Implied Volatility Driven Price Movements

Alperen Canbey

May 2022

Abstract

This paper proposes a model to explain the effect of implied volatility changes on underlying equity market. The implied volatility change results in rebalancing of delta hedges which puts pressure on the underlying equity and stock index prices. This feedback effect differs from the information-based explanations between the derivatives market and the underlying market. While price-demand elasticity and open interest on options amplify the magnitude of these flows; liquidity and the logarithmic distance between the strike price of the option and the underlying price weaken the impact. Depending on market positioning in options and the direction of the implied volatility change, the strike price can be a price magnet, or it can repel the price. Throughout the paper, simulations are conducted with different volatility functions to illustrate the effect. Pinning probability with different parameters is examined with Monte-Carlo simulations. In future work, pinning probability in small windows where the implied volatility changes drastically will be approximated mathematically. This study aims to illustrate the effect of market positioning of agents on option prices, especially during the volatility events are occurring.

Contents

1 Introduction

For investors, being decision-makers and decision-takers in financial markets makes the price discovery difficult and the job interesting enough. On top of that, after the release of equity and index options, another interesting question arose, which seeks an answer to whether the individual stocks and indexes are affected by the changes in the options market which are backed by these stocks and indexes. With the growing popularity and the usage of options, this question and its effect on the underlying market have gotten more and more significant. Besides the empirical research done so far, some theoretical models were developed to explain some phenomena caused by the interaction between the financial markets and the options markets. To explain the price-pinning caused by the time derivative of the delta of an option, Avellaneda and Lipkin (2003) [1] provide a diffusion model where the time component is dependent on the delta hedging activities of market-makers who are long options. Frey and Stremme (1997) [13] demonstrate how market volatility is affected under delta-hedging activities depending on both price and time. Jeannin, Iori, and Samuel (2008) [20] combine the previous two analyses to explain the price-pinning by the effects of a hedging-dependent drift term and a hedging-dependent volatility term.

This paper explains these feedback effects by investigating the changes in the implied volatility of options. In order to do this, we added an implied volatility term to the price evolution process of an equity and whenever there is a change in implied volatility it affects the underlying price in the amount which is required to rebalance the delta-hedges of the market-makers. Although this mechanism is expected to work whenever there is a change in IV, it is more obvious if the magnitude of the change is big. This yields possible explanations for sudden price movements near volatility events, such as elections, FOMC meetings, earning calls, and CPI announcements. The price uncertainty around earning announcements and the possibility of stock price jumps during these periods cause the IV to increase before earnings announcements and a sudden drop after the events (Dubinsky, Johannes, Kaeck, and Seeger (2019) [?] and Patell and Wolfson (1979) [?]). Andersen, Fusari, and Todorov (2016) [?] investigate the shifts in the left risk-neutral tail by analyzing the option surface of the weekly options and show that FOMC meetings are the events where shifts in the slope of the left tail occur.

To demonstrate the effects of implied volatility on the underlying equity prices, we need to state not only whether market makers are long options, but also where the option strike price is positioned compared to the underlying equity price. For market makers who are long options, whenever there is implied volatility compression in the market, the price moves towards the strike price with the rebalancing of the delta hedges, if they are short options, then the strike price of these options repels the market price. This is because, when market makers are long OTM options, compressed implied volatility means decreasing probability for price to end up ITM which decays the delta of the option, and market makers need to make price pressure towards to strike price while hedging their portfolio. It is expected that volatility events such as FOMC meetings and earning announcements are prone to large price movements independent from implied volatility moves during these periods. Nevertheless, Alexiou, Goyal, Kostakis, and Rompolis (2022) [?] document that equities that exhibit concave IV surface structure before earning announcements have a 1.65% higher absolute abnormal stock return compared to equities with non-concave IV curves. Although one can argue that concavity in the IV curve represents the expectations for post-earning announcement price moves, this paper can provide possible theoretical ground to explain high absolute returns after these events.

Since this paper provides explanations depending on the mechanics of market microstructure, there might be times that the price of an equity or an index might diverge from its efficient price because of these forces. Therefore, we assume that the market is not efficient in the short term, however, it is expected that the price goes to the level justified by the fundamentals after the mechanical forces diminish. Moreover, to be able to observe these price changes caused by the delta-hedging activity, we assume that the markets are not perfectly liquid as in Avellaneda and Lipkin (2003) [1] and Frey and Stremme (1997) [13]. Therefore, the open interest in options and the price demand elasticity, which is also explained by market liquidity, are the components that affect the magnitude of these delta-hedging flows while the market positioning of market makers determines the direction.

This paper is directly connected to the literature which explains the mentioned effect through the non-information-based microstructural mechanisms. Ni, Pearson, and Poteshman (2005) [26] and Golez and Jackwerth (2012) [16] show that option delta decay, caused by the time derivative of the delta, results in rebalancing of option market makers' delta hedges near the option expiration. This causes the underlying stock prices to cluster and stock index futures prices to pin at option strike prices near the option expiration. Price-pinning term defines the mechanism where the price ends up in a small neighborhood of the strike price at the option maturity. This happens, because whenever the price goes above the strike price rebalancing of delta hedges requires the market makers to sell the asset, and vice versa. The mechanism for price-pinning is modeled and simulated by Avellaneda and Lipkin (2003) [1] and a closed-form solution for market-pinning probability is suggested, while Krishnan and Nelken (2001) [21] propose a model for price-pinning where the mechanism is explained by using exogenous forces since the price dynamics are assumed to be based on a Brownian Bridge. Jeannin, Iori, and Samuel (2008) [20] extend the analysis of Avellaneda and Lipkin [1] and explain the price-pinning by combining the effects of a hedging-dependent drift term and a hedging-dependent volatility term. In this paper, price pinning does not define pinning on maturity where the price remains near the strike price. It defines the event where the price touches the given strike price at the end of determined windows where volatility changes drastically. Ni, Pearson, Poteshman, and White (2018) [27] studied the net gammas of the option positions to explain the negative correlation between the stock return volatility and these option gammas where the mechanism caused by the rebalancing of option market makers' delta hedges. The same mechanism is used by Barbon and Buraschi (2021) [4] to explain the emergence of intraday momentum and reversion occurring at both the individual stock and index levels. Anderegg, Ullmann, and Sornette (2022) [2] investigate the feedback effect occurring from delta hedging strategies in the FX spot market.

As this paper concerns the effect of the options markets on the underlying market, it is also related to the literature that explains this effect through informed trading and asymmetric information. Easley, O'Hara, and Srinivas (1998) [11] develop an asymmetric information model to explain why informed traders to trade in options markets and they present evidence for the informational role of the options volume in underlying markets. Glosten and Milgrom (1985) [15] discuss that insider trading and adverse selection are some reasons for the existence of the bid-ask spread and they show that the transaction prices convey information about future stock returns. Pan and Poteshman (2006) [29] provide evidence for the existence of an information-based channel linking the trading activity in the options market and the underlying market by using the options volume on puts and calls. Besides the price discovery, Ni, Pan, and Poteshman (2008) [25] investigate the effect of options demand on the subsequent realized volatility in the underlying market. Cremers and Weinbaum (2010) [7] investigate how the deviations from put-call parity can predict the information risk in the underlying stocks. The shape of the volatility skew is studied by Xing, Zhang, and Zhao (2010) [31] and Lin and Lu (2015) [22] to show its predictive power for future equity returns. Ge, Lin, and Pearson (2016) [14] introduce evidence that purchases of call options to open new positions are the strongest predictor of returns. Cremers, Fodor, Muravyev, and Weinbaum (2021) [6] show that option trading activity can predict future stock returns around corporate news events. Hu (2014) [18] and Omole and Şensoy (2022) [28] use option order imbalances to estimate the future underlying returns.

The rest of the paper is organized as follows. In section 2, the mechanism behind this feedback effect will be explained. In section 3, the model will be documented where we use the regular Black-Scholes equation with an extra implied volatility term. In section 4, the Monte-Carlo simulations illustrating this effect are conducted.

2 Mechanism

In this section, the hypotheses of this paper will be stated and the mechanisms that cause these phenomenons will be explained in plain English and shown mathematically.

- The evolution of implied volatility affects price evolution process through derivative market.
- Sudden increases and decreases of implied volatility may create market rallies or crushes especially in an illiquid environment.
- Over the course of the implied volatility compression or implied volatility upsurge, the strike price, where a huge amount of options are traded, behaves either as a price magnet or it pushes the price away from itself.

As discussed, market-makers and dealers have to delta hedge their positions in order to protect themselves from the market-risk. Delta-hedging depends on the delta of the option. This determines the price change of an option per unit change in the underlying price of the equity. Intuitively it represents an adjustment for the probability of an option ending up in-the-money. Delta of an option depends on some variables such as the underlying price of the equity, implied volatility and time to expiration. Therefore, changes in implied volatility affects the delta of an option as well as the hedging positions of market-makers and dealers. These changes may create buying or selling pressure depending on the direction of the change and the positioning of these market-makers.

For example, thinking of an equity with an underlying market mid-price at 90 dollar, an institutional investor sells 150 lots of European call options with a strike price 110 dollar. In each lot there are 100 contracts. Thus, the market-maker who bought these call options from the institutional investor has to hedge their position. Assuming the market-maker is only hedging the delta exposure of the trade, they have to sell the underlying equity by $150 * 100 * \Delta_{C,110,t}$, where $\Delta_{C,110,t}$ represents the delta of the call option at time t and where the equity price is 110 dollar. In accordance with the price-demand elasticity and the liquidity in the market, the price of the equity will be pushed to downside.

Now let's examine a case where the implied volatility of the call options decrease. Then, what happens? Intuitively, the probability of the price to make big moves decrease which fade the probability of these call options ending up in-the-money. This means the delta of the call option diminishes. Now, the delta exposure of the market-makers is less than what they were exposed to at the first place. Therefore, they need to re-buy some of the shares of the equity to re-hegde their position. This makes an upward pressure on the price.

To mathematically show this phenomenon, let's first define the price process where the underlying equity price follows Black Scholes model with time dependent volatility.

$$\frac{dS_t}{S_t} = rdt + \sigma_t dW_t.$$

where dW_t is a standard Brownian motion and $r > 0$ is the constant interest rate. The future realized volatility at time t is defined as

$$\hat{\sigma}_t^2 = \frac{1}{T-t}\int_t^T \sigma_\tau^2 d\tau \tag{1}$$

where σ_t^2 is the time-dependent volatility, and T is the maturity date. However, since the future realized volatility is not known, an estimate should be replaced with it. Defining a function F such that $F(t, S_t, K, \hat{\sigma}_t)$ gives the Black-Scholes price of a European Call option where the strike price K and the underlying price is S_t. Then the price of the call option can be written as

$$P_c = \mathbb{E}(F(T, S_T, K, \hat{\sigma}_T^2)). \tag{2}$$

Therefore we can define the implied volatility as $I_{t,K} = BS^{-}1(t, S_t, K, \hat{\sigma}_T^2)$. Since we work with a single strike throughout this paper, we use I_t from this point on.

Considering the market maker is long European call options with a strike price K and the time to maturity T when the underlying price is S_t, the directional effect of implied volatility change is:

$$
\begin{aligned}
\frac{\partial \Delta_t(S_t, I_t)}{\partial I_t} &= \frac{\partial N(\mathtt{d_{1,t}})}{\partial I_t} \\
&= n(\mathtt{d_{1,t}})\frac{\partial \mathtt{d_{1,t}}}{\partial I_t} \\
&= n(\mathtt{d_{1,t}})\Big(-\frac{1}{I_t^2\sqrt{T-t}}\Big(\ln\Big(\frac{S_t}{K}\Big) + r(T-t)\Big) + \frac{1}{2}\sqrt{T-t}\Big) \\
&= -n(\mathtt{d_{1,t}})\frac{1}{I_t}\mathtt{d_{2,t}}
\end{aligned}
$$

where it is well-known that $\Delta_t(S_t, I_t) = N(\mathtt{d_{1,t}})$.

$$
\begin{aligned}
\mathtt{d_{1,t}} &= \frac{1}{I_t\sqrt{T-t}}\Big(\ln\Big(\frac{S_t}{K}\Big) + \Big(r + \frac{1}{2}I_t^2\Big)(T-t)\Big), \\
\mathtt{d_{2,t}} &= \frac{1}{I_t\sqrt{T-t}}\Big(\ln\Big(\frac{S_t}{K}\Big) + \Big(r - \frac{1}{2}I_t^2\Big)(T-t)\Big).
\end{aligned}
$$

and all the process for $\mathtt{d_{1,t}}$, $\mathtt{d_{2,t}}$ and $\Delta_t(S_t, I_t)$ derived from Black Scholes option pricing equation $\frac{dS_t}{S_t} = rdt + \sigma_t dW_t$. N($\mathtt{d_{1,t}}$) and n($\mathtt{d_{1,t}}$) represents the cumulative and density distribution function of $\mathtt{d_{1,t}}$. Since $n(\mathtt{d_{1,t}})$ and I_t are always positive, we need to investigate the sign of $\mathtt{d_{2,t}}$ to assign the direction of the trade when the implied volatility decreases. As a reminder for the case that is investigated here, the market is positioned such that the market-maker sold call options.

To simplify and understand the logic, assume $r - \frac{1}{2}I_t^2 = 0$ around time t where the implied volatility change in the market is observed. Then,

- If $S_t < K$, then $\mathtt{d_{2,t}} < 0$, and it implies that $\frac{\partial \Delta_t(S_t, I_t)}{\partial I_t} > 0$. This means that the delta exposure of the market-maker decreases when volatility goes down. Therefore, during the time when volatility crushes the buying pressure of the market-maker may cause a rally. As volatility continues to decrease the price would raise until the strike price because if the price exceeds the strike price, then the conditions revert and there will be a selling pressure, which makes the strike price a magnet for underlying price.
- If $S_t > K$, then $\mathtt{d_{2,t}} > 0$, and it implies that $\frac{\partial \Delta_t(S_t, I_t)}{\partial I_t} < 0$. This means delta exposure of the market-makers goes up when the volatility goes down. Therefore, a reverse mechanism of the first case will be in play and the market-maker will sell the equity more, until $S_t = K$.

Thus, in such a positioning and with a compressing volatility, the strike price will work as a price magnet and depending on the the position of the strike price and the equity price, it is possible to see market rally or a market crush. If the volatility increases, then the mechanism would be the reverse and the price pressure would be away from the strike price.

Without assuming $r - \frac{1}{2}I_t^2 = 0$, the mechanism works with the same logic. However, this time the sign of $\mathtt{d_{2,t}}$ doesn't change at the point where $S_t = K$ but it changes in the neighborhood of the strike price as long as $(\frac{r}{I_t} - \frac{1}{2}I_t)\sqrt{T-t}$ is bounded by a constant C and the price will remain to converge somewhere between $K - C$ and $K + C$. Therefore, it is better to re-state the last hypothesis more precisely:

- Over the course of the volatility compression or volatility upsurge, a value in the neighborhood of the strike price behaves as a price magnet or it pushes the price away from itself.

3 Model

In this section, a new price evolution process will be defined to demonstrate the effect of implied volatility change, besides the noise and the trend in the market. Nevertheless, the hedging mechanism of option dealers will be assumed to remain the same. That is to say equity prices are assumed to be affected by implied volatility while option dealers will use $\mathtt{d_{1,t}}$ derived from the regular Black-Scholes price evolution process to delta-hedge their positions.

By abuse of notation we also define S_t as the new price process taking into account the trade size caused by the implied volatility change and price elasticity of different stocks. Then,

$$\frac{dS_t}{S_t} = EQ_t + rdt + \sigma_t dW_t,$$

where S_t is the stock price in the new process, E is the price-demand elasticity of a specific stock and Q is the trade size.

Assuming that the market makers are long c amount of call options (or put options):

$$Q_t = -c \frac{\partial \Delta_t(S_t, I_t)}{\partial I_t} dI_t.$$

Therefore, to show the price movement mechanism induced by the changes in volatility, a stochastic differential equation is assumed with an additional term which lead the price per unit change in volatility. Thus, the instantaneous change in price satisfies

$$\frac{dS_t}{S_t} = -Ec \frac{\partial \Delta(S_t, I_t)}{\partial I_t} dI_t + rdt + \sigma_t dW_t. \tag{3}$$

If the market makers are short c amount of call or put options:

$$\frac{dS_t}{S_t} = Ec \frac{\partial \Delta(S_t, I_t)}{\partial I_t} dI_t + rdt + \sigma_t dW_t.$$

where W_t is a standard Brownian motion.

A model with deterministic volatility process is assumed to better illustrate the effect of implied volatility on the price process. It is because, the focus of this paper is to explain the price movements happened in the windows where the implied volatility changes towards a direction strongly. Considering the log-moneyness process where we define it as $y_t = \ln(\frac{S_t}{K})$ and since the volatility and implied volatility process is deterministic, by applying Ito's formula:

$$dy_t = (r - \frac{1}{2}\sigma_t^2)dt + \sigma_t dW_t + cEn(\mathtt{d_{1,t}})\frac{1}{I_t}\mathtt{d_{2,t}}dI_t. \tag{4}$$

where

$$n(\mathtt{d_{1,t}})\frac{1}{I_t}\mathtt{d_{2,t}} = \frac{1}{\sqrt{2\pi}}e^{-\frac{\mathtt{d_{1,t}}^2}{2}}\Big(- \frac{1}{I_t^2\sqrt{T-t}}\Big(y_t + r(T-t) - \frac{1}{2}I_t^2(T-t)\Big)\Big).$$

To increase the generality of the variables, we write the log-moneyness evolution process defined in equation 4 in terms of non-dimensional variables. This will allow us to make consistent comparisons between the variables. For example, now we define a variable which not only defines the log-moneyness but also it scales the term in accordance with the implied volatility and time to maturity. Furthermore, the variables will be beneficial while finding a mathemeatical approximation of the pinning-probability. We will write the equation 4 in terms of the following terms:

$$\begin{aligned} z_t &= \frac{y_t}{I_t\sqrt{T}}, \\ s &= \frac{t}{T}. \end{aligned}$$

Applying Ito's Lemma and putting above variables into the equation to express the equation in terms of non-dimensional variables:

$$\begin{aligned}
dz_t &= \frac{dy_t}{I_t\sqrt{T}} - \frac{y_t}{I_t^2\sqrt{T}}dI_t, \\
dz_t &= \Big(- \frac{1}{I_t\sqrt{T}}\frac{cE}{2\pi}\frac{(y_t + r(T-t) - \frac{1}{2}I_t^2(T-t))}{I_t^2\sqrt{T-t}}e^{-\left(\frac{y_t+r(T-t)+\frac{1}{2}I_t^2(T-t)}{\sqrt{2}I_t\sqrt{T-t)}}\right)^2} - \frac{y_t}{I_t^2\sqrt{T}}\Big)dI_t + \frac{r-\frac{1}{2}\sigma_t^2}{I_t\sqrt{T}}dt + \frac{\sigma_t}{I_t\sqrt{T}}dW_t, \\
dz_t &= \Big(- \frac{cE}{2\pi}\frac{(z_t + \frac{r(T-t)}{I_t\sqrt{T}} - \frac{1}{2}\frac{I_t(T-t)}{\sqrt{T}})}{I_t^2\sqrt{T-t}}e^{-\left(\frac{y_t+r(T-t)+\frac{1}{2}I_t^2(T-t)}{\sqrt{2}I_t\sqrt{T-t)}}\right)^2} - \frac{z_t}{I_t}\Big)dI_t + \frac{r-\frac{1}{2}\sigma_t^2}{I_t\sqrt{T}}dt + \frac{\sigma_t}{I_t\sqrt{T}}dW_t, \\
dz_s &= \Big(- \frac{cE}{2\pi}\frac{(z_s + \frac{rT(1-s)}{I_s\sqrt{T}} - \frac{1}{2}\frac{I_sT(1-s)}{\sqrt{T}})}{I_s^2\sqrt{T(1-s)}}e^{-\left(\frac{y_s+rT(1-s)+\frac{1}{2}I_s^2T(1-s)}{\sqrt{2}I_s\sqrt{T(1-s))}}\right)^2} - \frac{z_s}{I_s}\Big)dI_s + \frac{r\sqrt{T}-\frac{1}{2}\sigma_t^2\sqrt{T}}{I_s}ds + \frac{\sigma_s}{I_s}dW_s, \\
dz_s &= \Big(- \frac{\beta_s(z_s + \alpha_s(1-s) - \Sigma_s(1-s))}{\sqrt{1-s}}e^{-\frac{(z_s+\alpha_s(1-s)+\Sigma_s(1-s))^2}{2(1-s)}} - \frac{z_s\sqrt{T}}{2\Sigma_s}\Big)dI_s + (\alpha_s - k_s^2\Sigma_s)ds + k_s dW_s.
\end{aligned}$$

where $ds = \frac{dt}{T}$, $0 < s < 1$, and $T^{-\frac{1}{2}}W_t = W_{\frac{t}{T}} = W_s$. The variables used above are as the following:

$$\begin{aligned} \beta_s &= \frac{cE}{\sqrt{2\pi I_s^4 T}}, \\ \alpha_s &= \frac{r\sqrt{T}}{I_s}, \\ \Sigma_s &= \frac{1}{2}I_s\sqrt{T}, \\ k_s &= \frac{\sigma_s}{I_s}. \end{aligned}$$

As the goal is to investigate the price movements happened in intervals where the implied volatility changes towards a direction strongly, a deterministic process for volatility will be assumed throughout the paper which will be useful for the illustration of simulations. Since the implied volatility is assumed as in Equation 2, writing the implied volatility process in terms of time:

$$\begin{aligned}
dI_t^2 &= \Big(\frac{1}{T-t}I_t^2 - \frac{1}{T-t}\sigma_t^2\Big)dt, \\
dI_t &= \frac{1}{2I_t}\frac{1}{T-t}\Big(I_t^2 - \sigma_t^2\Big)dt, \\
dI_s &= \frac{1}{\sqrt{T}}\frac{\Sigma_s(1-k_s)}{1-s}ds.
\end{aligned}$$

$$dz_s = \Big(-\frac{\tilde{\beta}_s(1-k_s)(z_s+\alpha_s(1-s)-\Sigma_s(1-s))}{(1-s)^{\frac{3}{2}}}e^{\frac{(z_s+\alpha_s(1-s)+\Sigma_s(1-s))^2}{2(1-s)}} - \frac{z_s(1-k_s)}{2(1-s)} + (\alpha_s - k_s^2\Sigma_s)\Big)ds + k_s dW_s$$

where

$$\tilde{\beta}_s = \frac{cE}{\sqrt{8\pi I_s^2 T}}. \tag{5}$$

Therefore, the parameters that will define the pinning-probability:

$$\begin{aligned}
z_0 &= \frac{y_0}{I_0\sqrt{T}}, \\
\tilde{\beta}_0 &= \frac{cE}{\sqrt{8\pi I_0^2 T}}, \\
\alpha_0 &= \frac{r\sqrt{T}}{I_0} \\
k_0 &= \frac{\sigma_0}{I_0}.
\end{aligned}$$

As mentioned, it is practical to make interpretations with these parameters. β_0 represents the effect of the number of options traded and the elasticity. Whenever either of them increases, the effect dependent on the volatility derivative of delta gets stronger. z_0 defines the initial non-dimensional logarithmic distance between the strike price and the underlying price which decreases the probability of pinning as the distance increases. α_0 defines the initial value of the drift parameter.

4 Simulations

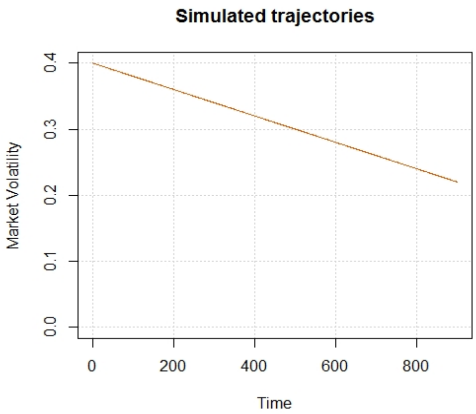
To see the effect of the implied volatility change on the price process, some simulations are conducted with different type of volatility functions. The time to maturity is taken as 0.1 year where it corresponds to 1000 discrete points in the simulations. The strike price is taken as 100 which is expected to behave as a price magnet or it can push the price away in accordance with the option positioning of market makers. The interest rate in the drift term is assumed to be 0.

4.1 Simulations with Deterministic Volatility

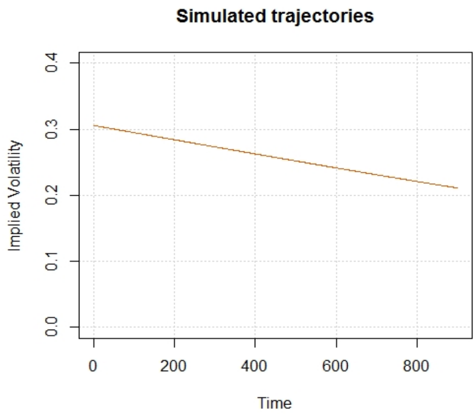
This part is documented to show the relation between the volatility and implied volatility which is driven from equation 2, and the behavior of the price evolution in accordance with the different deterministic volatility functions.

In Figure 1a, volatility is assumed to decrease from 0.4 to 0.2 and the corresponding levels of implied volatility is represented at 1b. Figure 2 illustrates the price evolution processes depending on the relation between the initial price (S_0) and the strike price (K) where the market makers have long position on options. In both of the graphs the strike price behaves as the price magnet. However, when the market makers are short options, then the mechanism revert and the strike price starts to put an outward pressure on the price as described in Figure 3.

In Appendix, the Figures from 11 to 16 represent the similar processes but under concave and convex volatility functions. One point to pay attention is that, although, the price process for the convex function in Figure 15 is still around the strike price, the effect is decreasing towards the end of the period which is consistent as the rate of volatility change diminishes.

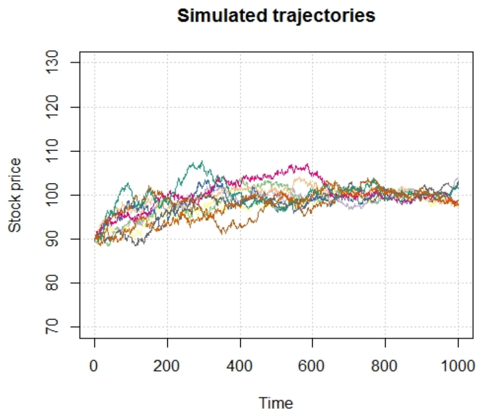


(a) Market Volatility

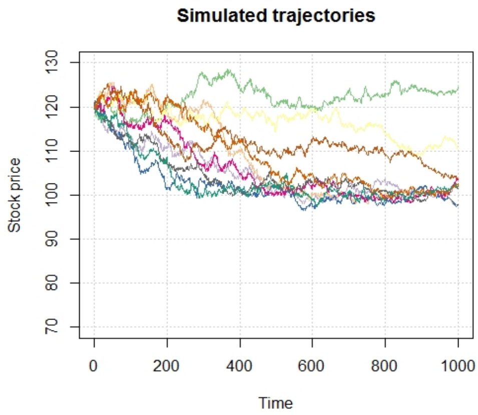


(b) Implied Volatility

Figure 1: Linear Volatility

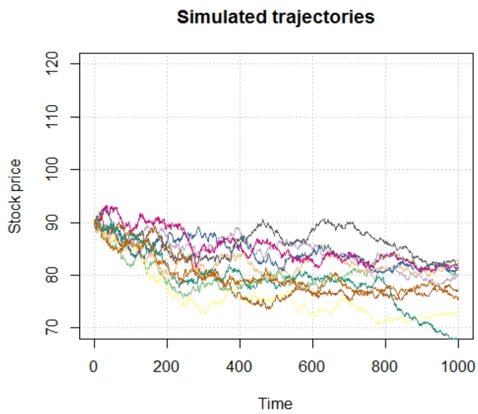


(a) $S_0 <$ K

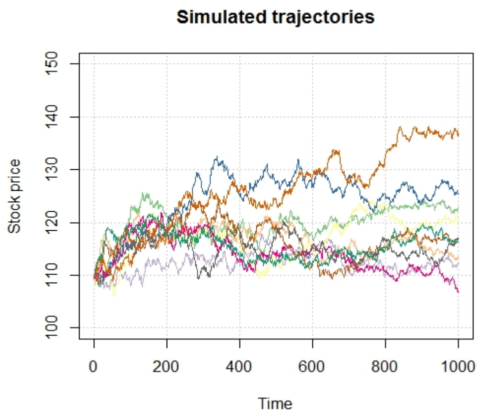


(b) $S_0 >$ K

Figure 2: Market Makers Long Options



(a) $S_0 <$ K



(b) $S_0 >$ K

Figure 3: Market Makers Short Options

4.2 Pinning Probability

As discussed in the Model part, the parameters affect the pinning probability, and they have meaningful interpretations of these probabilities. Throughout this section, the volatility change is assumed to be from 0.4 to 0.2. Since the pinning can be observed only in the case that the market makers are long options, this kind of positioning is assumed. Both Figures 4 and 5 are drawn by using Monte-Carlo simulations by using 100 points for the values in the x-axis and the probability calculations are made for each point out of 3000 simulations.

Pinning is assumed to happen if the log-price distance is smaller than 0.01 for the simulations. Nevertheless, theoretically $Pr[z(\tau) = 0|z(0) = z_0]$ defines the pinning probability, where τ is the time when volatility compression ends.

In Figure 4, $\tilde{\beta}_0$ is equalized to 3.15 which represents the power of the effect. The black line describes the pinning probability per the initial log distance when the implied volatility effect is in play while the red line illustrates the pinning probability when the implied volatility effect is muted. This graph clearly shows that the implied volatility effect increases the probability a lot when the volatility change is high. Moreover, it is observed that the pinning probability diminishes while the normalized log-distance increases.

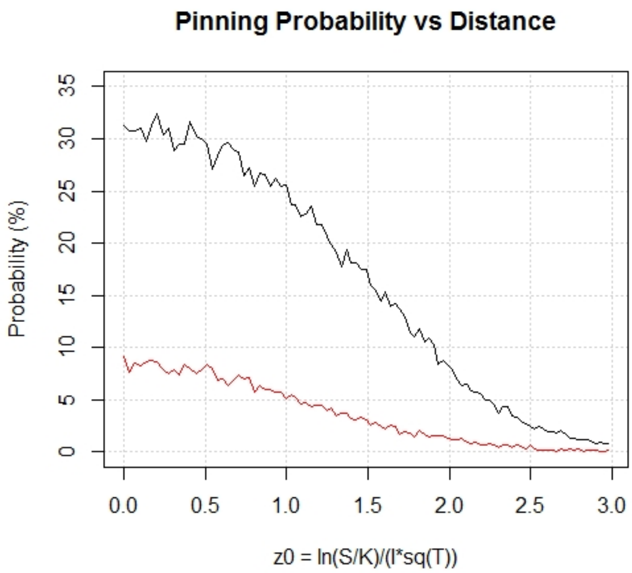


Figure 4: Pinning Probability vs Z_0

On the other hand, Figure 5 shows the effect of $\tilde{\beta}_0$ on the pinning probability where two different cases are illustrated where x" Consistent with the 4, as the normalized log-distance increases, the probability decreases. Since the $\tilde{\beta}_0$ represents the power of the effect, it is expected to see increments in probability as the $\tilde{\beta}_0$ increases. Similarly, we can comment on the components of $\tilde{\beta}_0$. As the total amount of the open interest in options increases, the power of the effect will be amplified. The same logic applies to the price-demand elasticity.

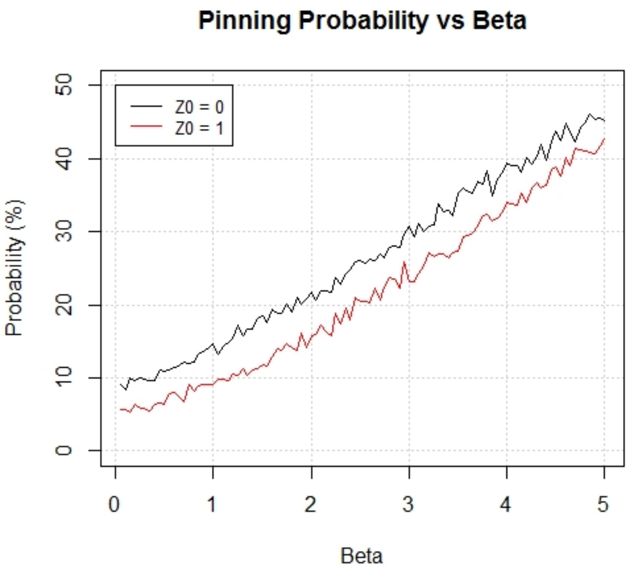
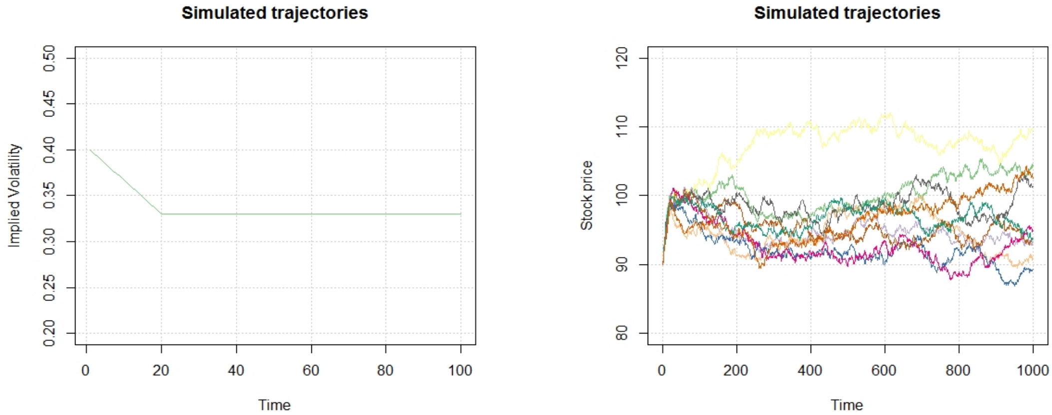


Figure 5: Pinning Probability vs $\tilde{\beta}_0$

4.3 Simulations to Show Local Effects



(a) Implied Volatility (b) Price Evolution

Figure 6: Market Makers Long Options

As the volatility compression during a month is not a common case in real life, it would be beneficial to look at small windows where volatility crushes. In this section, the simulations are conducted again in 0.1 year time period, however, the implied volatility change happens only in 0.002 years which corresponds to less than 1 day period. Figure 6 illustrates the time that the implied volatility decreases from 0.4 to 0.33 and the corresponding effect on the price evolution process. As observed, the compression in implied volatility causes a 1-day stock price rally before we observe a regular Black-Scholes price process. Figure 7 is drawn to show the difference between the effect in our model and the regular Black-Scholes model. Since the implied volatility changes in the first 20 periods in the simulation, the only difference is observed in these periods.

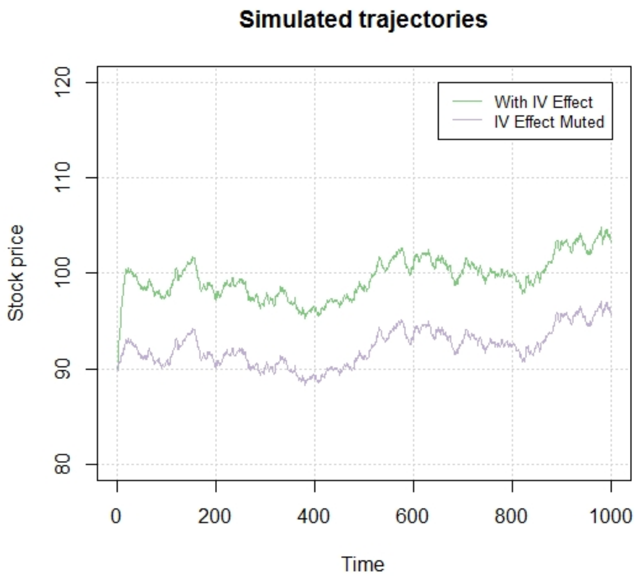


Figure 7: Implied Volatility Effect

Figures 8 and 9 illustrate the pinning probabilities following the change in normalized log distance and $\tilde{\beta}_0$, respectively. Both of the figures are drawn by using Monte-Carlo simulations where the values in the x-axis are simulated by 100 discrete points where the probability is calculated with 3000 simulations for each of these points. In Figure 8, $\tilde{\beta}_0$ is determined to be 4.7 and the implied volatility change is assumed to be from 0.4 to 0.33 for both figures.

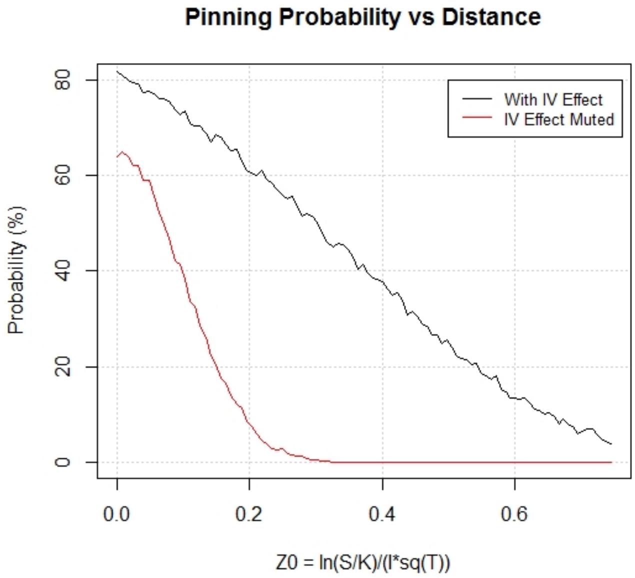


Figure 8: Pinning Probability vs z_0

In Figure 8, the black line represents the probability with the implied volatility effect on price while the red line illustrates the probability with the price process in the Black-Scholes model. As observed from the graph, implied volatility effect significantly increases the probability of pinning, and the normalized log distance is negatively correlated with the pinning probability. Figure 99 shows the dependence of the pinning probability on $\tilde{\beta}_0$ where the dependence on normalized distance is also illustrated by investigating the cases where $z_0 = 0.5$ and $z_0 = 0.75$.

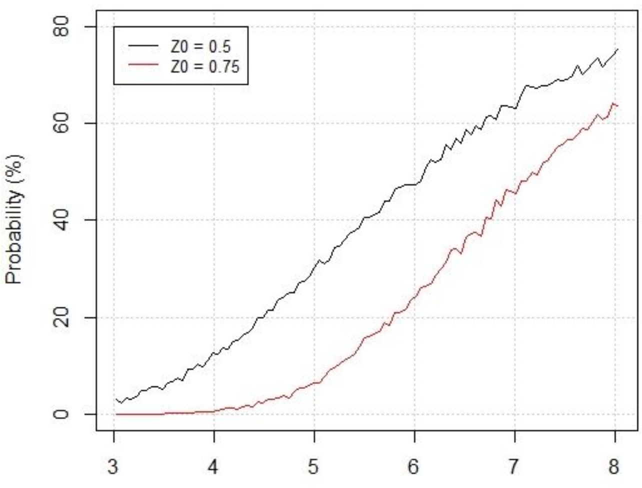
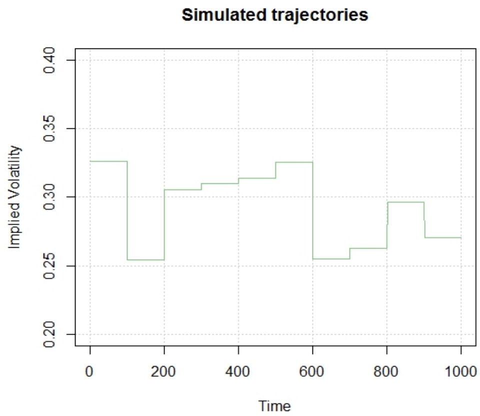


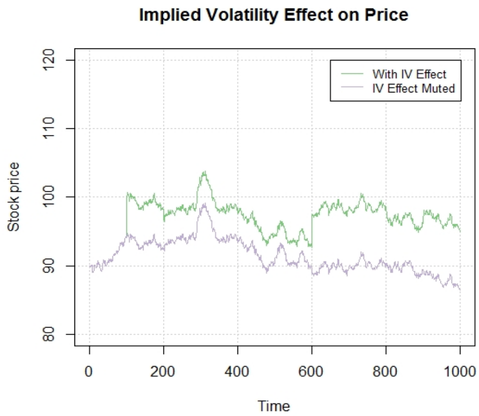
Figure 9: Pinning Probability vs $\tilde{\beta}_0$

4.4 Simulations with Stepwise Constant Implied Volatility

In this section, a stepwise constant implied volatility function is assumed to show the mechanism. Figure 10 illustrates the implied volatility function and corresponding price evolution process with a comparison with the Black-Scholes price process. Since the volatility crushes in periods 100 and 600 of the simulation the price is pulled by the strike price while in period 200 the mechanism works in the other direction.



(a) Stepwise Constant Implied Volatility



(b) Price Evolution

Figure 10: Effect of Implied Volatility where MMs are long options

5 Mathematical Approximation to Pinning Probability

[WORK IN PROGRESS]

6 Conclusion

[WORK IN PROGRESS]

References

[1] **Avellaneda, Marco and Michael D. Lipkin.** 2003. “A market-induced mechanism for stock pinning.” *Quantitative Finance.*

[2] **Anderegg, Benjamin, Florian Ulmann, Didier Sornette.** 2022. “The impact of option hedging on the spot market volatility.” *Journal of International Money and Finance.*

[3] **Ben-David, Itzhak, Francesco Franzoni, and Rabih Moussawi.** 2018. “Do etfs increase volatility?.” *Journal of Finance.*

[4] **Barbon, Andrea, Andrea Buraschi.** 2020. “Gamma fragility.” *SSRN Working Paper.*

[5] **Black, Fischer, and Myron Scholes.** 1973. “The pricing of options and corporate liabilities.” *Journal of Political Economy.*

[6] **Cremers, Martijn, Andrew Fodor, Dmitriy Muravyev, and David Weinbaum.** 2021. “Option Trading Activity, News Releases, and Stock Return Predictability.” *SSRN Working Paper.*

[7] **Cremers, Martijn, and David Weinbaum.** 2010. “Deviations from put–call parity and stock return predictability.” *Journal of Financial and Quantitative Analysis.*

[8] **Daniel, Kent, David Hirshleifer, and Avanhidar Subrahmanyam.** 1998. “A theory of overconfidence, self-attribution, and security market under and over-reactions.” *Journal of Finance.*

[9] **Duffie, Darrel., Jun Pan, and Kenneth Singleton.** 2000. “Transform analysis and asset pricing for affine jump-diffusions.” *Econometrica.*

[10] **Dupire, Bruno.** 1994. “Pricing with a smile.” *Risk.*

[11] **Easley, David, Maureen O’Hara, Pulle S. Srinivas.** 1998. “Option volume and stock prices: Evidence on where informed traders trade.” *Journal of Finance.*

[12] **Fama, Eugene F. and Kenneth F. French.** 1993. “Common risk factors in the returns on stocks and bonds.” *Journal of Financial Economics.*

[13] **Frey, Rudiger. and Alexander Stremme.** 1997. “Market volatility and feedback effects from dynamic hedging.” *Mathematical Finance.*

[14] **Ge, Li, Tse-Chun Lin, and Neil D. Pearson.** 2016. “Why does the option to stock volume ratio predict stock returns?.” *Journal of Financial Economics.*

[15] **Glosten, Lawrence, Paul Milgrom.** 1985. “Bid, ask, and transaction prices in a specialist market with heterogenously informed traders.” *Journal of Financial Economics.*

[16] **Golez, Benjamin and Jens Carsten Jackwerth.** 2012. “Pinning in the S&P 500 Futures.” *Journal of Financial Economics.*

[17] **Guillaume, Florence, and Wim Schoutens** 2014. "Heston Model: the Variance Swap Calibration" *Journal of Optimization Theory and Applications.*

[18] **Hu, Jianfeng.** 2014. "Does option trading convey stock price information?" *Journal of Financial Economics.*

[19] **Heston, Steven L.** 1993. "A closed-form solution for options with stochastic volatility with applications to bond and currency options." *Review of Financial Studies.*

[20] **Jeannin, Marc, Giulia Iori, and David Samuel.,** 2008. "Modeling stock pinning." *Quantitative Finance.*

[21] **Krishnan, H., Nelken, I.,** 2001. "The effect of stock pinning upon option prices." *RISK.*

[22] **Lin, Tse-Chun and Xiaolung Lu.** 2015. "Why Do Options Prices Predict Stock Returns? Evidence from Analyst Tipping." *Journal of Banking and Finance.*

[23] **Madan, Dilip, Peter Carr, and Eric Chang** 1998. "The variance gamma process and option pricing." *Review of Finance.*

[24] **Merton, Robert C.** 1973. "The theory of rational option pricing." *The Bell Journal of Economics and Management Science.*

[25] **Ni, Sophie Xiaoyan., Jun Pan, Allen M. Poteshman, and Joshua White.** 2008. "Volatility information trading in the option market." *Journal of Finance.*

[26] **Ni, Sophie Xiaoyan., Neil D. Pearson, Allen M. Poteshman, and Joshua White.** 2005. "Stock price clustering on option expiration dates." *Journal of Financial Economics.*

[27] **Ni, Sophie Xiaoyan., Neil D. Pearson, Allen M. Poteshman, and Joshua White.** 2018. "Does option trading have a pervasive impact on underlying stock prices?." *The Review of Financial Studies.*

[28] **Omole, John, and Ahmet Şensoy.** 2022. "Information content of order imbalance in the index options market." *International Review of Economics and Finance.*

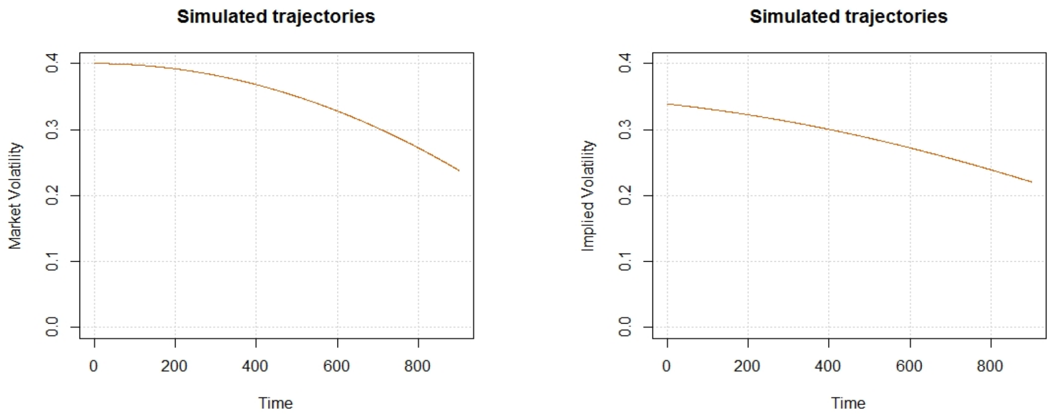
[29] **Pan, Jun, and Allen M. Poteshman.** 2006. "The information in option volume for future stock prices." *The Review of Financial Studies.*

[30] **Shum, Pauline, Walid Hejazi, Edgar Haryanto, and Arthur Rodier.** 2015. "Intraday share price volatility and leveraged etf rebalancing." *Review of Finance.*

[31] **Xing, Yuhang, Xiaoyan Zhang, and Ruhui Zhao.** 2010. "What Does the Individual Option Volatility Smirk Tell Us About Future Equity Returns?." *Journal of Financial and Quantitative Analysis.*

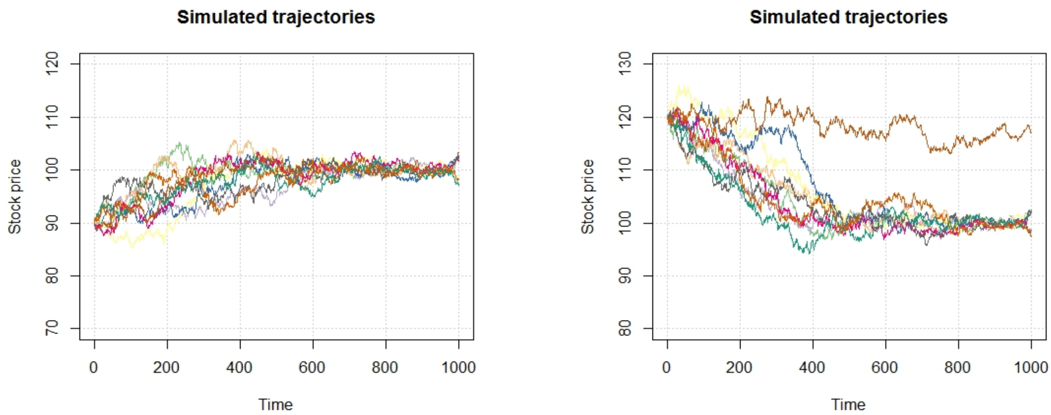
Appendix

A Figures and tables



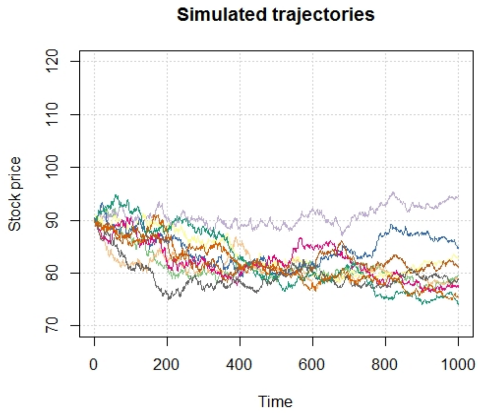
(a) Market Volatility (b) Implied Volatility

Figure 11: Concave Volatility

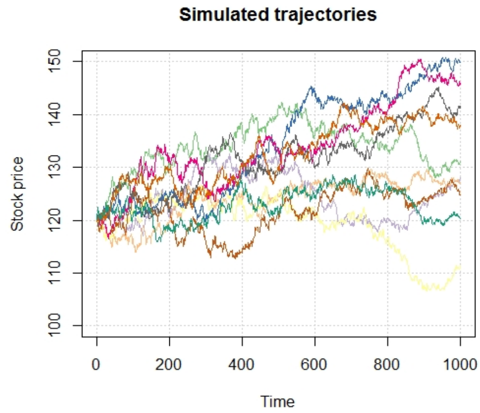


(a) S_0 < K (b) S_0 > K

Figure 12: Market Makers Long Options

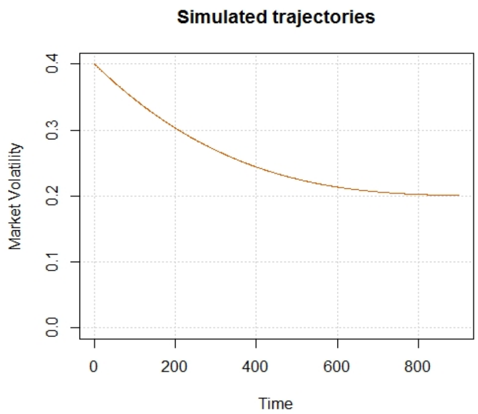


(a) S_0 < K

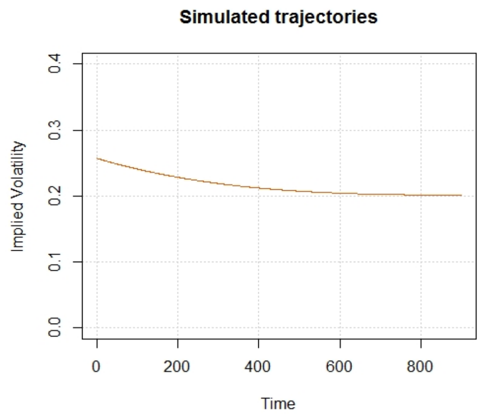


(b) S_0 > K

Figure 13: Market Makers Short Options

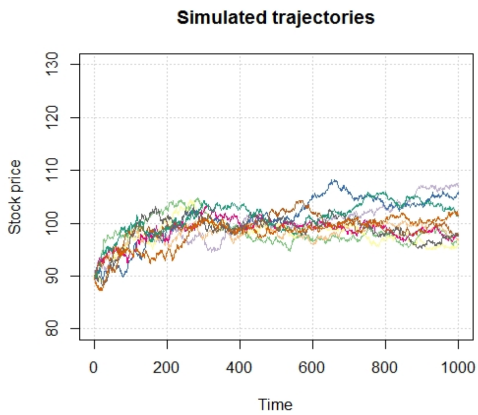


(a) Market Volatility

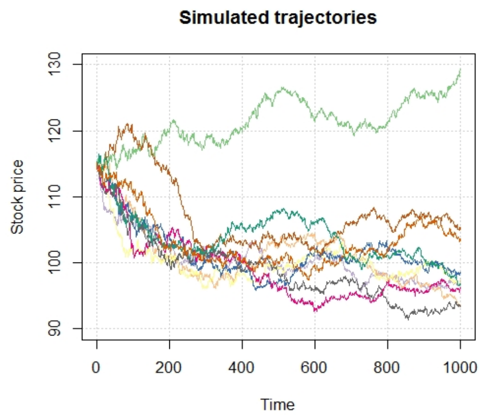


(b) Realized Volatility

Figure 14: Convex Volatility

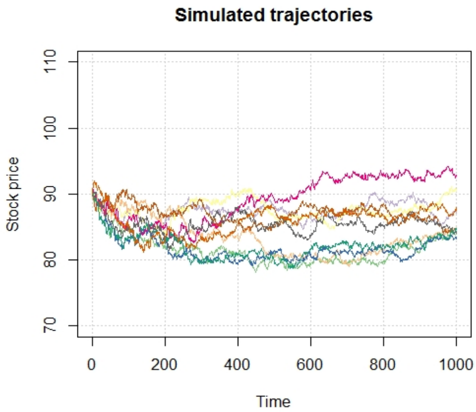


(a) S_0 < K

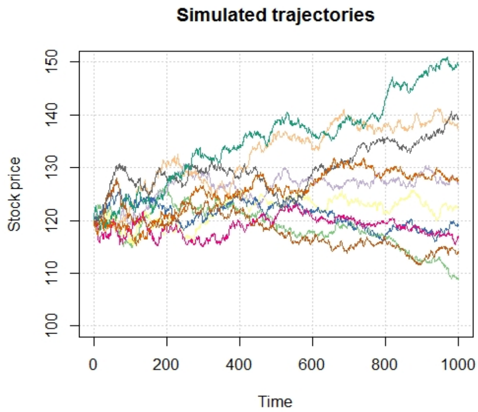


(b) S_0 > K

Figure 15: Market Makers Long Options



(a) S_0 < K



(b) S_0 > K

Figure 16: Market Makers Short Options