Detailed Description of Proposed Research:

Project 1: Nonparametric identification of production function

Objective: The objective is to provide nonparametric identification of production function when there are finite unobserved types of technologies given the panel data.

Context: Estimation of production function is one of the most important topics in empirical economics. Understanding how the input is related to the output is a fundamental issue in empirical industrial organization (see, for example, Ackerberg, Benkard, Berry, and Pakes, 2007). In empirical trade and macroeconomics, researchers are often interested in estimating production function to obtain a measure of total factor productivity to examine the effect of trade policy on productivity and to analyze the role of resource allocation on aggregate productivity (e.g., Pavcnik, 2002; Kasahara and Rodrigue, 2008; Hsieh and Klenow, 2009).

The Ordinary Least Squares (OLS) estimator of production function suffers from simultaneity bias (Marschak and Andrews, 1944). More recent literature attempts to address the simultaneity issue by employing dynamic panel approach (Arellano and Bond, 1991; Blundell and Bond, 1998; Blundell and Bond, 2000) or developing proxy variable approach (Olley and Pakes, 1996 (OP, hereafter); Levinsohn and Petrin, 2003 (LP, hereafter); Ackerberg, Caves, and Frazer, 2015, (ACF, hereafter); Wooldridge, 2009), the latter of which is now widely used in empirical applications.

Despite their popularity, however, potential identification issues of proxy variable approach have been pointed out in the literature (e.g., ACF). Gandhi, Navarro, and Rivers (2016, GNR hereafter) argue that, if the firm's decision follows a Markovian strategy, then the conditional moment restriction implied by proxy variable approach may not provide enough restriction for non-parametrically identifying gross production function. GNR exploit the first order condition with respect to flexible input under profit maximization and establish the identification of production function without making any functional form assumption.

Almost all existing identification results are derived under the assumption that production function is identical across firms except for the Hicks neutral terms. Formal non-parametric identification result for production function is not available in more realistic case, where production functions differ across firms.

Methodology: We consider a finite mixture specification in which there are J distinct time-varying production technologies, denoted by

$$Y_{it} = F_t^j(X_{it})e^{\omega_{it}+\epsilon_{it}},\ \omega_{it} = h^j(\omega_{it-1}) + \eta_{it},\ \eta_{it} \overset{iid}{\sim} g_\eta^j(\eta_{it}),\ \text{and}\ \epsilon_{it} \overset{iid}{\sim} g_\epsilon^j(\epsilon_{it}), \qquad (1)$$

where Y_{it} is an output, $X_{it} = (L_{it}, K_{it}, M_{it})'$ is a vector of inputs (labor, capital, and materials), ω_{it} follows a first-order Markov process, and ϵ_{it} is an idiosyncratic productivity shock. Each firm belongs to the j-th type with probability π^j. Econometricians do not observe the type of firms. Without making any functional form assumption on F_t^j, we establish nonparametric identification of the model structure $\{\{F_t^j\}_{t=1}^T, \pi^j, h^j, g_\eta^j, g_\epsilon^j\}_{j=1}^J$ when we have panel data $\{\{Y_{it}, X_{it}\}_{t=1}^T\}_{i=1}^n$ with fixed T.

Assume that L_{it} and K_{it} are pre-determined before observing ω_{it} while M_{it} is chosen flexibly after observing ω_{it} but before observing ϵ_{it}. It is possible to establish identification under the alternative assumption that L_{it} is also flexibly chosen. The material demand function is given by $\mathbb{M}_t^j(L_{it}, K_{it}, \omega_{it}) = \arg\max_{M_{it}} E_\epsilon{}^j[F_t^j(L_{it}, K_{it}, M_{it})e^{\omega_{it}+\epsilon}|\mathcal{I}_{it}] - P_{m,t}M_{it}$, where $\mathcal{I}_{it}$ is the information set at the beginning of period t and $P_{m,t}$ is the price of materials relative to output. $\mathbb{M}_t^j(L_{it}, K_{it}, \omega_{it})$ is strictly increasing in ω_{it}, leading to the positive correlation between material M_{it} and productivity shock ω_{it} which is the source of endogeneity bias in the OLS estimator.

To deal with this endogeneity bias, we use the first order condition for the material choice as the additional constraint. Specifically, rearranging the first order condition for the material choice gives the following material share equation:

$$S_{it}^m = G_{m,t}^j(X_{it})E[e^\epsilon|\text{type=j}]e^{-\epsilon_{it}} \tag{2}$$

with $S_{it}^m := \frac{P_{m,t}M_{it}}{Y_{it}}$ and $G_{m,t}^j(X_{it}) := \frac{\partial F_t^j(X_{it})/\partial M}{F_t^j(X_{it})/M_{it}}$. In Cobb-Douglas case with $F_t^j(X_t) = e^{\beta_0^j + \beta_\ell^j \ell_{it} + \beta_k^j k_{it} + \beta_m^j m_{it}}$, this material share equation (2) is written in logarithm as $\ln S_{it}^m = \ln \beta_m^j + \ln E[e^\epsilon|\text{type=j}] - \epsilon_{it}$ so that $\beta_m^j = G_{m,t}^j(X_{it})$. Importantly, (2) does not contain ω_{it} in its expression; as a result, the constraint implied by the material share equation is free from the endogeneity concern. GNR shows that, when there is only one type of firms (i.e., $J = 1$), it is possible to nonparametrically identify the model structure from the moment conditions implied by (1)-(2).

To show the nonparametric identification when $J > 1$, we follow three steps. First, we show that $(Y_{it}, X_{it}, S_{it}^m)$ follows the first-order Markov process within each type under the primitive assumptions on production function so that we can write the joint probability distribution of observed data $\{Y_t, X_t, S_t^m\}_{t=1}^T$ as a finite mixture of J type-specific probability distribution: $\mathrm{P}(\{Y_t, X_t, S_t^m\}_{t=1}^T) = \sum_{j=1}^J \pi^j \mathrm{P}_1^j(Y_1, X_1, S_1^m)$ $\times \prod_{t=2}^T \mathrm{P}_t^j(Y_t, X_t, S_t^m|Y_{t-1}, X_{t-1}, S_{t-1}^m)$. Second, we extend the argument of Kasahara and Shimotsu (2009) and Hu and Shum (2013) to show that type-specific joint probability distributions $\{\pi^j, \mathrm{P}_1^j, \mathrm{P}_t^j\}_{j=1}^J$ in this finite mixture can be non-parametrically identified. Third, once type-specific joint probability distribution is non-parametrically identified, the identification of model structure $\{\{F_t^j\}_{t=1}^T, \pi^j, h^j, g_\eta^j, g_\epsilon^j\}$ for each j follows from the identification argument of GNR.

Establishing the first-order Markov property is key for identification. To see this, suppose that a possible set of values which $W_t := (Y_t, X_t, S_t^m)$ can take is finite and given by $|\mathcal{W}|$. In the absence of the first-order Markov property, we have $\mathrm{P}(\{W_t\}_{t=1}^T) = \sum_{j=1}^J \pi^j \mathrm{P}_1^j(W_1) \prod_{t=2}^T \mathrm{P}_t^j(W_t|\{W_{t-s}\}_{s=1}^{t-1})$ where $\mathrm{P}(\{W_t\}_{t=1}^T)$ is directly identified from the data while the right-hand side contains the unknowns. Evaluating this equation at all possible points of $\{W_t\}_{t=1}^T$ gives $|\mathcal{W}|^T - 1$ restrictions but the number of unknowns are of order of $J \times |\mathcal{W}|^T$ so that identification is not possible. Under the first-order Markov process, we have $\mathrm{P}(\{W_t\}_{t=1}^T) = \sum_{j=1}^J \pi^j \mathrm{P}_1^j(W_1) \prod_{t=2}^T \mathrm{P}_t^j(W_t|W_{t-1})$, and the number of unknowns are of the order of $J \times (1 + |\mathcal{W}| + (T-1)|\mathcal{W}|^2)$ which is smaller than $|\mathcal{W}|^T - 1$ when $T \geq 3$ and $|\mathcal{W}|$ is sufficiently large. Counting the number of restrictions and the number of unknowns is not the proof of identification but it gives important intuition. In particular, the length of panel data T plays an important role because the number of restrictions increases exponentially with T while the number of unknowns increases only linear with T. In our preliminary investigation, we find that $T \geq 4$ is required to constructively prove the non-parametric identification of the model structure.

Project 2: Estimation of production function

Objective: To propose estimation procedures for production function under unobserved heterogeneity and to develop the estimation software for public use.

Context: Estimation of production function is important in various empirical applications, where production function is typically specified using the Cobb-Douglas form as $y_{it} = \beta_0 + \beta_m m_{it} + \beta_\ell \ell_{it} + \beta_k k_{it} + \omega_{it} + \epsilon_{it}$, where the coefficient $(\beta_m, \beta_\ell, \beta_k)$ is assumed to be common across i's within "narrowly defined" industry.

Methodology: The setup is similar to the one in the first project but we specify the model parametrically. There are J unobserved types. We assume that J is known but, in principle, we may empirically determine J by hypothesis testing based on Kasahara and Shimotsu (2015). We specify the j-th type of production function as

$$y_{it} = \beta_0^j + \beta_m^j m_{it} + \beta_\ell^j \ell_{it} + \beta_k^j k_{it} + \omega_{it} + \epsilon_{it}^m \quad \text{with} \quad \omega_{it} = \rho^j \omega_{it-1} + \eta_{it}, \tag{3}$$

where the coefficient $(\beta_0^j, \beta_m^j, \beta_\ell^j, \beta_k^j, \rho^j)'$ and the distribution of η_{it} and ϵ_{it}^m differ across types. In estimation, we assume that both material and labour inputs are flexibly chosen. Then, the material share equation (2) and the corresponding labor share equation in logarithm are given by

$$s^m_{it} = \ln\beta^j_m + \log E[e^{\epsilon^m}|\text{type=j}] - \epsilon^m_{it} \quad \text{and} \quad s^\ell_{it} = \ln\beta^j_\ell + \log E[e^{\epsilon^\ell}|\text{type=j}] - \epsilon^\ell_{it}, \tag{4}$$

where $s^m_{it} = \log S^m_{it}$ and $s^\ell_{it} = \log(\frac{P_{\ell,t}L_{it}}{Y_{it}})$ is the logarithm of the ratio of wage bills to output. Here, motivated by the timing argument of ACF, the superscript m and ℓ in ϵ_{it} in this specification captures the possible discrepancy in the timing of input choices between material and labor. Equation (4) suggests that, if Hicks-neutral technology term is the only source of permanent unobserved heterogeneity in production function, then the material share after controlling for capital, labor, and materials should not exhibit any serial correlation. Using the panel of Japanese manufacturing firms that belongs to machine industry, we find that the serial correlation of the material share is very high at 0.95 and that, even after controlling for capital, labor, and materials, the majority of variation in the material shares can be explained by the firm-specific persistent component rather than the idiosyncratic component, suggesting the importance of unobserved heterogeneity in production technologies.

We propose a two-stage maximum likelihood estimator. Let $s_{it} = (s^m_{it}, s^\ell_{it})'$ and $\theta^j = (\theta^j_1, \theta^j_2)$. Because $\mathrm{P}^j_t(m_{it}, \ell_{it}|y_{it}, s_{it}) = 1$ holds from $m_{it} = y_{it} + s^m_{it} - \ln P_{m,t}$ and $\ell_{it} = y_{it} + s^\ell_{it} - \ln P_{\ell,t}$, we write the density function of $\{y_{it}, k_{it}, s_{it}\}^T_{t=1}$ for type j as $\mathrm{f}^j_t(\{y_{it}, k_{it}, s_{it}\}^T_{t=1}) = L_{1i}(\theta^j_1)L_{2i}(\theta^j_1, \theta^j_2)$, where $L_{1i}(\theta^j_1) = \prod^T_{t=1}\mathrm{f}^j_t(s_{it}; \theta^j_1)$ represents the type-specific density function of $\{s_{it}\}^T_{t=1}$ associated with (4) while $L_{2i}(\theta^j_1, \theta^j_2) = \mathrm{f}^j_1(y_{i1}, k_{it}|s_{i1}; \theta^j)\prod^T_{t=2}\mathrm{f}^j_t(y_{it}|s_{it}, k_{it}, y_{it-1}, s_{it-1}, k_{it-1}; \theta^j)\mathrm{f}_t(k_{it}|y_{it-1}, s_{it-1}, k_{it-1}; \theta^j)$ represents the type-specific density function of $\{y_{it}, k_{it}\}^T_{t=1}$ conditional on $\{s_{it}\}^T_{t=1}$.

In the first stage, we estimate the parameter θ^j_1 that can be identified from (4). Assume that $(\epsilon^m_{it}, \epsilon^\ell_{it})' \overset{iid}{\sim} N(0, \Sigma^j)$ conditional on being type j with $\Sigma^j = \begin{pmatrix} (\sigma^j_m)^2 & \rho^j_{m\ell}\sigma^j_m\sigma^j_\ell \\ \rho^j_{m\ell}\sigma^j_m\sigma^j_\ell & (\sigma^j_\ell)^2 \end{pmatrix}$. We estimate $\boldsymbol{\theta}_1 = (\theta^1_1, ..., \theta^J_1)$ with $\theta^j_1 = (\beta^j_m, \beta^j_\ell, \lambda^j_{11}, \lambda^j_{21}, \lambda^j_{22})'$ and π^j's by maximizing the marginal log-likelihood function $\sum^n_{i=1}\ln\left(\sum^J_{j=1}\pi^j L_{1i}(\theta^j_1)\right)$ with $L_{1i}(\theta^j_1) := \prod^T_{t=1}\frac{1}{\sqrt{1-(\rho^j_{m\ell})^2}\sigma^j_m\sigma^j_\ell}\phi\left(\frac{\epsilon^{m*}_{it}(\theta^j_1)}{\sigma^j_m}\right)\phi\left(\frac{\epsilon^{\ell*}_{it}(\theta^j_1) - \rho^j_{m\ell}(\sigma^j_\ell/\sigma^j_m)\epsilon^{m*}_{it}(\theta^j_1)}{\sqrt{1-(\rho^j_{m\ell})^2}\sigma^j_\ell}\right)$, where $\epsilon^{m*}_{it}(\theta^j_1) = -s^m_{it} + \ln\beta^j_m + 0.5(\sigma^j_m)^2$ and $\epsilon^{\ell*}_{it}(\theta^j_1) = -s^\ell_{it} + \ln\beta^j_\ell + 0.5(1-\rho^2_{m\ell})(\sigma^j_\ell)^2$.

In the second stage, given the first-stage estimate $\hat{\boldsymbol{\theta}}_1$, we estimate θ^j_2's and π^j's by maximizing the log-likelihood function $\sum^n_{i=1}\log\left(\sum^J_{j=1}\pi^j L_{1i}(\hat\theta^j_1)L_{2i}(\hat\theta^j_1, \theta^j_2)\right)$. We derive $L_{2i}(\theta^j_1, \theta^j_2)$ as follows. Because $m_{it} = y_{it} + s^m_{it} - \ln P_{m,t}$ and $\ell_{it} = y_{it} + s^\ell_{it} - \ln P_{\ell,t}$, from (3), we have $\omega_{it} = \omega^*_{it}(\theta^j_1, \theta^j_2) = (1-\beta^j_m-\beta^j_\ell)y_{it}-\beta^j_m s^m_{it}-\beta^j_\ell s^\ell_{it}-\epsilon^{m*}_{it}(\theta^j_1)-\tilde{\beta}^j_t-\beta^j_k k_{it}$, where $\tilde{\beta}^j_t = \beta^j_0+\beta^j_m \ln P_{m,t}+\beta^j_\ell \ln P_{\ell,t}$. Therefore, by the change of variables, we can relate the density function of y_{it} conditional on s^m_{it}, s^ℓ_{it}, and k_{it} to the density function of ω_{it} as $\mathrm{f}^j_t(y_{it}|s^j_{it}, k_{it}) = (1-\beta^j_m-\beta^j_\ell)\mathrm{f}^j_t(\omega_{it})$. Using the change of variables and the monotonicity of material and demand functions with respect to ω_{it}, we have $L_{2i}(\theta^j_2, \theta^j_1) = (1-\beta^j_m-\beta^j_\ell)^T \mathrm{f}_1(\omega_{i1}, k_{i1}; \theta^j_1, \theta^j_2) \prod_{t=2}^T \mathrm{f}^j_t(\omega_{it}|\omega_{it-1}; \theta^j_1, \theta^j_2)\mathrm{f}^j_t(k_{it}|k_{it-1}, \omega_{it-1}; \theta^j_1, \theta^j_2)$, where $\mathrm{f}_1(\omega_{i1}, k_{i1}; \theta^j_1, \theta^j_2)$ and $\mathrm{f}^j_t(k_{it}|k_{it-1}, \omega_{it-1}; \theta^j_1, \theta^j_2)$ are parametrically but flexibly specified using the type-specific multivariate normal distribution with the second-order polynomials while $\mathrm{f}^j_t(\omega_{it}|\omega_{it-1}; \theta^j_1, \theta^j_2) = (1/\sigma^j_\eta)\phi\left((\omega^*_{it}(\theta^j_1, \theta^j_2) - \rho^j \omega^*_{it-1}(\theta^j_1, \theta^j_2))/\sigma^j_\eta\right)$ under the specification in (3) with $\eta_{it} \overset{iid}{\sim} N(0, (\sigma^j_\eta)^2)$ conditional on being type j.

We also consider an alternative estimation procedure based on classification when T is relatively long. Specifically, given the first-stage estimates, we may construct posterior type probability of being type j for firm i as $\hat{\pi}^j_i = \frac{\hat{\pi}^j L_{1i}(\hat{\theta}^j_1)}{\sum_{k=1}^J \hat{\pi}^k L_{1i}(\hat{\theta}^k_1)}$ and take the highest posterior probability as the type for firm i. This will allow us to classify each firm into one of the J types, leading to J data sets, within each of which production technologies are homogenous; we may estimate $\hat{\theta}^j_2$ by the standard GMM estimation procedure using the subsample of firms that are classified as the j-th type.

In our preliminary investigateion, we have estimated (3) under the classification-based method using the Japanese publicly-traded manufacturing firms between 1980 and 2007 and find that the estimated serial correlation in ex-post shocks of random coefficients model (3) with $J = 3$ is substantially lower than that of homogenous model with $J = 1$. We also find that the correlation between estimated productivity and investment is different across different types of firms, where the correlation is stronger among a type of firms with capital intensive production technology.

We plan to develop free open-access software implementing our proposed method in R, Stata, and Matlab. Using the developed software, we will analyze the Japanese manufacturing census panel data for 1986-2010 which contains more than 3 million plant-time observations, and illustrate that ignoring heterogeneity in production functions lead to a large bias in the source of aggregate technological change via resource allocation across different productivities. We will further decompose the source of aggregate productivity growth into the reallocation across different types of technologies (e.g., low vs. high capital-labor ratios), which will provide important insight on how the resource allocations across different types of firms will contribute to the aggregate productivity growth within narrowly defined industry.