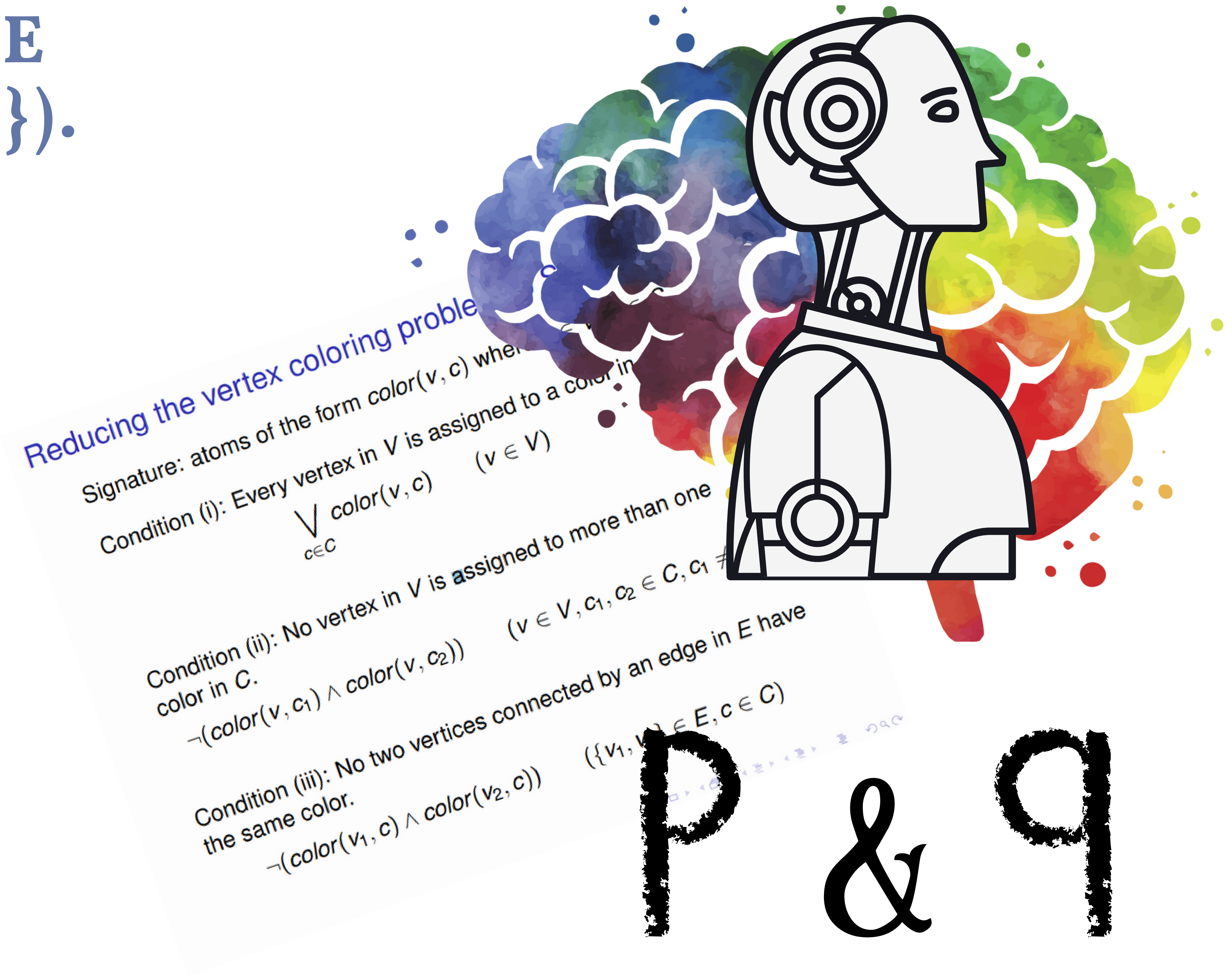
WRITE OUT THESE FORMULAE FOR THE GRAPH ({0, 1}, {{0, 1}}).

Atoms:

r0: coloring the 0th vertex with red
b0: coloring the 0th vertex with blue
r1: coloring the 1st vertex to red
b1: coloring the 1st vertex with blue

3 CONDITION TO SATISFY FOR VERTEX COLORING



- Every vertex is assigned to a color

  ((r0 v b0) ^ (r1 v b1))

- Every vertex is assigned to a color

  (¬(r0 ^ b0) ^ ¬(r1 ^ b1))

- Every vertex is assigned to a color

  (¬(r0 ^ r1) ^ ¬(b0 ^ b1))

THE FORMULAE GATHERED FROM THESE:

((r0 v b0) ^ (r1 v b1)) ∧ (¬(r0 ^ b0) ^ ¬(r1 ^ b1)) ∧ (¬(r0 ^ r1) ^ ¬(b0 ^ b1))

=

(r0 v b0) ^ (r1 v b1) ∧ ¬(r0 ^ b0) ^ ¬(r1 ^ b1) ∧ ¬(r0 ^ r1) ^ ¬(b0 ^ b1)

CONVERT TO CNF FORMAT

To obtain a CNF format, first we need to convert our formulae into form of conjunctions of simple disjunctions. To do this, we need to acquire clauses from our formulae. First we'll distribute all the negations and obtain clauses from second and third condition.

1ST CONDITION

((r0 v b0) ^ (r1 v b1))

=

{r0, b0}, {r1,b1}

This is already a conjunction of simple disjunctions. Thus we can directly convert it to clauses

2ND CONDITION

We distributed negation to gather conjunction of simple disjunctions

(¬(r0 ^ b0) ^ ¬(r1 ^ b1)) = ((¬r0 v ¬b0) ^ (¬r1 v ¬b1))

= {¬r0, ¬b0}, {¬r1,¬b1}

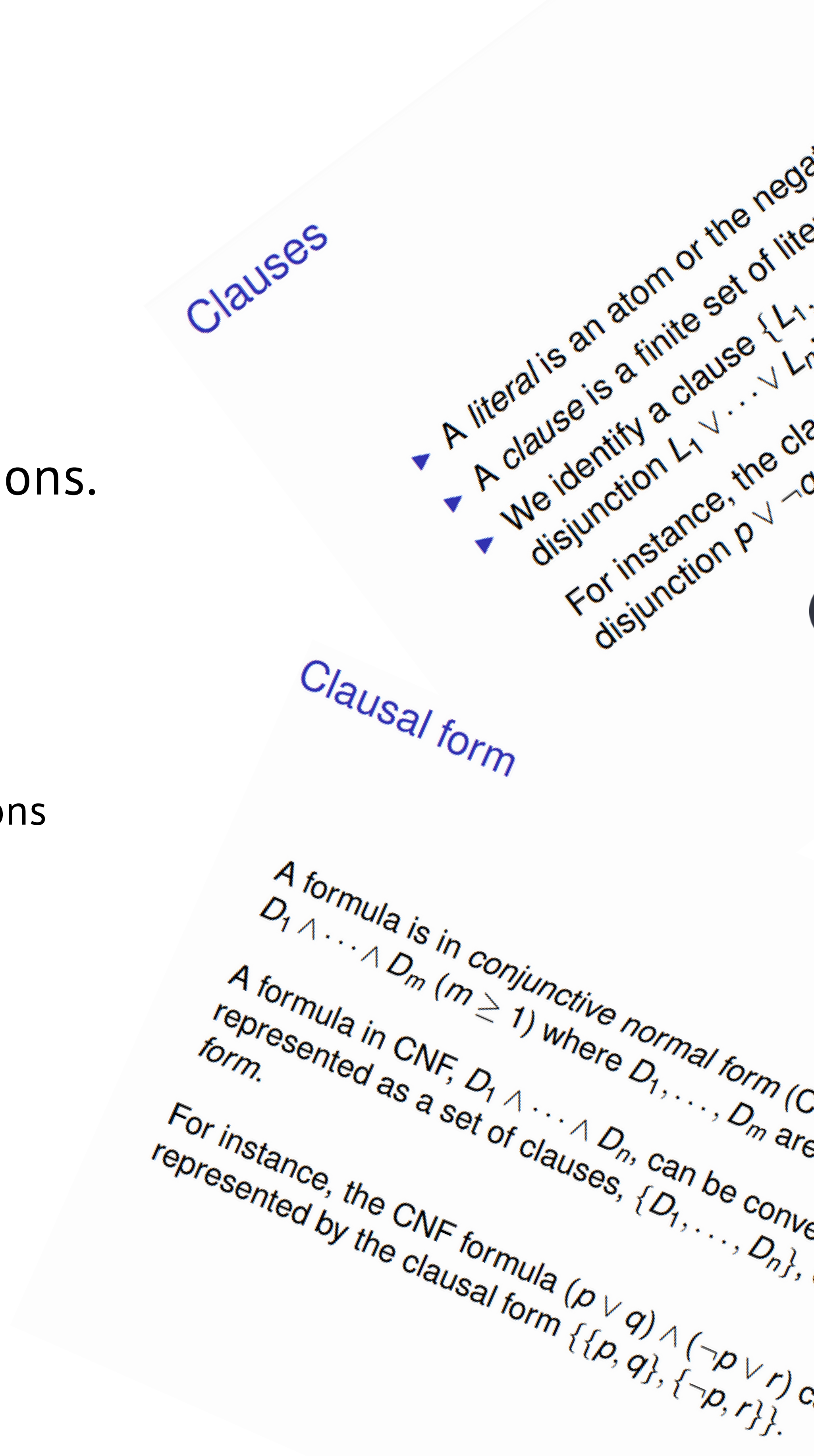
2ND CONDITION

(¬(r0 ^ r1) ^ ¬(b0 ^ b1)) = ((¬r0 v ¬r1) ^ (¬b0 v ¬b1))

= {¬r0, ¬r1}, {¬b0,¬b1}

{{r0, b0}, {r1,b1} ,{¬r0, ¬b0}, {¬r1,¬b1},{¬r0, ¬r1}, {¬b0,¬b1}}

All of the disjunctions combined with logical and, thus we combined clauses



Clauses

- A *literal* is an atom or the negation of an atom.
- A *clause* is a finite set of literals.
- We identify a clause $\{L_1,\dots,L_n\}$ with the simple disjunction $L_1 \vee \dots \vee L_n$; the *empty clause* is identified by $\perp$.

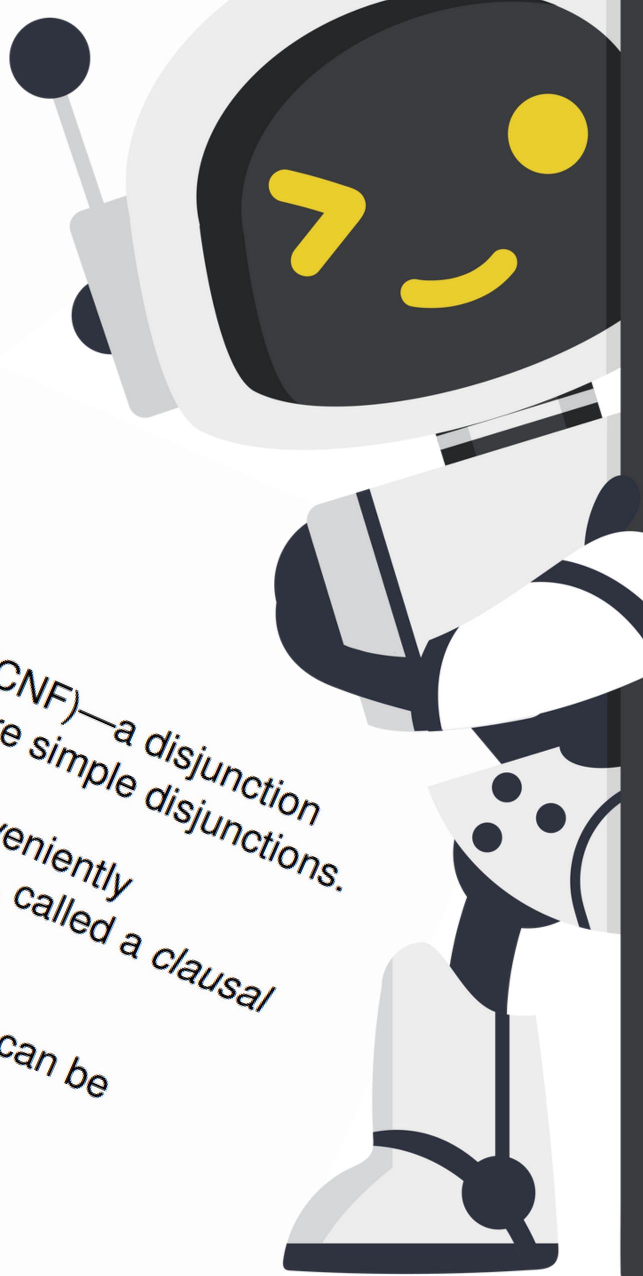
For instance, the clause $\{p, \neg q\}$ is identified by the simple disjunction $p \vee \neg q$.

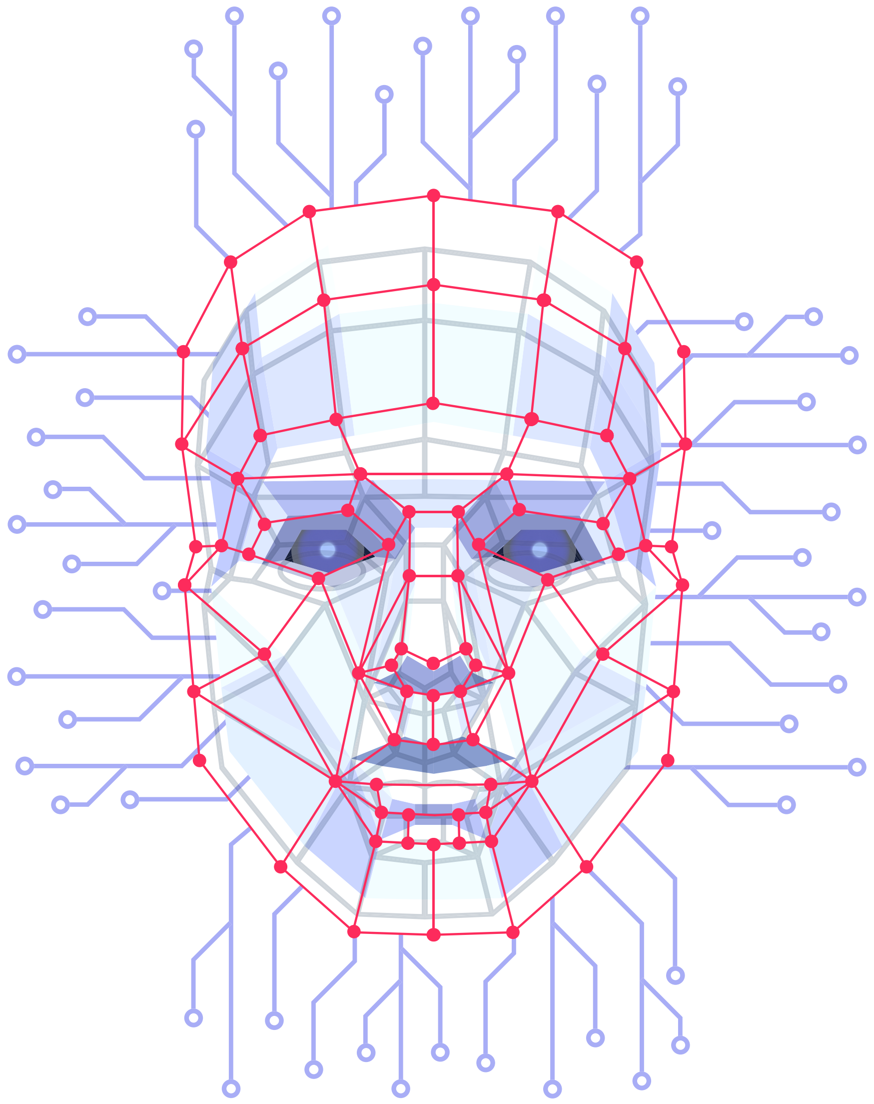
Clausal form

A formula is in *conjunctive normal form* (CNF)—a disjunction $D_1 \wedge \dots \wedge D_m$ ($m \geq 1$) where $D_1,\dots,D_m$ are simple disjunctions.

A formula in CNF, $D_1 \wedge \dots \wedge D_n$, can be conveniently represented as a set of clauses, $\{D_1,\dots,D_n\}$, called a clausal form.

For instance, the CNF formula $(p \vee q) \wedge (\neg p \vee r)$ can be represented by the clausal form $\{\{p,q\},\{\neg p, r\}\}$.





DIMACS CNF FORMAT

DIMACS CNF FORMAT IS THE TEXTUAL REPRESENTATION OF CNF FORMAT TO BE USED AS A TESTING FORMAT FOR SAT SOLVERS. IN DIMACS CNF FORMAT, ATOMS ARE REPRESENTED WITH POSITIVE INTEGERS AND WHEN WE NEGATE AN ITEM WE NEGATE THE ASSIGNED POSITIVE INTEGER FOR THAT ATOM. FOR INSTANCE IF P0 IS 1, ¬P0 IS -1.

LINK TO GET MORE INSIGHT ABOUT DIMACS CNF FORMAT

FOR VERTEX COLORING

Lets assign r0 to 1, b0 to 2, r1 to 3 and b1 to 4

Due to the formatting in the first row we need to write:
p cnf <number of variables> <number of clauses>

SINCE WE HAVE 4 VARIABLE AND 6 CLAUSES

```
p cnf 4 6
1 2 0
3 4 0
-1 -2 0
-3 -4 0
-1 -3 0
-2 -4 0
```

2nd row represents the first clause which is {r0, b0}. 0 at the end represents end of clause (in other words close the clause and add logical and to form conjunction).

⊞ Running Minisat

Verdict: *SATISFIABLE*

SATISFIABLE

-1 2 3 -4 0

ONLINE SAT SOLVER GIVES THIS SOLUTION WHICH CONFORM ALL THREE CONDITIONS OF VERTEX COVER PROBLEM. -1, 2, 3, -4 MEAN ASSIGN BLUE TO 0TH VERTEX AND RED TO 1ST INDEX