Algorithms
Dynamic Programming Group Project (75 pts)

This project requires you to first theoretically solve the dynamic programming problem below and then write a program that implements your solution. [1] **You are not allowed to use the internet or consult any references. The only people you can work with on this project are your group members.**

1. *Problem Description*:

   The Astronomy club is faced with the following algorithmic problem. There are n consecutive astronomical events they could observe on a particular night that occur exactly one minute apart. Thus event j occurs at minute j. Also, event j occurs at integer coordinate d_j in the sky (we're assuming the sky is one-dimensional). The telescope's initial position at minute 0 is assumed to be coordinate 0 and the club is required to observe the last event n (occurring at minute n). The catch here is that the telescope can only be moved one coordinate per minute. So, at minute 1, the telescope can be moved to coordinate location 1 or -1 (or it could remain at location 0).

   The optimization problem you have to solve is: given the coordinates of each of the n events, find a viewable subset of *maximum* size, subject to the requirement that it should contain event n.

   Example: In the example below, the optimal solution is to observe events {1,3,6,9}. Note that the telescope has time to move from one event in this set to the next event moving at one coordinate location per minute:

Event	1	2	3	4	5	6	7	8	9
Coordinate	1	-4	-1	4	5	-4	6	7	-2

2. *Deliverables*: Please submit all of the items requested below in a single PDF file on Canvas. Please include your group number and the names of all of the students in the group.

   (a) [40 pts] Theory: Devise an efficient dynamic programming algorithm that finds an optimal solution. Demonstrate each of the following steps:

      i. [20 pts] Develop appropriate notation and describe a recursive solution to the problem. This can take the form of a mathematical expression or recursive pseudocode, but should be accompanied by a brief explanation (typically a few carefully-written sentences).

      ii. [10 pts] Write pseudocode for a dynamic programming algorithm. As discussed in class, this can use loops or recursion. Either way, the idea is that you are storing intermediate results (describe the data structure you are using to store these).

      iii. [10 pts] Use pseudocode to describe a traceback algorithm that returns the decision at each minute (i.e., -1, 0, or $+1$) in an optimal solution. If actual code expresses the idea well, you could simply paste it here.

   (b) [10 pts] Theory: Derive the complexity of your algorithm in terms of n.

   (c) [15 pts] Implementation: Implement your dynamic programming and traceback algorithms and include your (well-written and documented) code. If you were unable to get your code to compile/run, please state this clearly. We may choose a few groups and ask them to demonstrate that their code works.

[1] The problem has been adapted from the text by Kleinberg and Tardos

(d) [10 pts] Implementation: Demonstrate that your code works correctly by showing its results on the following instance.

Event	1	2	3	4	5	6	7	8	9	10
Coordinate	0	0	0	4	3	3	8	-1	6	8

Output Format

Your output format should consist of three lines.

- The first line gives the maximum number of astronomical events observed.
- The second line gives the events observed.
- The third line gives the direction of the telescope movement at each step.

For the example given on the previous page, a possible output could be:

```
4
1 3 6 9
+1 -1 -1 -1 -1 -1 +1 +1 0
```

(e) Group effort percentages must be submitted individually by each group member in response to a separate question on Canvas; i.e., do not include as part of this report.