Omit i subscripts...

$$u(c_t) = \frac{1}{1-\gamma} c^{1-\gamma}$$
$$u_c(c_t) = c_t^{-\gamma}$$

Retirement (FOC for T$_{ret}$-1:T-2)

Adapted from KV2010 ???:

$$V_t(a_{t+1}, \tilde{y}) = \max_{c_t, a_{t+1}} u(c_t) + \beta \mathbb{E}_t[V_{t+1}(a_{t+2}, \tilde{y})]$$
$$\text{s.t. } c_t = (1+r)a_t + P(\tilde{y}) - \frac{\xi_t}{\xi_{t+1}} a_{t+1}$$

$$\frac{\partial u(c_t)}{\partial a_{t+1}} = -\frac{\xi_t}{\xi_{t+1}} c_t^{-\gamma}$$

$$\frac{\partial V_{t+1}(\cdot)}{\partial a_{t+1}} = (1+r) c_{t+1}^{-\gamma}$$

$$\Rightarrow c_t^{-\gamma} = \beta \frac{\xi_{t+1}}{\xi_t} (1+r) c_{t+1}^{-\gamma}$$

$$\Rightarrow c_t = \left(\beta \frac{\xi_{t+1}}{\xi_t} (1+r) \right)^{-1/\gamma} c_{t+1}$$

FOC for retirement:

$$\frac{\xi_t}{\xi_{t+1}} c_t^{-\gamma} = \beta \frac{\xi_t}{\xi_{t+1}} (1+r) c_{t+1}^{-\gamma}$$

Following Giovanni's notes, would have:

$$V_t(a_{t+1}, \tilde{y}) = \max_{c_t, a_{t+1}} u(c_t) + \beta \frac{\xi_{t+1}}{\xi_t} \mathbb{E}_t[V_{t+1}(a_{t+2}, \tilde{y})]$$
$$\text{s.t. } c_t = (1+r)a_t + P(\tilde{y}) - a_{t+1}$$

$$\Rightarrow c_t^{-\gamma} = \beta \frac{\xi_{t+1}}{\xi_t} (1+r) c_{t+1}^{-\gamma}$$

$$\Rightarrow c_t = \left(\beta \frac{\xi_{t+1}}{\xi_t} (1+r) \right)^{-1/\gamma} c_{t+1}$$

Working years (FOC for T=1:T$_{ret}$-2)

$$
\begin{aligned}
V_t(a_{t+1}, z_t) &= \max_{c_t, a_{t+1}} u(c_t) + \beta \mathbb{E}_t[V_{t+1}(a_{t+2}, z_{t+1})] \\
\text{s.t. } c_t &= (1+r)a_t + y_t - a_{t+1} \\
&\Rightarrow c_t^{-\gamma} = \beta(1+r)\mathbb{E}_t[c_{t+1}^{-\gamma}]
\end{aligned}
$$

$$
\begin{aligned}
\mathbb{E}_t[c_{t+1}^{-\gamma}] &= \mathbb{E}_t[((1+r)a_{t+1} + y_{t+1} - a_{t+2})^{-\gamma}] \\
&= \mathbb{E}_t[((1+r)a_{t+1} + \exp(\kappa_{t+1} + z_{t+1} + \varepsilon_{t+1}) - a_{t+2})^{-\gamma}] \\
&= \sum_{z_{t+1}|z_t} \sum_{\varepsilon_t} \pi_{z,t}(z_{t+1}|z_t)\pi_\varepsilon(\varepsilon_{t+1}) \left((1+r)a_{t+1} + e^{(\kappa_{t+1}+z_{t+1}+\varepsilon_{t+1})} - a_{t+2}\right)^{-\gamma}
\end{aligned}
$$