Homophily and Common Knowledge in a Threshold Model of Collective Action

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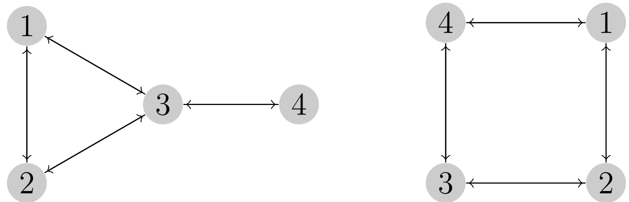
1 Introduction

We consider a model in which agents decide to commit to participate a collective action such as a revolt based on their inference on the behaviour of other members of the society. The model is based on the collective action framework developed in *Chwe, Michael (2000)* [2]. An agent's commitment to the action requires the inferred quantity of agents that will be committing to the action to be above the threshold of the agent. The inference mechanism, and the decision to commit or not, is instantaneous for all members of the society, thus the model is not a typical (dynamic) diffusion model widely covered in the literature (Centola and Macy (2007)) [1]. The key point of inference is the formation of the common knowledge between each given pair of agents regarding each other's action.

Agents can only observe the threshold values of the others that they have communication links with. For agent i to infer that agent j will commit, i has to infer (through her links) that there are sufficient (that is above the threshold of agent j) number of agents linked with j that will be committing to the action. This multiplicity of inference, while introducing a computational complexity, differentiates the application of the model to rather interesting situations.



(a) Kite (b) Square

Figure 1:

For a simple illustration of the effects of communication structure on collective action, consider two societies with communication networks as in Figure 1 where thresholds of agents are identical and equal to 3. Our inference mechanism suggests that society structured as a *kite* will have agents $1, 2$ and 3 will commit to the collective action while agent 4 will not. The intuition is that agents 1,2 and 3 can form the common knowledge that if each of them commits others will have sufficient incentive to commit themselves, and vice versa. However, for agent 4, which can observe only agent 3, such inference is not possible, that is, 4 does not have enough information to guarantee that 3 will commit to the collective action, thus 4 herself will not commit. On the other hand in the society that is configured as a *square*, shown in figure 1b, the collective action will not emerge at all. Although each agent still observes two others (like the members of the triangle in the previous society), they know that their neighbors are not directly linked, thus the common knowledge will not form. Note that both societies have same number of communication links between agents.

Collective action models generally focus on the diffusion of participation decisions of agents given a jump start by a few agents with zero or very low thresholds. Chwe's model is more adequate for modeling the collective action problems in which diffusion is rather difficult as in the case of strikes or political protests against authoritarian regimes. In the cascading or diffusion models, agents have time to observe their neighbors while the agents who have already decided to participate do not bear any costs before the remaining agents join. In reality, once a few agents signal that they are strikers or rebels, firms or directors readily punish them In our model, the collective action decision is simultaneously taken by all agents in the society.

2 Model

There is a finite set of agents $N = \{1, 2, , ..., n\}$. Each agent $i \in N$ chooses an action $a_i \in \{r, s\}$ where r is the risky action and s is the safe action. Each agent i has an idiosyncratic private threshold $\Theta_i \in \{1, 2, .., n\}$. Given agent i's threshold Θ_i and every agent's actions $\mathbf{a} = (a_1, a_2, ..., a_n)$, an agent's utility is given by

$$U_i = \begin{cases} 0, & \text{if } a_i = s \\ 1, & \text{if } a_i = r \wedge \{j \in N : a_j = r\} \geq \Theta_i \\ -z, & \text{if } a_i = r \wedge \{j \in N : a_j = r\} < \Theta_i \end{cases}$$

where $-z < 0$ is the penalty the agent gets if she revolts and less than Θ_i people join her. Thus, an agent will only revolt if the total number of revolters is greater or equal to her threshold.

We argue that differentiating the underlying social networks from random to small world networks does not suffice to capture the "social" dimension of collective action problems. People are different than computer agents in simulations, they tend to associate with alikes. Homophily is fundamental in social networks. Communities form up as agents link up with similar agents.

In the collective action (revolt) setting, the similarities of agents manifest themselves in terms of distribution of thresholds.The communication network is undirected and represent small world type communication technology in which people have close and distant contacts. The network structure for common knowledge is then readily formed by the formation of cliques. A subset of agents $M \subset N$ revolts when (i) they share common knowledge of the thresholds and states (s or r) and (ii) their thresholds are lower than or equal to the size of the subset M.

Our work is related to Siegel (2009) [4] as that paper considers "Village (Clique) Networks". The emphasis is different though as [4] vary the clique size and average connectivity across cliques rather than homophily parameter. More importantly the underlying idea is still the dynamic diffusion.

The closest study is Korkmaz et. al. (2019) [3] in which authors vary homophily in a "facebook model" of collective action but still in a dynamic setting. Our paper is different as we find facebook model rather unpersuasive as an underlying platform for common knowledge formation and focus on one-time and spontaneous model of collective action given homophily in a homophily based small-world network

3 Results and Discussion

3.1 Homophily

Society involve two or more communities. These communities differ in terms of their average threshold levels; say one community is more radical with a low level of average thresholds and the other is more conservative with a high level of average thresholds. Within the communities we allow a certain dispersion of individual thresholds.

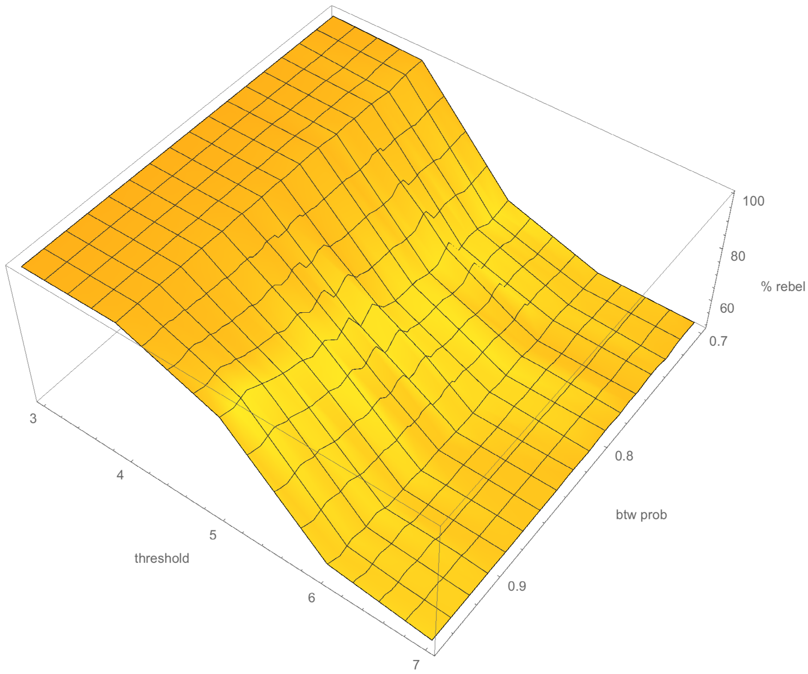


Figure 2: Homophily and Thresholds

Each community is also linked with the others through "weak", across community, links. A community can be more or less cohesive depending on the intensity of these "weak" links that individual actors keep intact. These weak links can be formed up in various contexts such as random meetings in parties or over internet forums.

Our main contributions cover the interactions of community structures and threshold distributions. We vary the degree of homophily as well as the threshold levels within and across the communities.

We partition a society with 100 agents into two equal groups. The degree of each agent is set to 10 and the threshold of first group is fixed at 3. We check the effects of two parameters on the percentage of agents that commit to the collective action. We change the in-between probability (probability of having a link within one's own group relative to an agent outside the group) of agents from 70 percent to 95 percent with 1 percent intervals. In-between probability is a proxy for the degree of homophily. We also change the threshold value of the second group from 3 to 7 with unit intervals. Above setting gave us 150 parameter combinations to test. For each parameter setting we generate 10 random homophily graphs and calculate the percentage of agents that commit to the collective action.

Figure 2 illustrates the results of the simulation. We find that homophily enhances collective action. The degree of homophily has a non-linear effect in conjunction with varying threshold levels in one of the communities.

3.2 WhatsApp Model

In the WhatsApp model, cliques of equal size have connections across each other. The main difference with random or small world network is that cliques of a given size is guaranteed to form up. Obviously if the threshold is less than the clique size then the collective action is also forthcoming. Figure 3 is an example of a such network.

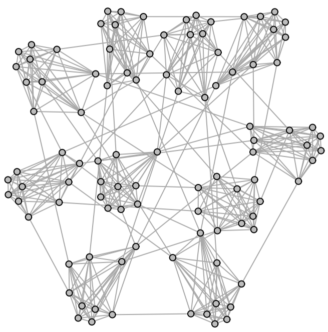


Figure 3: Clique Connection Network

The distribution of thresholds matter in such a setting. Let assume that there are two groups with low (L) and high (H) thresholds. The degree of homophily p_h, is related to the likelihood that the L agents are linked to other L agents in their cliques. Given 100 agents and 10 cliques of size 10 we vary the degree of the homophily and the number of connections across cliques. We take begin with

$L = 4$ and $H = 8$ and vary p_h as increments of 0.1 from 0 to 1. Keeping the average threshold constant we change threshold parameters as $L = 3$ and $H = 9$ Generating 100 simulations we calculate collective action success rates in WAM, as the Revolt Rate in the Figure 4.

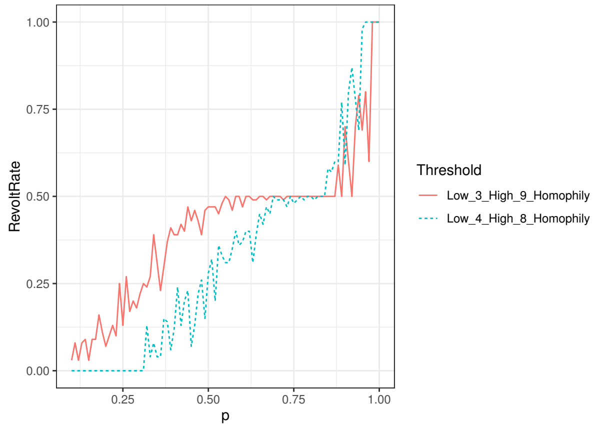


Figure 4: Revolt Rates in WAM Model

In order to illustrate why homophily matters, we mix low and high threshold agents in each clique. We do the same simulations. Revolt rates decline dramatically, especially for low levels of probability of within clique connection. The results are in Figure 5.

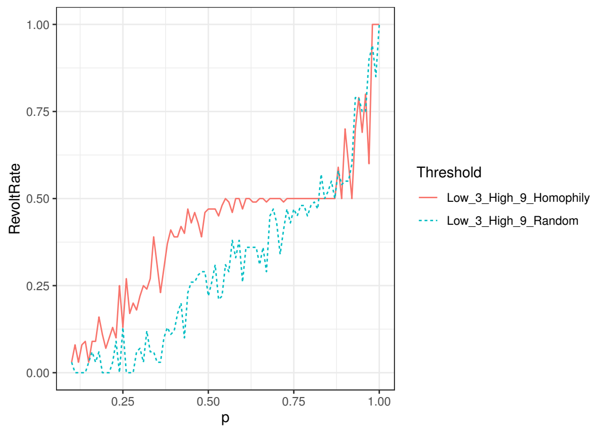


Figure 5: Revolt Rates Without Homophily in WAM Model

We further take two sets of agents with low and high thresholds. In this setting the maximum value of the low threshold agents is 10 and the maximum value of the high threshold agents is 20. We draw 10 threshold distributions of 50 from the low thresholds and 50 from the high thresholds. In each step average threshold in the network is incremented upwards. We also vary the degree of homophily from 0.1 to 1 with an increment of 0.1. We run 100 simulations and take the average revolt rate in the WAM (islands model) networks.

Figure 6 demonstrates the results.

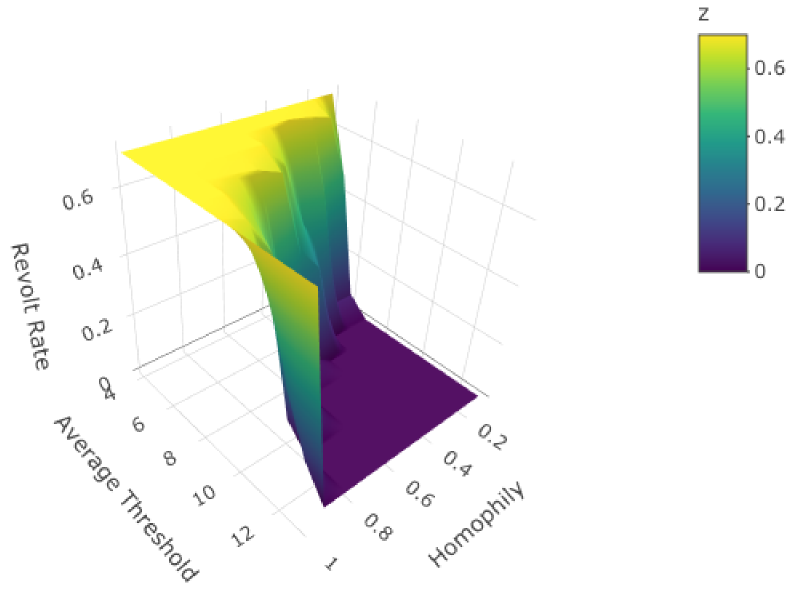


Figure 6: Revolt Rates with Varying Homophily and Average Threshold in WAM Model

4 Discussion

We agree that common knowledge is essential for the collective action to be successful. We embed the communication network through which the common knowledge can emerge in more plausible social settings. We emphasize the importance of homophily and the WhatsApp groups type configurations. Our simulation results suggest that indeed these are important in terms of general success rate of collective action.

References

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