PHYS307 - Applied Modern Physics

Spring 2021

Experiment 3 - Franck Hertz Experiment

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Section 1 - Introduction

In this experiment, our aim is to demonstrate the discrete energy levels of atoms and determine the first excitation energy level of a Mercury atom.[1] As a result of this experiment, we should arrive at the conclusion of existence of quantum nature in atoms as suggested in the Bohr atom model and that the energy levels of electrons are indeed discrete.

Bohr atom model is a model is the first model explains atomic structures using quantum theory.[2] In earlier atom models, electrons were modeled such that they have continuous orbits. However in Bohr model, they can only exist in certain levels. In his theory, Bohr stated three postulates.[3]

- Electrons exist in orbits that are called stationary orbits. This orbits have discrete levels and electrons are not allowed to exist anywhere else. Moreover, while in these stationary orbits, electrons do not emit energy.
- Angular momentum of electrons are quantized. They should be integer multiples of reduced Planck's constant. Once in the lowest energy state, which is also the state that is closest to the nucleus, electrons can't lose anymore energy or move closer to the nucleus.
- Change in energy levels happens with the emission or absorption of energy. This energy is transferred with photons whose frequency is determined by the energy transferred.

Using the second postulate, the allowed energy levels for electrons can be found by following these steps. Bohr's postulate and de Broglie wavelength is given as

$$mvr = n\hbar \equiv L \tag{1}$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} \tag{2}$$

Also for an electron in orbit, we have coulomb force which should be equal to the centripetal force

$$\frac{e^2}{r^2} = m\frac{v^2}{r} \implies v = \frac{e^2}{n\hbar} \tag{3}$$

Using these equation we find

$$v = \frac{e^2}{n\hbar} \tag{4}$$

$$r = \frac{n^2\hbar^2}{me^2} \tag{5}$$

By calculating total energy (i.e kinetic energy + potential energy) we obtain

$$E = \frac{1}{2}mv^2 - \frac{e^2}{r} = \frac{1}{2}mc^2\left(\frac{e^2}{\hbar c}\right)^2\frac{1}{n^2} \tag{6}$$

which simplifies into

$$E_n = -13.6eV\ \frac{1}{n^2} \qquad n = 1, 2, 3... \tag{7}$$

In this experiment, one of our aims was to demonstrate the discrete energy levels which is one of the predictions made by Bohr' atom model. Therefore, this experiment acts as an experimental confirmations to the discretized energy levels suggested by Bohr model.[1]

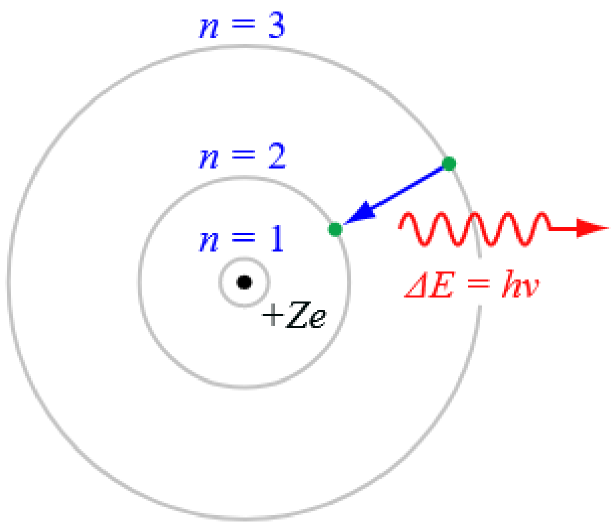

Figure 1: A representation of Bohr model

Apart from the Bohr atom model, there are some physical concepts that are closely related with this experiment with requires some explanation beforehand. Since it is the very beginning of the experiment, let us begin with thermionic emission.

Thermionic emission is the process of ejection of electrons from an electrode by the help of thermal energy.[4] Electrons are bounded to the electrode with a potential energy.[5] Just like in the Photoelectric Effect, electrons can gain enough energy so that they can get released from this potential. In thermionic emission, this provided energy is in terms of heat energy. In the experiment, thermionic emission happens in the cathode of the vacuum tube when the cathode is heated and electrons are released into the tube, then accelerated towards anode.
Another important concept is the contact potential. Contact potential is a potential which arises from the difference of the work function of cathode and anode metals in contact.[6] In our case, work function of anode is higher than that of cathode, this difference causes a shift in the current-voltage curves we obtain.[7]

We should also discuss about ionization potential and excitation potential. Excitation potential is the amount of an energy electron requires so that it moves to the higher energy level, since electrons are not stable in their excited state level, this process reverses and the electrons return to their original state and emit a photon with an energy equal to the energy difference between higher and lower state. Whereas the ionization potential is the potential required to release the electron completely thus producing an ion, once ionized, electron does not return to its original state so it does not emit a photon. In our experiment, excitation potential will be the difference voltage between successive peaks (or troughs) in our current vs voltage plot.

Finally, the last concept is elastic/inelastic collision. Inside the vacuum tube, electrons released from the cathode collide with the mercury atoms found in the mercury vapor. There are two types of collisions in this case. First type is the elastic collision, in an elastic collision, total kinetic energy of the bodies is conserved and it is not converted into any other type of energy. On the contrary, in inelastic collisions kinetic energy of the bodies is not conserved and it is converted into another forms of energy such as heat due to the friction. However, we should note that momentum is conserved in both cases. If the electrons have an energy below a certain threshold, they collide elastically with mercury atoms. Since the mass of the mercury atoms are considerably greater than of the electrons, just like the truck and tennis ball example given in elementary mechanics, mercury atom is not effected in collision and electron just changes its direction while preserving its total kinetic energy.[1] When the electron is above that threshold, the collisions become inelastic. When electrons have enough energy, their collision with atoms can cause electrons in the atom to change state since the collision provides enough energy. Therefore, some of the kinetic energy is lost in the collision which makes it inelastic. This doesn't have to happen only once, electrons might have enough energy that they may cause another such excitations once they lose some of their energy in initial collision.

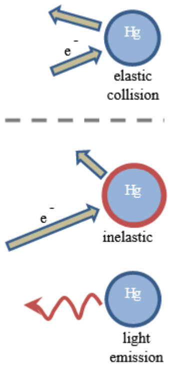

Figure 2: Representation of elastic and inelastic collisions

Section 2 - Experimental Details

In this experiment, the equipment that will be used are like following

- Hg filled vacuum tube (Franck-Hertz tube)
- Electric oven
- Control unit with power supplies and current amplifier
- Digital thermometer
- NiCr-Ni thermocouple
- Dual Channel 30 MHz oscilloscope

The vacuum tube and electric oven is enclosed in a single assembly. In the vacuum tube, we have 3 electrodes, first one is the cathode which emits electrons when heated. Second electrode is the anode which is used to accelerate the electrons by applying a potential difference with respect to cathode. Final electrode is the collecting electrode which has again a potential difference with respect to anode but this time it is used to decelerate the electrons so that only electrons with certain energy reaches it. The cathode in the tube has a filament near it so it can be indirectly heated (around 900 K) by the filament to produce thermionic emission. This entire tube is inside an electric oven so that it can be heated to a temperature around 440 K but smaller than 473 K.[7] This is to produce Hg vapor inside the tube. The temperature inside the oven is measured by a thermocouple which is connected to a digital thermometer.

Then we have the control unit. Control unit is a combination of multiple power supplies and a DC amplifier. As we can see on the Franck-Hertz tube and oven structure, there are multiple connections since there are 3 different potentials applied. First is for heating the filament near cathode, second is the potential difference between anode and cathode and it is used to accelerate the electrons. The final one is for the collector-anode voltage which decelerates the electrons. Finally, the DC amplifier is used to amplify the current signal from the collector. Since this value is very small, this device needs to be sensitive.[1]

Our final equipment is an oscilloscope. The oscilloscope is a device that visualizes signals in time domain. In this experiment, we will be using the oscilloscope in the X-Y mode meaning that the signals will be plotted against each other. For our case, our signals are voltage and current. Channel 1 (CH1) will be measuring the acceleration voltage and will be plotted in X axis whereas Channel 2 (CH2) will be measures the current formed by free electrons and will be plotted in Y axis.

Assuming all connections are made properly, the experiment begins by heating the vacuum tube using electric oven until the desired temperature is reached. Then we should set and apply potential difference on anode/cathode pair. In order to commence thermionic emission, we should also increase the filament voltage so that it heats the cathode. Also, gain of the collector current should be set so that it is amplified into an measurable magnitude. Then we should see the plot of voltage and current on the oscilloscope screen. After we see the plot, then we can gather the data from the oscilloscope screen taking the axis scales into consideration.

As stated above, oscilloscope will be reading a voltage and a current value. The voltage will be of the accelerating voltage between anode and cathode, whereas the current will be of the collector and cathode, since the circuit is completed when the electrons which are released from the cathode reach the collector.

When reading voltage data from the oscilloscope, we should expect successive peaks with 4.9 V in between. However, even though the spacing remains to be 4.9 V, we do observe a shift in the values that is the first peak doesn't occur at 4.9 V but it is a little bit shifted. This is because of the contact potential of the anode and cathode plates. As discussed in the introduction section, the difference in the work functions of metals leads to a potential difference between those two electrodes which results with a shift in our data.

Section 3 - Data & Measurement

Following tables shows us the locations of the maxima and minima of the current in terms of accelerating potential. In the tables, voltage differences between these maxima and minima along with their mean is also tabulated.

Filament Voltage = 6V Collector Voltage = 2V Oven Temperature = 180-150 °C		
	Maxima at (V)	Minima at (V)
Voltage at first min/max [V]	8.5	11.0
Voltage at second min/max [V]	13.0	16.0
Voltage at third min/max [V]	18.0	21.0
Voltage at fourth min/max [V]	N/A	N/A
Voltage difference between the first and the second min/max [V]	4.5	5.0
Voltage difference between second and third min/max [V]	5.0	5.0
Mean of voltage difference between the min/max [V]	4.75	5

Figure 3: This table shows the data obtained with the first experiment configuration

Filament Voltage = 5V Collector Voltage = 1V Oven Temperature = 176-197°C		
	Maxima at (V)	Minima at (V)
Voltage at first min/max [V]	8.5	10.5
Voltage at second min/max [V]	13.0	15.0
Voltage at third min/max [V]	18.0	20.0
Voltage at fourth min/max [V]	N/A	N/A
Voltage difference between the first and the second min/max [V]	4.5	4.5
Voltage difference between second and third min/max [V]	5.0	5.0
Mean of voltage difference between the min/max [V]	4.75	4.75

Figure 4: This table shows the data obtained with the second experiment configuration

Different configurations was achieved by changing the oven temperature along with the collector voltage and the heating filament voltage. In the tables above, missing data points were marked as not available.

	First Configuration	Second Configuration
Excited Energy Level	7.61E-19	7.61E-19

Figure 5: This table shows the excited energy levels of Mercury found using peaks

Section 4 - Graphs

Here are the plots for our data, red dots represents the data obtained in the experiment and the blue curves shows the curves fitted for the data.

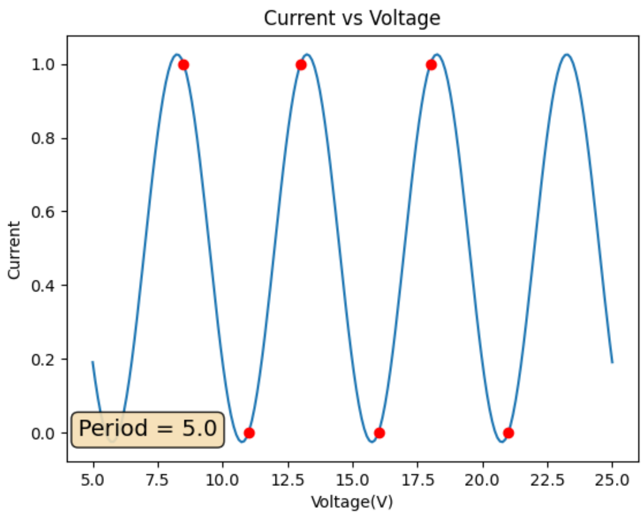

Figure 6: This is the plot for the first experiment configuration, period of the curve fit is shown in the box

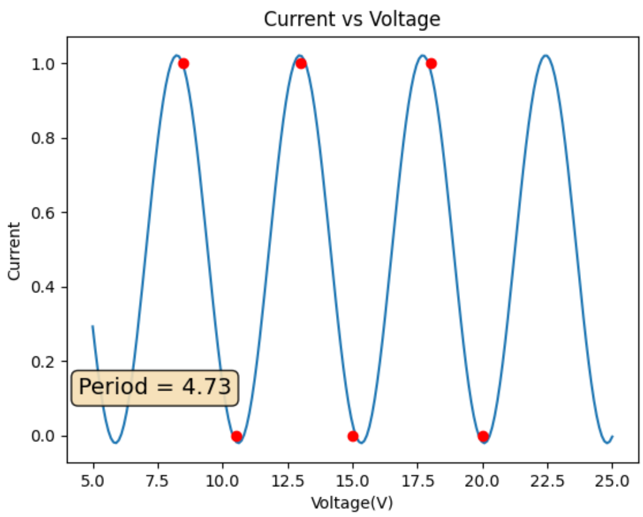

Figure 7: This is the plot for the second experiment configuration, period of the curve fit is shown in the box

These plots were obtained using a Python script that will be attached in the end of this report. In the script, CURVE_FIT function was used to determine the parameters of a sine function which describes our data. Then this sine function was used to find the period.

Section 5 - Calculations

Now we shall be calculating the parameters that might be useful in the interpretation of our data. These calculations will be performed for each of the two datasets individually. For these calculations I will be using the maxima of the plot.

First Dataset

First, let us calculate the first excited energy level of Hg atom. Let V_{max-ij} denote the j-th maximum of i-th set and V_{min-ij} denote the j-th minimum of the i-th set. Then

$$V_{max-12} - V_{max-11} = 13.0 - 8.5 = 4.5V \tag{8}$$

$$V_{max-13} - V_{max-12} = 18.0 - 13.0 = 5.0V \tag{9}$$

and their average is

$$V_{mean} = \frac{5.0 + 4.5}{2} = 4.75V \tag{10}$$

In order to calculate the excitation energy we use the relation between electric potential and electrical potential energy which is

$$E = qV \implies E = e * V_{mean} \tag{11}$$

where **e** denotes charge of the electron. Therefore

$$E_{excitation} = 4.75\ eV \tag{12}$$

by the definition of electron volt. This value can be express in Joules as following

$$E_{excitation} = 1.60217662 * 10^{-19}[C] * 4.75[V] = 7,61 * 10^{-19}\ J \tag{13}$$

The population standard deviation is given as

$$\sigma = \sqrt{\frac{1}{N}\Sigma_i(x_i - \mu)^2} \tag{14}$$

where

- N = 2 is the number of data points
- x_i is the data
- Population mean of the data

Then the standard deviation becomes

$$\sigma = \sqrt{\frac{1}{2}\left[(5.0 - 4.75)^2 + (4.5 - 4.75)^2\right]} = 0.25V \tag{15}$$

The true percent error is given as

$$\epsilon_t = \left|\frac{true - experimental}{true}\right| 100\% \tag{16}$$

which in our case becomes

$$\epsilon_t = \left|\frac{4.9 - 4.75}{4.9}\right| 100\% \approx 3\% \tag{17}$$

As stated in the introduction and experimental details part, there is supposed to be a shift in our experimental values due to the contact potential. It can be calculated. In order to obtain this value, we should find the difference between mean excitation energy and the voltage which the first peak occurs.

$$E_{excitation} = 7,61 * 10^{-19}\ J \tag{18}$$

$$\text{Contact Potential} = \frac{E_{excitation}}{e} - V_{max-11} = 4.75 - 8.5 = -3.75\ V \tag{19}$$

Finally, using the period of the curve fit, which is equal to the voltage difference between two consecutive maxima, we can calculate the energy of the first excited level.

$$E_{first} = T[V] * Chrage[C] \tag{20}$$

$$5 * 1.60217662 * 10^{-19} = 8,01 * 10^{-19}\ Joule \tag{21}$$

Second Dataset

Now we should follow the same steps for the second dataset. However we should note that, both dataset have the same values for maxima, their only difference is the period given by the curve fit.

$$V_{max-22} - V_{max-21} = 13.0 - 8.5 = 4.5V \tag{22}$$

$$V_{max-23} - V_{max-22} = 18.0 - 13.0 = 5.0V \tag{23}$$

and their average is

$$V_{mean} = \frac{5.0 + 4.5}{2} = 4.75V \tag{24}$$

This values lead to the excitation energy

$$E_{excitation} = 4.75\ eV \tag{25}$$

or when expressed in terms of Joules

$$E_{excitation} = 1.60217662 * 10^{-19}[C] * 4.75[V] = 7,61 * 10^{-19}\ J \tag{26}$$

Standard deviation calculated for these values are

$$\sigma = \sqrt{\frac{1}{2}\left[(5.0 - 4.75)^2 + (4.5 - 4.75)^2\right]} = 0.25V \tag{27}$$

and the true percent error is calculated as

$$\epsilon_t = \left|\frac{4.9 - 4.75}{4.9}\right| 100\% \approx 3\% \tag{28}$$

We should again calculate the contact potential with the same method as the first part

$$E_{excitation} = 7,61 * 10^{-19}\ J \tag{29}$$

$$\text{Contact Potential} = \frac{E_{excitation}}{e} - V_{max-21} = 4.75 - 8.5 = -3.75\ V \tag{30}$$

Finally, using the period of the curve fit, which is equal to the voltage difference between two consecutive maxima, we can calculate the energy of the first excited level.

$$E_{first} = T[V] * Chrage[C] \tag{31}$$

$$4.73 * 1.60217662 * 10^{-19} = 7,57 * 10^{-19}\ Joule \tag{32}$$

Section 6 - Discussion & Conclusion

In this part we will be discussing the data, calculations and results obtained through this experiment. The

most important thing to note is that the data for the peaks of the two configurations were the same even though their values for the minima were different. This caused the calculations in the previous chapter to be almost identical. However since the minima values were different, curve fit function has found different parameters for these two configurations, hence we had a different period value even though the calculated mean, standard deviation, error etc. were identical.

In the first configuration, the first peak was found in 8.5V rather than about 4.9V due shift caused by the contact potential. Also we have calculated our experimental mean difference to be 4.75V between the successive peaks where our theoretical value was 4.9V. Since we calculated our error to be 3%, we can say that this result is acceptable. Then using the curve fitting function, we've found a sine curve that fits this data. The period of this curve was calculated to be 5V, by multiplying this voltage value with the electron charge we've obtained the energy value for the first excited state of Mercury atom to be $8,01 * 10^{-19}$ J.

When the same steps were performed for the second configuration, we've found the contact potential and experimental mean difference between successive peaks to be 8.5V and 4.75V respectively. Which has again lead to an error of 3%. This time, due to the differences in minima, the curve fitting function resulted with a different period value which leads to an energy value of $7,57 * 10^{-19}$ J.

These data we've obtained are acceptable results in terms of their error. Since we've seen an oscillation in the current values we can say that we've demonstrated the discrete energy levels of atoms which was one of the purposes of this experiment. Also, with our calculations we've found the first excitation energy level of a Hg atom which was again one of the purposes of this experiment, therefore we can consider this as a successfully conducted experiment.

There are two things to note about this experiment. In this experiment we've only recorded excitations to the first excited states. An indicator of this would be the distance between extrema, if there were other excitation levels there would be extrema at other intervals which wouldn't have the same periodicity with the first energy level. Also we can say that this is also the case because energy of the electrons in this experiment is not high enough to excite the atoms into higher energy levels. Another point is the existence of multiple peaks in our plots. As discussed in the previous sections, if the electrons has enough energy such that they still have enough energy to cause another excitation in a new collision after the first collision, they will continue to do so. Therefore these secondary extrema is the implication of such electrons.

Finally, we should discuss the possible causes of errors in this experiment. Most important fact to note is that this experiment is highly temperature dependent. Therefore, we can say that fluctuations in the temperature of oven and the heating filament might lead to different results since they have effect on thermionic emission and density of the mercury vapor.

Another possible source of error is the errors caused by equipments. We are using devices such as oscilloscope and power supplies which naturally introduce errors of their own due to numerous factors. One possible error to note is the error of the current amplifier, since the currents measured are very low, this device should be very sensitive so any error in this device will effect the results greatly.

Lastly, there are errors caused by humans. While making a measurement from oscilloscope, we are obtaining the data visually, therefore this part of the experiment might have errors introduced by the experimenter.

Addendum

Script for the first dataset

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# Define x and y dataset
xdata = np.array([8.5, 11.0, 13.0, 16.0, 18.0, 21.0]) # Dataset 1
ydata = np.array([1.0, 0.0, 1.0, 0.0, 1.0, 0.0])

# Define the function for the fit
def func(x,p1,p2,p3):
    return p1*np.sin(x*p2+p3)+0.5

# Start fittinf, ( the results are kept in sonuc array)
sonuc, pcov = curve_fit(func, xdata, ydata, p0 = (0.2,1.2,1.0))

# Gather the results of fitting
p1 = sonuc[0]
p2 = sonuc[1]
p3 = sonuc[2]

# Period of the sine function
period = 2*np.pi/p2

# Recalculation of sine function with linear density 200
# between 5 and 25 on x-axis

xaxis = np.linspace(5,25,200)
curve_y = func(xaxis, p1, p2, p3)

```

```
fig, ax = plt.subplots()
# Add the fit curve to the plot
plt.plot(xaxis, curve_y, '-')
# Add the data to the plot
plt.plot(xdata, ydata, 'ro')
plt.title('Current vs Voltage')
plt.xlabel('Voltage(V)')
plt.ylabel('Current')

# Show period on plot
props = dict(boxstyle='round', facecolor='wheat', alpha=0.9)
textstr = 'Period = {}'.format(period)
ax.text(0.02, 0.1, textstr, transform=ax.transAxes, fontsize=14,
        verticalalignment='top', bbox=props)

# Show the plot
plt.show()
```

Script for the second dataset

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit

# Define x and y dataset
xdata = np.array([8.5, 10.5, 13.0, 15.0, 18.0, 20.0]) # Dataset 2
ydata = np.array([1.0, 0.0, 1.0, 0.0, 1.0, 0.0])

# Define the function for the fit
def func(x,p1,p2,p3):
    return p1*np.sin(x*p2+p3)+0.5

# Start fittinf, ( the results are kept in sonuc array)
sonuc, pcov = curve_fit(func, xdata, ydata, p0 = (0.2,1.2,1.0))

# Gather the results of fitting
p1 = sonuc[0]
p2 = sonuc[1]
p3 = sonuc[2]

# Period of the sine function
period = 2*np.pi/p2

# Recalculation of sine function with linear density 200
# between 5 and 25 on x-axis

xaxis = np.linspace(5,25,200)
curve_y = func(xaxis, p1, p2, p3)

fig, ax = plt.subplots()
# Add the fit curve to the plot
plt.plot(xaxis, curve_y, '-')
# Add the data to the plot
plt.plot(xdata, ydata, 'ro')
```

```
plt.title('Current vs Voltage')
plt.xlabel('Voltage(V)')
plt.ylabel('Current')

# Show period on plot
props = dict(boxstyle='round', facecolor='wheat', alpha=0.9)
textstr = 'Period = {:.2f}'.format(period)
ax.text(0.02, 0.2, textstr, transform=ax.transAxes, fontsize=14,
        verticalalignment='top', bbox=props)

# Show the plot
plt.show()
```

References

[1] Franck hertz experiment - lab manual.

[2] Bohr model. https://www.britannica.com/science/Bohr-model. Access Date: May 15, 2021.

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[7] Franck hertz experiment - lecture notes.