PHYS307 - Applied Modern Physics

Spring 2021

Experiment 4 - Electron Diffraction

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Submission date: May 30, 2021

Introduction

In the early 1900s, the existence of matter waves was proposed by Louis de Broglie.[1] This theory suggests that all matter exhbibits wave properties causing a wave-particle duality in the nature. In this experiment our purpose will be to see the wave-like behaviour of electrons by diffracting them using graphite and find the interplanar spacings of graphite crystal using the equations related with diffraction (Bragg's Law).[1]

This theory associates waves with regular matter, one of the most important property of this waves are the associated wavelengths. It was well known at the time that energy of a photon is given by the equation

$$E = h\nu = \frac{hc}{\lambda} \tag{1}$$

where:

- h: Planck's constant
- ν: Frequency of the light
- c: Speed of light in vacuum
- λ: Wavelength of the light

Moreover, Albert Einstein suggested that the energy of a system is related with its mass and its momentum as following[2]

$$E = \sqrt{(mc^2)^2 + (pc)^2} \tag{2}$$

Since photons are massless particles, we can establish a relation between the wavelength of a photon and its momentum as following

$$\frac{hc}{\lambda} = \sqrt{(pc)^2} \tag{3}$$

$$p = \frac{h}{\lambda} \iff \lambda = \frac{h}{p} \tag{4}$$

What de Broglie suggested was this equation was not applicable to only photons but also the waves associated with massive particles such as electrons. In this case momentum can be expanded as in its classical form to obtain

$$\lambda = \frac{h}{mv} \tag{5}$$

In our experiment, the electrons will be accelerated with a known potential difference, therefore we can express their momentum thus their wavelength in terms of this accelerating potential. By definition of the unit volt, a particle with charge of one Coulomb will gain one Joule of kinetic energy accelerating under this potential.[3] We can use this knowledge with the charge of electrons and non-relativistic kinetic energy formulation (relativistic effects are negligible in this experiment due to the low operating voltages[4]) to obtain

$$KE = \frac{1}{2}mv^2 = eV_A \tag{6}$$

where:

- m: Mass of electron
- v: Speed of electron
- e: electron charge
- V_A: Anode voltage

$$KE = \frac{p^2}{2m} = eV_A \tag{7}$$

$$\frac{h^2}{2m\lambda^2} = eV_A \tag{8}$$

Thus

$$\lambda = \sqrt{\frac{h^2}{2meV_A}} \tag{9}$$

which evaluates into

$$\lambda = \sqrt{\frac{150}{V_A}} \tag{10}$$

Since we have established a relation between momentum (or accelerating voltage) and wavelength of the waves we can now establish an other important relation in this experiment, which relates the lattice spacing and radius of the rings in the interference pattern given a wavelength.

Throughout this experiment, Thomson's method is used which utilizes a thin film with randomly oriented crystals(polycrystalline graphite in this case) to create a diffraction pattern.[1] The graphite crystal used in this experiment has hexagonal shaped lattice arrangement which can be visualised as following

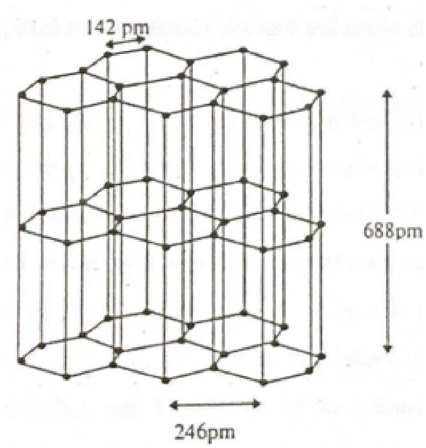

Figure 1: Lattice structure of graphite crystal

Considering the crystallographic planes in graphite by projecting the strucure into the plane we can see the lattice spacings

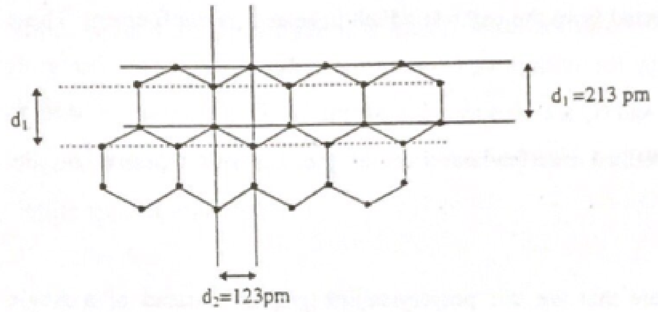

Figure 2: Crystallographic planes and lattice spacings for the first two interference rings

Derivation of Bragg's Law is based on the fact that waves can either construct or destruct each other when interfering depending on their phase. When diffracted from a crystal, if the path difference between these diffracted waves is an integer multiple of their wavelength, they interfere constructively.

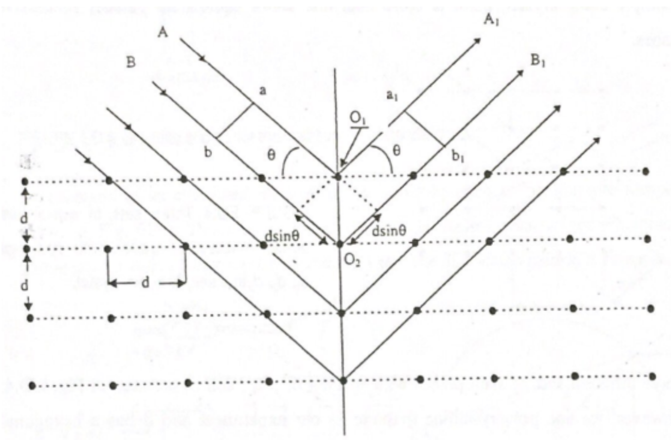

Figure 3: Diffracted waves and their path differences along with the lattice spacings

Using this knowledge, Bragg's Law can be derived as

$$2d\sin(\theta) = n\lambda \tag{11}$$

where:

- d: Lattice spacing
- n: Order of diffraction
- λ: Wavelength of the diffracted wave

Now we should consider our diffraction tube and its geometry to work out the actual relation. Consider the following figure

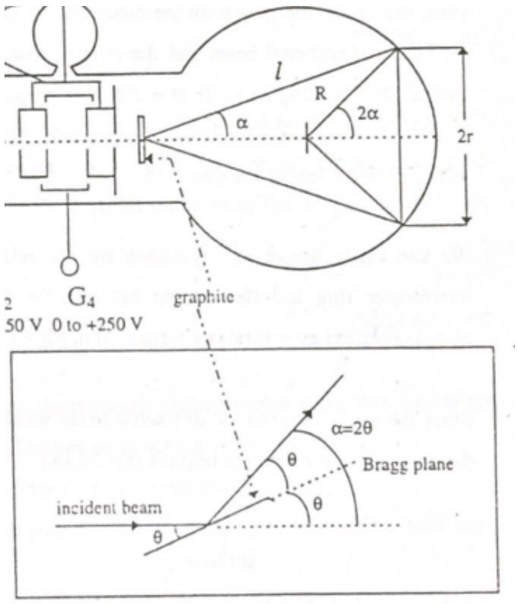

Figure 4: Geometry of the diffracted wave and spherical tube

In this figure, it can easily be seen that

$$\alpha = 2\theta \implies \sin(2\alpha) = \sin(4\theta) \tag{12}$$

Using the half angle formula for sine on this equation would yield

$$\sin(2\gamma) = 2\sin(\gamma)\cos(\gamma) \tag{13}$$
$$\sin(4\theta) = 2\sin(2\theta)\cos(2\theta) \tag{14}$$

Hence if we apply small angle approximation to this equation ($\cos(\theta) \approx 1$ for $\theta << 1$) we would obtain

$$\sin(4\theta) = 2\sin(2\theta) \tag{15}$$

Applying the half angle formula once more to **Equation-15** would lead to

$$\sin(4\theta) = 2\sin(2\theta) = 4\sin(\theta)\cos(\theta) \tag{16}$$

Finally, small angle approximation on cosine term is applied once more to yield

$$\sin(4\theta) = 4\sin(\theta) \tag{17}$$

Since ($\alpha = 2\theta$) we can write the following using **Figure-4**

$$\sin(2\alpha) = \frac{r}{R} \implies \sin(4\theta) = \frac{r}{R} \tag{18}$$

Now using **Equation-17** to transform **Equation-18** into

$$4\sin(\theta) = \frac{r}{R} \implies \sin(\theta) = \frac{r}{4R} \tag{19}$$

Now we can substitute this expression into Bragg's Law given in **Equation-11** to get

$$2d\frac{r}{4R} = n\lambda \tag{20}$$

Which can be rearranged into

$$r = \frac{2R}{d}n\lambda \tag{21}$$

which is the relationship between lattice spacing, radius of the interference pattern ring and wavelength of electron waves as desired.

Experimental Details

The list of equipment which will be used in this experiment are like following

- Spherical Diffraction Tube
- Digital Multimeter
- High Tension Probe (30kV)
- 10MΩ Resistor
- Power Supply for Accelerating Voltage
- Power supply for Collimator
- Vernier Caliper

In the following figure, we can see a section view of the spherical diffraction tube which is the main part of the experiment.

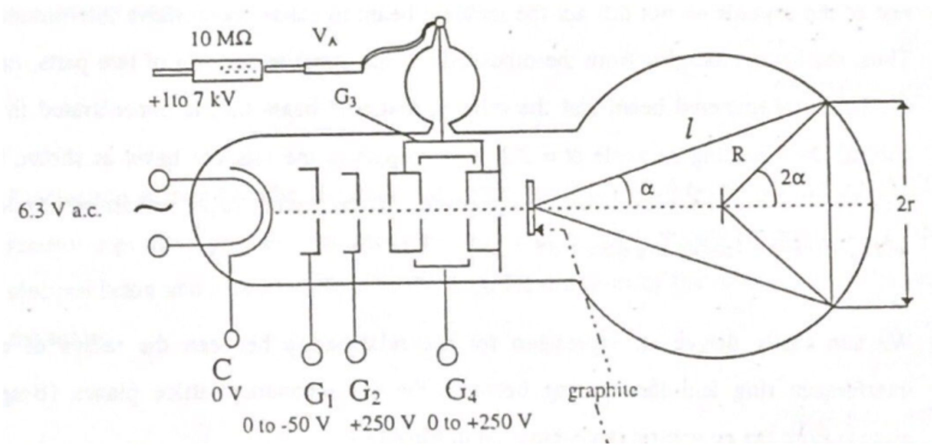

Figure 5: Section View of the Spherical Diffraction Tube

Here we can see a number of different components. Leftmost is the filament and the cathode, when provided a voltage, the filament heats up which in turn heats the cathode so the cathode emits electrons. These electrons then are accelerated towards the anode by the accelerating potential which is provided by the respective power supply. While they are accelerating, the electron beam is collimated with the grids **G1**, **G2**, **G4** which are energized by the collimator power supply. Collimator grids focuses the electron beam so that the interference pattern on the screen are sharp which makes the measurement easier and more accurate. These accelerated electrons are now incident upon a layer of polycrystalline graphite. The graphite crystal now diffracts the initial electron beam. Due to its structure, orientation of the crystals in the graphite are random which creates a diffracted electron beam in a conical shape. This conically shaped beam is directed towards the end of the tube. This end of the spherical tube is coated with a fluorescent material, when hit by the incoming electrons, this material emits light so we can observe the location of incidence. This enables us to observe the interference pattern, which is the main reason to use a fluorescent coating inside the tube. Then the Vernier caliper is used to measure the radius of the patterns on the screen which in turn is noted to the data table with the respective anode voltage which is read from the multimeter.

An important point about this experimental configuration is that we are using a spherically shaped tube. As mentioned, the graphite crystal creates a conically shaped diffracted beam. Since the base of this cone is a circular shape, if we were to project this beam onto a flat surface, there would be distortions and the resulting measurements might not be accurate (for example distortions on the map of the Earth). So this choice of geometry provides a surface to observe the interference pattern with as little distortion as possible.

Data & Measurement

		First Ring			Second Ring		
Anode Voltage [V]	Wavelength [Å]	Inner Radius [cm]	Outer Radius [cm]	Average Radius [cm]	Inner Radius [cm]	Outer Radius [cm]	Average Radius [cm]
4000	0.1936	1.33	1.39	1.36	2.22	2.41	2.32
4200	0.1890	1.31	1.38	1.35	2.17	2.37	2.27
4400	0.1846	1.25	1.36	1.31	2.11	2.32	2.22
4600	0.1806	1.22	1.33	1.28	2.10	2.29	2.20
4800	0.1768	1.17	1.29	1.23	2.04	2.25	2.15
5000	0.1732	1.16	1.27	1.22	2.01	2.22	2.12
5200	0.1698	1.14	1.24	1.19	1.99	2.18	2.09
5400	0.1667	1.12	1.22	1.17	1.95	2.16	2.06
5600	0.1637	1.09	1.20	1.15	1.91	2.11	2.01
5800	0.1608	1.08	1.18	1.13	1.86	2.06	1.96
6000	0.1581	1.06	1.17	1.12	1.84	2.01	1.93

Figure 6: This table shows the data obtained in the experiment. Data consists of changing outer and inner ring size with respect to anode voltage, also respective wavelengths are tabulated.

In the **Figure-6**, wavelengths of the electron waves were found using the following relation as derived in the introduction section.

$$\lambda = \sqrt{\frac{150}{V_A}} \tag{22}$$

This equation yields λ in the units of Angstrom when provided the anode voltage V_A in the units of V. For example, for the first row of the table

$$\lambda = \sqrt{\frac{150}{4000}} = 0.1936 \text{ Å} \tag{23}$$

Also, in order to increase the accuracy of the measurements, the radii of the inner and outer circles were averaged using

$$r_{avg} = \frac{r_{in} + r_{out}}{2} \tag{24}$$

Using these averaged value of the radii, the following plot was obtained using a Python script.

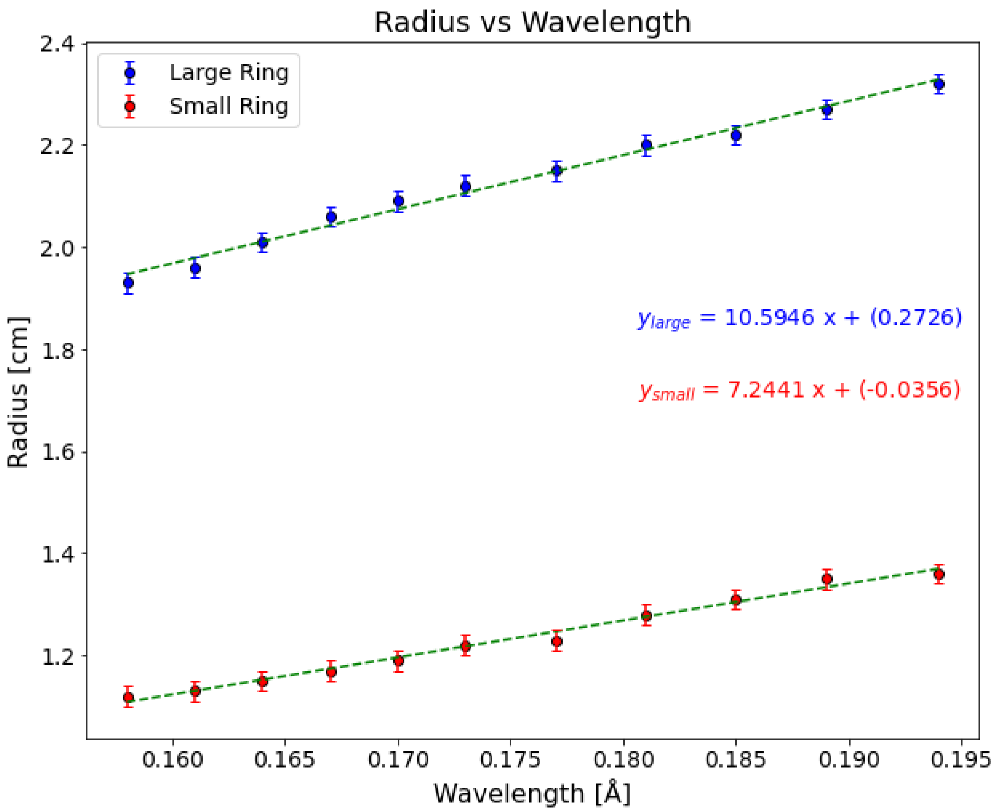

Figure 7: Average radius of both rings rings with respect to wavelength (with an error of 0.02 cm) and the equations for the lines that were fitted to the data.

In **Figure-7**, data for the large and small ring is plotted with their respective errors. In this figure, also the equation for the result of the linear regression analysis is shown with matching colors. Plots of these fitted lines are shown in dashed green.

Using the equations of the fitted curves we can make an estimation of the d1 & d2 spacings of graphite. We can see that the slope of these lines yield

$$\text{Slope } = \frac{r_{avg}}{\lambda} \tag{25}$$

we can use this knowledge in

$$r = \frac{2R}{d} n\lambda \implies \frac{r}{\lambda} = \frac{2nR}{d} \tag{26}$$

where:

- R: Radius of the spherical part of the diffraction tube (R = 6.35 cm)
- n: Order of diffraction (n = 1 in our case)
- d: Lattice spacing

Let us proceed with the calculation of the large ring

$$\text{Slope } = \frac{r_{avg}}{\lambda} = 10.5946 \left[\frac{cm}{\text{Å}}\right] \tag{27}$$

Then

$$10.5946 \left[\frac{cm}{\text{Å}}\right] = \frac{2nR}{d} = \frac{2 * 6.35\,[cm] * 1}{d} \tag{28}$$

$$d = \frac{2 * 6.35\,[cm]}{10.5946 \left[\frac{cm}{\text{Å}}\right]} \tag{29}$$

$$d_{2Exp} = d_{\text{Large Ring}} \approx 1.1987\,[\text{Å}] \tag{30}$$

Likewise, we can repeat the same procedure for the small ring with

$$\frac{r_{avg}}{\lambda} = 7.2441 \tag{31}$$

which leads to

$$d = \frac{2 * 6.35\,[cm]}{7.2441 \left[\frac{cm}{\text{Å}}\right]} \tag{32}$$

$$d_{1Exp} = d_{\text{Small Ring}} \approx 1.7531\,[\text{Å}] \tag{33}$$

Discussion & Conclusion

By looking at the outcome of the experiment, we can deduce that this experiment was successful in its attempt to prove the wave properties of electron. In our previous experiments, we've shown the particle nature of the electrons that were emitted from the cathode with same method. Yet, in this experiment we've observed a ring pattern on the fluorescent screen which is a result of the interference of the diffracted waves. Since interference and diffraction is a characteristic of waves rather than particles we can say that these electrons also exhibit this kind of behaviour. Therefore, we were able to prove the aforementioned wave properties of electrons.

The results of this experiment appears to be consistent with the theory. By looking at the equation

$$r = \frac{2R}{d} n\lambda \tag{34}$$

we can easily see that, given the wavelength with radius of the spherical tube and order of diffraction as constants, the radius of the ring pattern is inversely proportional to the lattice spacing. The calculations in the **Data & Measurement** section shows that this is the case indeed. We've calculated a larger lattice spacing for the small ring and smaller lattice spacing for the large ring. Yet, just like in every experiment, our results have an error margin as well. The theoretical lattice spacings for polycrystalline graphite is given as following[1]

$$d_1 = 213\ pm\ = 2.13\text{Å} \tag{35}$$

$$d_2 = 123\ pm\ = 1.23\text{Å} \tag{36}$$

Recalling our experimental results as

$$d_{1Exp} = 1.7531\ [\text{Å}] \tag{37}$$

$$d_{2Exp} = 1.1987\ [\text{Å}] \tag{38}$$

We can calculate the true percentage error using

$$\epsilon_t = \left| \frac{experiment - true}{true} \right| 100\% \tag{39}$$

Therefore the error on the first lattice spacing is

$$\epsilon_t 1 = \left| \frac{1.7531 - 2.13}{2.13} \right| 100\% = 17.69\% \tag{40}$$

and on the second lattice spacing is

$$\epsilon_t 2 = \left| \frac{1.1987 - 1.23}{1.23} \right| 100\% = 2.54\% \tag{41}$$

Even though the error on the second lattice spacing appears to be small, the first lattice spacing introduced a large error. This is most likely the result of the distortions caused by the geometry of the beams, even though the spherical tube compensates for a portion of this condition, it wasn't able to eliminate the whole error. As the radius of the circles increases this distortion becomes more and more apparent. Therefore, a larger error is found for the larger rings.[5]

Other than this error caused by the geometry, there are errors that introduced by humans and equipment. A possible error caused by humans can be the errors in the measurement using vernier caliper, which directly effects the data obtained. Other than the human errors, equipments used in the experiment can also introduce some error. For example, the graphite crystal is most likely not perfect, this causes the actual diffraction pattern to be slightly different from that of the theory. Moreover, the electronic equipments used in the measurement might have error, such as the multimeter and power supply. An error in these will effect the acquired data and calculations.

Appendix

Script used to plot the data and apply linear regression

```
# Import necessary modules
```

```
import numpy as np
import matplotlib.pyplot as plt

# Numpy arrays for data
wavelength = np.array([0.194, 0.189, 0.185, 0.181, 0.177, 0.173, 0.170, 0.167, 0.164,
    0.161, 0.158])

firstRing = np.array([1.36, 1.35, 1.31, 1.28, 1.23, 1.22, 1.19, 1.17, 1.15, 1.13, 1.12])

secondRing = np.array([2.32, 2.27, 2.22, 2.20, 2.15, 2.12, 2.09, 2.06, 2.01, 1.96, 1.93])

# First degree polynomial fit
p1 = np.polyfit(wavelength,firstRing,1)
p2 = np.polyfit(wavelength,secondRing,1)

# Create figure to plot
fig, ax = plt.subplots(figsize = (10,8))

# Display the equations of the linear fit on the figure
ax.text(0.8, 0.6, '$y_{large}$' + ' = {:.4f} x + ({:.4f})'.format(p2[0],p2[1]),
    horizontalalignment='center', verticalalignment='center', transform=ax.transAxes,
    color = 'blue', fontsize = 14)
ax.text(0.8, 0.5, '$y_{small}$' + ' = {:.4f} x + ({:.4f})'.format(p1[0],p1[1]),
    horizontalalignment='center', verticalalignment='center', transform=ax.transAxes,
    color = 'red', fontsize = 14)

# Errorbar plot for the data and line plot for the linear fit of the respective dataset
ax.errorbar(wavelength,secondRing, yerr = 0.02, fmt = 'bo', ecolor = 'b', capsize = 3,
    label = 'Large Ring', barsabove = True, markeredgewidth = 1, markeredgecolor = 'k')
ax.plot(wavelength,np.polyval(p1,wavelength), linestyle = 'dashed', color = 'g')

ax.errorbar(wavelength,firstRing, yerr = 0.02, fmt = 'ro', ecolor = 'r', capsize = 3,
    label = 'Small Ring', barsabove = True, markeredgewidth = 1, markeredgecolor = 'k')
ax.plot(wavelength,np.polyval(p2,wavelength), linestyle = 'dashed', color = 'g')

# Plot settings, labels, title etc.
ax.set_title('Radius vs Wavelength', fontsize = 18)
ax.set_ylabel('Radius [cm]', fontsize = 16)
ax.set_xlabel('Wavelength Å[]', fontsize = 16)
plt.xticks(fontsize = 14)
plt.yticks(fontsize = 14)
ax.legend(fontsize = 14)
fig.savefig('plot.png',facecolor = 'w', bbox_inches = 'tight')
```

References

[1] The diffraction of electrons by graphite - lab manual.

[2] Mass-energy equivalence. https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence. Access Date: May 30, 2021.

[3] Volt. https://en.wikipedia.org/wiki/Volt. Access Date: May 30, 2021.

[4] The electron diffraction - lecture notes.

[5] On the diffraction of electrons through a graphite foil. https://openwetware.org/wiki/Physics307L:People/Klimov/Electron_Diffraction_Final. Access Date: May 30, 2021.