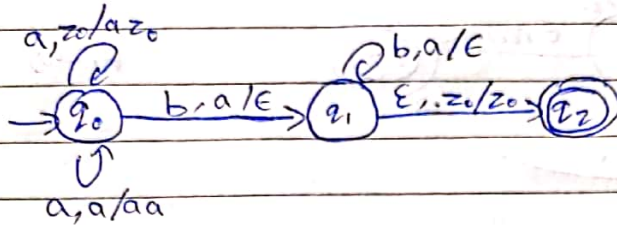


Tutorial 10

Generate and verify PDA for following -

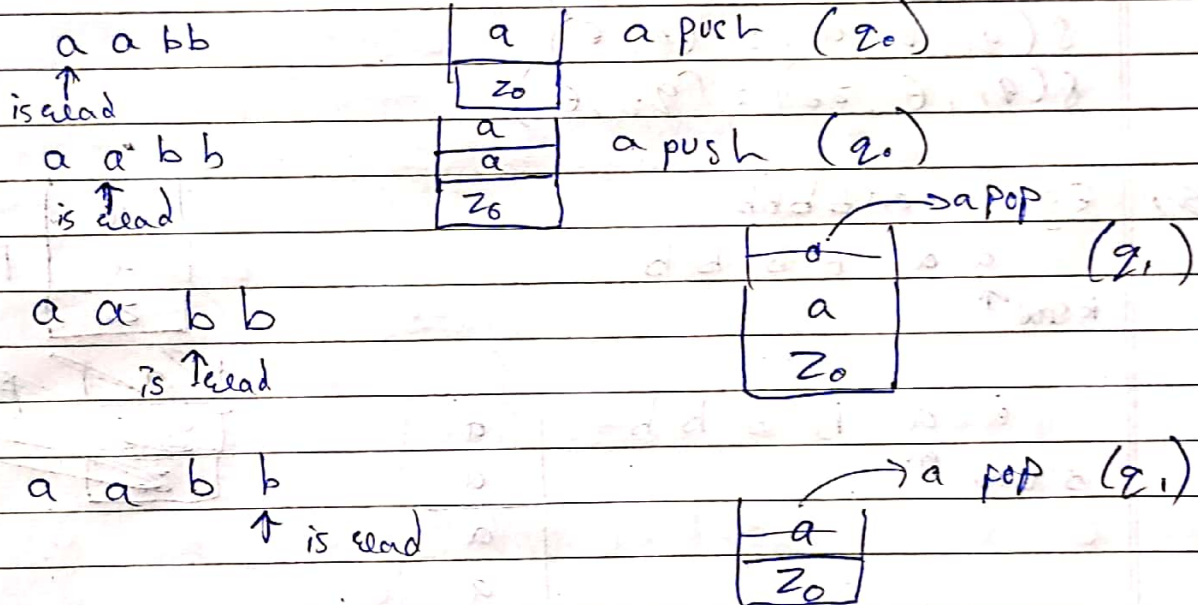
1.  $L = \{a^n b^n \mid n > 0\}$



B245

Let  $w = aabb$

Then as per PDA,



Finally: stack  $\Rightarrow \begin{bmatrix} z_0 \end{bmatrix}$ , state  $q_2$

$\therefore$  transition functions -

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

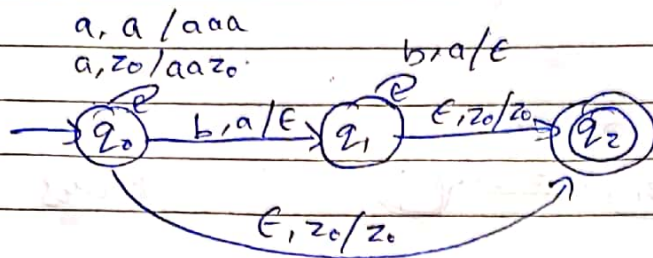
$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$$

$$2. L = \{a^n b^{2n} \mid n \geq 0\}$$



BWS

$$\delta(q_0, a, z_0) = (q_0, aaz_0)$$

$$\delta(q_0, a, a) = (q_0, aaa)$$

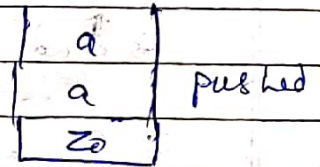
$$\delta(q_0, b, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, a) = (q_1, \epsilon)$$

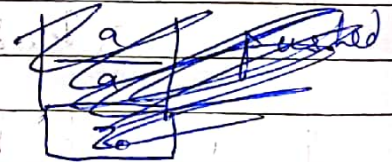
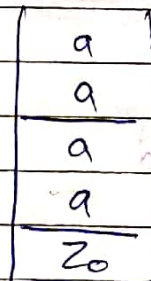
$$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$$

Ex.  $aaabbbb$

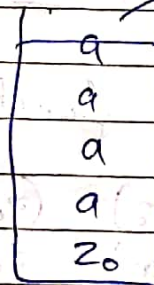
$a \ a \ b \ b \ b \ b$   
 is read  $\uparrow$



$a \ a \ b \ b \ b \ b$   
 is read  $\uparrow$

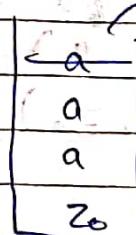


$a \ a \ b \ b \ b \ b$   
 is read  $\uparrow$



pop

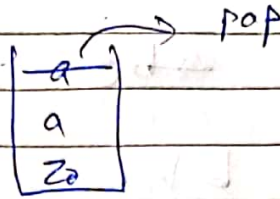
$a \ a \ b \ b \ b \ b$   
 is read  $\uparrow$



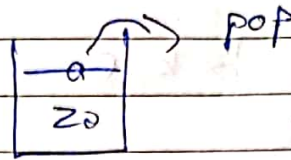
pop



a a b b b b  
is read ↑

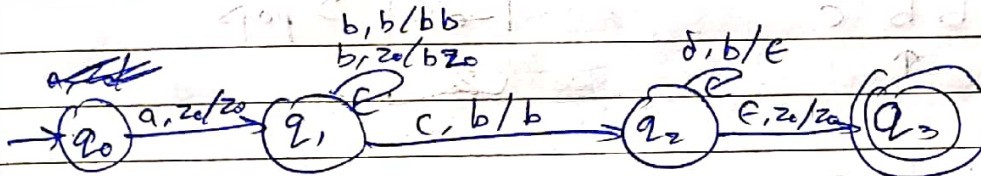


a a b b b b  
is read ↑



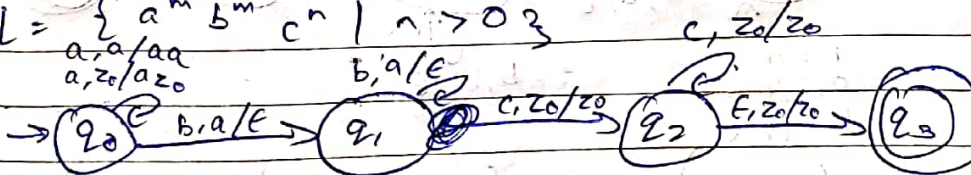
final stack z0

3.  $L = \{ a b^n c d^n \mid n \geq 0 \}$



$$\begin{aligned} \delta(q_0, a, z_0) &= (q_1, z_0) \\ \delta(q_1, b, z_0) &= (q_1, b z_0) \\ \delta(q_1, b, b) &= (q_1, b b) \\ \delta(q_1, c, b) &= (q_2, b) \\ \delta(q_2, d, b) &= (q_2, \epsilon) \\ \delta(q_2, \epsilon, z_0) &= (q_3, z_0) \end{aligned}$$

4.  $L = \{ a^m b^m c^n \mid n \geq 0 \}$



$$\begin{aligned} \delta(q_0, a, z_0) &= (q_0, a z_0) \\ \delta(q_0, a, a) &= (q_0, a a) \\ \delta(q_0, b, a) &= (q_1, \epsilon) \\ \delta(q_1, b, a) &= (q_1, \epsilon) \\ \delta(q_1, c, z_0) &= (q_2, z_0) \\ \delta(q_2, c, z_0) &= (q_2, z_0) \\ \delta(q_2, \epsilon, z_0) &= (q_3, \epsilon) \end{aligned}$$

eg. ~~abc~~ aa b b c

a a b b c

↑

a a b b c

↑

a

pushed

[z<sub>0</sub>]

a
a
z <sub>0</sub>

 pushed

a a b b c

↑

a
a
z <sub>0</sub>

 pop

a a b b c

↑

a
z <sub>0</sub>

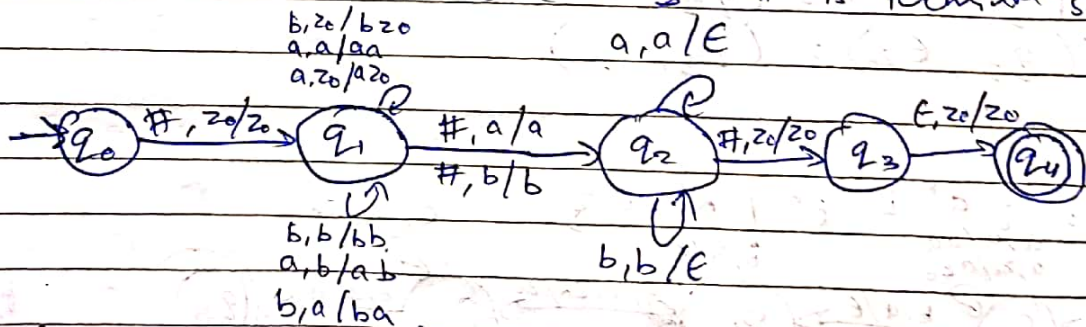
 pop

a a b b c

↑

[z<sub>0</sub>]

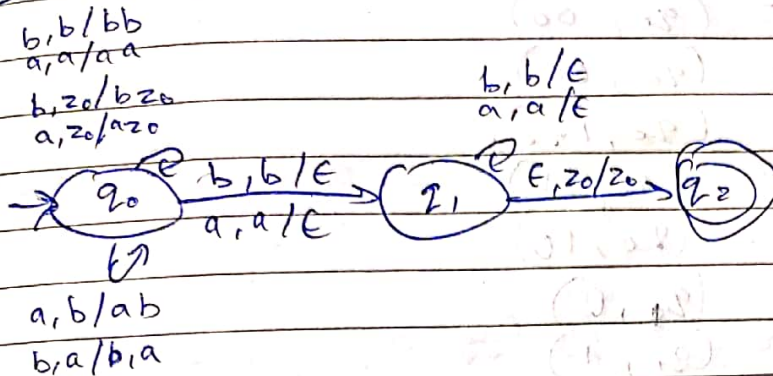
do nothing

final stack [z<sub>0</sub>]S.  $L = \{ \# w \# w^R \# \mid w \in (a+b)^* \}$  # is terminal symbol.

$$\begin{aligned}
 \delta(q_0, \#, z_0) &= (q_1, z_0) & \delta(q_1, \#, a) &= (q_2, a) \\
 \delta(q_1, a, z_0) &= (q_1, a z_0) & \delta(q_1, \#, b) &= (q_2, b) \\
 \delta(q_1, b, z_0) &= (q_1, b z_0) & \delta(q_2, a, a) &= (q_2, \epsilon) \\
 \delta(q_1, a, a) &= (q_1, a a) & \delta(q_2, b, b) &= (q_2, \epsilon) \\
 \delta(q_1, a, b) &= (q_1, a b) & \delta(q_2, \#, z_0) &= (q_3, z_0) \\
 \delta(q_1, b, b) &= (q_1, b b) & \delta(q_3, \epsilon, z_0) &= (q_4, z_0) \\
 \delta(q_1, b, a) &= (q_1, b a)
 \end{aligned}$$



6.  $L = \{ ww^R \mid w \in (a+b)^* \}$



B245

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, a, b) = (q_0, ab)$$

$$\delta(q_0, b, z_0) = (q_0, bz_0)$$

$$\delta(q_0, b, b) = (q_0, bb)$$

$$\delta(q_0, b, a) = (q_0, ba)$$

$$\delta(q_0, b, \epsilon) = (q_1, \epsilon)$$

$$\delta(q_0, a, a) = (q_1, \epsilon)$$

$$\delta(q_1, a, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, b) = (q_1, \epsilon)$$

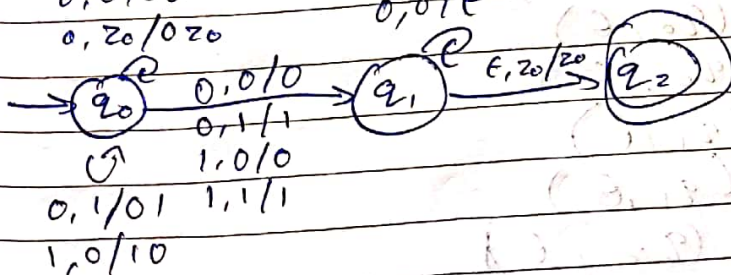
$$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$$

7. Odd length palindrome over  $\{0, 1\}^*$

$$= w c w^R, \quad c = 0 \text{ or } 1$$

$$\begin{aligned}
 &1, 1/11 \\
 &1, z_0/1z_0 \\
 &0, 0/00 \\
 &0, z_0/0z_0
 \end{aligned}$$

$$\begin{aligned}
 &1, 1/\epsilon \\
 &0, 0/\epsilon
 \end{aligned}$$



B245

$$\delta(q_0, 0, z_0) = (q_0, 0z_0)$$

$$\delta(q_0, 0, 0) = (q_0, 00)$$

$$\delta(q_0, 0, 1) = (q_0, 01)$$

$$\delta(q_0, 1, z_0) = (q_0, 1z_0)$$

$$\delta(q_0, 1, 1) = (q_0, 11)$$

$$\delta(q_0, 1, 0) = (q_0, 10)$$

$$\delta(q_0, 0, 0) = (q_1, 0)$$

$$\delta(q_0, 0, 1) = (q_1, 1)$$

$$\delta(q_0, 1, 0) = (q_1, 0)$$

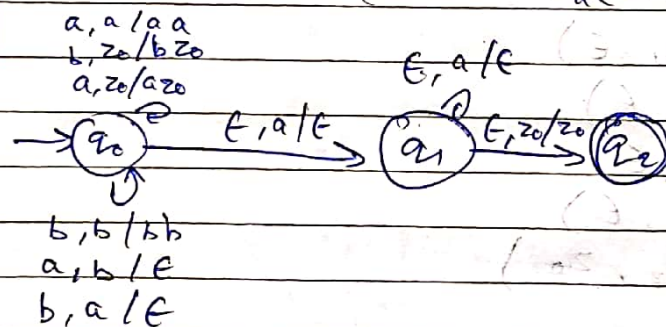
$$\delta(q_0, 1, 1) = (q_1, 1)$$

$$\delta(q_1, 1, 1) = (q_1, \epsilon)$$

$$\delta(q_1, 0, 0) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$$

8.  $L = \{w \mid w \in (a+b)^*, n_a(w) > n_b(w)\}$



B24 S

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, a, a) = (q_0, aa)$$

$$\delta(q_0, b, z_0) = (q_0, bz_0)$$

$$\delta(q_0, b, b) = (q_0, bb)$$

$$\delta(q_0, a, b) = (q_0, \epsilon)$$

$$\delta(q_0, b, a) = (q_0, \epsilon)$$

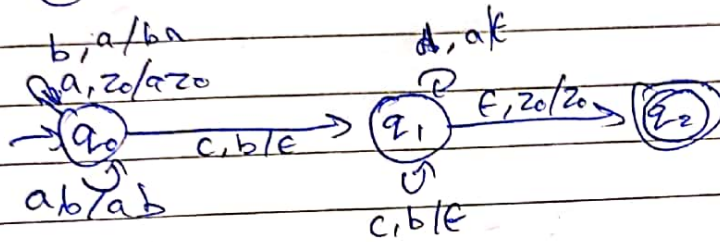
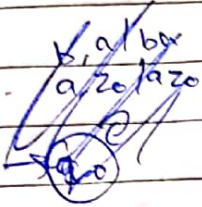
$$\delta(q_0, \epsilon, a) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, a) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$$



$$9. \quad L = \{(ab)^n (cd)^m\} \quad n \geq 1$$



BZUS

$$\delta(q_0, a, z_0) = (q_0, az_0)$$

$$\delta(q_0, b, a) = (q_0, ba)$$

$$\delta(q_0, a, b) = (q_0, ab)$$

$$\delta(q_0, c, b) = (q_1, \epsilon)$$

$$\delta(q_1, d, a) = (q_1, \epsilon)$$

$$\delta(q_1, c, b) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$$