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CSC508 Data Structures

Topic 13 : Graph 1

Compiled & edited by: Zahid Zainal

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Recap

- Hashing
- Hash Method
- Collision Resolution
- Load Factor

Topic Structure

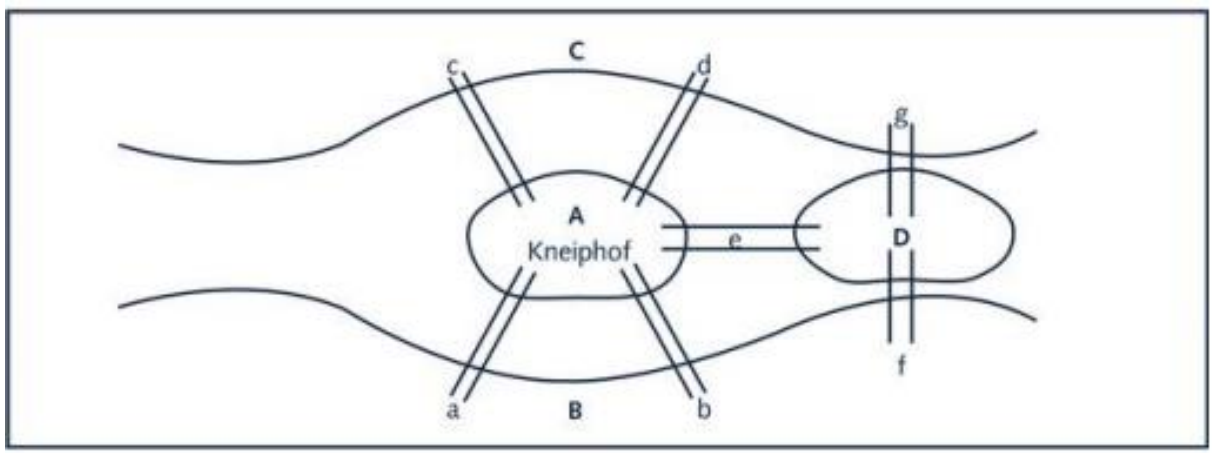
- Kőnigsberg Bridge Problem
- Graph Definition
- Graph Representations

Learning Outcomes

- At the end of this lesson, students should be able to:
  - Explain the concept of graph data structure
  - Describe graph representations
  - Describe graph traversals

Kőnigsberg Bridge Problem

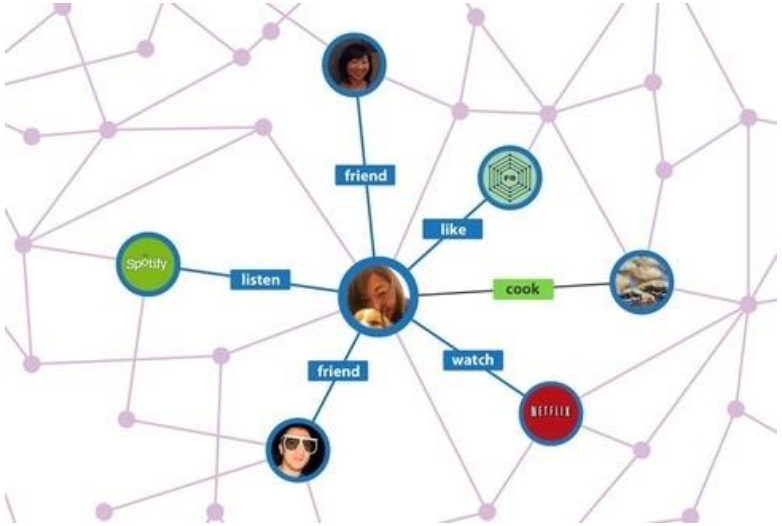
- River Pregel (Pregolya) flows around the island Kneiphof
  - Divides into two
  - River has four land areas (A, B,C, D)
  - Bridges are labeled a, b, c, d, e, f, g
- Problem : Starting at one land area, is it possible to walk across all the bridges exactly once and return to the starting land area?
- In 1736, Euler represented Königsberg bridge problem as graph

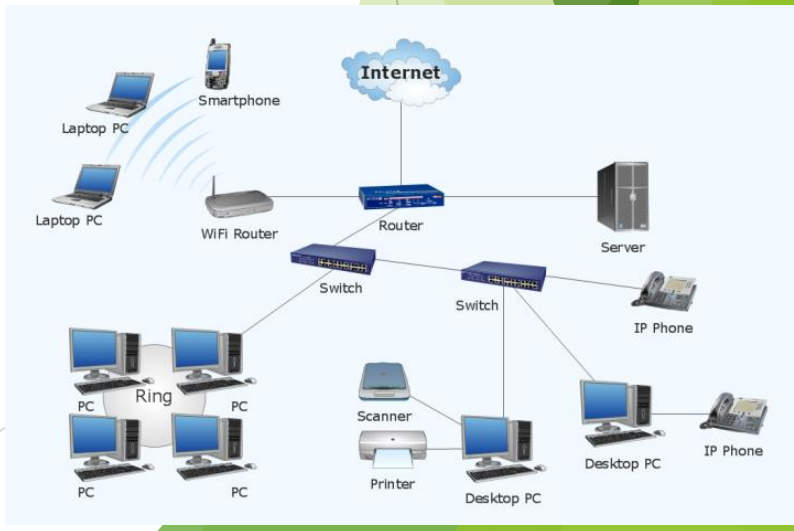


The birth of graph theory!!!

What is Graph?

- A data structure that consists of a set of vertices (nodes) and a set of edges (links, or arcs) that relate the nodes to each other
- Edges describe relationships among the objects (represented by vertices)
- A useful structure to represent non-linear relationships between objects/entities, like connectivity, dependency, interactivity, etc.
  - used in the analysis of electrical circuits and network communications, finding shortest route between two places, and in applications related to project planning, linguistics, genetics, social sciences, and many more





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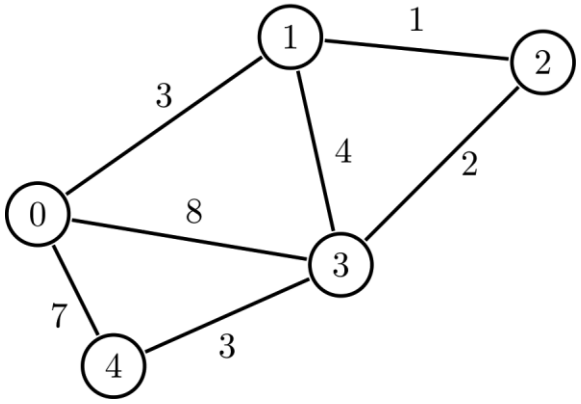
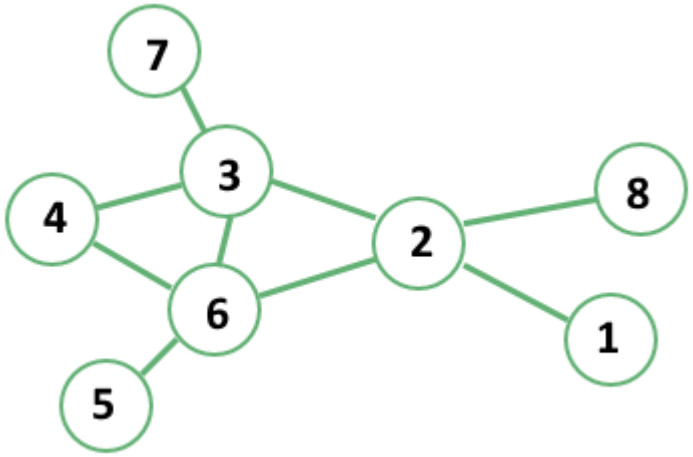
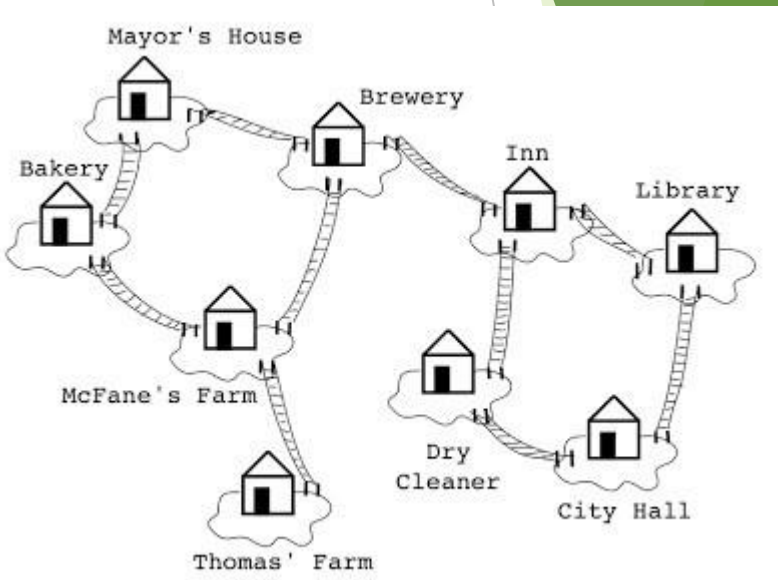
Graph Definitions & Notations

- A graph G is a pair, G = (V, E),
  - where V is a finite nonempty set, called the set of vertices of G, and $E \subseteq V \times V$
  - E are the pair of elements of V. E is called the set of edge
- Let V(G) denote the set of vertices, and E(G) denote the set of edges of a graph G. If the elements of E(G) are ordered pairs, g is called a directed graph or digraph; Otherwise, g is called an undirected graph
- In an undirected graph, the pairs (u, v) and (v, u) represent the same edge

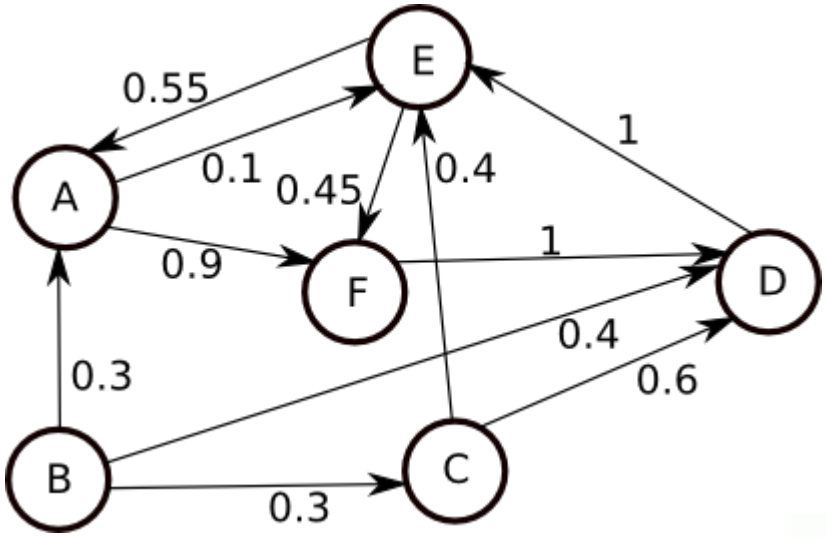
Samples of Undirected Graphs

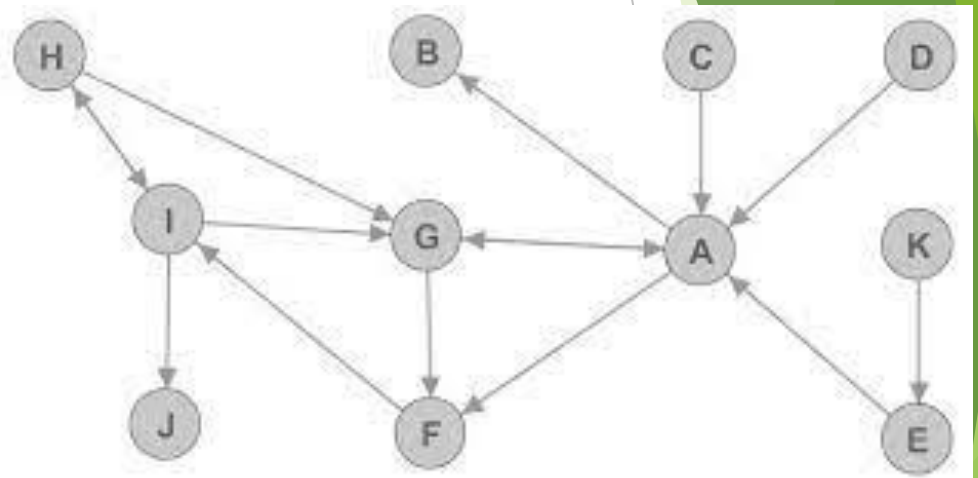


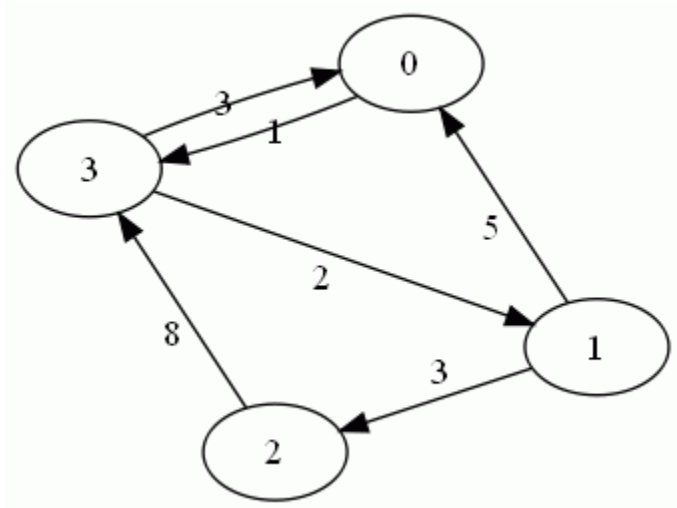




Samples of Directed Graphs







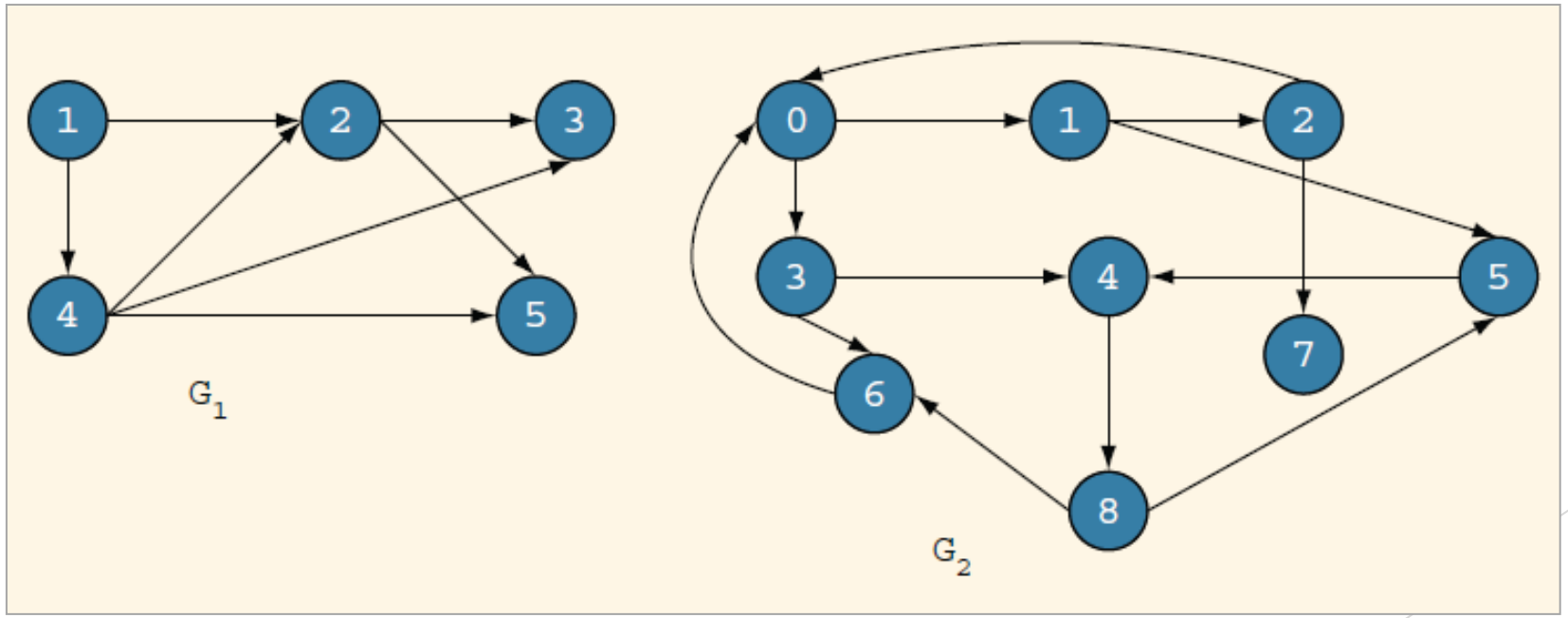
Graph Definitions & Notations (cont.)

$V(G_1) = \{1, 2, 3, 4, 5\}$

$V(G_2) = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

$E(G_1) = \{(1, 2), (1, 4), (2, 3), (2, 5), (4, 2), (4, 3), (4, 5)\}$

$E(G_2) = \{(0, 1), (0, 3), (1, 2), (1, 5), (2, 0), (2, 7), (3, 4), (3, 6), (4, 8), (5, 4), (6, 0), (8, 5), (8, 6),\}$



Graph Representation

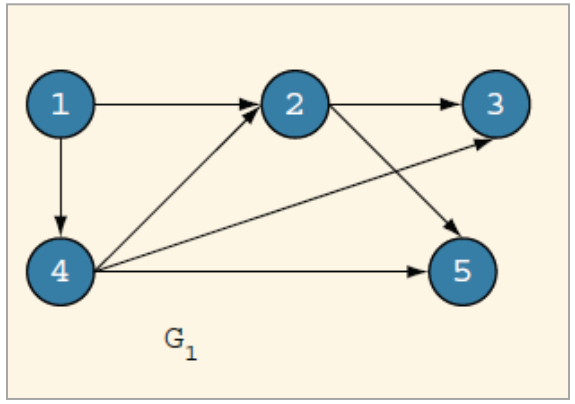
- Graph can be represented in two ways:
  - Adjacency matrix
  - Adjacency list

Adjacency Matrix

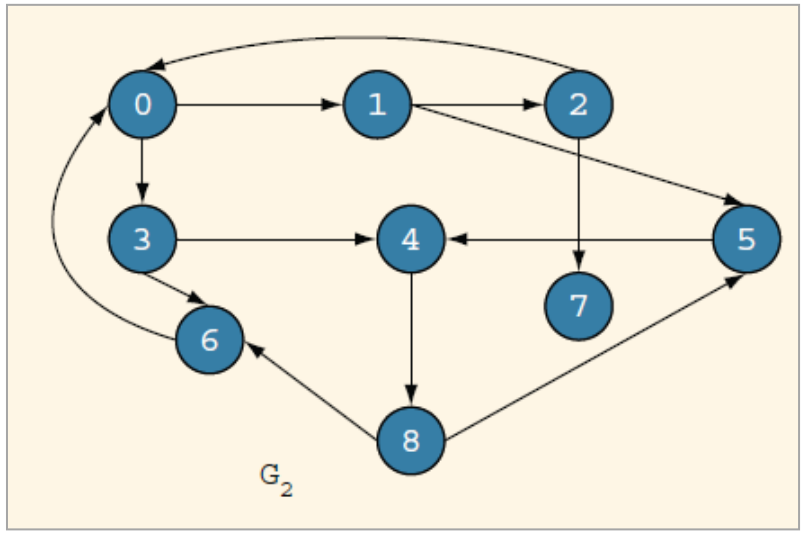
- Let G be a graph with n vertices, where n > 0
- Let V(G) = {v1 , v2 , ..., vn }
- The adjacency matrix A_G is a two-dimensional n × n matrix such that the (i, j)th entry of A_G is 1 if there is an edge from v_i to v_j; otherwise, the (i, j)th entry is zero

$$A_G(i,j) = \begin{cases} 1 & \text{if } (\nu_i, \nu_j) \in E(G) \\ 0 & \text{otherwise} \end{cases}$$

Adjacency Matrix - Samples



$$A_{G_1} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

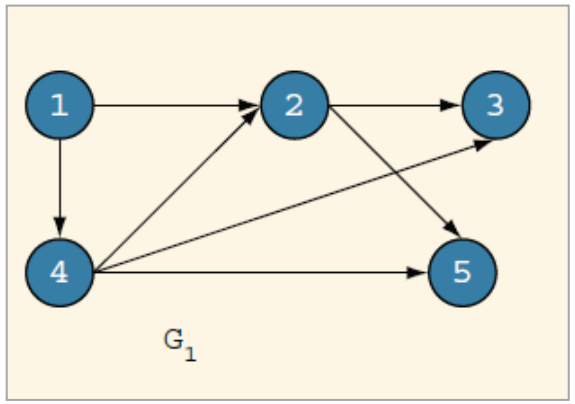


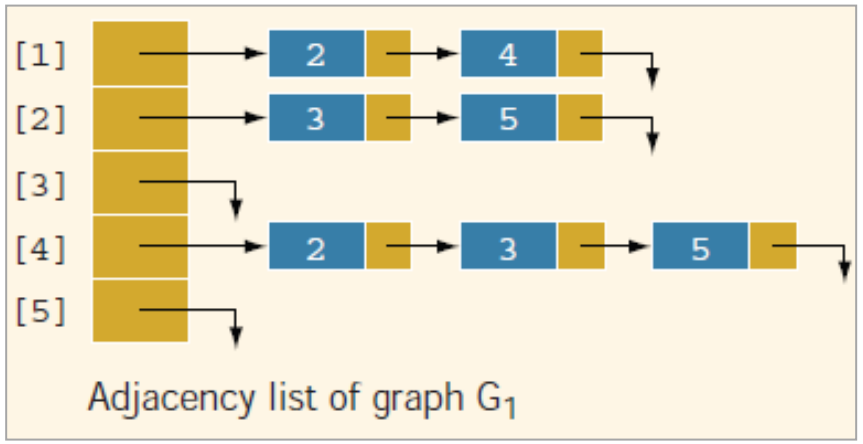
$$A_{G_2} = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

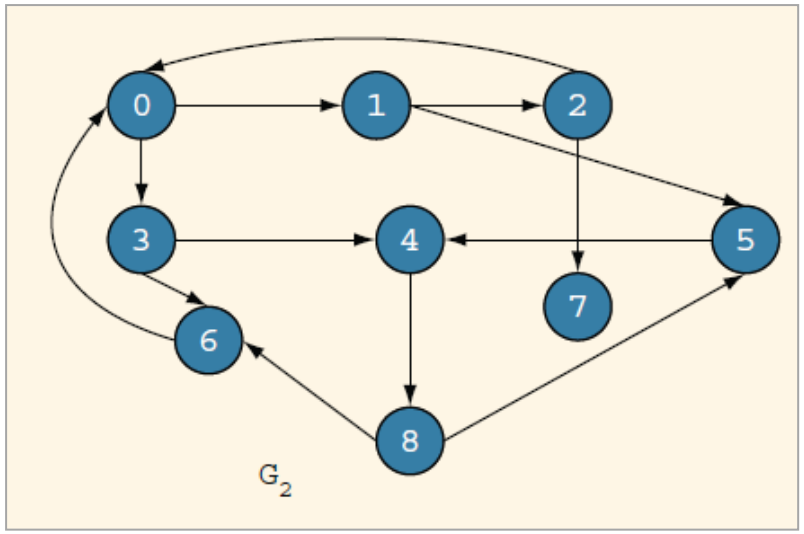
Adjacency List

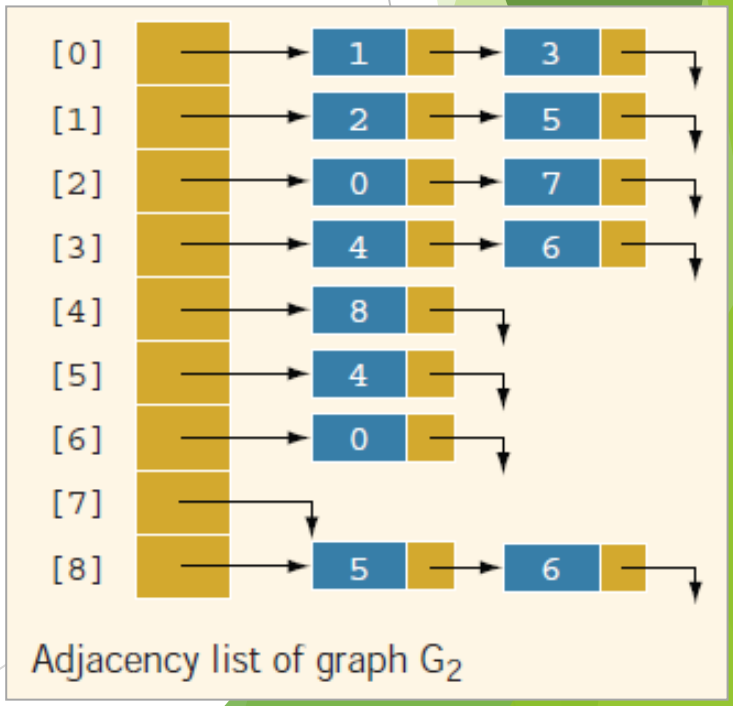
- In adjacency list representation, corresponding to each vertex, v, is a linked list such that each node of the linked list contains the vertex u, such that (v, u) $\in$ E(G)
- Array, A, of size n, such that A[i] is a reference variable pointing to address of first node of linked list containing the vertices to which v_i is adjacent
- Each node has two components, (vertex and link)
- Component vertex contains index of vertex adjacent to vertex i

Adjacency List - samples









References

- Carrano, F. & Savitch, W. 2005. *Data Structures and Abstractions with Java, 2nd ed. Prentice-Hall.*
- Malik D.S, & Nair P.S., Data Structures Using Java, Thomson Course Technology, 2003.
- Rada Mihalcea, CSCE 3110 Data Structures and Algorithm Analysis notes, U of North Texas.

TEST 2

- Date :
- Time :
- Topic Covered
  - Recursion
  - Sorting Algorithm
  - Searching
  - Hashing
  - Graph – part 1