PHYS307 - Applied Modern Physics

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Experiment 7 - Stern-Gerlach Experiment

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Introduction

In this experiment, our aim was to demonstrate the quantisation of the spatial orientation of angular momentum[1]

An electrically charged spinning particle can be modelled with a magnetic dipole.[2] This magnetic dipole has an associated dipole moment that is proportional to the spin of the particle and the relation is as following

$$\vec{\mu} = \gamma \vec{S} \tag{1}$$

where:

- μ: Magnetic Dipole Moment
- S: Spin Angular Momentum
- γ: Gyromagnetic Ratio

Just like an electric dipole, a magnetic dipole experiences torque in the presence of the associated field. This torque tends to align the direction of dipole parallel to the magnetic field. The torque on dipole applied by the magnetic field is given as

$$\vec{\tau} = \vec{\mu} \times \vec{B} \tag{2}$$

In Stern-Gerlach Experiment, neutral atoms are used but due to an unpaired electron the atoms still have magnetic dipole characteristics. As a magnetic dipole, the atoms experiences the torque mentioned previously in a magnetic field. However if the external magnetic field is not uniform, the atoms experience not only a torque but also a force[2] given by the expression

$$\vec{F} = \nabla(\vec{\mu} \cdot \vec{B}) \tag{3}$$

According to this expression, the non uniform magnetic field

$$\vec{B} = -\alpha x \hat{i} + (B_0 + \alpha z)\hat{k} \tag{4}$$

applies the force

$$\vec{F} = \gamma\alpha(-S_x \hat{i} + S_z \hat{k}) \tag{5}$$

on the atom. It can be shown that the x-component of the spin angular momentum oscillates rapidly and averages to zero (Larmor Precession[2]) therefore we can consider only the forces in the z direction.

Under the assumption that the magnetic force directly effects the position on the screen where atoms strike (deflection is determined by applied force), we would expect to see a continuous image on the screen according to the classical theories. However, quantisation of angular momentum (hence quantisation of S_z) causes the beams to split into $2s+1$ discrete beams which strikes the screen on different positions. For the case in the Stern-Gerlach Experiment, the unpaired electron in the silver atom has $s = 1/2$ so the beam would split into two parts.

The orientation of the magnet while performing this experiment determines the axis of measurement in which the measurements are made. For example, previously we had a magnetic force dominantly in the z direction which lead to a measurement of the quantised spin in z direction. These measurements can be replicated in different directions by using magnets with different alignments.

Experimental Details

In this experiment, following equipment is used

- Oven
- Non-Uniform Magnetic Field Generator (Magnetic Poles)
- Collimator
- Detector Screen

The experiment starts with the heating of the silver metal. When heated by the oven, metal plate begins to emit silver atoms. These emitted atoms then passes through a collimator which produces a beam of atoms. This beam of atom then arrives at a non-uniform magnetic field generator, which is a pair of magnetic poles shaped specifically that the magnetic field varies spatially. Finally, the beam of atoms arrive at the screen that shows the point where atoms strike.

The experiment will be conducted in 4 different configurations. In the first configuration, we will use a single magnet and align it with the z-axis.

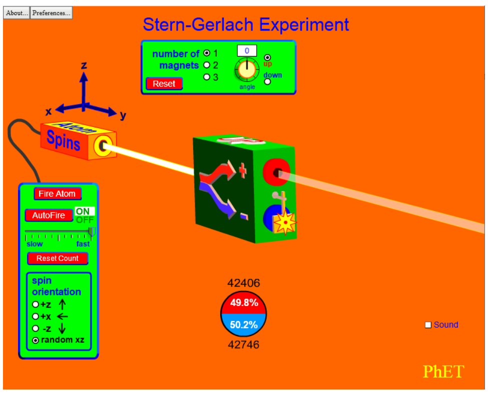

Figure 1: Setup for the first configuration, measuring the spin projection in z-axis

In the second configuration a second magnet will be placed after the first one with the same alignment (first sequential).

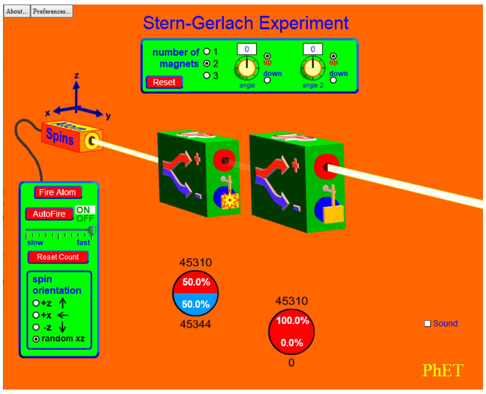

Figure 2: Setup for the second configuration, first sequential set of Stern-Gerlach Experiment

In the third configuration, the second magnet that is aligned with z-axis will be replaced with a magnet aligned with x-axis.

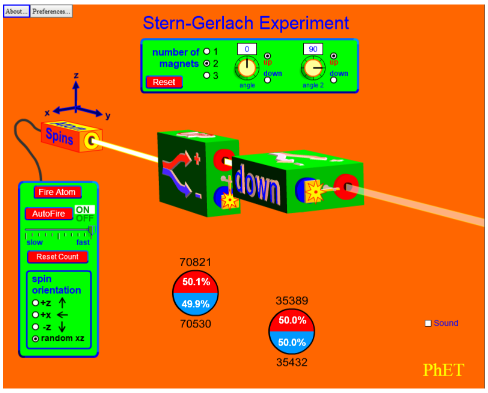

Figure 3: Setup for the third Configuration, second sequential set of Stern-Gerlach Experiment

For the final configuration, a third magnet aligned with z-axis will be placed after the magnets in the previous configuration so the final order of magnets will be ZXZ in the order of from atom gun to the screen.

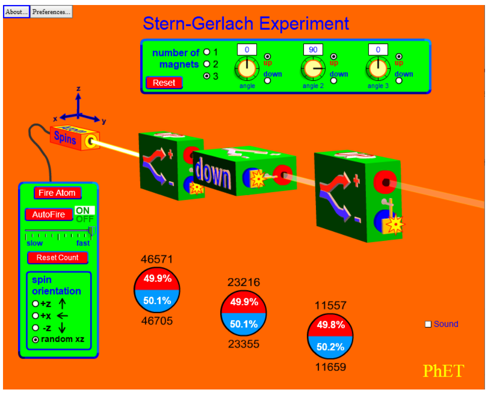

Figure 4: Setup for the fourth configuration, second sequential set of Stern-Gerlach Experiment

We should note that during the simulation the spin orientation of atoms were chosen to be random, which would be the case if we repeated this experiment with a real atom gun. Otherwise the initial spin orientation would change the results of the experiment.

Data & Measurement

The data obtained from this experiment is visible on the figure of each respective configuration. In Figure-1 we can see that half of the atoms were deflected in negative z direction and half of them were deflected in the positive z direction which is what we expected as spins of the atoms were distributed randomly.

When the experiment is performed with two identically aligned magnets as shown in Figure-2, the output of the first magnet is fed into the input of the second magnet. As the output of the first magnet is chosen to be the atoms that deflected in positive z direction, all the atoms that arrive to the second magnet should have the same spin. In the second magnet all the atoms deflect in a single direction which is the same with previous deflection as all atoms with opposite spin have been blocked in the first magnet.

In the third configuration shown in the Figure-3 we are making measurements in two different directions, namely in z and x axes. This time, the atoms deflected in positive z direction in the first magnet keeps their components in x direction so the first magnet doesn't effect the output of the second magnet and in each magnet, atoms split into two possible directions equally.

In the fourth configuration shown in Figure-4 the results seem to be contradicting with the previous findings as atoms split into two beams in the third magnet which is aligned with z-axis even though they were supposed to be filtered by the first magnet. This is a result of the uncertainty principle.[1] Uncertainty principle forbids the measurement of angular momentum in two perpendicular directions at the same time. Due to this reason, the measurement made in second magnet removes the measurement made in the first magnet thus the atoms arriving at the third magnet split into two beams as they never passed through the first magnet in the first place.

Discussion & Conclusion

References

[1] Stern-gerlach experiment. `https://en.wikipedia.org/wiki/Stern%E2%80%93Gerlach_experiment`. Access Date: July 11, 2021.

[2] David J. Griffiths. *Introduction to Quantum Mechanics*. Cambridge University Press, 2018.