PHYS307 - Applied Modern Physics

Fall 2020

Experiment 2 - Atomic Spectra

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Section 1 - Introduction

In this experiment, our aim is to observe the characteristic spectral lines of sodium metal in visible range and calculate the respective wavelengths.[1] Then we shall observe the fine structure splitting of the spectral lines which is a result of the electron spin and relativistic corrections.[2]

Atomic spectra is the unique spectrum (i.e range of discrete values in some sense) of electromagnetic radiation that is emitted or absorbed by an atom due to the state transitions of its electrons. As stated in the Bohr atom model, electrons can only exist in discrete energy levels. This model had some flaws in the sense that it only applied to hydrogen atom and its prediction of motion of electrons were not entirely true.[3] So it was later revised by Schrödinger who arrived at a more accurate description of these energy levels. Yet, Bohr's model was accurate about the energies of emitted/absorbed photons stating that "energy of the photon is the difference between two allowed orbits."[3] Hence the transitions between these levels leads to emission or absorption of photons with certain amounts of energy. Therefore, unique atomic structures of each atom leads to a unique set of electronic transitions, which in return makes up the unique atomic spectra that is characteristic to an atom.

So far we've only discussed the mechanism that is responsible for the line spectra of atoms, but the word spectrum in physics has a wider meaning which demands definition. Spectrum is usually related with electromagnetic radiation, in fact, every possible frequency/wavelength of electromagnetic radiation creates the electromagnetic spectrum spanning from radio waves to gamma rays[i]. This is an example of continuous spectrum, which is one of the 3 different types of spectrum which will be discussed later. As its name suggests, in continuous spectrum there are no definite changes of color meaning that colors just fade into each other. We can observe continuous spectrum in the blackbody radiation. To give an example, when a metal is heated enough-it is not necessary though, it just makes easier to detect-it begins to glow as its temperature increases. If we put a prism in front of this light, we will be able observe the continuous spectrum.

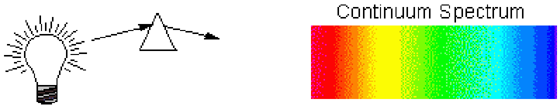

Figure 1: Incandescent bulb which operates with the principles of blackbody radiation. It produces a continuous spectrum.

[i]For the sake of simplicity, my discussion will concern only the visible range from now on.

Another type of spectrum is the emission spectrum. When atoms are separated to a rarefied gas form such that their interactions are quite diminished, we can observe the characteristic behaviour of these atoms rather than their collective behaviour.[4] This condition creates a suitable environment to observe emission spectra which is the result of the state transitions as discussed above. In this type of spectrum, emitted light has discrete values and these discrete values appear as bright lines, since only certain frequencies of photons will be emitted, the rest of the spectrum will be dark (note that this is not a perfect dark since matter will continue to emit blackbody radiation in all cases, yet the intensities are negligible). The state transitions mentioned can be achieved by passing electric current through the gas. This is the operating principle for gas discharge lamps which is also used in our experiment. Just as in the continuous spectrum, the light emitted by the gas can be transformed into a spectrum by using a prism or diffraction grating.

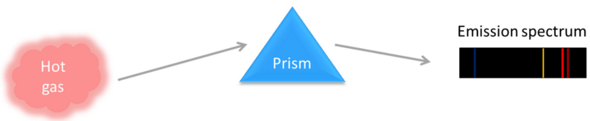

Figure 2: Excited gas emits light at discrete values, which can be observed after diffraction as emission spectrum.

Finally, we have the absorption spectrum. This is very similar to the emission spectrum in terms of its properties. Only difference is the direction of energy. In this case, the gas actually uses certain frequencies in the continuous spectrum in order to achieve state transitions. This results with dark lines in the continuous spectrum. Note that this dark lines should exactly correspond to bright lines in the emission spectrum. Absorption spectra can be achieved by passing white light through cold gas. Since white light is made up of continuous spectrum, frequencies absorbed by the cold gas will be visible as an absorption spectrum after passing through a prism or grating. A common place where we can observe this is the absorption spectra of the cold gases in the universe. This spectrum can be used to determine the composition of celestial objects.

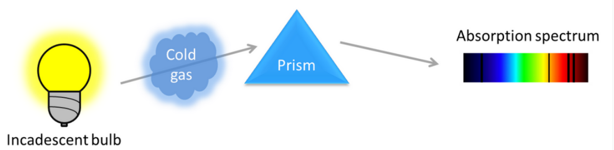

Figure 3: Cold gas absorbs certain frequencies from continuous spectrum, hence the resulting spectrum has dark line

As we have stated numerously, the light emitted by the sources, be it cold gas, hot gas or just hot metal, has to be separated into its components so that we can discern different colors. This can be achieved either by a prism or a diffraction grating. Prisms, separate the light into its components by the principle of dispersion. Meaning that, different wavelengths of light is refracted with different refractive indices in a material.[5] Another method is diffraction. This is achieved by using a diffraction grating. Diffraction gratings operate on the principle of diffraction and interference. Diffraction phenomena occurs when a wave passes through a barrier or opening.[6] The barriers that waves passes through acts as a source that produces waves which move at different directions for the space beyond itself. When we have many of these barriers, their resulting waves can interfere with each other either destructively or constructively. A diffraction grating exploits this property. These barriers are separated from each other by a known value. Meaning that waves produced by different barriers emerge from different positions. This implies that waves have to travel different distances in order to arrive at a point. This path difference is related with the spacing of the barriers.

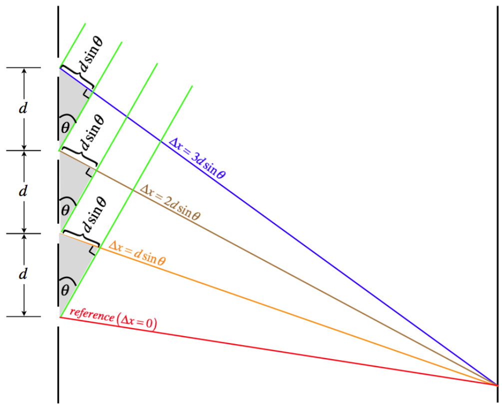

Figure 4: Trigonometric relation between path difference and spacing

Since waves arrive at the barrier simultaneously they should be also produced simultaneously, however due to this path difference. A set of waves traveling in a direction has different phases, this results with interference. If this phase difference is equal to the wavelength (or integer multiple) of the respective wave, then peaks would coincide with peaks and troughs would coincide with troughs which would imply constructive interference. Whereas if this phase difference is a multiple of half a wavelength then peaks would coincide with troughs and vice versa which would result with destructive interference. Starting with the trigonometric properties shown on the Figure-4 we can arrive at following

$$n\lambda = d * \sin(\theta) \quad n = \pm 1, \pm 2... \tag{1}$$

In this equation, n corresponds to the order of diffraction. Since one wavelength difference or two wavelength difference of phase will still lead to constructive interference, we should include the multiples as well. It is already shown that d is the spacing between barriers. The angle θ is the angle which we should expect to observe a particular wavelength of light.[7] So far we've only discussed monochromatic waves, however same principles apply for non-chromatic waves as well. Non-monochromatic waves are comprised of waves of different wavelengths. Looking at the Equation-1 we can see that different wavelengths of light will result with a different angle θ in the calculation so we should expect constructive interference to occur at different positions. This leads to the fact that diffraction grating acts as a dispersive element.[8]

In the discussion of line spectra, there is an other important point that needs to be discussed. Which is the existance of fine structure. Fine structure is defined as the splitting of the line spectra due to the spin-orbit coupling.[1] Spin-orbit coupling or in other words internal Zeeman effect is the interaction between orbital magnetic moment and the intrinsic spin magnetic moment.[9] This interaction is more dominant in larger atoms.[3] If we consider the motion in the electrons frame of reference, we can say that the nucleus moves around the electron. Since nucleus is a composition of charged particles we can think of it as a single charged particle. Since this charged particle is moving in a loop, it can be thought as a current loop. By the laws of electrodynamics, this loop creates a magnetic field. Spin property of the electron can be thought as a tiny bar magnet.[10] Therefore it has a magnetic moment. Magnetic field created by the motion of the nucleus interacts with this magnetic moment so there exists and associated energy with this interaction. This energy related with the interaction is a correction factor to our previously obtained energy levels. However, electrons does not have a single spin, in different configurations of electrons different combinations of electrons with different spins can occur. Therefore energy related with the magnetic moment has different possible values. Since we have different energy values depending on the spin of electron, our previously defined energy states split into two. This results with the fine structure.

Section 2 - Experimental Details

Our experiment setup, consists of following devices/objects:

- Sodium Arc Lamp
- Diffraction Grating
- Telescope
- Power Supply
- Vernier Scale
- Collimator

Our experiment begins with the light produced by the Sodium Arc Lamp. This lamp belongs to a wider class known as gas-discharge lamps, as the name suggests, sodium is one of the various elements that can be used as a gas inside this lamp. These lamps produce light by using the energy level transitions of sodium. These transitions are triggered by electric current which is provided by our power supply. One thing we should note about this lamp is that they require are pre-heating in order to operate as desired. The lamps are usually filled with a mixture called Penning mixture.[11] When the temperature is below a certain limit, sodium is not properly vaporised so this Penning mixture (mainly a mixture of argon and neon) produces light of its own atomic properties. When the lamp heats gradually, sodium vaporizes and begins to show its characteristic color.

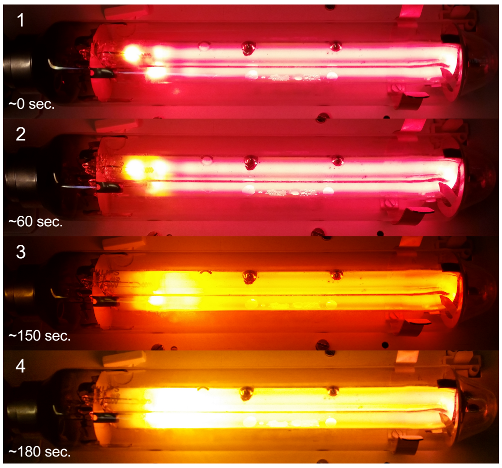

Figure 5: Sodium lamp during the process of heating

In the discussion of the characteristic color, we've discussed that there are multiple spectral lines for sodium atom which corresponds to different colors of light. Yet, we are only seeing one of them which is yellow (yellow/orange). This is because some of the wavelengths are more dominant in terms of intensities. Therefore a particular color dominates the visible output of the lamp. We can also observe this on the spectra, we should expect some of the lines to be brighter than others. In this case, we should expect yellow/orange line to be brighter than for example, blue line.

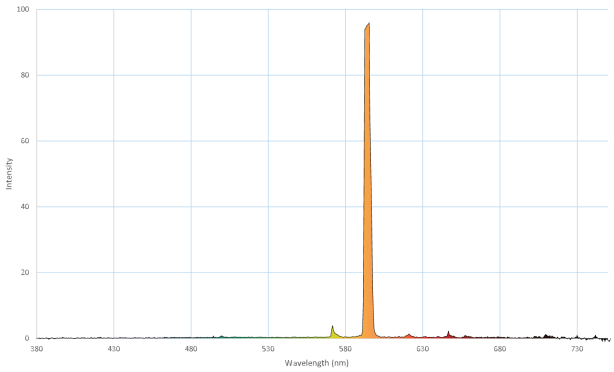

Figure 6: Wavelengths of light and their respective intensities as emitted by sodium lamp

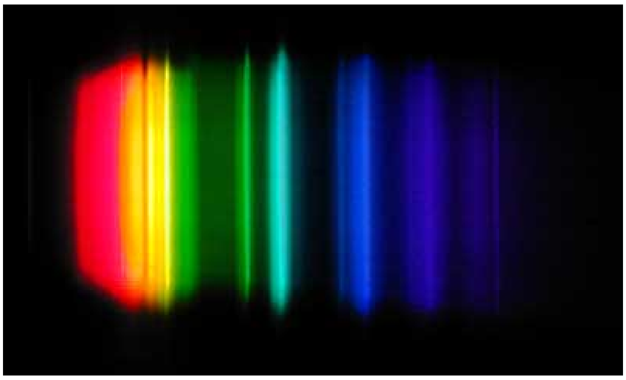

Figure 7: Spectral lines of the sodium lamp, relative brightness can easily be observed

Light emitted from the sodium lamp then arrives at the collimator. Collimator is an equipment, most likely made from a lens, that is used align the directions of the light beams so that angle of incidence is homogeneous through the surface of the grating. Light emitted from the lamp propagates in all directions, however after passing through the collimator they are aligned so that they are as parallel as possible to each other.

After the collimator, light arrives to the diffraction grating. As discussed in the previous section, diffraction grating is an equipment that has equally spaced slits. Since we are using a transmissive grating for this experiment it wouldn't be wrong to specify slits but there are also reflective gratings which has ridges instead of slits. In our case the grating spacing is chosen as 16666Å. When light hits diffraction grating, it separates into its wavelength components. As we've specified before, a prism could have been used as well. However, there are certain disadvantages of using a prism. One of those disadvantages is the fact that light has to travel through the prism, this implies that light can be absorbed or scattered, both of these are undesired side effects. Moreover, prisms operate by the principle of dispersion rather than diffraction like gratings. Meaning that, prisms might have wavelength specific characteristics. This might mean nonlinear resolution over the spectrum.[12]

Our last two equipments are angular vernier scale and telescope. Angular vernier scale is used to measure the angular deflection of our eyepiece with respect to a given reference point. This device can make measurements on the order second of arc. For comparison, second of arc is approximately the angle subtended by a coin that has a diameter of 18mm over a distance of 4 kilometers.[13] Finally, the telescope is an optical eyepiece that enables us to look on the grating by magnifying over it. It stands on a rotating stand so that we can look at different angles.

Since we have covered our equipment, we can now discuss the actual experiment. This experiment consists of two main parts.

In the first part, our objective is to observe the first order spectral lines. In order to achieve this, we should look for the first instance of each color in the telescope. Due to the symmetry of diffraction, we should expect to see the same pattern on each side of the zero axis. When we see the desired spectral line in the eyepiece, we should note its angular position and this procedure should be repeated for both sides of the zero line axis. At this point we should recall that angular positions are with respect to 180°. Therefore, in order to obtain actual angular displacement, we should calculate the absolute difference of the angle for the spectral line with the 180°. Since we are using data on the both side of the axis, we should average these values. This averaging procedure should improve our measurements. After we obtain the average value for angular displacement, all that is left to perform is to calculate the wavelength using Equation-1 noting that we only looked for first order diffractions in this part.

In the second part of the experiment we aim to observe the doublet lines of the sodium atom. Normally, these doublets should exist for all spectral lines. Yet, due to our lack of resolving power we are not able to observe them. Resolving power power is given as

$$R = Nn \tag{2}$$

where N is the total number of slit lines and n is the order of diffraction.[9] Since we have only one diffraction grating, the only possibility left is to increase the order if diffraction. To achieve this we should look for the recurring instances of spectral lines (i.e second occurrence of color green etc). Also, in order to increase the resolving power even further, we tilt our diffraction grating by an angle of 30°. When we look at these lines we should see that they are split into two very close lines. Those are the doublets we wanted to observe. In this part, we should note that we are not required to observe every color that was in the part one of experiment. This is due to the fact that as order of diffraction increases, intensity decreases. Therefore this would make some of the lines hard to see. As discussed above, brightest spectral line is yellow, and two close colors which are red and green are also relatively bright. However blue has a low relative intensity thus, it was not observed in this part of the experiment.

Section 3 - Data & Measurement

Part - 1

In the first part of the experiment, our aim was to observe the first order diffraction spectrum of sodium, and then calculate the wavelengths of emission lines by measuring diffraction angles. We have our data in *DMS* format.
These angle measurements are the angular deflection from 0° of the scale, hence their are absolute measurements. However due to the placement of our setup we should be using relative angular displacements from

Color of the line observed	Order number **(n=1)**	Angular Position of Left Image θ_L	Angular Position of Right Image θ_R	Average Angular Position of Image θ	$\sin\theta$	Experimental Wavelength λ (Å)	Theoretical Wavelength λ_t (Å)	Percentage Error (%)	Maximum Rel. Error in Wavelength $\Delta\lambda$ (Å)
Violet	1	$163^\circ\ 22'$	$196^\circ\ 36'$	16.617	0.28597	4765.9	4668.5	2.08	140.37
Green-blue	1	$161^\circ\ 37'$	$198^\circ\ 21'$	18.367	0.31512	5251.8	5153.6	1.90	154.89
Green	1	$159^\circ\ 41'$	$200^\circ\ 17'$	20.301	0.34695	5782.3	5688.2	1.65	170.90
Yellow	1	$158^\circ\ 56'$	$201^\circ\ 2'$	21.035	0.35894	5982.1	5895.9	1.46	177.13
Red	1	$157^\circ\ 58'$	$202^\circ\ 00'$	22.015	0.37485	6247.2	6160.7	1.40	185.06

Table 1: This table shows the measured angle data of the respective colors (first order) along with their averages corresponding wavelengths.

a reference point of 180°, hence the actual angular measurements are

$$\theta_{actual} = \left|180 - \theta_{L/R}\right| \tag{3}$$

Since there are two symmetric diffraction patterns about the zero axis, we will have two angular measurements. This symmetry is actually beneficial in terms of our measurements, by averaging two values, we can obtain a better result. After averaging actual values for both left and right measurements we then calculate sine of that angle. Now we have everything required to use diffraction formula which is the following

$$n\lambda = d * \sin(\theta) \tag{4}$$

where

$$n = 1$$
$$d = 16666\,\text{Å}$$

After obtaining the experimental wavelength by this formula, we can compare it to theoretical wavelength for the respective colors and calculate error using following formulas

$$\epsilon = \frac{|\lambda_{true} - \lambda_{exp}|}{\lambda_{true}} 100\% \tag{5}$$

$$\Delta\lambda = \lambda * \sqrt{\left(\frac{\Delta\theta}{\theta}\right)^2 + \left(\frac{\Delta d}{d}\right)^2} \tag{6}$$

If we were to do one of the calculations explicitly, it would be like the following

$$157^\circ\, 58' \approx 157.98^\circ \implies \theta_{L\_Actual} = 180 - 157.97 = 22.03^\circ$$
$$202^\circ = 202^\circ \implies \theta_{R\_Actual} = 202 - 180 = 22^\circ$$

which averages to

$$avg = \frac{22.02 + 22}{2} = 22.015^\circ$$
$$\sin(22.015) \approx 0.37485$$

Then

$$1 * \lambda = 16666 * 0.37485 \implies \lambda \approx 6247.2\text{Å}$$

In the calculation above, 10^{-10} multiplier was dropped since the desired wavelength has also the same units with grating constant.
Then using the theoretical values we can calculate experimental errors using Equation-5 and Equation-6 as following

$$\epsilon = \frac{|6247.2 - 6160.7|}{6160.7} 100\% \approx 1.40$$

$$\Delta\lambda = 6160.7 * \sqrt{\left(\frac{0.03}{22.015}\right)^2 + \left(\frac{500 * 10^{-10}}{16666 * 10^{-10}}\right)^2} \approx 185.06$$

Part - 2

For this part of the experiment, we aimed to observe the fine structure splitting in the spectral lines. Similarly we are given our data in *DMS* format. This time, we don't need to correct for angular offset of the vernier scale as in the previous part but this time the diffraction grating is tilted 30° so that it provides increased resolving power. The modified diffraction equation is

$$n\lambda = d * (\sin(\theta) + \sin(\theta_{tilt})) \tag{7}$$

where

$$\theta_{tilt} = 30\,^\circ$$
$$d = 16666\,\text{Å}$$

If we are to calculate one of the data explicitly

$$\theta_{d1} = 34^\circ\, 6'\, 48'' \approx 34.1133^\circ$$
$$\theta_{d2} = 34^\circ\, 11'\, 21'' \approx 34.1891^\circ$$
$$\sin(\theta_{d1}) = \sin(34.1133) \approx 0.560831$$
$$\sin(\theta_{d2}) = \sin(34.1891) \approx 0.561926$$

By using Equation-7 we can calculate following

$$\lambda_1 = \left(\frac{16666}{3}\right) * (0.560831 + 0.5) = 5893.3\text{Å}$$

$$\lambda_2 = \left(\frac{16666}{3}\right) * (0.561926 + 0.5) = 5899.3\text{Å}$$

which leads t0

$$\Delta\lambda = 6.08\text{Å} \tag{8}$$

Since we already know the theoretical value for the wavelength difference we can calculate the percentage error of the wavelength differences by using an equation in the same form of Equation-5

$$\epsilon = \frac{|6.08 - 6.0|}{6.0} 100\% \approx 1.33$$

Color observed	Order number (n)	Angle of the left doublet θ_{d1}	Angle of the right doublet θ_{d2}	$\sin\theta_{d1}$	$\sin\theta_{d2}$	λ_1 (Å)	λ_2 (Å)	$\Delta\lambda = \lambda_1 - \lambda_2$ (Å)	Theoretical $\Delta\lambda$(Å)	Percentage error of $\Delta\lambda$ (%)
Red	2	13° 50′	13° 52′ 49″	0.239098	0.239893	6158.9	6165.5	6.63	6.5	1.98
Red	3	37° 29′	37° 34′ 8″	0.608530	0.609714	6158.2	6164.8	6.60	6.5	1.21
Yellow	2	11° 58′ 24″	12° 00′ 59″	0.207456	0.208191	5895.2	5901.3	6.12	6.0	2.08
Yellow	3	34° 6′ 48″	34° 11′ 21″	0.560831	0.561926	5893.3	5899.3	6.08	6.0	1.33
Green	2	6° 49′ 60″	6° 52′ 21″	0.118981	0.119660	5157.9	5163.6	5.65	5.6	0.99
Green	3	25° 21′ 12″	25° 25′ 2″	0.428199	0.429206	5156.4	5162.0	5.59	5.6	0.06

Table 2: This table shows the angular displacements of the observed doublets and their order. Also calculated experimental wavelength and error with respect to theoretical value is shown.

Section 4 - Discussion & Conclusion

Experimental values that we've obtained in both part one and part two appears to be highly accurate given their percentage errors. For both parts percentage error did not exceed two percent. In terms of agreement with the theory, it would be safe to say that our findings are in the same direction with our expectations for both parts.
In the first part, we used the diffraction equation to calculate the wavelength of light which appeared on a given angular deflection. For example, for a line we observed at position 163°22′ (and its symmetric counterpart) we calculated the expected wavelength to be 4765.9Å. Theoretical expectation was 4668.5Å. For this measurement, our value is within the 2 percent of expected wavelength so we might conclude that it is an accurate measurement.

Similarly for second part as visible on Table-2. Wavelength differences between split lines are consistent with what we expected in theory. There is something important to note in here. For a given color we can see that error decreases as we go from second order diffraction to third order diffraction. This might possibly be related with the fact that lines become more distinguishable as order increases since resolving power also increases. However, after some point there will be diminishing return. As order continues to increase, lines will become less and less apparent therefore, the decrease in error due to the more distinguishable split lines might be canceled by error which increases due to the measurement error made while measuring less bright lines.

As every other experiment there were certain aspects of this experiments that were vulnerable to error that we should discuss. The most obvious error is human error. Due to the nature of this experiment, almost every measurement was made by humans. We should note that it is not simply a matter of reading a value from a measurement device, in this experiment human involvement is particularly high. Experimenter has to align the telescope to the color which is seen on the eyepiece, this task is prone to error since humans might not be able to do a perfect alignment given that boundaries are not perfectly visible, also there is the parallax effect which can effect measurements greatly. Moreover, proper placement of diffraction grating and reading of the vernier scale is also prone to human error. Along with these there are also some other possible human errors that could effect the end result such as improper handling of the equipment. For example, dirt on the diffraction grating could lead to undesired results.

We should also discuss the environmental errors, since this experiment involves light, we can say that it is particularly inviting to experimental errors caused by environmental factors. Any external light source can become a false data point on this experiment, therefore we should be particularly careful while conducting the experiment about external light sources. For the errors discussed until now, we can design an experimental setup which utilizes automated measuring devices with calibration capabilities which can work inside a light proof enclosure. This would minimize errors from environmental factors and humans.

We can also talk about the errors due to the imperfections in experimental setup and non ideal equipment. If we are to give examples, we can say that any manufacturing imperfection (non homogeneous spacing etc) on diffraction grating will lead to experimental errors. Also, as previously mentioned our light source doesn't contain only sodium gas. This means that, in addition to possible errors that might be caused by the temperature of the lamp we can also suggest that some other elements existing in the lamp can produce spectral lines. Only way to eliminate error of this category is to choose high quality equipment for the experiment.

Finally, there are the calculation errors. As we can see, our calculations required many significant digits. Even a slightest error be it caused by computers (round-off, truncation etc) or pen and paper, will propagate and become larger in the end result. There is nothing much that can be done for this matter. Only suggestion might be to be more careful about numerical errors in computations hoping that improvements over other factors will diminish the overall error.

In conclusion, this experiment proved us that electrons can only exist in certain energy states. During transitions, the energy difference between these states create a characteristic spectral line for that particular atom. Also, we can predict colors of these spectral lines by using the energy wavelength relation for photons. Moreover, we can also conclude that even in the absence of external fields/effects/factors, certain self-interactions can occur in an atom such as spin orbit coupling that leads to other characteristic behaviours. One example to this is the fine structure that we observed in the second part. Most of the characteristics we discussed so far are result of the electronic configuration for an atom. This configuration includes, different energy levels and electron counts. This variation between atoms causes spectra of a certain element to be different than one another.

References

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Addendum

Here is one of the scripts that I've used for calculation of the necessary values that we filled into table 2.

```
pkg load mapping

% Data
leftAngles = [13 50 0; 37 29 0; 11 58 24; 34 6 48; 6 50 0; 25 21 12];
rightAngles = [13 52 49; 37 34 8; 12 00 59; 34 11 21; 6 52 21; 25 25 2];
% Conversion from DMS to degrees
leftAngles = dms2degrees(leftAngles);
rightAngles = dms2degrees(rightAngles);

% Orders of diffraction
orders = transpose([2 3 2 3 2 3]);

% Calculates sines
leftSines = sind(leftAngles);
rightSines = sind(rightAngles);

% Find wavelength
lambdaOne = (16666./orders).*(leftSines + sind(30));
lambdaTwo = (16666./orders).*(rightSines + sind(30));
lambdaDiff = abs(lambdaOne - lambdaTwo);

theoreticals = transpose([6.5 6.5 6.0 6.0 5.6 5.6]);

% Calculate error
errors = (abs(lambdaDiff - theoreticals)./theoreticals)*100
```