Graduation Project Topics
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In all of the following projects, your task is to implement a polynomial-time heuristic algorithm for the pertinent problem, which is in general NP-hard. You are required to make a comprehensive literature review and identify the existing approaches. You may then opt to implement an existing polynomial-time heuristic algorithm, if any, for the problem. If not and/or you prefer not to do so, then you need to propose your own polynomial-time heuristic algorithm and implement it. You are also required to present the computational complexity analysis of the algorithm you have implemented. In addition, you need to make a numerical comparative evaluation of your proposed algorithm via extensive simulations. You are also required to prepare a graphical user interface that demonstrates the execution of your algorithm.

1. **Maximum Independent Sequence Problem**

   Given a graph $G = (V, E)$, the *maximum independent sequence problem* is to find an independent sequence for G, that is, a sequence $v_1, v_2, \dots, v_m$ of independent vertices of G such that, for all $i < m$, a vertex $\overline{v_i} \in V$ exists which is adjacent to v_{i+1} but is not adjacent to any v_j for $j \leq i$ and the length of the sequence (*m*) is minimized.

2. **Computing the Zero Forcing Number of a Graph**

   Let *G* be a graph with each vertex initially colored either black or white. From the initial coloring, vertices change color according to the *color-change rule:* if *u* is a black vertex and exactly one neighbor *v* of *u* is white, then change the color of *v* to black. When the color change rule is applied to *u* to change the color of *v*, we say *u forces v* and write *u*→*v*. Given an initial coloring of *G*, the *derived set* is the set of vertices colored black after the color change rule is applied until no more changes are possible. In an initial black-white coloring of a graph *G*, if the set of black vertices *Z* has derived set that is all the vertices of *G*, we say *Z* is a *zero forcing set* for *G.* A zero forcing set with the minimum number of vertices is called an *optimal zero forcing set,* and this minimum size of a zero forcing set for a graph *G* is the *zero forcing number* of the graph, denoted by *Z(G).*

3. Defective Coloring

Proper coloring refers to an assignment of colors or labels to vertices such that no adjacent vertices have the same color. In defective coloring, on the other hand, vertices are allowed to have neighbors of the same color to a certain extent. Given a positive integer *k* and a non-negative integer *d,* a *(k,d)-coloring* of a graph *G* is a coloring of its vertices with *k* colors such that each vertex *v* has at most *d* neighbors having the same color as vertex *v*. Hence, *(k, 0)-coloring* is equivalent to proper vertex coloring. The minimum number of colors *k* required for which *G* is *(k, d)*-colorable is called the *d-defective chromatic number*.

4. Precoloring Extension

Precoloring extension is the problem of extending a graph coloring of a subset of the vertices of a graph, with a given set of colors, to a coloring of the whole graph that does not assign the same color to any two adjacent vertices. In particular, given a graph with some of the vertices a preassigned color, the precoloring extension problem is to find whether this coloring can be extended to a proper *k*-coloring of the graph.

5. Complete Coloring and Achromatic Number of a Graph

A *complete coloring* of a graph G is a proper vertex coloring of *G* having the property that for every two distinct colors *i* and *j* used in the coloring, there exist adjacent vertices of *G* colored *i* and *j.* A complete coloring in which *k* colors are used is a *complete k-coloring*. The largest positive integer *k* for which G has a complete k-coloring is the *achromatic number* of *G*.

6. T-Coloring

Given a set T of nonnegative integers containing 0, a T-coloring of a graph *G=(V, E)* is a function $c:V(G)\to N$ that maps each vertex to a positive integer (color) such that if *u* and *w* are adjacent, then $|c(u) - c(w)| \notin T$. In other words, the absolute value of the difference between two colors of adjacent vertices must not belong to fixed *T*. The *T-chromatic number* is the minimum number of colors that can be used in a T-coloring of *G*.

7. Roman Domination Number

A Roman dominating function of a graph $G=(V, E)$ is a function $f : V \rightarrow \{0, 1, 2\}$ such that every vertex x with $f(x)=0$ is adjacent to at least one vertex y with $f(y)=2$. The weight of a Roman dominating function is defined to be $f(v) = \sum_{x \in V} f(x)$, and the minimum weight of a Roman dominating function on a graph *G* is called the *Roman domination number* of *G*.

8. Connected Domination Number

A *dominating set* of a graph $G=(V, E)$ is a subset *D* of the vertices *V* such that every vertex not in *D* is adjacent to at least one member of *D*. A *connected dominating set* is a dominating set *D* such that the vertices in *D* induce a connected subgraph of *G*.

9. Maximum Triangle Packing

Given a graph $G=(V, E)$, a *triangle packing* for *G* is a collection $V_1, V_2, \ldots, V_k$ of disjoint subsets of *V*, each containing exactly three vertices such that for each $V_i = \{u_i, v_i, w_i\}$, $1 \leq i \leq k$, all three of the edges $u_i v_i$, $v_i w_i$, $u_i w_i$ belong to *E*. The *maximum triangle packing* problem aims to find a triangle packing, that is, disjoint subsets V_i, with minimum size.

10. Minimum Cut Cover

Given a graph $G=(V, E)$, a *cut set* is a subset $V_i \subseteq V$ such that, for each edge $uv \in E$, either $u \in V_i$ and $v \notin V_i$ or $u \notin V_i$ and $v \in V_i$. *Minimum cut cover* problem is to find a collection $V_1, V_2, \ldots, V_m$ of cut sets with minimum cardinality *m*.

11. Minimum Steiner Tree

Given a complete graph $G=(V, E)$, a metric given by edge weights $s: E \rightarrow N$ and a subset $S \subseteq V$ of required vertices, the *minimum Steiner tree* problem is to find a Steiner tree, that is, a subtree of *G* that includes all the vertices in *S*, where the sum of the edge weights in the Steiner tree is minimum.