

Homework #2

Implementation:

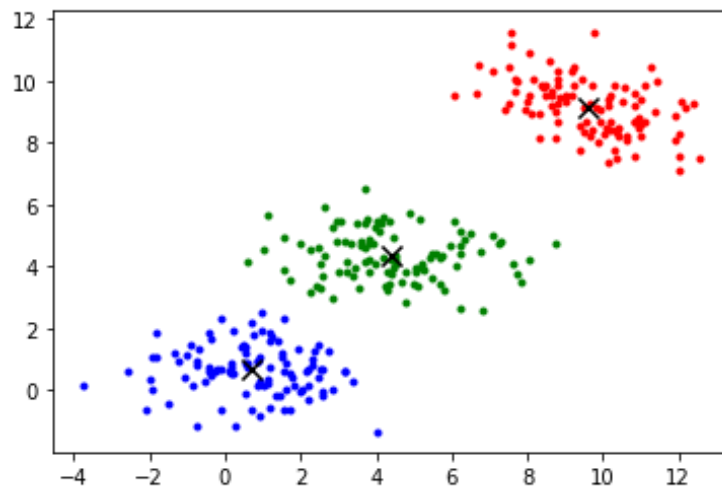
I have only used the information on the PDF to implement my code.

I have utilized NumPy's random number generator functions to initialize μ , Σ , and π , setting the seed to 0:

- I have picked three random points from the data to initialize μ .
- I have first created three 2x2 arrays with random numbers from a uniform distribution over $[0, 1)$ and multiplied them with their transposes to initialize Σ .
- I have picked three random numbers from a uniform distribution over $[0, 1)$ to initialize π and divided each one by their sum to set their sum equal to 1.

Then I have defined functions to perform the steps of Expectation-Maximization algorithm for Gaussian mixture model based on the information on the PDF. I have set the threshold value as 10^{-8} . Then I have run the algorithm. After 52 iterations, the algorithm converged as the difference between the current and the previous value of the log likelihood was less than the threshold. Then I have assigned labels to each data point according to their γ values and plotted them with the estimated π values of their clusters.

Plot:



Estimated results:

$$\mu_1 = [0.702083 \quad 0.661362]$$

$$\Sigma_1 = \begin{bmatrix} 2.11797 & -0.0997327 \\ -0.0997327 & 0.640837 \end{bmatrix}$$

$$\mu_2 = [9.60516 \quad 9.16836]$$

$$\Sigma_2 = \begin{bmatrix} 2.01245 & -0.641668 \\ -0.641668 & 0.821711 \end{bmatrix}$$

$$\mu_3 = [4.37905 \quad 4.35184]$$

$$\Sigma_3 = \begin{bmatrix} 2.7479 & -0.119232 \\ -0.119232 & 0.618065 \end{bmatrix}$$