ELECTRONIC ENGINEERING DEPARTMENT

MATH 214 NUMERICAL METHODS

2020 – 2021 FALL

FINAL PROJECT

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1. Problem Definition and Formulation

The problem is, calculating the Current flows through the diode, voltage on the diode, voltage on the resistor and voltage on the inductor for 25 milliseconds and 2.5 milliseconds timesteps with using current voltage relation in fprdata.dat given data set. The circuit which analyzed in this project is given at Figure 1.

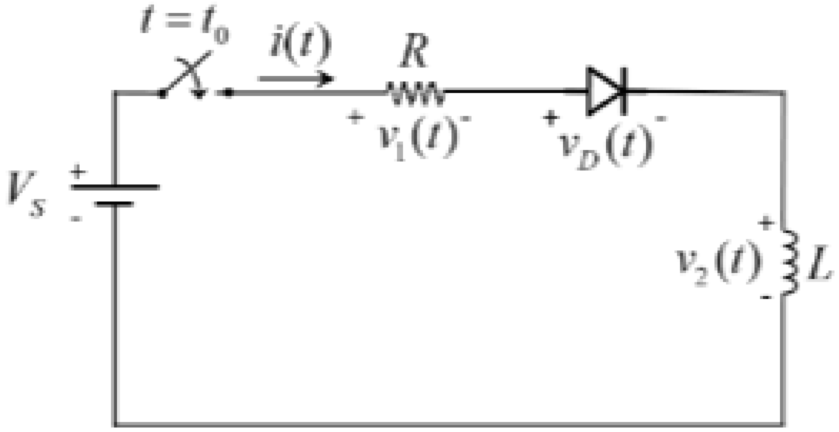

Figure 1. The circuit which used in this project

In Figure 1, R is 14.2 Ω, L is 0.98 H and V_s is 2 V values are given.

I. Current-voltage Relation on diode

The current on the diode is depends on voltage on the diode as shown in equation (1).

$$I_D = I_S(e^{\frac{V_D}{nV_T}} - 1) \tag{1}$$

In this equation, I_D is current on diode, V_D is voltage on diode, V_T is thermal voltage and I_S is reverse saturation current.

For very small values '-1' at the formula can be negligible. So the new formula is given in equation (2).

$$I_D = I_S e^{\frac{V_D}{nV_T}} \tag{2}$$

Since V_D, V_T, n and I_s are constants, it can be said that the current on the diode and the voltage relation is exponential function such like in equation (3).

$$I_D = be^{(aV_D)} \tag{3}$$

Since the current and voltage values at certain points on diode are given, a graph between current and voltage can be plotted with using given data set. With using that graph, an exponential least squares polynomial can be generated and this new polynomial can be used for finding the voltage on the diode for specific current values.

In equation (3), if natural logarithm(ln) taken for both sides of the equation, the equation will turn into a linear equation.

$$\ln(I_D) = \ln(b) + aV_D \tag{4}$$

Then, linear least squares polynomial can be applied to equation (4). The formula of linear least square polynomial is:

$$P = a_1 + a_0 x \tag{5}$$

In this case a_1 is ln(b) and a_0 is a.

The formulas of coefficients at linear least squares polynomial are given at equation (6) and (7).

$$\ln(b) = \frac{(\sum_{i=1}^{m} {V_D}_i^2 \cdot \sum_{i=1}^{m} \ln{(I_D)_i}) - (\sum_{i=1}^{m} {V_D}_i \ln(I_D)_i \cdot \sum_{i=1}^{m} {V_D}_i)}{m\left(\sum_{i=1}^{m} {V_D}_i^2\right) - \left(\sum_{i=1}^{m} {V_D}_i\right)^2} \tag{6}$$

$$a = \frac{m.\sum_{i=1}^{m} {V_D}_i \ln(I_D)_i - \left(\sum_{i=1}^{m} \ln(I_D)_i \cdot \sum_{i=1}^{m} {V_D}_i\right)}{m\left(\sum_{i=1}^{m} {V_D}_i^2\right) - \left(\sum_{i=1}^{m} {V_D}_i\right)^2} \tag{7}$$

In here m is amount of data which is 5 in this case.

After finding the coefficients, b can be found from using ln(b) and exponential least squares polynomial which given at equation (3) can be applied.

II. Creating an initial value problem

If Kirchhoff's voltage law applied in Figure 1, the equation at shown in equation (8) can be found.

$$V_S - V_1 - V_2 - V_D = 0 \tag{8}$$

From Ohm's law, voltage on the resistor and inductor can be written as shown in equation (9).

$$V_S - RI_D - \frac{dI_D}{dt}L - V_D = 0 \tag{9}$$

Since V_D has a relation with I_D, it can be written as follows:

$$\ln(I_D) = \ln(be^{aV_D})$$

$$\ln(I_D) = \ln(b) + aV_D$$

$$\mathrm{aV_D} = \ln\left(\frac{I_D}{b}\right)$$

$$\mathrm{V_D} = \frac{\ln\left(\frac{I_D}{b}\right)}{a} \tag{10}$$

After equation (10) is implemented into equation (9), equation (11) occurs.

$$V_S - RI_D - \frac{dI_D}{dt}L - \frac{\ln\left(\frac{I_D}{b}\right)}{a} = 0 \tag{11}$$

$$\frac{dI_D}{dt} = \frac{V_s - I_D R - \frac{\ln\left(\frac{I_D}{b}\right)}{a}}{L} \tag{12}$$

Now, Euler's method can be applied for solving the first order differential equation which given at equation (12).

III. Solving differential equation

Since $I_D = 0$ and $V_D = 0$ at t=0 values are given, the equation (12) became an initial value problem and it can be solved easily with using Euler's method.

Euler's Method

Let there be a differential equation such like this:

$$\frac{dy}{dx} = f(x,y) \tag{13}$$

Euler's method is based on a simple idea. Assuming the slope of the graph of given function is constant at given interval. The derivative and the y value can be found by this assumption.

By using equation (10), the Euler's method formula can be written as:

$$y_i^0 = y_{i-1} + f(x_{i-1}, y_{i-1})h \tag{14}$$

i=step number, h=rate of change of x

in this case, y is I_D and x is time.

After this Variables are implemented to code, all the current values are found. Then, V_1 can be found from Ohm's law and V_D can be found from exponential least squares polynomial.

$$V_1 = I_D R$$

$$\mathrm{V_D} = \frac{\ln\left(\frac{I_D}{b}\right)}{a}$$

After finding V_1 and V_D values, V_2 can easily found by equation (15).

$$V_2 = V_S - V_D - V_1 \tag{15}$$

2. Code and Inputs

All results and figures are generated by using code which given at the appendix section. The input data fprdata.dat is imported. After importing the data, summation of $\ln(I_D)$, summation of $\mathrm{V_D}$, summation of $V_D \ln(I_D)$, summation of V_D^2 are calculated for applying least squares polynomial. After finding These summations, the coefficients of linear least squares polynomial are calculated by using equation (6) and equation (7). After finding coefficients of linear least squares polynomial, b coefficient is found by using ln(b). Then, the exponential least squares polynomial is created as putting the variables at equation (3). After the least squares polynomial is found, errors between given data and exponential least squares polynomial are calculated and graph of exponential least squares polynomial is plotted.

An initial value problem is created using equation (12) and solved by using Euler's method at h=25 and h=2.5 milliseconds step sizes. After solving this, the current values (I_D) are found. Then, by using current values, voltage on the diode, voltage on the resistor and voltage on the inductor values are found and graphs of this voltages are plotted for comparison.

3. Project Results and Discussions

A) Calculating the Exponential Least Squares Polynomial

In this subsection the graph of the exponential least squares polynomial is given in Figure 2 and the coefficients of exponential least square polynomial are given as follows:

$$a = 21.2313$$
$$\ln(b) = -27.3296$$
$$b = 1.3517 \times 10^{-12}$$

So the exponential least squares polynomial is:

$$I_D = 1.3517 \times 10^{-12} \times e^{21.2313V_D}$$

And the error values between exponential least squares polynomial and given data set is given at Table 1.

Table 1. error values of least squares polynomial

acutal current	current by least squares method	error
0.000000023994400	0.000000023570645	0.017660560463059
0.000000054703000	0.000000055105686	0.007361316963999
0.001159870000000	0.001188199723263	0.024424912501796
0.190309000000000	0.202432541475187	0.063704509377837
0.421568000000000	0.390948593712329	0.072632188134942

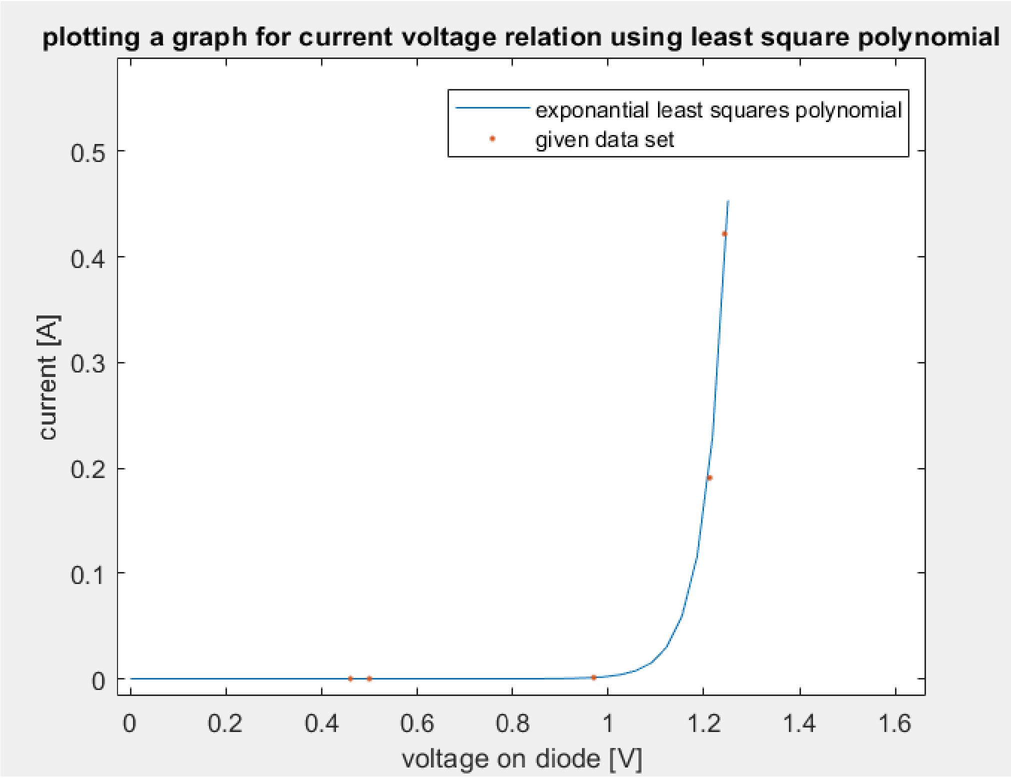

Figure 2. comparison of exponential least squares polynomial and given data graphs

B) Solving Initial Value Problem

In this subsection, the results of current and voltage graphs on diode which found by Euler's method is given in Figure 3 and comparison off all current voltage relation graphs is given in Figure 4. Since small step sizes means more accurate results, it can be expected to h=0.0025 give more accurate results. From Figure 3 and Figure 4, it can be told that this assumption is true.

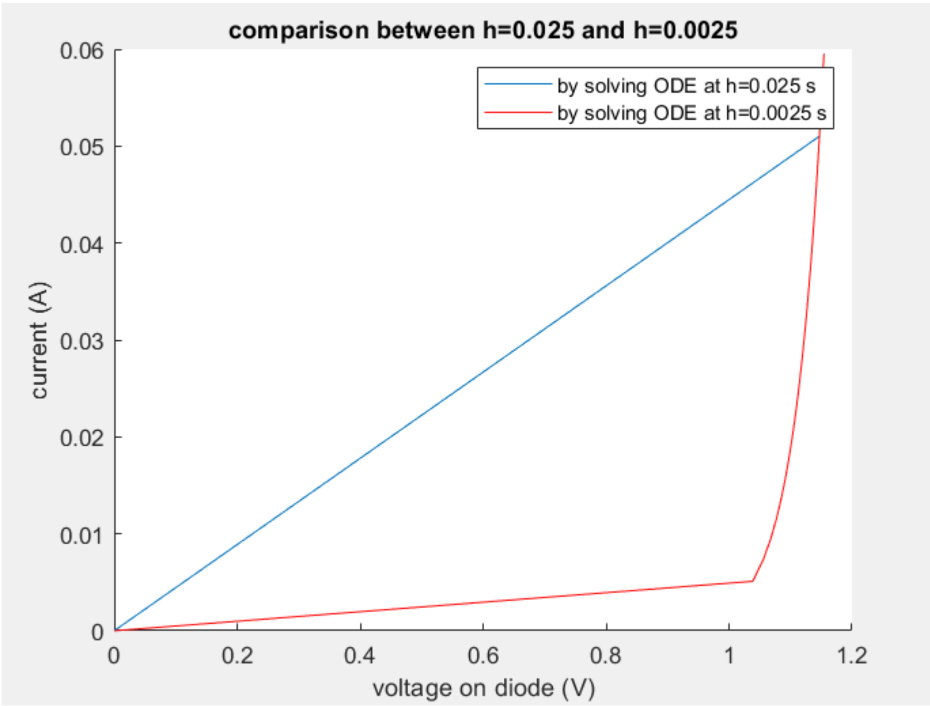

Figure 3. voltage and current relation comparison for different step sizes

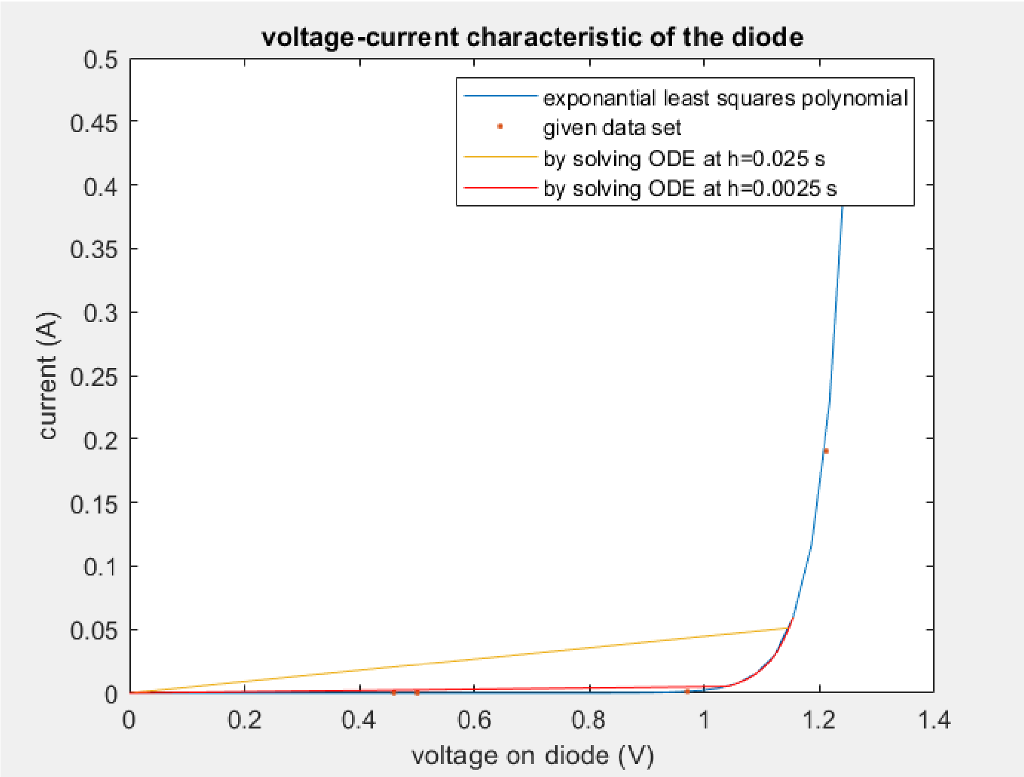

Figure 4. comparison of all voltage current relations on diode

C) Finding Voltage on The Inductor and Resistor

In this subsection, The voltage on the inductor versus time graph is given in Figure 5, Voltage on the Resistor graph is given in Figure 6, Voltage on the diode versus time graph is given in Figure 7 and current versus time graph is given in Figure 8.

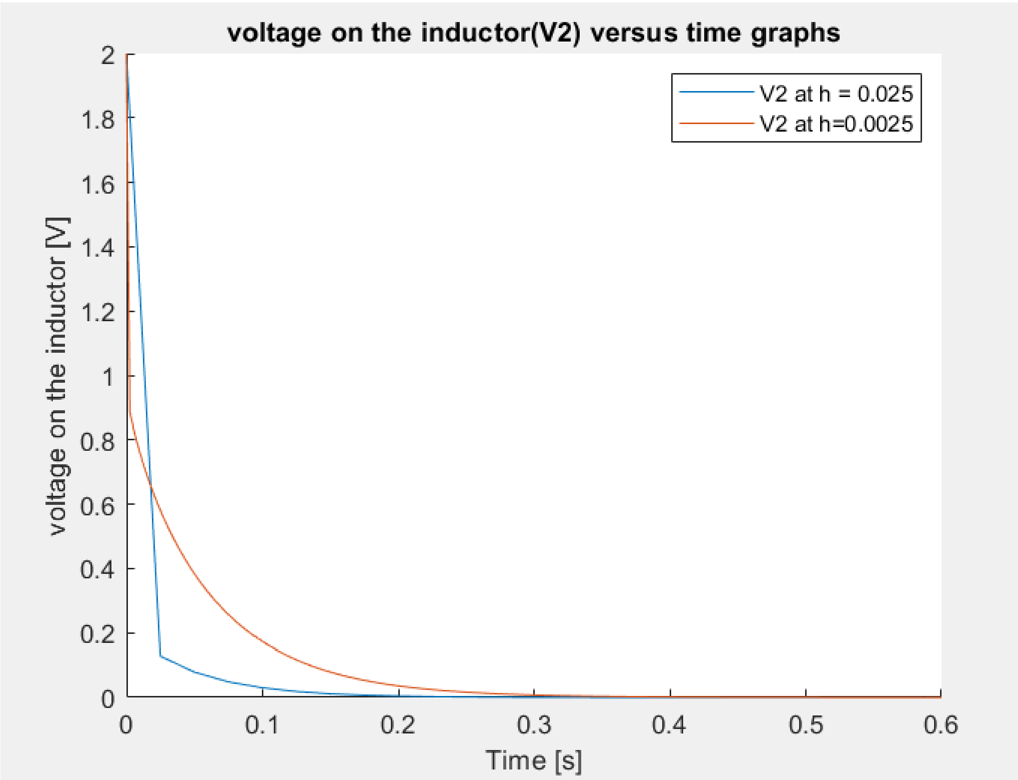

Figure 5. Voltage on the inductor versus time graph for different step sizes

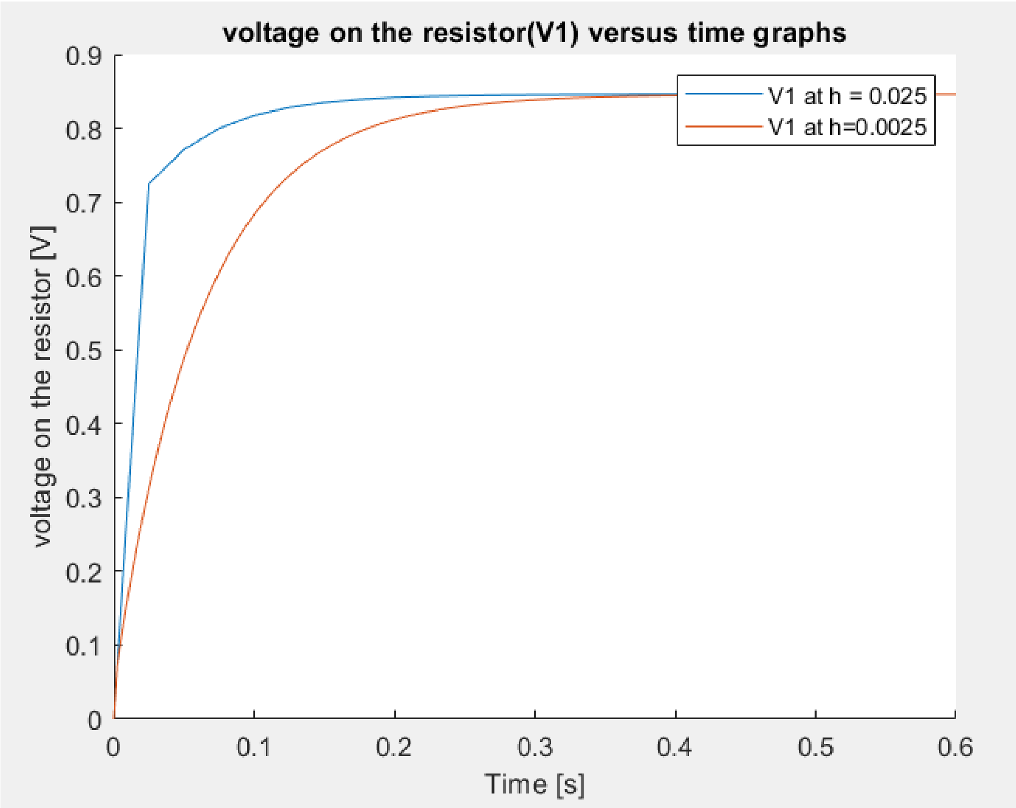

Figure 6. Voltage on the resistor versus time graph for different step sizes

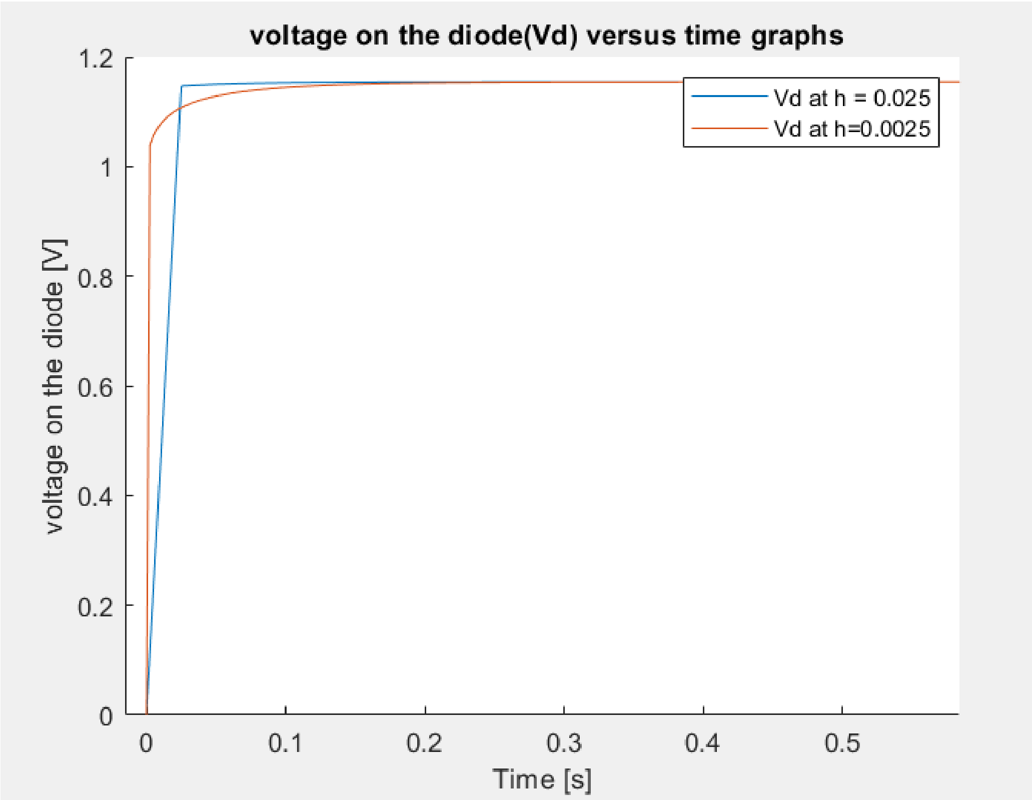

Figure 7. Voltage on the diode versus time graph for different step sizes

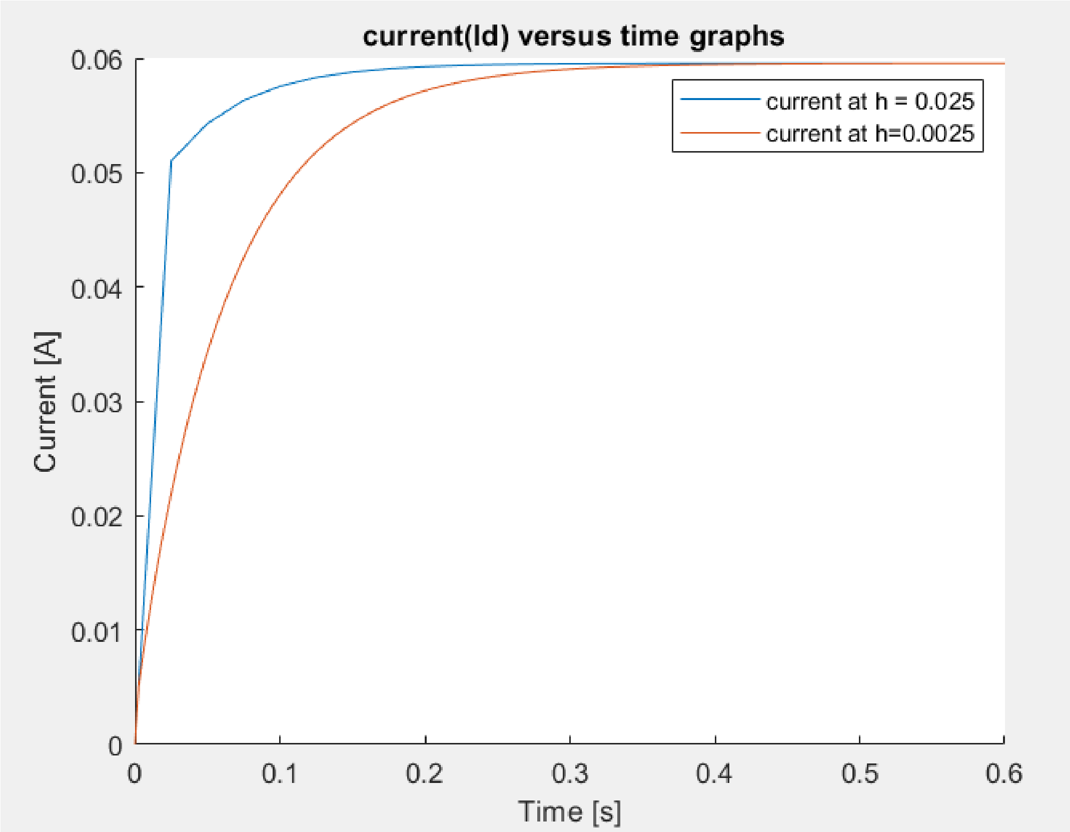

Figure 8. current versus time graph for different step sizes

4. Conclusion

It can be concluded that whenever the time step size decrease, the accuracy of results is increase. From all of the graphs, it can be said that the small time step sizes gives more proper results.

Exponential least squares polynomial method and Euler's method are used in this project for solving the problem. Since the current and voltage o the diode relation is an exponential function and little data set about this relation is given, Exponential least squares polynomial method looks like very decent way to finding an equation between current and voltage on the diode. Because all the circumstances are perfect for this.

The Euler's method is used for solving initial value problem in this project, Unlike the high order Runge-Kutta methods, Euler's method is not provide high accuracy rates, especially it can be seen at h=0.025 seconds time step size. But despite all this, Euler's method is the simplest way to solve an initial value problem and it still gives us reliable results. And that is why Euler's method is chosen for this problem.

5. Appendix

The code of the project is given below:

```
clear all;clc;
load fprdata.dat
```

```
%% fitting the exponential least squares polynomial
%ln(y)=ln(b)+ax
m=5;
%sum of x
sum_x=0;
for i=1:m
   sum_x = sum_x + fprdata(i,1);
end

% %sum Of y
% sum_y=0;
% for i=1:m
%    sum_y = sum_y + fprdata(i,2);
% end

%sum of ln(y)
sum_lny=0;
for i=1:m
   sum_lny = sum_lny + log(fprdata(i,2));
end

%sum Of x^2
sum_x2=0;
for i=1:m
   sum_x2 = sum_x2 + fprdata(i,1)^2;
end

%sum of x*ln(y)
sum_xlny=0;
for i=1:m
   sum_xlny = sum_xlny + fprdata(i,1)*log(fprdata(i,2));
end

a=((m*sum_xlny)-(sum_x*sum_lny))/(m*sum_x2-sum_x^2);

lnb=((sum_x2*sum_lny)-(sum_xlny*sum_x))/(m*sum_x2-sum_x^2);
b = exp(lnb);

Id_function = @(x) b * exp(a * x);
as=linspace(0,1.25,40);
 plot(as,Id_function(as));
hold on;
plot(fprdata(:,1),fprdata(:,2),".");
xlabel("voltage on diode [V]");
ylabel("current [A]");
legend("exponantial least squares polynomial","given data set");
title("plotting a graph for current voltage relation using least
square polynomial");

%finding error values
   fprintf("acutal current      current by least squares method
error\n");
for i=1:5
   error=abs(fprdata(i,2)-Id_function(fprdata(i,1)))/fprdata(i,2);
   fprintf("%15.15f \t %15.15f \t \t
%15.15f\n",fprdata(i,2),Id_function(fprdata(i,1)),error);
```

```
end


%% finding current and voltages
Vs=2;
R=14.2;
L=0.98;
h=0.025;


V1(1)=0;
Id(1)=0;
Vd(1)=0;


for i=1:0.6/h
    Id(i+1) = Id(i) + h * ( Vs - V1(i) - Vd(i) ) / L;
    Vd(i+1) = log(Id(i+1)/b)/a;
    V1(i+1) = Id(i+1)*R;
end



%Vs=V1+V2+Vd
%V2=Vs-V1-Vd
for i=1:(0.6/h)+1 %adding plus 1 because since we use i+1 notation
at other variables its length must be same with them
V2(i)=Vs - V1(i )- Vd(i);
end

h=0.0025;

V1_2(1)=0;
Id_2(1)=0;
Vd_2(1)=0;

for i=1:0.600/h
    Id_2(i+1) = Id_2(i) + h * ( Vs - V1_2(i) - Vd_2(i) ) / L;
    Vd_2(i+1) = log(Id_2(i+1)/b)/a;
    V1_2(i+1) = Id_2(i+1)*R;
end


%Vs=V1+V2+Vd
%V2=Vs-V1-Vd
for i=1:(0.6/h)+1 %adding plus 1 because since we use i+1 notation
at other variables its length must be same with them
V2_2(i)=Vs - V1_2(i) - Vd_2(i);
end

figure;
hold on;
plot(Vd,Id);
plot(Vd_2,Id_2,"-r");
xlabel("voltage on diode (V)");
ylabel("current (A)");
```

```
legend("by solving ODE at h=0.025 s","by solving ODE at h=0.0025
s");
title("comparison between h=0.025 and h=0.0025");
hold off;

figure;
plot(as,Id_function(as));
hold on;
plot(fprdata(:,1),fprdata(:,2),".");
plot(Vd,Id);
plot(Vd_2,Id_2,"-r");
xlabel("voltage on diode (V)");
ylabel("current (A)");
legend("exponantial least squares polynomial","given data set","by
solving ODE at h=0.025 s","by solving ODE at h=0.0025 s");
title("voltage-current characteristic of the diode");
%creating time arrays for plotting graphs versus time
time_1(1)=0;
time_2(1)=0;

for i=1:0.6/0.025
   time_1(i+1)= time_1(i) + 0.025;
end

for i=1:0.6/0.0025
   time_2(i+1)=time_2(i) + 0.0025;
end
%plotting current graph
figure;
title(" current(Id) versus time graphs");
hold on;
plot(time_1,Id);
plot(time_2,Id_2);
hold off;
legend("current at h = 0.025","current at h=0.0025");
xlabel("Time [s]");
ylabel("Current [A]");
%plotting Vd graph
figure;
title(" voltage on the diode(Vd) versus time graphs");
hold on;
plot(time_1,Vd);
plot(time_2,Vd_2);
hold off;
legend("Vd at h = 0.025","Vd at h=0.0025");
xlabel("Time [s]");
ylabel("voltage on the diode [V]");
%plotting V1 vs time graph
figure;
title("voltage on the resistor(V1) versus time graphs");
hold on;
plot(time_1,V1);
plot(time_2,V1_2);
xlabel("Time [s]");
ylabel("voltage on the resistor [V]");
hold off;
```

```
legend("V1 at h = 0.025","V1 at h=0.0025");
%plotting V2 vs time graphs
figure;
title("voltage on the inductor(V2) versus time graphs");
hold on;
plot(time_1,V2);
plot(time_2,V2_2);
hold off;
legend("V2 at h = 0.025","V2 at h=0.0025");
xlabel("Time [s]");
ylabel("voltage on the inductor [V]");
```