impressed voltage is 11.997032
derivative of current at t=0.390000 is: $((0.842805)-0.842452)/0.010=0.035316$
impressed voltage is 11.997432

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derivative of current at t=0.400000 is:((0.843111)-0.842805)/0.010=0.030552
impressed voltage is 11.997778
derivative of current at t=0.410000 is:((0.843375)-0.843111)/0.010=0.026431
impressed voltage is 11.998078
derivative of current at t=0.420000 is:((0.843604)-0.843375)/0.010=0.022866
impressed voltage is 11.998337
derivative of current at t=0.430000 is:((0.843802)-0.843604)/0.010=0.019781
impressed voltage is 11.998562
derivative of current at t=0.440000 is:((0.843973)-0.843802)/0.010=0.017113
impressed voltage is 11.998756
derivative of current at t=0.450000 is:((0.844121)-0.843973)/0.010=0.014805
impressed voltage is 11.998923
derivative of current at t=0.460000 is:((0.844249)-0.844121)/0.010=0.012808
impressed voltage is 11.999069
derivative of current at t=0.470000 is:((0.844360)-0.844249)/0.010=0.011080
impressed voltage is 11.999194
derivative of current at t=0.480000 is:((0.844456)-0.844360)/0.010=0.009585
impressed voltage is 11.999303
derivative of current at t=0.490000 is:((0.844539)-0.844456)/0.010=0.008292
impressed voltage is 11.999397
derivative of current at t=0.500000 is:((0.844610)-0.844539)/0.010=0.007174
impressed voltage is 11.999478
derivative of current at t=0.510000 is:((0.844672)-0.844610)/0.010=0.006206
impressed voltage is 11.999549
derivative of current at t=0.520000 is:((0.844726)-0.844672)/0.010=0.005369
impressed voltage is 11.999610
derivative of current at t=0.530000 is:((0.844773)-0.844726)/0.010=0.004645
impressed voltage is 11.999662
derivative of current at t=0.540000 is:((0.844813)-0.844773)/0.010=0.004018
impressed voltage is 11.999708
derivative of current at t=0.550000 is:((0.844847)-0.844813)/0.010=0.003476
impressed voltage is 11.999747
derivative of current at t=0.560000 is:((0.844878)-0.844847)/0.010=0.003007
impressed voltage is 11.999781
derivative of current at t=0.570000 is:((0.844904)-0.844878)/0.010=0.002602
impressed voltage is 11.999811
derivative of current at t=0.580000 is:((0.844926)-0.844904)/0.010=0.002251
impressed voltage is 11.999836
derivative of current at t=0.590000 is:((0.844946)-0.844926)/0.010=0.001947
impressed voltage is 11.999858

```

backward-difference formula

```

fprintf("\nbackward difference formula\n\n");
fprintf("current1\n");
for i=2:1:9
    backward1(i)=(current1(i,2)-current1(i-1,2))/0.0750;
    fprintf("derivative of current at t=%f is:((%f)-
%f)/0.0750=%f\n",current1(i,1),current1(i,2),current1(i-1,2),backward1(i));
    E1_backward(i)= 0.98*backward1(i)+14.2*current1(i,2);
    fprintf("impressed voltage is %f\n",E1_backward(i));
end
fprintf("\n current2\n");
for i=2:1:13
    backward2(i)=(current2(i,2)-current2(i-1,2))/0.050;
    fprintf("derivative of current at t=%f is:((%f)-
%f)/0.050=%f\n",current2(i,1),current2(i,2),current2(i-1,2),backward2(i));
    E2_backward(i)= 0.98*backward2(i)+14.2*current2(i,2);

```



```

    fprintf("impressed voltage is %f\n",E2_backward(i));
end
fprintf("\ncurrent3\n");
for i=2:1:25
    backward3(i)=(current3(i,2)-current3(i-1,2))/0.025;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.025=%f\n",current3(i,1),current3(i,2),current3(i-1,2),backward3(i));
    E3_backward(i)= 0.98*backward3(i)+14.2*current3(i,2);
    fprintf("impressed voltage is %f\n",E3_backward(i));
end
fprintf("\ncurrent4\n");
for i=2:1:61
    backward4(i)=(current4(i,2)-current4(i-1,2))/0.010;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.010=%f\n",current4(i,1),current4(i,2),current4(i-1,2),backward4(i));
    E4_backward(i)= 0.98*backward4(i)+14.2*current4(i,2);
    fprintf("impressed voltage is %f\n",E4_backward(i));
end

```

The formula of backward difference method is

$$f'(x_a) = \frac{f(x_a) - f(x_a - \Delta t)}{\Delta t}$$

For the current1, the $\Delta t=0.075$, current2 $\Delta t=0.05$, current3 $\Delta t=0.025$, current4 $\Delta t=0.01$

In backward difference method, for calculating the derivative of a point, another point which is smaller than concerned point must be known. Since there are no point which smaller than first point, the derivative at the first step of the data set can not be determined.

For applying backward difference method, the derivative of current1 must be determined and stored inside of an array. Then, with using derivative of current1, impressed voltage can be calculated by applying according to the given formula.

$$\mathcal{E}(t) = L \frac{d}{dt} i(t) + i(t)R$$

Applying this algorithm for current2, current3 and current4 too, gives us the following results:

backward difference formula

```

current1
derivative of current at t=0.075000 is:((0.593746)-0.100000)/0.0750=6.583282
impressed voltage is 14.882812
derivative of current at t=0.150000 is:((0.760295)-0.593746)/0.0750=2.220647
impressed voltage is 12.972419
derivative of current at t=0.225000 is:((0.816474)-0.760295)/0.0750=0.749060
impressed voltage is 12.328013
derivative of current at t=0.300000 is:((0.835424)-0.816474)/0.0750=0.252670
impressed voltage is 12.110644
derivative of current at t=0.375000 is:((0.841817)-0.835424)/0.0750=0.085230
impressed voltage is 12.037322
derivative of current at t=0.450000 is:((0.843973)-0.841817)/0.0750=0.028749
impressed voltage is 12.012589
derivative of current at t=0.525000 is:((0.844700)-0.843973)/0.0750=0.009698
impressed voltage is 12.004247
derivative of current at t=0.600000 is:((0.844946)-0.844700)/0.0750=0.003271

```

impressed voltage is 12.001432

current2

derivative of current at $t=0.050000$ is: $((0.484030)-0.100000)/0.050=7.680607$

impressed voltage is 14.400226

derivative of current at $t=0.100000$ is: $((0.670121)-0.484030)/0.050=3.721805$

impressed voltage is 13.163082

derivative of current at $t=0.150000$ is: $((0.760295)-0.670121)/0.050=1.803482$

impressed voltage is 12.563597

derivative of current at $t=0.200000$ is: $((0.803990)-0.760295)/0.050=0.873916$

impressed voltage is 12.273103

derivative of current at $t=0.250000$ is: $((0.825164)-0.803990)/0.050=0.423475$

impressed voltage is 12.132338

derivative of current at $t=0.300000$ is: $((0.835424)-0.825164)/0.050=0.205204$

impressed voltage is 12.064127

derivative of current at $t=0.350000$ is: $((0.840396)-0.835424)/0.050=0.099436$

impressed voltage is 12.031074

derivative of current at $t=0.400000$ is: $((0.842805)-0.840396)/0.050=0.048184$

impressed voltage is 12.015058

derivative of current at $t=0.450000$ is: $((0.843973)-0.842805)/0.050=0.023349$

impressed voltage is 12.007297

derivative of current at $t=0.500000$ is: $((0.844539)-0.843973)/0.050=0.011314$

impressed voltage is 12.003536

derivative of current at $t=0.550000$ is: $((0.844813)-0.844539)/0.050=0.005482$

impressed voltage is 12.001713

derivative of current at $t=0.600000$ is: $((0.844946)-0.844813)/0.050=0.002657$

impressed voltage is 12.000830

current3

derivative of current at $t=0.025000$ is: $((0.326418)-0.100000)/0.025=9.056722$

impressed voltage is 13.510724

derivative of current at $t=0.050000$ is: $((0.484030)-0.326418)/0.025=6.304492$

impressed voltage is 13.051633

derivative of current at $t=0.075000$ is: $((0.593746)-0.484030)/0.025=4.388632$

impressed voltage is 12.732054

derivative of current at $t=0.100000$ is: $((0.670121)-0.593746)/0.025=3.054979$

impressed voltage is 12.509592

derivative of current at $t=0.125000$ is: $((0.723286)-0.670121)/0.025=2.126607$

impressed voltage is 12.354733

derivative of current at $t=0.150000$ is: $((0.760295)-0.723286)/0.025=1.480356$

impressed voltage is 12.246934

derivative of current at $t=0.175000$ is: $((0.786057)-0.760295)/0.025=1.030494$

impressed voltage is 12.171893

derivative of current at $t=0.200000$ is: $((0.803990)-0.786057)/0.025=0.717339$

impressed voltage is 12.119657

derivative of current at $t=0.225000$ is: $((0.816474)-0.803990)/0.025=0.499348$

impressed voltage is 12.083295

derivative of current at $t=0.250000$ is: $((0.825164)-0.816474)/0.025=0.347602$

impressed voltage is 12.057982

derivative of current at $t=0.275000$ is: $((0.831214)-0.825164)/0.025=0.241970$

impressed voltage is 12.040362

derivative of current at $t=0.300000$ is: $((0.835424)-0.831214)/0.025=0.168438$

impressed voltage is 12.028097

derivative of current at $t=0.325000$ is: $((0.838356)-0.835424)/0.025=0.117252$

impressed voltage is 12.019558

derivative of current at $t=0.350000$ is: $((0.840396)-0.838356)/0.025=0.081620$

impressed voltage is 12.013615

derivative of current at $t=0.375000$ is: $((0.841817)-0.840396)/0.025=0.056817$

impressed voltage is 12.009477
derivative of current at t=0.400000 is: $((0.842805)-0.841817)/0.025=0.039551$
impressed voltage is 12.006597
derivative of current at t=0.425000 is: $((0.843494)-0.842805)/0.025=0.027532$
impressed voltage is 12.004593
derivative of current at t=0.450000 is: $((0.843973)-0.843494)/0.025=0.019165$
impressed voltage is 12.003197
derivative of current at t=0.475000 is: $((0.844306)-0.843973)/0.025=0.013341$
impressed voltage is 12.002225
derivative of current at t=0.500000 is: $((0.844539)-0.844306)/0.025=0.009287$
impressed voltage is 12.001549
derivative of current at t=0.525000 is: $((0.844700)-0.844539)/0.025=0.006465$
impressed voltage is 12.001078
derivative of current at t=0.550000 is: $((0.844813)-0.844700)/0.025=0.004500$
impressed voltage is 12.000751
derivative of current at t=0.575000 is: $((0.844891)-0.844813)/0.025=0.003133$
impressed voltage is 12.000523
derivative of current at t=0.600000 is: $((0.844946)-0.844891)/0.025=0.002181$
impressed voltage is 12.000364

current4

derivative of current at t=0.010000 is: $((0.200502)-0.100000)/0.010=10.050213$
impressed voltage is 12.696339
derivative of current at t=0.020000 is: $((0.287448)-0.200502)/0.010=8.694545$
impressed voltage is 12.602410
derivative of current at t=0.030000 is: $((0.362665)-0.287448)/0.010=7.521743$
impressed voltage is 12.521151
derivative of current at t=0.040000 is: $((0.427736)-0.362665)/0.010=6.507139$
impressed voltage is 12.450853
derivative of current at t=0.050000 is: $((0.484030)-0.427736)/0.010=5.629395$
impressed voltage is 12.390038
derivative of current at t=0.060000 is: $((0.532731)-0.484030)/0.010=4.870049$
impressed voltage is 12.337426
derivative of current at t=0.070000 is: $((0.574862)-0.532731)/0.010=4.213131$
impressed voltage is 12.291911
derivative of current at t=0.080000 is: $((0.611310)-0.574862)/0.010=3.644824$
impressed voltage is 12.252535
derivative of current at t=0.090000 is: $((0.642842)-0.611310)/0.010=3.153176$
impressed voltage is 12.218471
derivative of current at t=0.100000 is: $((0.670121)-0.642842)/0.010=2.727846$
impressed voltage is 12.189001
derivative of current at t=0.110000 is: $((0.693719)-0.670121)/0.010=2.359888$
impressed voltage is 12.163507
derivative of current at t=0.120000 is: $((0.714135)-0.693719)/0.010=2.041564$
impressed voltage is 12.141452
derivative of current at t=0.130000 is: $((0.731797)-0.714135)/0.010=1.766179$
impressed voltage is 12.122371
derivative of current at t=0.140000 is: $((0.747076)-0.731797)/0.010=1.527940$
impressed voltage is 12.105865
derivative of current at t=0.150000 is: $((0.760295)-0.747076)/0.010=1.321837$
impressed voltage is 12.091585
derivative of current at t=0.160000 is: $((0.771730)-0.760295)/0.010=1.143535$
impressed voltage is 12.079231
derivative of current at t=0.170000 is: $((0.781623)-0.771730)/0.010=0.989284$
impressed voltage is 12.068544
derivative of current at t=0.180000 is: $((0.790181)-0.781623)/0.010=0.855840$
impressed voltage is 12.059298
derivative of current at t=0.190000 is: $((0.797585)-0.790181)/0.010=0.740396$

impressed voltage is 12.051299
derivative of current at t=0.200000 is: $((0.803990) - 0.797585) / 0.010 = 0.640525$
impressed voltage is 12.044379
derivative of current at t=0.210000 is: $((0.809532) - 0.803990) / 0.010 = 0.554125$
impressed voltage is 12.038393
derivative of current at t=0.220000 is: $((0.814326) - 0.809532) / 0.010 = 0.479379$
impressed voltage is 12.033214
derivative of current at t=0.230000 is: $((0.818473) - 0.814326) / 0.010 = 0.414716$
impressed voltage is 12.028734
derivative of current at t=0.240000 is: $((0.822060) - 0.818473) / 0.010 = 0.358775$
impressed voltage is 12.024858
derivative of current at t=0.250000 is: $((0.825164) - 0.822060) / 0.010 = 0.310380$
impressed voltage is 12.021505
derivative of current at t=0.260000 is: $((0.827849) - 0.825164) / 0.010 = 0.268513$
impressed voltage is 12.018604
derivative of current at t=0.270000 is: $((0.830172) - 0.827849) / 0.010 = 0.232294$
impressed voltage is 12.016095
derivative of current at t=0.280000 is: $((0.832182) - 0.830172) / 0.010 = 0.200960$
impressed voltage is 12.013924
derivative of current at t=0.290000 is: $((0.833920) - 0.832182) / 0.010 = 0.173852$
impressed voltage is 12.012046
derivative of current at t=0.300000 is: $((0.835424) - 0.833920) / 0.010 = 0.150401$
impressed voltage is 12.010421
derivative of current at t=0.310000 is: $((0.836726) - 0.835424) / 0.010 = 0.130114$
impressed voltage is 12.009015
derivative of current at t=0.320000 is: $((0.837851) - 0.836726) / 0.010 = 0.112563$
impressed voltage is 12.007799
derivative of current at t=0.330000 is: $((0.838825) - 0.837851) / 0.010 = 0.097379$
impressed voltage is 12.006747
derivative of current at t=0.340000 is: $((0.839667) - 0.838825) / 0.010 = 0.084244$
impressed voltage is 12.005837
derivative of current at t=0.350000 is: $((0.840396) - 0.839667) / 0.010 = 0.072880$
impressed voltage is 12.005050
derivative of current at t=0.360000 is: $((0.841027) - 0.840396) / 0.010 = 0.063050$
impressed voltage is 12.004368
derivative of current at t=0.370000 is: $((0.841572) - 0.841027) / 0.010 = 0.054545$
impressed voltage is 12.003779
derivative of current at t=0.380000 is: $((0.842044) - 0.841572) / 0.010 = 0.047187$
impressed voltage is 12.003269
derivative of current at t=0.390000 is: $((0.842452) - 0.842044) / 0.010 = 0.040822$
impressed voltage is 12.002828
derivative of current at t=0.400000 is: $((0.842805) - 0.842452) / 0.010 = 0.035316$
impressed voltage is 12.002447
derivative of current at t=0.410000 is: $((0.843111) - 0.842805) / 0.010 = 0.030552$
impressed voltage is 12.002117
derivative of current at t=0.420000 is: $((0.843375) - 0.843111) / 0.010 = 0.026431$
impressed voltage is 12.001831
derivative of current at t=0.430000 is: $((0.843604) - 0.843375) / 0.010 = 0.022866$
impressed voltage is 12.001584
derivative of current at t=0.440000 is: $((0.843802) - 0.843604) / 0.010 = 0.019781$
impressed voltage is 12.001371
derivative of current at t=0.450000 is: $((0.843973) - 0.843802) / 0.010 = 0.017113$
impressed voltage is 12.001186
derivative of current at t=0.460000 is: $((0.844121) - 0.843973) / 0.010 = 0.014805$
impressed voltage is 12.001026
derivative of current at t=0.470000 is: $((0.844249) - 0.844121) / 0.010 = 0.012808$
impressed voltage is 12.000887
derivative of current at t=0.480000 is: $((0.844360) - 0.844249) / 0.010 = 0.011080$

```

impressed voltage is 12.000768
derivative of current at t=0.490000 is:((0.844456)-0.844360)/0.010=0.009585
impressed voltage is 12.000664
derivative of current at t=0.500000 is:((0.844539)-0.844456)/0.010=0.008292
impressed voltage is 12.000575
derivative of current at t=0.510000 is:((0.844610)-0.844539)/0.010=0.007174
impressed voltage is 12.000497
derivative of current at t=0.520000 is:((0.844672)-0.844610)/0.010=0.006206
impressed voltage is 12.000430
derivative of current at t=0.530000 is:((0.844726)-0.844672)/0.010=0.005369
impressed voltage is 12.000372
derivative of current at t=0.540000 is:((0.844773)-0.844726)/0.010=0.004645
impressed voltage is 12.000322
derivative of current at t=0.550000 is:((0.844813)-0.844773)/0.010=0.004018
impressed voltage is 12.000278
derivative of current at t=0.560000 is:((0.844847)-0.844813)/0.010=0.003476
impressed voltage is 12.000241
derivative of current at t=0.570000 is:((0.844878)-0.844847)/0.010=0.003007
impressed voltage is 12.000208
derivative of current at t=0.580000 is:((0.844904)-0.844878)/0.010=0.002602
impressed voltage is 12.000180
derivative of current at t=0.590000 is:((0.844926)-0.844904)/0.010=0.002251
impressed voltage is 12.000156
derivative of current at t=0.600000 is:((0.844946)-0.844926)/0.010=0.001947
impressed voltage is 12.000135

```

Three Point midpoint formula

```

fprintf("\n\n three point midpoint formula\n");
fprintf("\ncurrent1\n\n");
h=0.075;
threepoint1(1)=(1/(2*h))*(-3*current1(1,2)+4*current1(2,2)-current1(3,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*%f+4*%f-
%f)=%f\n",current1(1,1),2*h,current1(1,2),current1(2,2),current1(3,2),threepoint1(1));
E1_threepoint(1)=threepoint1(1)*0.98+14.2*current1(1,2);
fprintf("impressed voltage is %f\n",E1_threepoint(1));
for i=2:1:8
    threepoint1(i)=(1/(2*h))*(current1(i+1,2)-current1(i-1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current1(i,1),2*h,current1(i+1,2),current1(i-1,2),threepoint1(i));
    E1_threepoint(i)=threepoint1(i)*0.98+14.2*current1(i,2);
    fprintf("impressed voltage is %f\n",E1_threepoint(i));
end
h=-0.075;
threepoint1(9)=(1/(2*h))*(-3*current1(9,2)+4*current1(8,2)-current1(7,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*%f+4*%f-
%f)=%f\n",current1(9,1),2*h,current1(9,2),current1(8,2),current1(7,2),threepoint1(9));
E1_threepoint(9)=threepoint1(9)*0.98+14.2*current1(9,2);
fprintf("impressed voltage is %f\n",E1_threepoint(9));

fprintf("\ncurrent2\n\n");
h=0.05;
threepoint2(1)=(1/(2*h))*(-3*current2(1,2)+4*current2(2,2)-current2(3,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*%f+4*%f-
%f)=%f\n",current2(1,1),2*h,current2(1,2),current2(2,2),current2(3,2),threepoint2(1));
E2_threepoint(1)=threepoint2(1)*0.98+14.2*current2(1,2);

```

```

fprintf("impressed voltage is %f\n",E2_threepoint(1));
for i=2:1:12
    threepoint2(i)=(1/(2*h))*(current2(i+1,2)-current2(i-1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current2(i,1),2*h,current2(i+1,2),current2(i-1,2),threepoint2(i));
    E2_threepoint(i)=threepoint2(i)*0.98+14.2*current2(i,2);
    fprintf("impressed voltage is %f\n",E2_threepoint(i));
end
h=-0.05;
threepoint2(13)=(1/(2*h))*(-3*current2(13,2)+4*current2(12,2)-current2(11,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*f+4*f-
%f)=%f\n",current2(13,1),2*h,current2(13,2),current2(12,2),current2(11,2),threepoint2(13));
E2_threepoint(13)=threepoint2(13)*0.98+14.2*current2(13,2);
fprintf("impressed voltage is %f\n",E2_threepoint(13));

fprintf("\ncurrent3\n\n");
h=0.025;
threepoint3(1)=(1/(2*h))*(-3*current3(1,2)+4*current3(2,2)-current3(3,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*f+4*f-
%f)=%f\n",current3(1,1),2*h,current3(1,2),current3(2,2),current3(3,2),threepoint3(1));
E3_threepoint(1)=threepoint3(1)*0.98+14.2*current3(1,2);
fprintf("impressed voltage is %f\n",E3_threepoint(1));
for i=2:1:24
    threepoint3(i)=(1/(2*h))*(current3(i+1,2)-current3(i-1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current3(i,1),2*h,current3(i+1,2),current3(i-1,2),threepoint3(i));
    E3_threepoint(i)=threepoint3(i)*0.98+14.2*current3(i,2);
    fprintf("impressed voltage is %f\n",E3_threepoint(i));
end
h=-0.025;
threepoint3(25)=(1/(2*h))*(-3*current3(24,2)+4*current3(23,2)-current3(22,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*f+4*f-
%f)=%f\n",current3(25,1),2*h,current3(24,2),current3(23,2),current3(22,2),threepoint3(25));
E3_threepoint(25)=threepoint3(25)*0.98+14.2*current3(25,2);
fprintf("impressed voltage is %f\n",E3_threepoint(25));

fprintf("\ncurrent4\n\n");
h=0.010;
threepoint4(1)=(1/(2*h))*(-3*current4(1,2)+4*current4(2,2)-current4(3,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*f+4*f-
%f)=%f\n",current4(1,1),2*h,current4(1,2),current4(2,2),current4(3,2),threepoint4(1));
E4_threepoint(1)=threepoint4(1)*0.98+14.2*current4(1,2);
fprintf("impressed voltage is %f\n",E4_threepoint(1));
for i=2:1:60
    threepoint4(i)=(1/(2*h))*(current4(i+1,2)-current4(i-1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current4(i,1),2*h,current4(i+1,2),current4(i-1,2),threepoint4(i));
    E4_threepoint(i)=threepoint4(i)*0.98+14.2*current4(i,2);
    fprintf("impressed voltage is %f\n",E4_threepoint(i));
end
h=-0.010;
threepoint4(61)=(1/(2*h))*(-3*current4(60,2)+4*current4(59,2)-current4(58,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-3*f+4*f-
%f)=%f\n",current4(61,1),2*h,current4(60,2),current4(59,2),current4(58,2),threepoint4(61));

```

```
E4_threepoint(61)=threepoint4(61)*0.98+14.2*current4(61,2);
fprintf("impressed voltage is %f\n",E4_threepoint(61));
```

There is no exception for three point method; unlike the forward difference and backward difference methods, in three point methods the derivatives of all points can be determined including first point and last point of data sets.

There is two formulas for calculating derivative using three point method. If the derivative at the first or the last point is wanted, we can use three point endpoint formula. The formula of three point endpoint method is:

$$f'(x_a) = \frac{-3f(x_a) + 4f(x_a + \Delta t) - f(x_a + 2\Delta t)}{2\Delta t}$$

Else If the derivative of another point is wanting, we can apply three point midpoint formula. The formula three point midpoint method is :

$$f'(x_a) = \frac{f(x_a + \Delta t) - f(x_a - \Delta t)}{\Delta t}$$

For the current1, the $\Delta t=0.075$, current2 $\Delta t=0.05$, current3 $\Delta t=0.025$, current4 $\Delta t=0.01$

For applying the three point method, firstly the first point of the data is must be determined by applying three point endpoint formula. After that, derivative of other points except the last point of the data set can be determined with applying the three point midpoint formula. And finally the derivative at the last point of data set can be determined by applying three point endpoint formula again. But at this time the difference between steps(Δt) is taken as $-\Delta t$, therefore, the formula of the three point endpoint method at the last point of the data set can be write like this:

$$f'(x_a) = \frac{-3f(x_a) + 4f(x_a - \Delta t) - f(x_a - 2\Delta t)}{-2\Delta t}$$

And after finding the derivatives, the impressed voltage can be calculated according to given formula:

$$\mathcal{E}(t) = L \frac{d}{dt} i(t) + i(t)R$$

Applying this method for current1, current2, current3 and current4 gives us the following results:

```
three point midpoint formula

current1

derivative of current at t=0.000000 is:(1/0.150000)*(-3*0.100000+4*0.593746-0.760295)=8.764599
impressed voltage is 10.009307
derivative of current at t=0.075000 is:1/0.150000*(0.760295-0.100000)=4.401965
impressed voltage is 12.745121
derivative of current at t=0.150000 is:1/0.150000*(0.816474-0.593746)=1.484854
impressed voltage is 12.251341
derivative of current at t=0.225000 is:1/0.150000*(0.835424-0.760295)=0.500865
impressed voltage is 12.084781
derivative of current at t=0.300000 is:1/0.150000*(0.841817-0.816474)=0.168950
```

impressed voltage is 12.028598
derivative of current at $t=0.375000$ is: $1/0.150000*(0.843973-0.835424)=0.056990$
impressed voltage is 12.009647
derivative of current at $t=0.450000$ is: $1/0.150000*(0.844700-0.841817)=0.019223$
impressed voltage is 12.003254
derivative of current at $t=0.525000$ is: $1/0.150000*(0.844946-0.843973)=0.006484$
impressed voltage is 12.001098
derivative of current at $t=0.600000$ is: $(1/-0.150000)*(-3*0.844946+4*0.844700-0.843973)=0.000058$
impressed voltage is 11.998283

current2

derivative of current at $t=0.000000$ is: $(1/0.100000)*(-3*0.100000+4*0.484030-0.670121)=9.660008$
impressed voltage is 10.886808
derivative of current at $t=0.050000$ is: $1/0.100000*(0.670121-0.100000)=5.701206$
impressed voltage is 12.460413
derivative of current at $t=0.100000$ is: $1/0.100000*(0.760295-0.484030)=2.762643$
impressed voltage is 12.223103
derivative of current at $t=0.150000$ is: $1/0.100000*(0.803990-0.670121)=1.338699$
impressed voltage is 12.108109
derivative of current at $t=0.200000$ is: $1/0.100000*(0.825164-0.760295)=0.648696$
impressed voltage is 12.052387
derivative of current at $t=0.250000$ is: $1/0.100000*(0.835424-0.803990)=0.314340$
impressed voltage is 12.025385
derivative of current at $t=0.300000$ is: $1/0.100000*(0.840396-0.825164)=0.152320$
impressed voltage is 12.012301
derivative of current at $t=0.350000$ is: $1/0.100000*(0.842805-0.835424)=0.073810$
impressed voltage is 12.005961
derivative of current at $t=0.400000$ is: $1/0.100000*(0.843973-0.840396)=0.035766$
impressed voltage is 12.002888
derivative of current at $t=0.450000$ is: $1/0.100000*(0.844539-0.842805)=0.017331$
impressed voltage is 12.001400
derivative of current at $t=0.500000$ is: $1/0.100000*(0.844813-0.843973)=0.008398$
impressed voltage is 12.000678
derivative of current at $t=0.550000$ is: $1/0.100000*(0.844946-0.844539)=0.004070$
impressed voltage is 12.000329
derivative of current at $t=0.600000$ is: $(1/-0.100000)*(-3*0.844946+4*0.844813-0.844539)=0.001244$
impressed voltage is 11.999446

current3

derivative of current at $t=0.000000$ is: $(1/0.050000)*(-3*0.100000+4*0.326418-0.484030)=10.432837$
impressed voltage is 11.644181
derivative of current at $t=0.025000$ is: $1/0.050000*(0.484030-0.100000)=7.680607$
impressed voltage is 12.162131
derivative of current at $t=0.050000$ is: $1/0.050000*(0.593746-0.326418)=5.346562$
impressed voltage is 12.112862
derivative of current at $t=0.075000$ is: $1/0.050000*(0.670121-0.484030)=3.721805$
impressed voltage is 12.078564
derivative of current at $t=0.100000$ is: $1/0.050000*(0.723286-0.593746)=2.590793$
impressed voltage is 12.054690
derivative of current at $t=0.125000$ is: $1/0.050000*(0.760295-0.670121)=1.803482$
impressed voltage is 12.038070
derivative of current at $t=0.150000$ is: $1/0.050000*(0.786057-0.723286)=1.255425$
impressed voltage is 12.026501

derivative of current at t=0.175000 is: $1/0.050000 \times (0.803990 - 0.760295) = 0.873916$
 impressed voltage is 12.018448
 derivative of current at t=0.200000 is: $1/0.050000 \times (0.816474 - 0.786057) = 0.608343$
 impressed voltage is 12.012842
 derivative of current at t=0.225000 is: $1/0.050000 \times (0.825164 - 0.803990) = 0.423475$
 impressed voltage is 12.008939
 derivative of current at t=0.250000 is: $1/0.050000 \times (0.831214 - 0.816474) = 0.294786$
 impressed voltage is 12.006223
 derivative of current at t=0.275000 is: $1/0.050000 \times (0.835424 - 0.825164) = 0.205204$
 impressed voltage is 12.004332
 derivative of current at t=0.300000 is: $1/0.050000 \times (0.838356 - 0.831214) = 0.142845$
 impressed voltage is 12.003015
 derivative of current at t=0.325000 is: $1/0.050000 \times (0.840396 - 0.835424) = 0.099436$
 impressed voltage is 12.002099
 derivative of current at t=0.350000 is: $1/0.050000 \times (0.841817 - 0.838356) = 0.069219$
 impressed voltage is 12.001461
 derivative of current at t=0.375000 is: $1/0.050000 \times (0.842805 - 0.840396) = 0.048184$
 impressed voltage is 12.001017
 derivative of current at t=0.400000 is: $1/0.050000 \times (0.843494 - 0.841817) = 0.033541$
 impressed voltage is 12.000708
 derivative of current at t=0.425000 is: $1/0.050000 \times (0.843973 - 0.842805) = 0.023349$
 impressed voltage is 12.000493
 derivative of current at t=0.450000 is: $1/0.050000 \times (0.844306 - 0.843494) = 0.016253$
 impressed voltage is 12.000343
 derivative of current at t=0.475000 is: $1/0.050000 \times (0.844539 - 0.843973) = 0.011314$
 impressed voltage is 12.000239
 derivative of current at t=0.500000 is: $1/0.050000 \times (0.844700 - 0.844306) = 0.007876$
 impressed voltage is 12.000166
 derivative of current at t=0.525000 is: $1/0.050000 \times (0.844813 - 0.844539) = 0.005482$
 impressed voltage is 12.000116
 derivative of current at t=0.550000 is: $1/0.050000 \times (0.844891 - 0.844700) = 0.003816$
 impressed voltage is 12.000081
 derivative of current at t=0.575000 is: $1/0.050000 \times (0.844946 - 0.844813) = 0.002657$
 impressed voltage is 12.000056
 derivative of current at t=0.600000 is: $(1/-0.050000) \times (-3 \times 0.844891 + 4 \times 0.844813 - 0.844700) = 0.002449$
 impressed voltage is 12.000627

current4

derivative of current at t=0.000000 is: $(1/0.020000) \times (-3 \times 0.100000 + 4 \times 0.200502 - 0.287448) = 10.728047$
 impressed voltage is 11.933486
 derivative of current at t=0.010000 is: $1/0.020000 \times (0.287448 - 0.100000) = 9.372379$
 impressed voltage is 12.032062
 derivative of current at t=0.020000 is: $1/0.020000 \times (0.362665 - 0.200502) = 8.108144$
 impressed voltage is 12.027737
 derivative of current at t=0.030000 is: $1/0.020000 \times (0.427736 - 0.287448) = 7.014441$
 impressed voltage is 12.023995
 derivative of current at t=0.040000 is: $1/0.020000 \times (0.484030 - 0.362665) = 6.068267$
 impressed voltage is 12.020759
 derivative of current at t=0.050000 is: $1/0.020000 \times (0.532731 - 0.427736) = 5.249722$
 impressed voltage is 12.017959
 derivative of current at t=0.060000 is: $1/0.020000 \times (0.574862 - 0.484030) = 4.541590$
 impressed voltage is 12.015536
 derivative of current at t=0.070000 is: $1/0.020000 \times (0.611310 - 0.532731) = 3.928977$
 impressed voltage is 12.013441
 derivative of current at t=0.080000 is: $1/0.020000 \times (0.642842 - 0.574862) = 3.399000$

impressed voltage is 12.011628
derivative of current at t=0.090000 is: $1/0.020000 \times (0.670121 - 0.611310) = 2.940511$
impressed voltage is 12.010059
derivative of current at t=0.100000 is: $1/0.020000 \times (0.693719 - 0.642842) = 2.543867$
impressed voltage is 12.008702
derivative of current at t=0.110000 is: $1/0.020000 \times (0.714135 - 0.670121) = 2.200726$
impressed voltage is 12.007528
derivative of current at t=0.120000 is: $1/0.020000 \times (0.731797 - 0.693719) = 1.903871$
impressed voltage is 12.006513
derivative of current at t=0.130000 is: $1/0.020000 \times (0.747076 - 0.714135) = 1.647059$
impressed voltage is 12.005634
derivative of current at t=0.140000 is: $1/0.020000 \times (0.760295 - 0.731797) = 1.424888$
impressed voltage is 12.004874
derivative of current at t=0.150000 is: $1/0.020000 \times (0.771730 - 0.747076) = 1.232686$
impressed voltage is 12.004217
derivative of current at t=0.160000 is: $1/0.020000 \times (0.781623 - 0.760295) = 1.066410$
impressed voltage is 12.003648
derivative of current at t=0.170000 is: $1/0.020000 \times (0.790181 - 0.771730) = 0.922562$
impressed voltage is 12.003156
derivative of current at t=0.180000 is: $1/0.020000 \times (0.797585 - 0.781623) = 0.798118$
impressed voltage is 12.002730
derivative of current at t=0.190000 is: $1/0.020000 \times (0.803990 - 0.790181) = 0.690461$
impressed voltage is 12.002362
derivative of current at t=0.200000 is: $1/0.020000 \times (0.809532 - 0.797585) = 0.597325$
impressed voltage is 12.002043
derivative of current at t=0.210000 is: $1/0.020000 \times (0.814326 - 0.803990) = 0.516752$
impressed voltage is 12.001768
derivative of current at t=0.220000 is: $1/0.020000 \times (0.818473 - 0.809532) = 0.447048$
impressed voltage is 12.001529
derivative of current at t=0.230000 is: $1/0.020000 \times (0.822060 - 0.814326) = 0.386746$
impressed voltage is 12.001323
derivative of current at t=0.240000 is: $1/0.020000 \times (0.825164 - 0.818473) = 0.334578$
impressed voltage is 12.001145
derivative of current at t=0.250000 is: $1/0.020000 \times (0.827849 - 0.822060) = 0.289447$
impressed voltage is 12.000990
derivative of current at t=0.260000 is: $1/0.020000 \times (0.830172 - 0.825164) = 0.250403$
impressed voltage is 12.000857
derivative of current at t=0.270000 is: $1/0.020000 \times (0.832182 - 0.827849) = 0.216627$
impressed voltage is 12.000741
derivative of current at t=0.280000 is: $1/0.020000 \times (0.833920 - 0.830172) = 0.187406$
impressed voltage is 12.000641
derivative of current at t=0.290000 is: $1/0.020000 \times (0.835424 - 0.832182) = 0.162127$
impressed voltage is 12.000555
derivative of current at t=0.300000 is: $1/0.020000 \times (0.836726 - 0.833920) = 0.140258$
impressed voltage is 12.000480
derivative of current at t=0.310000 is: $1/0.020000 \times (0.837851 - 0.835424) = 0.121338$
impressed voltage is 12.000415
derivative of current at t=0.320000 is: $1/0.020000 \times (0.838825 - 0.836726) = 0.104971$
impressed voltage is 12.000359
derivative of current at t=0.330000 is: $1/0.020000 \times (0.839667 - 0.837851) = 0.090812$
impressed voltage is 12.000311
derivative of current at t=0.340000 is: $1/0.020000 \times (0.840396 - 0.838825) = 0.078562$
impressed voltage is 12.000269
derivative of current at t=0.350000 is: $1/0.020000 \times (0.841027 - 0.839667) = 0.067965$
impressed voltage is 12.000232
derivative of current at t=0.360000 is: $1/0.020000 \times (0.841572 - 0.840396) = 0.058797$
impressed voltage is 12.000201
derivative of current at t=0.370000 is: $1/0.020000 \times (0.842044 - 0.841027) = 0.050866$

```

impressed voltage is 12.000174
derivative of current at t=0.380000 is:1/0.020000*(0.842452-0.841572)=0.044005
impressed voltage is 12.000151
derivative of current at t=0.390000 is:1/0.020000*(0.842805-0.842044)=0.038069
impressed voltage is 12.000130
derivative of current at t=0.400000 is:1/0.020000*(0.843111-0.842452)=0.032934
impressed voltage is 12.000113
derivative of current at t=0.410000 is:1/0.020000*(0.843375-0.842805)=0.028491
impressed voltage is 12.000097
derivative of current at t=0.420000 is:1/0.020000*(0.843604-0.843111)=0.024648
impressed voltage is 12.000084
derivative of current at t=0.430000 is:1/0.020000*(0.843802-0.843375)=0.021323
impressed voltage is 12.000073
derivative of current at t=0.440000 is:1/0.020000*(0.843973-0.843604)=0.018447
impressed voltage is 12.000063
derivative of current at t=0.450000 is:1/0.020000*(0.844121-0.843802)=0.015959
impressed voltage is 12.000055
derivative of current at t=0.460000 is:1/0.020000*(0.844249-0.843973)=0.013806
impressed voltage is 12.000047
derivative of current at t=0.470000 is:1/0.020000*(0.844360-0.844121)=0.011944
impressed voltage is 12.000041
derivative of current at t=0.480000 is:1/0.020000*(0.844456-0.844249)=0.010333
impressed voltage is 12.000035
derivative of current at t=0.490000 is:1/0.020000*(0.844539-0.844360)=0.008939
impressed voltage is 12.000031
derivative of current at t=0.500000 is:1/0.020000*(0.844610-0.844456)=0.007733
impressed voltage is 12.000026
derivative of current at t=0.510000 is:1/0.020000*(0.844672-0.844539)=0.006690
impressed voltage is 12.000023
derivative of current at t=0.520000 is:1/0.020000*(0.844726-0.844610)=0.005788
impressed voltage is 12.000020
derivative of current at t=0.530000 is:1/0.020000*(0.844773-0.844672)=0.005007
impressed voltage is 12.000017
derivative of current at t=0.540000 is:1/0.020000*(0.844813-0.844726)=0.004332
impressed voltage is 12.000015
derivative of current at t=0.550000 is:1/0.020000*(0.844847-0.844773)=0.003747
impressed voltage is 12.000013
derivative of current at t=0.560000 is:1/0.020000*(0.844878-0.844813)=0.003242
impressed voltage is 12.000011
derivative of current at t=0.570000 is:1/0.020000*(0.844904-0.844847)=0.002805
impressed voltage is 12.000010
derivative of current at t=0.580000 is:1/0.020000*(0.844926-0.844878)=0.002426
impressed voltage is 12.000008
derivative of current at t=0.590000 is:1/0.020000*(0.844946-0.844904)=0.002099
impressed voltage is 12.000007
derivative of current at t=0.600000 is:(1/-0.020000)*(-3*0.844926+4*0.844904-
0.844878)=0.002075
impressed voltage is 12.000260

```

plotting graphs

```

subplot(3,1,2);
hold on
grid on;
plot(current1(2:end,1),backward1(2:end),'-x');
plot(current2(2:end,1),backward2(2:end),'-o');
plot(current3(2:end,1),backward3(2:end),'-s');
plot(current4(2:end,1),backward4(2:end),'-');

```

```

legend('derivative of current1','derivative of current2','derivative of current3','derivative
of current4');
xlabel('time');
ylabel('derivative of current');
title('derivative of current versus time graph in backward-difference formula');
hold off
subplot(3,1,1);
hold on
grid on;
plot(current1(1:end-1,1),forward1,'-x');
plot(current2(1:end-1,1),forward2,'-o');
plot(current3(1:end-1,1),forward3,'-s');
plot(current4(1:end-1,1),forward4,'-');
legend('derivative of current1','derivative of current2','derivative of current3','derivative
of current4');
xlabel('time');
ylabel('derivative of current');
title('derivative of current versus time graph in forward-difference formula');
hold off;
subplot(3,1,3);
hold on;
grid on;
plot(current1(:,1),threepoint1,'-x');
plot(current2(:,1),threepoint2,'-o');
plot(current3(:,1),threepoint3,'-s');
plot(current4(:,1),threepoint4,'-');
legend('derivative of current1','derivative of current2','derivative of current3','derivative
of current4');
xlabel('time');
ylabel('derivative of current');
title('derivative of current versus time graph in threepoint-difference formula');
hold off;

figure(2);
subplot(3,1,1);
hold on;
grid on;
plot(current1(1:end-1,1),E1_forward,'-x');
plot(current2(1:end-1,1),E2_forward,'-o');
plot(current3(1:end-1,1),E3_forward,'-s');
plot(current4(1:end-1,1),E4_forward,'-');
legend('E(t) for current1','E(t) for current2','E(t) for current3','E(t) for current4');
xlabel('time');
ylabel('voltage');
title('impressed voltage versus time graph in forward-difference formula');
hold off;

subplot(3,1,2);
hold on;
grid on;
plot(current1(2:end,1),E1_backward(2:end),'-x');
plot(current2(2:end,1),E2_backward(2:end),'-o');
plot(current3(2:end,1),E3_backward(2:end),'-s');
plot(current4(2:end,1),E4_backward(2:end),'-');
legend('E(t) for current1','E(t) for current2','E(t) for current3','E(t) for current4');
xlabel('time');
ylabel('voltage');

```

```

title('impressed voltage versus time graph in backward-difference formula');
hold off;

subplot(3,1,3);
hold on;
grid on;
plot(current1(:,1),E1_threepoint,'-x');
plot(current2(:,1),E2_threepoint,'-o');
plot(current3(:,1),E3_threepoint,'-s');
plot(current4(:,1),E4_threepoint,'-');
legend('E(t) for current1','E(t) for current2','E(t) for current3','E(t) for current4');
xlabel('time');
ylabel('voltage');
title('impressed voltage versus time graph in threepoint-difference formula');
hold off;

```

If we plot the graphs for derivatives of the currents relative to time for each method and then make another figure and plot the voltage values relative to time for each method, we can make an analysis for methods and currents.

The results of the plots are as follows:

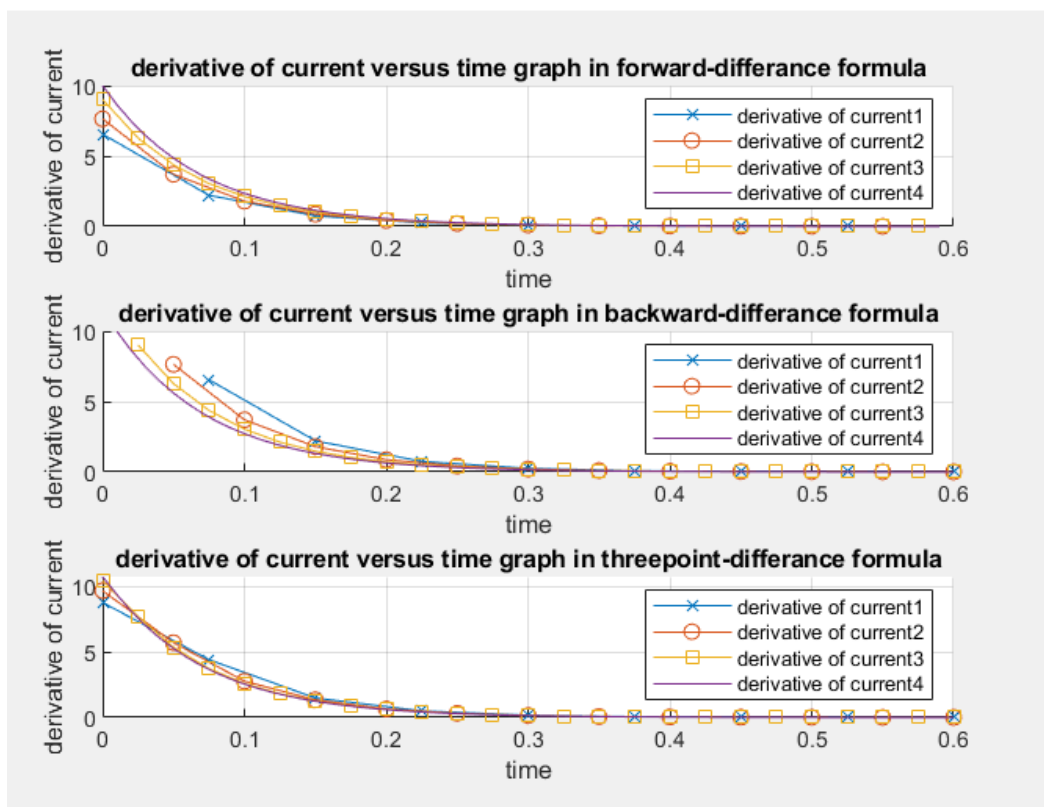


Figure 2: Derivative of Current Versus Time Graph For Each Method

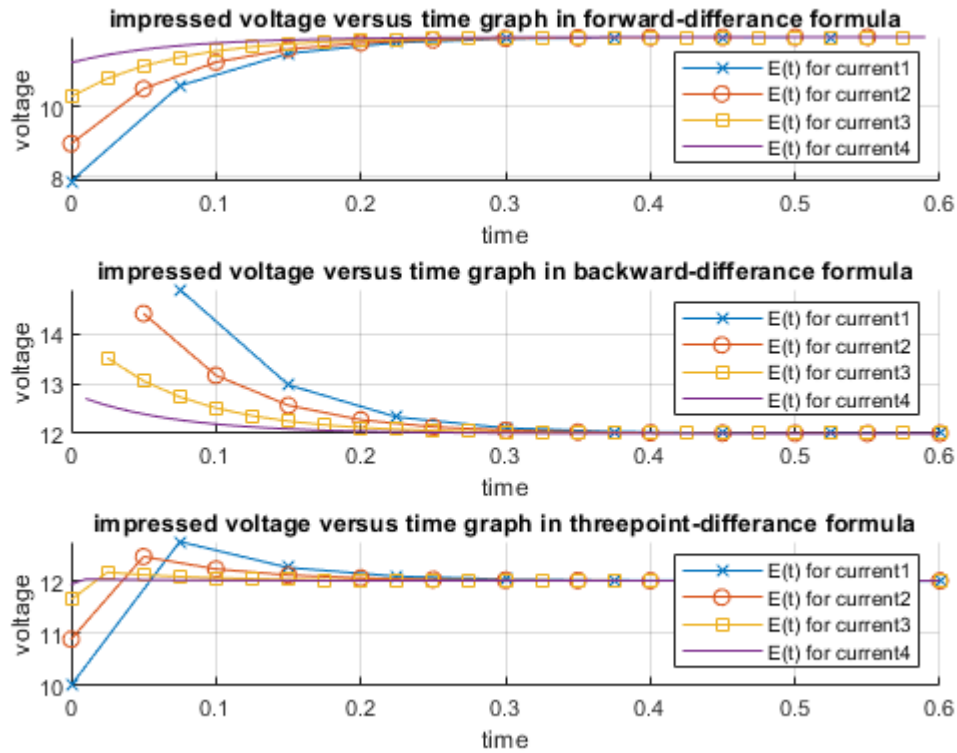


Figure 3: Impressed Voltage Versus Time Graph For Each Method

error analysis

```
fprintf("\nerror analysis\n\n");
fprintf("errors for forward-difference method\n\n");
fprintf("error of current1 is : %f\n", norm(E1_forward-12)/norm(12));
fprintf("error of current2 is : %f\n", norm(E2_forward-12)/norm(12));
fprintf("error of current3 is : %f\n", norm(E3_forward-12)/norm(12));
fprintf("error of current4 is : %f\n", norm(E4_forward-12)/norm(12));
fprintf("\n\nerrors for backward difference method\n\n");
fprintf("error of current1 is : %f\n", norm((E1_backward(2:end)-12))/norm(12));
fprintf("error of current2 is : %f\n", norm(E2_backward(2:end)-12)/norm(12));
fprintf("error of current3 is : %f\n", norm(E3_backward(2:end)-12)/norm(12));
fprintf("error of current4 is : %f\n", norm(E4_backward(2:end)-12)/norm(12));
fprintf("\n\nerrors for threepoint midpoint method\n\n");
fprintf("error of current1 is : %f\n", norm(E1_threepoint-12)/norm(12));
fprintf("error of current2 is : %f\n", norm(E2_threepoint-12)/norm(12));
fprintf("error of current3 is : %f\n", norm(E3_threepoint-12)/norm(12));
fprintf("error of current4 is : %f\n", norm(E4_threepoint-12)/norm(12));
fprintf("\n\nerrors for threepoint midpoint method without endpoints\n\n");
fprintf("error of current1 is : %f\n", norm(E1_threepoint(2:end-1)-12)/norm(12));
fprintf("error of current2 is : %f\n", norm(E2_threepoint(2:end-1)-12)/norm(12));
fprintf("error of current3 is : %f\n", norm(E3_threepoint(2:end-1)-12)/norm(12));
fprintf("error of current4 is : %f\n", norm(E4_threepoint(2:end-1)-12)/norm(12));
```

When we analyze the results of impressed voltages at each method, we can see that in all methods, the impressed voltage value is approaches to 12. Therefore we can guess that the actual value of the impressed voltage is 12.

Since we have a guess for the actual value of the impressed voltage, we can make an error analysis calculation to see the relative errors for each method and each current value.

The formula of relative error is:

$$error = \frac{x_{measured} - x_{real}}{x_{real}}$$

We can use norm function for taking the magnitude of vector, so the formula is

$$error = \frac{norm(x_{measured} - x_{real})}{norm(x_{real})}$$

So the results of error analysis are following:

error analysis

errors for forward-difference method

error of current1 is : 0.365451

error of current2 is : 0.290845

error of current3 is : 0.197838

error of current4 is : 0.121415

errors for backward difference method

error of current1 is : 0.255191

error of current2 is : 0.228658

error of current3 is : 0.175356

error of current4 is : 0.115691

errors for threepoint midpoint method

error of current1 is : 0.178523

error of current2 is : 0.102613

error of current3 is : 0.035120

error of current4 is : 0.007688

errors for threepoint midpoint method without endpoints

error of current1 is : 0.065959

error of current2 is : 0.043861

error of current3 is : 0.018819

error of current4 is : 0.005327

IV. Conclusion and Remarks

a) Converge analysis with respect to the step size Δt

The step sizes of data sets are:

current1=0.075 s

current2=0.050 s

current3=0.025 s

current4=0.01 s

If we rank these four currents by step sizes:

$$current1 > current2 > current3 > current4$$

And if we look at Figure 3, we can see that current4 which has the lowest step size has the most decent curve, and current1 which has the largest step size has the most irregular one.

And also, if we look at the errors, we can see that in each method, current4 has the lowest error rate and current1 has the largest error rate. Therefore we can say that small step size means more accurate results.

b) Converge analysis with respect to order of the method

If we look at the error rates, we can see that three point midpoint method is the most accurate method because it has the lowest error rates. Backward difference method is the second most accurate method and forward difference method is the least accurate one because of the high error rates.

If we look at the Figure 3, we can see that in forward difference method, the voltage values are smaller than expected at first, however after some time, the values are started to converge towards 12 which is the estimated real value.

But unlike the forward difference, in backward difference the voltage values are higher than expected, however after some time, the values are started to converge towards 12.

Knowing this, we can say that forward difference and backward difference are opposite approximations which converge towards the same results.

In the end, we can say that we observed the expected converge rates. Three point midpoint is the most accurate one, and forward difference and backward difference methods has relatively the same rates. But we could not expect that the converge rate in the backward difference method is the more accurate than the forward difference method.

Question: if the end point formula is not used, how the accuracy effected?

If we look at the results of three point midpoint formula, we can see that especially the first point of the data set has the most inaccurate error rate.

```
errors for threepoint midpoint method with endpoints
```

```
error of current1 is : 0.178523
```

```
error of current2 is : 0.102613
```

```
error of current3 is : 0.035120
```

```
error of current4 is : 0.007688
```

```
errors for threepoint midpoint method without endpoints
```

```
error of current1 is : 0.065959
```

```
error of current2 is : 0.043861
```

```
error of current3 is : 0.018819
```

```
error of current4 is : 0.005327
```

if we analyze the results we can easily say that three point endpoint method is more accurate when we do not use end points.

Appendix

The full code is given below:

```
clear all;  
clc;  
format long;
```



```

load current1.dat
load current2.dat
load current3.dat
load current4.dat
%% forward-difference formula
for i=1:1:8
    forward1(i)=(current1(i+1,2)-current1(i,2))/0.0750;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.0750=%f\n",current1(i,1),current1(i+1,2),current1(i,2),forward1(i));
    E1_forward(i)= 0.98*forward1(i)+14.2*current1(i,2);
    fprintf("impressed voltage is %f\n",E1_forward(i));
end
fprintf("\n current2\n");
for i=1:1:12
    forward2(i)=(current2(i+1,2)-current2(i,2))/0.050;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.050=%f\n",current2(i,1),current2(i+1,2),current2(i,2),forward2(i));
    E2_forward(i)= 0.98*forward2(i)+14.2*current2(i,2);
    fprintf("impressed voltage is %f\n",E2_forward(i));
end
fprintf("\ncurrent3\n");
for i=1:1:24
    forward3(i)=(current3(i+1,2)-current3(i,2))/0.025;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.025=%f\n",current3(i,1),current3(i+1,2),current3(i,2),forward3(i));
    E3_forward(i)= 0.98*forward3(i)+14.2*current3(i,2);
    fprintf("impressed voltage is %f\n",E3_forward(i));
end
fprintf("\ncurrent4\n");
for i=1:1:60
    forward4(i)=(current4(i+1,2)-current4(i,2))/0.010;
    fprintf("derivative of current at t=%f is:((%f)-%f)/0.010=%f\n",current4(i,1),current4(i+1,2),current4(i,2),forward4(i));
    E4_forward(i)= 0.98*forward4(i)+14.2*current4(i,2);
    fprintf("impressed voltage is %f\n",E4_forward(i));
end

%% backward-difference formula
fprintf("\nbackward difference formula\n\n");
fprintf("current1\n");
for i=2:1:9
    backward1(i)=(current1(i,2)-current1(i-1,2))/0.0750;

```

```

    fprintf("derivative of current at t=%f is: ((%f)-
%f)/0.0750=%f\n",current1(i,1),current1(i,2),current1(i-
1,2),backward1(i));
    E1_backward(i)= 0.98*backward1(i)+14.2*current1(i,2);
    fprintf("impressed voltage is %f\n",E1_backward(i));
end
fprintf("\n current2\n");
for i=2:1:13
    backward2(i)=(current2(i,2)-current2(i-1,2))/0.050;
    fprintf("derivative of current at t=%f is: ((%f)-
%f)/0.050=%f\n",current2(i,1),current2(i,2),current2(i-
1,2),backward2(i));
    E2_backward(i)= 0.98*backward2(i)+14.2*current2(i,2);
    fprintf("impressed voltage is %f\n",E2_backward(i));
end
fprintf("\ncurrent3\n");
for i=2:1:25
    backward3(i)=(current3(i,2)-current3(i-1,2))/0.025;
    fprintf("derivative of current at t=%f is: ((%f)-
%f)/0.025=%f\n",current3(i,1),current3(i,2),current3(i-
1,2),backward3(i));
    E3_backward(i)= 0.98*backward3(i)+14.2*current3(i,2);
    fprintf("impressed voltage is %f\n",E3_backward(i));
end
fprintf("\ncurrent4\n");
for i=2:1:61
    backward4(i)=(current4(i,2)-current4(i-1,2))/0.010;
    fprintf("derivative of current at t=%f is: ((%f)-
%f)/0.010=%f\n",current4(i,1),current4(i,2),current4(i-
1,2),backward4(i));
    E4_backward(i)= 0.98*backward4(i)+14.2*current4(i,2);
    fprintf("impressed voltage is %f\n",E4_backward(i));
end
%% Three point midpoint formula
fprintf("\n\n three point midpoint formula\n");
fprintf("\ncurrent1\n\n");
h=0.075;
threepoint1(1)=(1/(2*h))*(-
3*current1(1,2)+4*current1(2,2)-current1(3,2));
fprintf("derivative of current at t=%f is: (1/%f)*(-
3*%f+4*%f-
%f)=%f\n",current1(1,1),2*h,current1(1,2),current1(2,2),cu
rrent1(3,2),threepoint1(1));
E1_threepoint(1)=threepoint1(1)*0.98+14.2*current1(1,2);
fprintf("impressed voltage is %f\n",E1_threepoint(1));
for i=2:1:8

```

```

    threepoint1(i)=(1/(2*h))*(current1(i+1,2)-current1(i-
1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current1(i,1),2*h,current1(i+1,2),current1(i-
1,2),threepoint1(i));

E1_threepoint(i)=threepoint1(i)*0.98+14.2*current1(i,2);
    fprintf("impressed voltage is %f\n",E1_threepoint(i));
end
h=-0.075;
threepoint1(9)=(1/(2*h))*(-
3*current1(9,2)+4*current1(8,2)-current1(7,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-
3*%f+4*%f-
%f)=%f\n",current1(9,1),2*h,current1(9,2),current1(8,2),cu
rrent1(7,2),threepoint1(9));
    E1_threepoint(9)=threepoint1(9)*0.98+14.2*current1(9,2);
    fprintf("impressed voltage is %f\n",E1_threepoint(9));

fprintf("\ncurrent2\n\n");
h=0.05;
threepoint2(1)=(1/(2*h))*(-
3*current2(1,2)+4*current2(2,2)-current2(3,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-
3*%f+4*%f-
%f)=%f\n",current2(1,1),2*h,current2(1,2),current2(2,2),cu
rrent2(3,2),threepoint2(1));
E2_threepoint(1)=threepoint2(1)*0.98+14.2*current2(1,2);
fprintf("impressed voltage is %f\n",E2_threepoint(1));
for i=2:1:12
    threepoint2(i)=(1/(2*h))*(current2(i+1,2)-
current2(i-1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current2(i,1),2*h,current2(i+1,2),current2(i-
1,2),threepoint2(i));

E2_threepoint(i)=threepoint2(i)*0.98+14.2*current2(i,2);
    fprintf("impressed voltage is %f\n",E2_threepoint(i));
end
h=-0.05;
threepoint2(13)=(1/(2*h))*(-
3*current2(13,2)+4*current2(12,2)-current2(11,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-
3*%f+4*%f-

```

```

%f)=%f\n",current2(13,1),2*h,current2(13,2),current2(12,2)
,current2(11,2),threepoint2(13));
E2_threepoint(13)=threepoint2(13)*0.98+14.2*current2(13,2)
;
fprintf("impressed voltage is %f\n",E2_threepoint(13));

fprintf("\ncurrent3\n\n");
h=0.025;
threepoint3(1)=(1/(2*h))*(-
3*current3(1,2)+4*current3(2,2)-current3(3,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-
3*%f+4*%f-
%f)=%f\n",current3(1,1),2*h,current3(1,2),current3(2,2),cu
rrent3(3,2),threepoint3(1));
E3_threepoint(1)=threepoint3(1)*0.98+14.2*current3(1,2);
fprintf("impressed voltage is %f\n",E3_threepoint(1));
for i=2:1:24
    threepoint3(i)=(1/(2*h))*(current3(i+1,2)-
current3(i-1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current3(i,1),2*h,current3(i+1,2),current3(i-
1,2),threepoint3(i));

E3_threepoint(i)=threepoint3(i)*0.98+14.2*current3(i,2);
    fprintf("impressed voltage is %f\n",E3_threepoint(i));
end
h=-0.025;
threepoint3(25)=(1/(2*h))*(-
3*current3(24,2)+4*current3(23,2)-current3(22,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-
3*%f+4*%f-
%f)=%f\n",current3(25,1),2*h,current3(24,2),current3(23,2)
,current3(22,2),threepoint3(25));
E3_threepoint(25)=threepoint3(25)*0.98+14.2*current3(25,2)
;
fprintf("impressed voltage is %f\n",E3_threepoint(25));

fprintf("\ncurrent4\n\n");
h=0.010;
threepoint4(1)=(1/(2*h))*(-
3*current4(1,2)+4*current4(2,2)-current4(3,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-
3*%f+4*%f-

```

```

%f)=%f\n",current4(1,1),2*h,current4(1,2),current4(2,2),cu
rrent4(3,2),threepoint4(1));
E4_threepoint(1)=threepoint4(1)*0.98+14.2*current4(1,2);
fprintf("impressed voltage is %f\n",E4_threepoint(1));
for i=2:1:60
    threepoint4(i)=(1/(2*h))*(current4(i+1,2)-
current4(i-1,2));
    fprintf("derivative of current at t=%f is:1/%f*(%f-
%f)=%f\n",current4(i,1),2*h,current4(i+1,2),current4(i-
1,2),threepoint4(i));

E4_threepoint(i)=threepoint4(i)*0.98+14.2*current4(i,2);
    fprintf("impressed voltage is %f\n",E4_threepoint(i));
end
h=-0.010;
threepoint4(61)=(1/(2*h))*(-
3*current4(60,2)+4*current4(59,2)-current4(58,2));
fprintf("derivative of current at t=%f is:(1/%f)*(-
3*%f+4*%f-
%f)=%f\n",current4(61,1),2*h,current4(60,2),current4(59,2)
,current4(58,2),threepoint4(61));
E4_threepoint(61)=threepoint4(61)*0.98+14.2*current4(61,2)
;
fprintf("impressed voltage is %f\n",E4_threepoint(61));

%% plotting graphs
figure(1)
subplot(3,1,2);
hold on
grid on;
plot(current1(2:end,1),backward1(2:end),'-x');
plot(current2(2:end,1),backward2(2:end),'-o');
plot(current3(2:end,1),backward3(2:end),'-s');
plot(current4(2:end,1),backward4(2:end),'-');
legend('derivative of current1','derivative of
current2','derivative of current3','derivative of
current4');
xlabel('time');
ylabel('derivative of current');
title('derivative of current versus time graph in
backward-difference formula');
hold off
subplot(3,1,1);
hold on
grid on;
plot(current1(1:end-1,1),forward1,'-x');

```

```

plot(current2(1:end-1,1),forward2,'-o');
plot(current3(1:end-1,1),forward3,'-s');
plot(current4(1:end-1,1),forward4,'-');
legend('derivative of current1','derivative of
current2','derivative of current3','derivative of
current4');
xlabel('time');
ylabel('derivative of current');
title('derivative of current versus time graph in forward-
difference formula');
hold off;
subplot(3,1,3);
hold on;
grid on;
plot(current1(:,1),threepoint1,'-x');
plot(current2(:,1),threepoint2,'-o');
plot(current3(:,1),threepoint3,'-s');
plot(current4(:,1),threepoint4,'-');
legend('derivative of current1','derivative of
current2','derivative of current3','derivative of
current4');
xlabel('time');
ylabel('derivative of current');
title('derivative of current versus time graph in
threepoint-difference formula');
hold off;

```

```

figure(2);
subplot(3,1,1);
hold on;
grid on;
plot(current1(1:end-1,1),E1_forward,'-x');
plot(current2(1:end-1,1),E2_forward,'-o');
plot(current3(1:end-1,1),E3_forward,'-s');
plot(current4(1:end-1,1),E4_forward,'-');
legend('E(t) for current1','E(t) for current2','E(t) for
current3','E(t) for current4');
xlabel('time');
ylabel('voltage');
title('impressed voltage versus time graph in forward-
difference formula');
hold off;

subplot(3,1,2);
hold on;
grid on;

```

```

plot(current1(2:end,1),E1_backward(2:end),'-x');
plot(current2(2:end,1),E2_backward(2:end),'-o');
plot(current3(2:end,1),E3_backward(2:end),'-s');
plot(current4(2:end,1),E4_backward(2:end),'-');
legend('E(t) for current1','E(t) for current2','E(t) for
current3','E(t) for current4');
xlabel('time');
ylabel('voltage');
title('impressed voltage versus time graph in backward-
difference formula');
hold off;

subplot(3,1,3);
hold on;
grid on;
plot(current1(:,1),E1_threepoint,'-x');
plot(current2(:,1),E2_threepoint,'-o');
plot(current3(:,1),E3_threepoint,'-s');
plot(current4(:,1),E4_threepoint,'-');
legend('E(t) for current1','E(t) for current2','E(t) for
current3','E(t) for current4');
xlabel('time');
ylabel('voltage');
title('impressed voltage versus time graph in threepoint-
difference formula');
hold off;
%% error analysis
fprintf("\nerror analysis\n\n");
fprintf("errors for forward-difference method\n");
fprintf("error of current1 is : %f\n",norm(E1_forward-
12)/norm(12));
fprintf("error of current2 is : %f\n",norm(E2_forward-
12)/norm(12));
fprintf("error of current3 is : %f\n",norm(E3_forward-
12)/norm(12));
fprintf("error of current4 is : %f\n",norm(E4_forward-
12)/norm(12));
fprintf("\n\nerrors for backward difference method\n");
fprintf("error of current1 is :
%f\n",norm((E1_backward(2:end)-12))/norm(12));
fprintf("error of current2 is :
%f\n",norm(E2_backward(2:end)-12)/norm(12));
fprintf("error of current3 is :
%f\n",norm(E3_backward(2:end)-12)/norm(12));
fprintf("error of current4 is :
%f\n",norm(E4_backward(2:end)-12)/norm(12));
fprintf("\n\nerrors for threepoint midpoint method\n");

```

```
fprintf("error of current1 is : %f\n",norm(E1_threepoint-12)/norm(12));
fprintf("error of current2 is : %f\n",norm(E2_threepoint-12)/norm(12));
fprintf("error of current3 is : %f\n",norm(E3_threepoint-12)/norm(12));
fprintf("error of current4 is : %f\n",norm(E4_threepoint-12)/norm(12));
fprintf("\n\nerrors for threepoint midpoint method without endpoints\n");
fprintf("error of current1 is : %f\n",norm(E1_threepoint(2:end-1)-12)/norm(12));
fprintf("error of current2 is : %f\n",norm(E2_threepoint(2:end-1)-12)/norm(12));
fprintf("error of current3 is : %f\n",norm(E3_threepoint(2:end-1)-12)/norm(12));
fprintf("error of current4 is : %f\n",norm(E4_threepoint(2:end-1)-12)/norm(12));
```