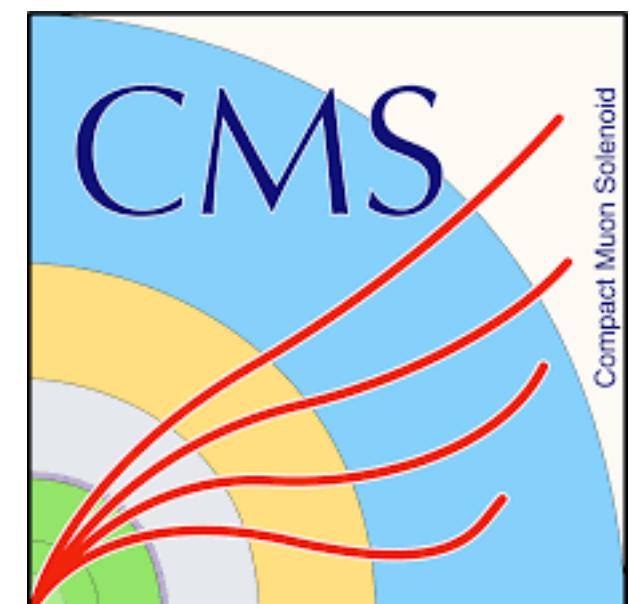
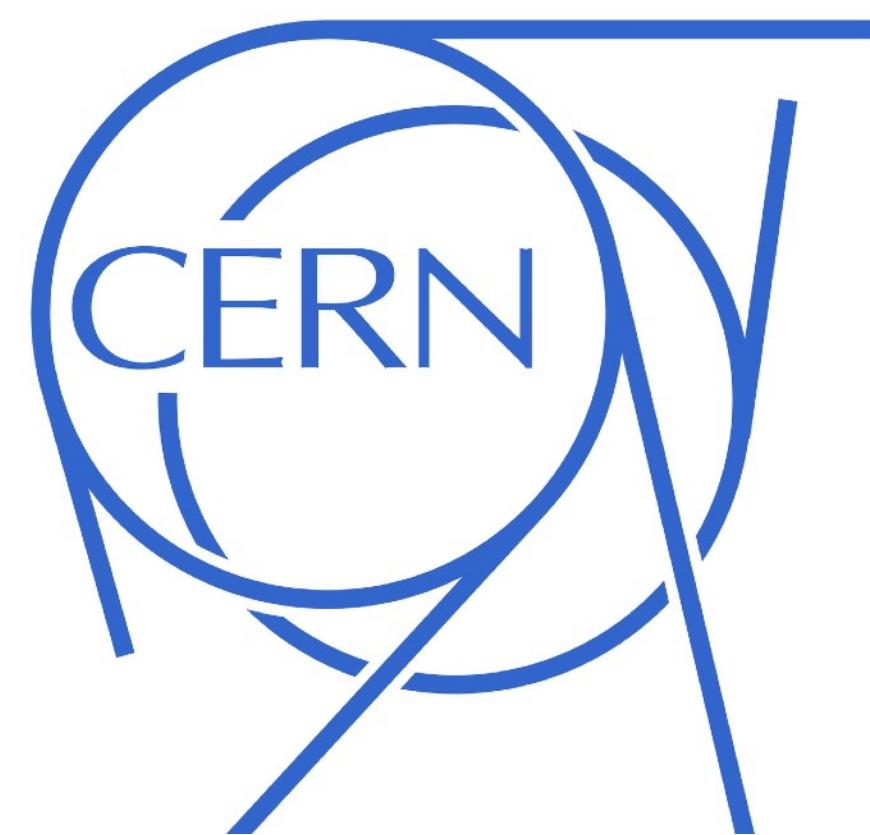


# INTRO TO MACHINE LEARNING IN NUCLEAR PHYSICS

MICHELLE KUCHERA  
DAVIDSON COLLEGE

JOINT ICTP-IAEA SCHOOL ON AI FOR NUCLEAR, PLASMA, AND  
FUSION SCIENCE

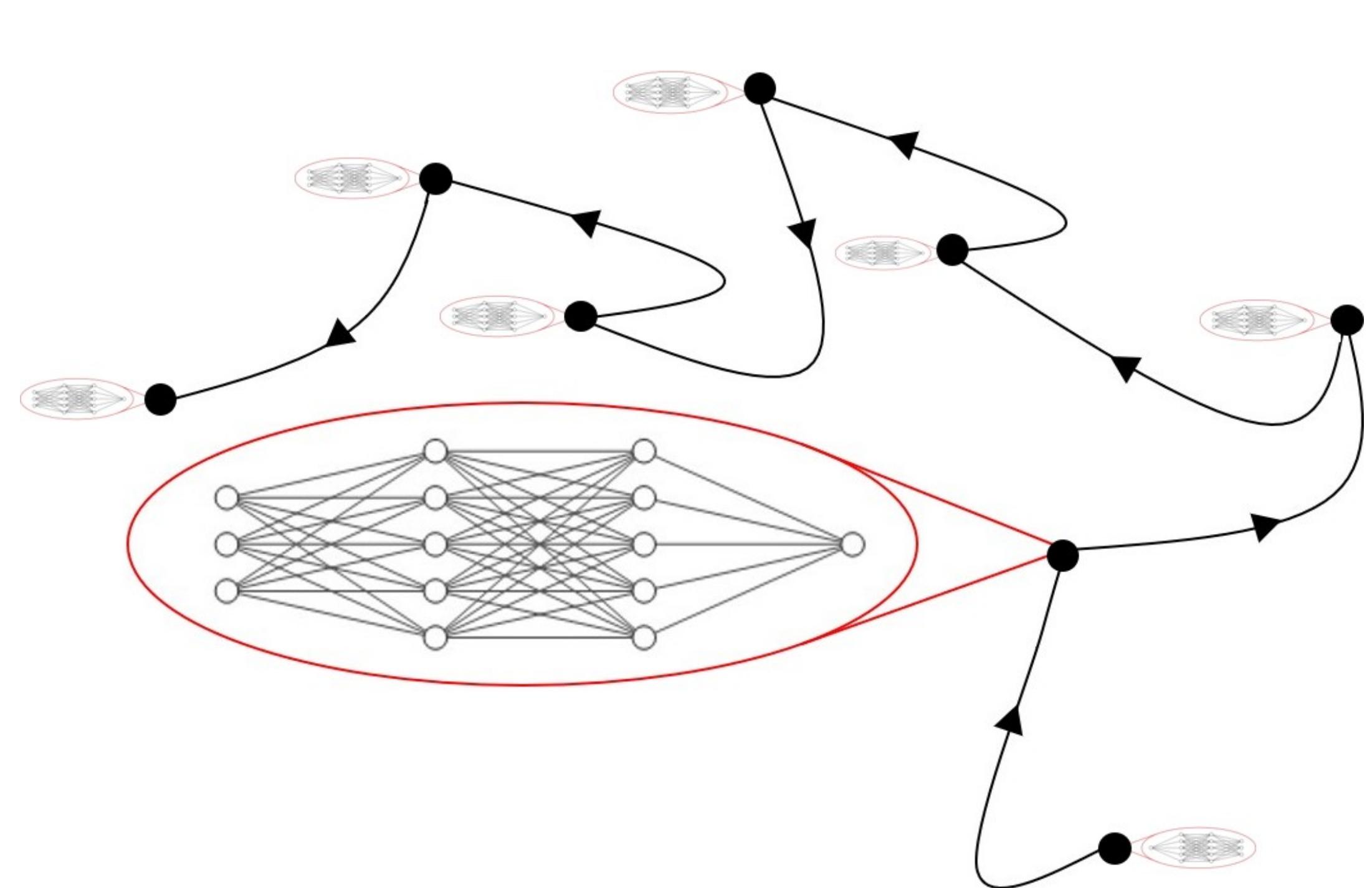
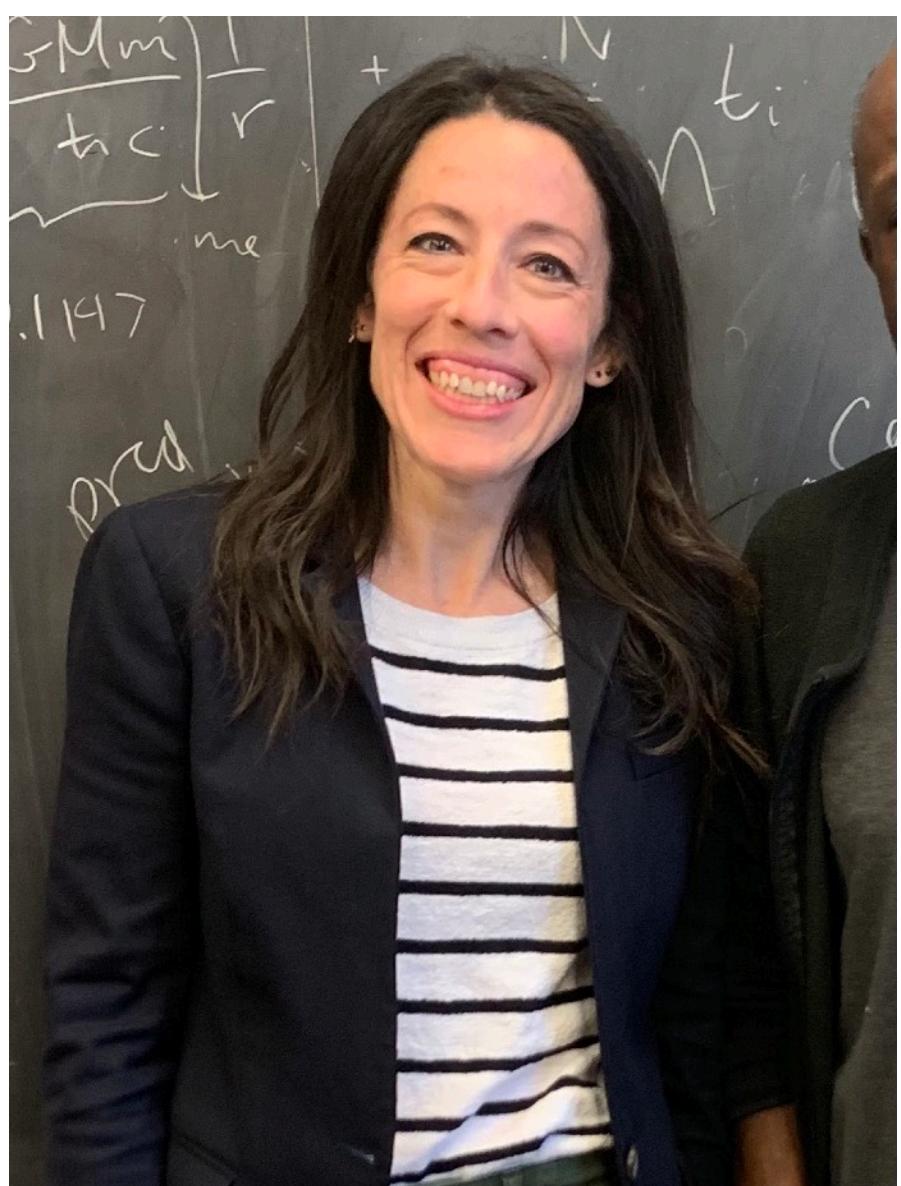
ICTP  
22 MAY 2023



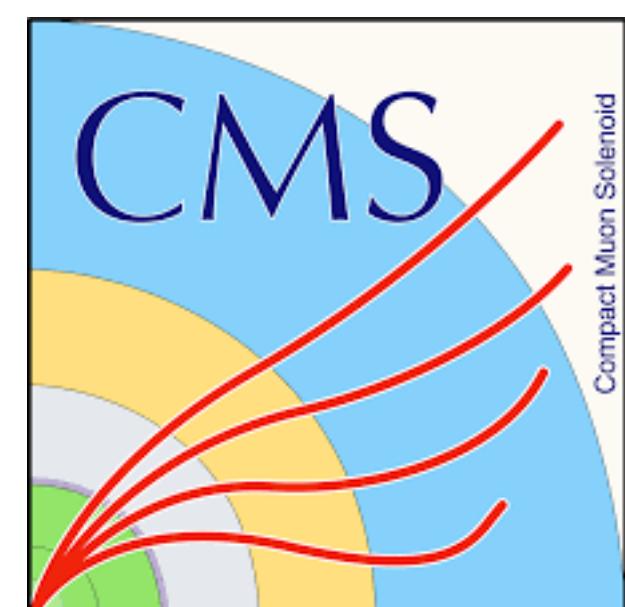
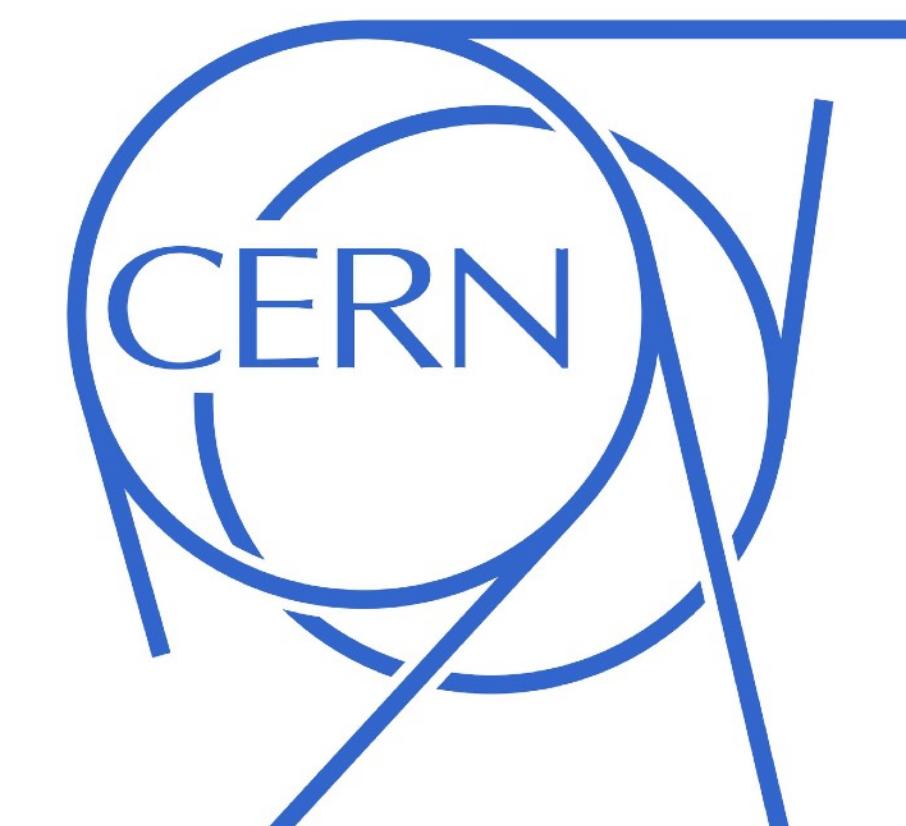
MICHELLE KUCHERA

B.S., M.S. PHYSICS

M.S., PH.D. COMPUTATIONAL SCIENCE



**ALPhA**  
DAVIDSON COLLEGE



**Jefferson Lab**



**DAVIDSON**



# SESSION 1 TOPICS

- Nuclear physics motivation
- Gradient descent on graphs
- Neural Networks

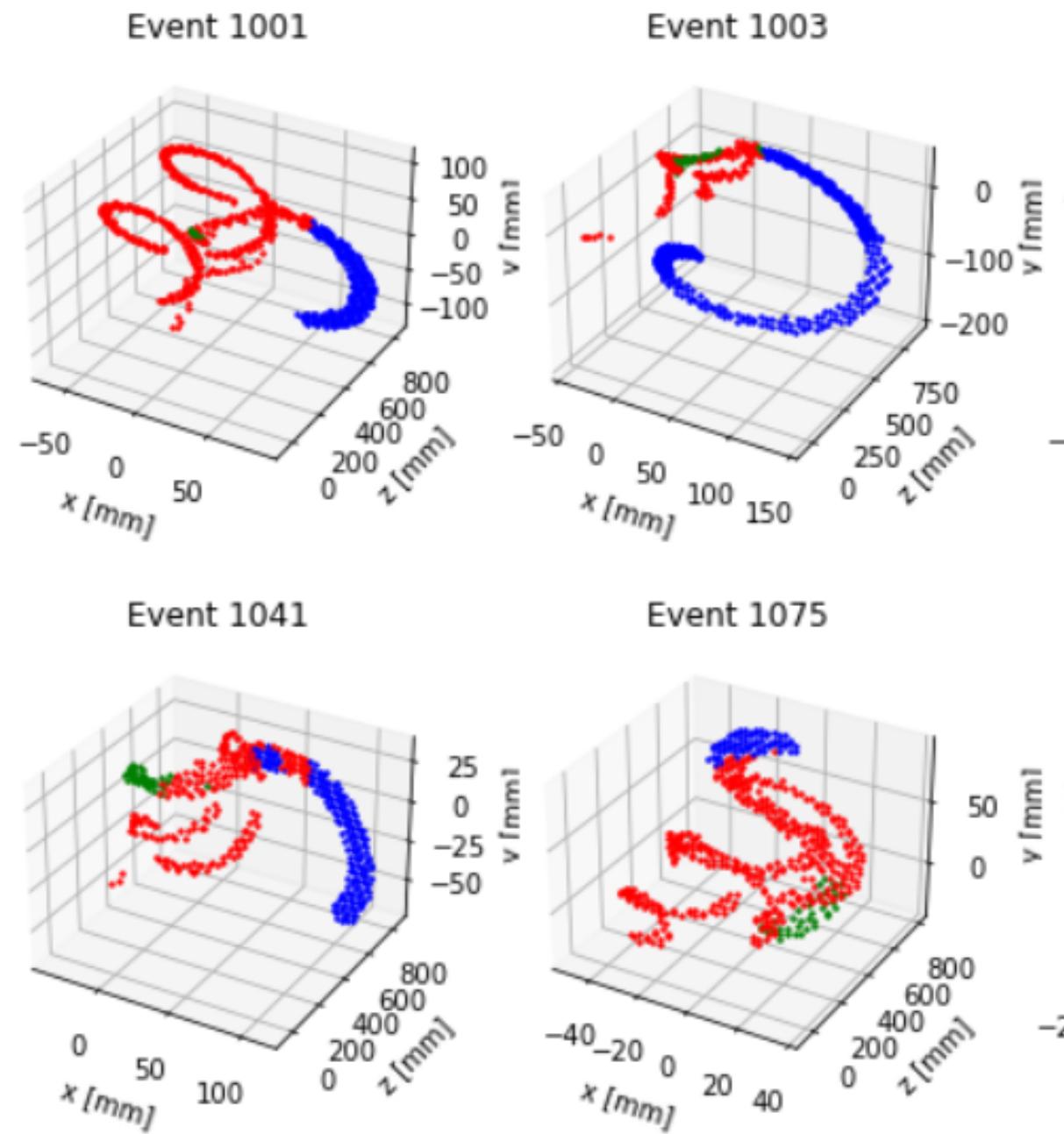
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MICHELLE KUCHERA  
DAVIDSON COLLEGE

JOINT ICTP-IAEA SCHOOL ON AI FOR NUCLEAR, PLASMA, AND  
FUSION SCIENCE

ICTP  
22 MAY 2023

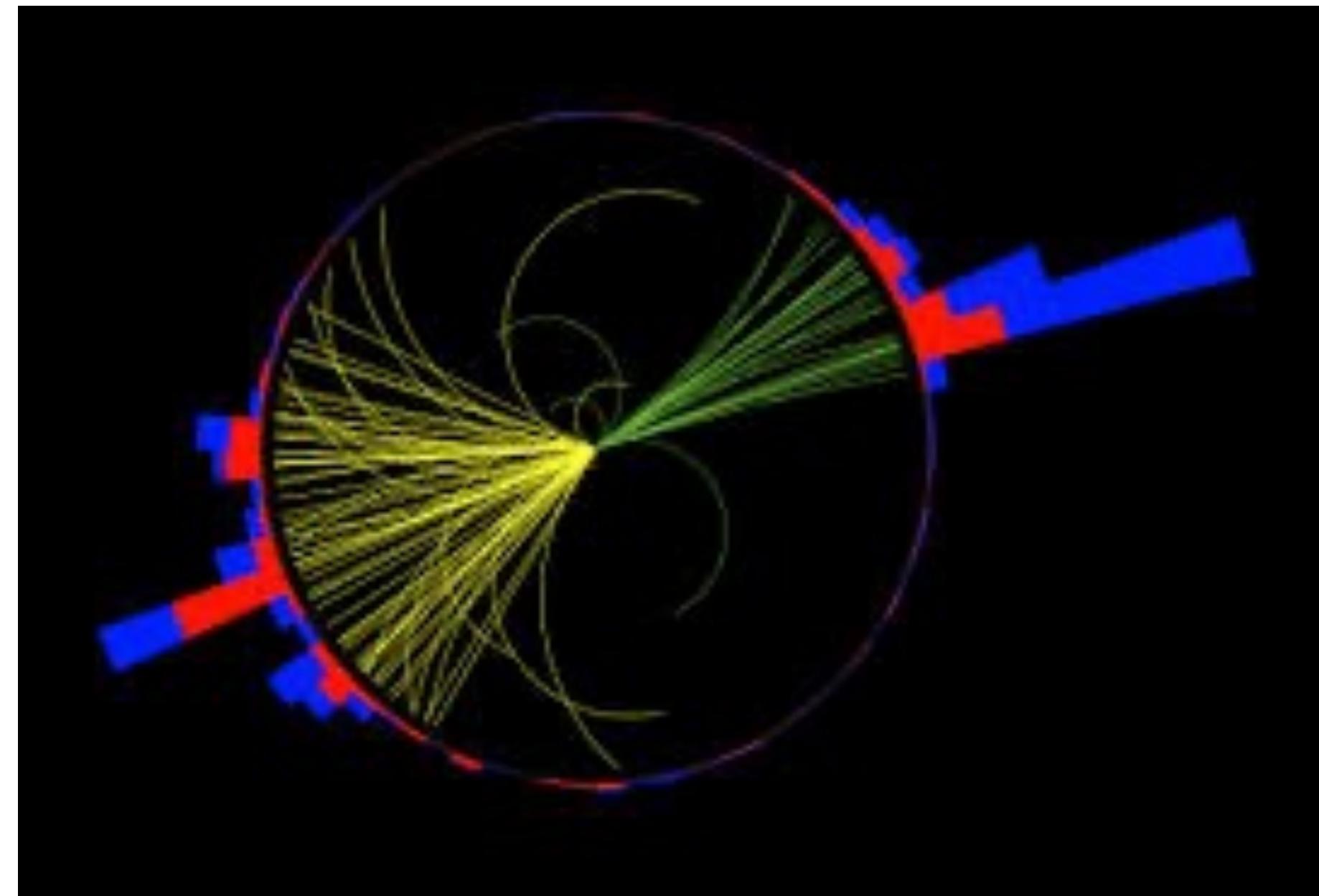
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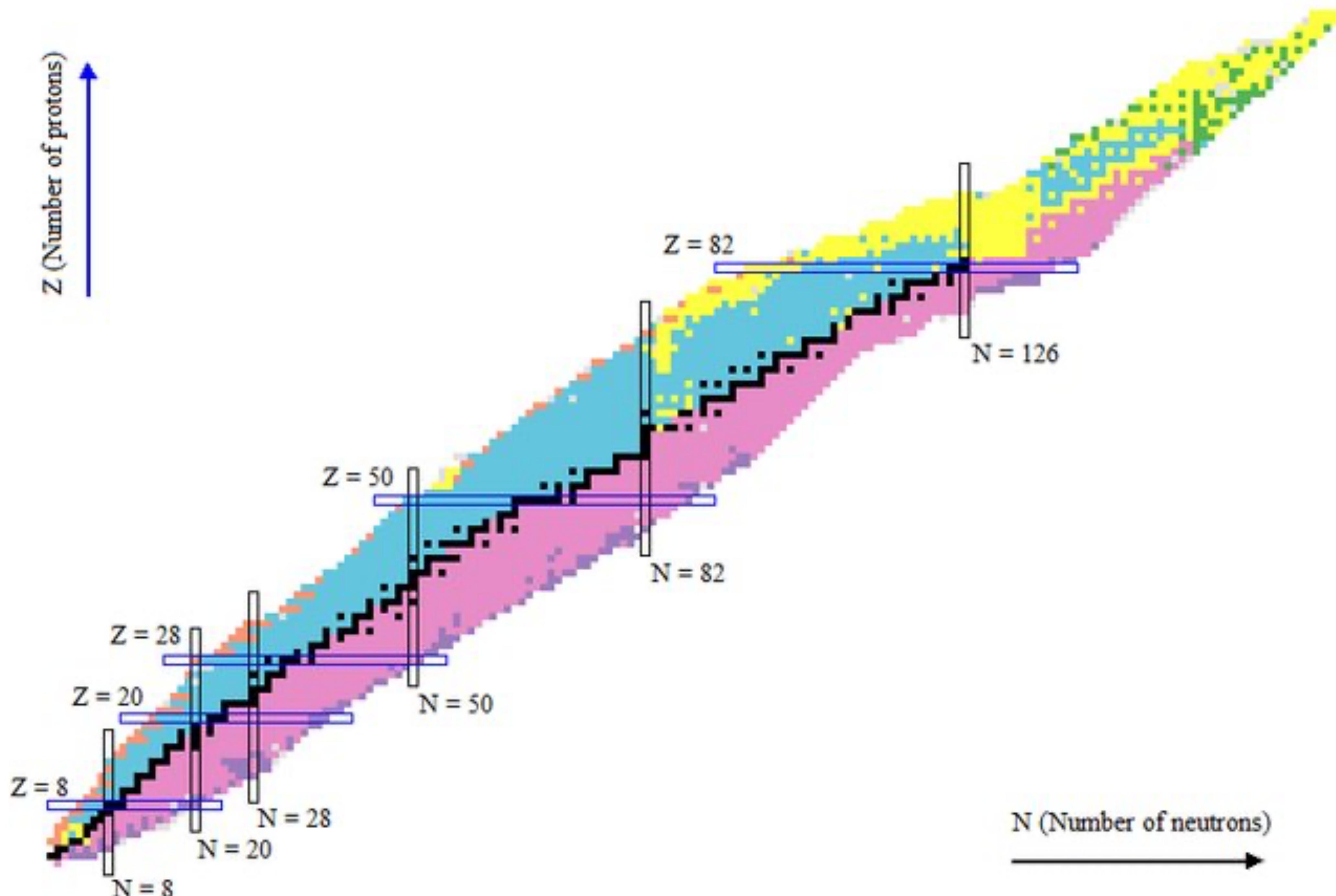
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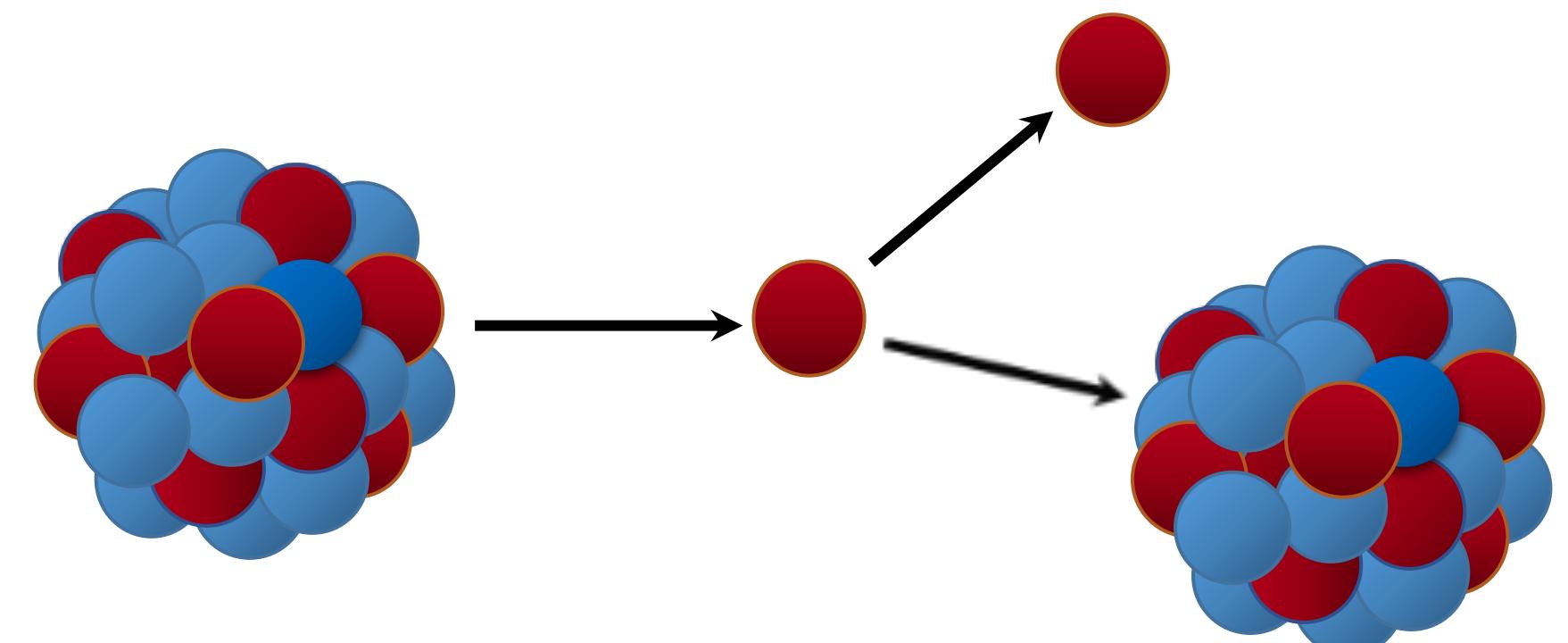
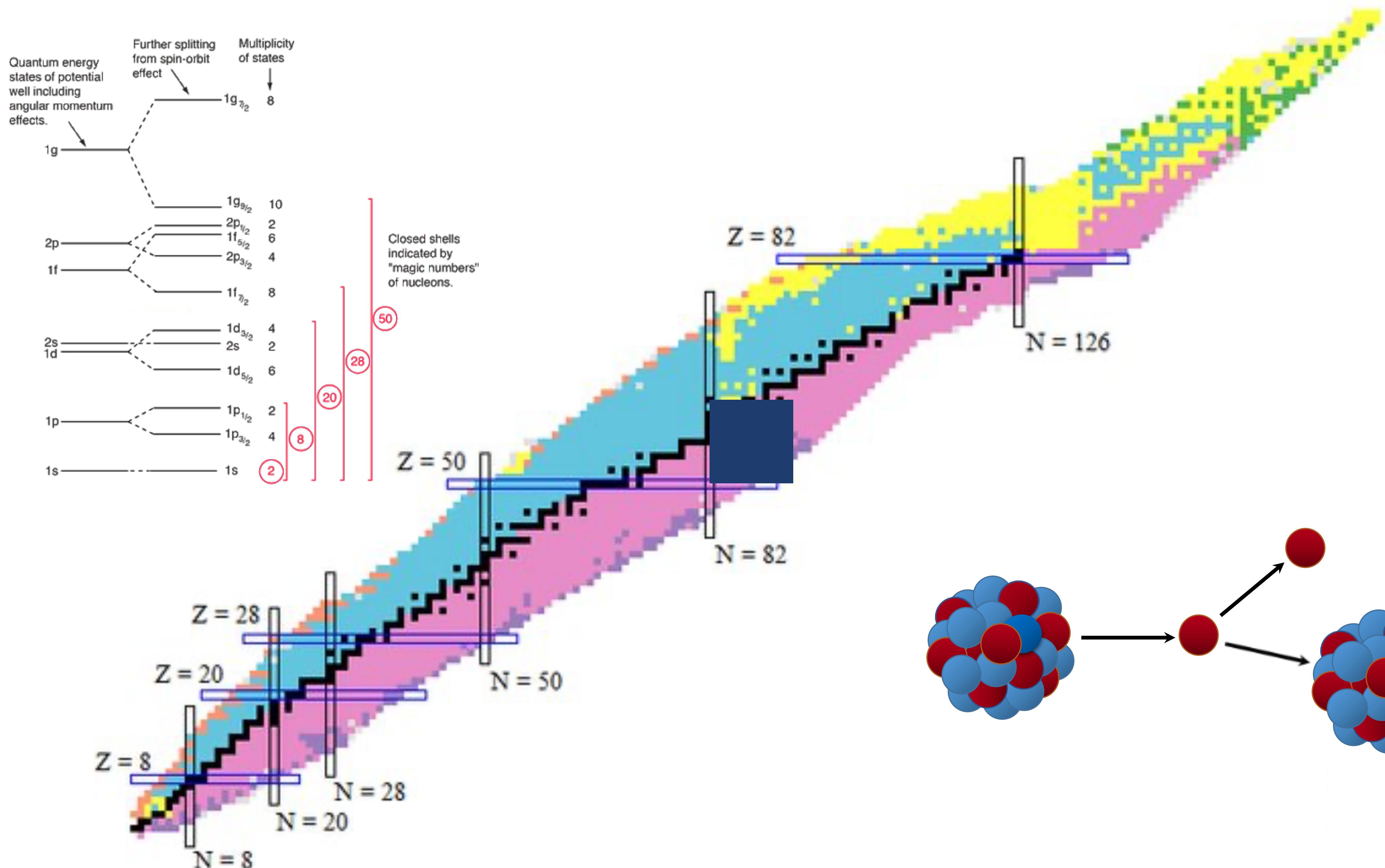
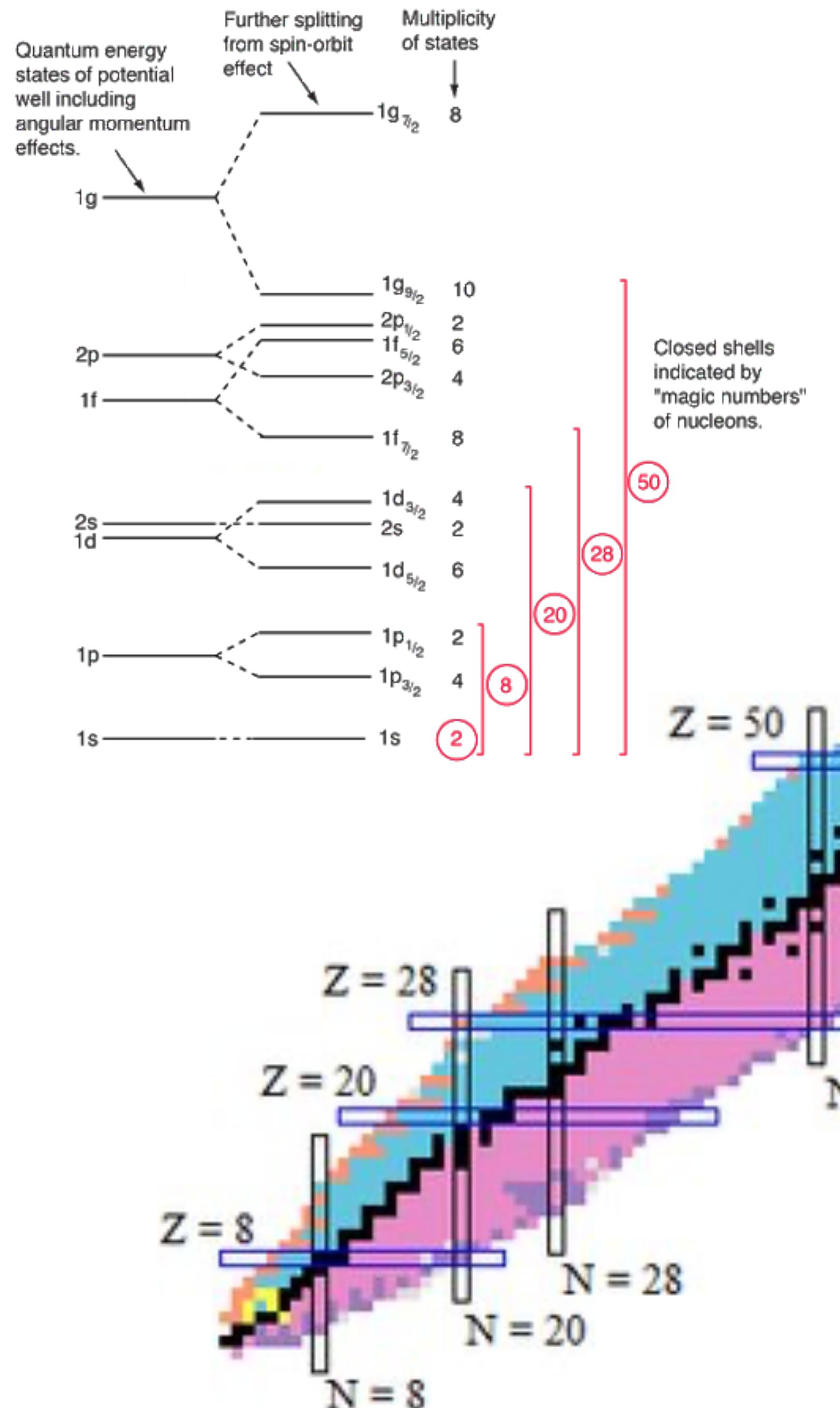


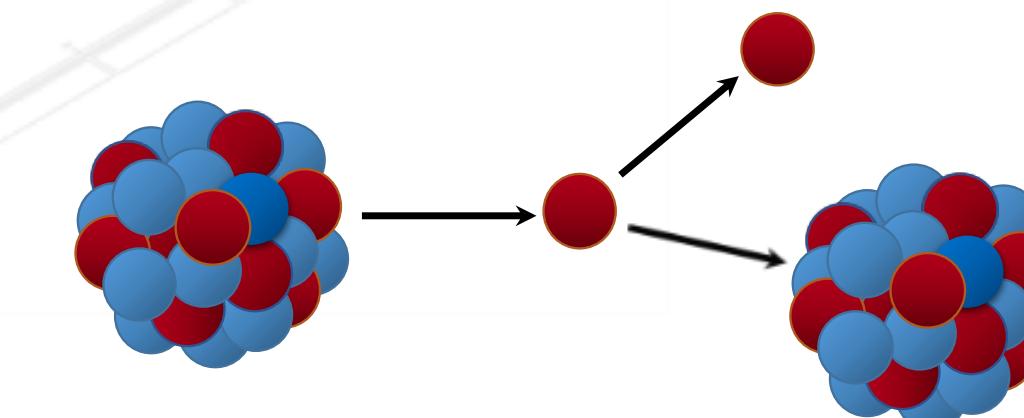
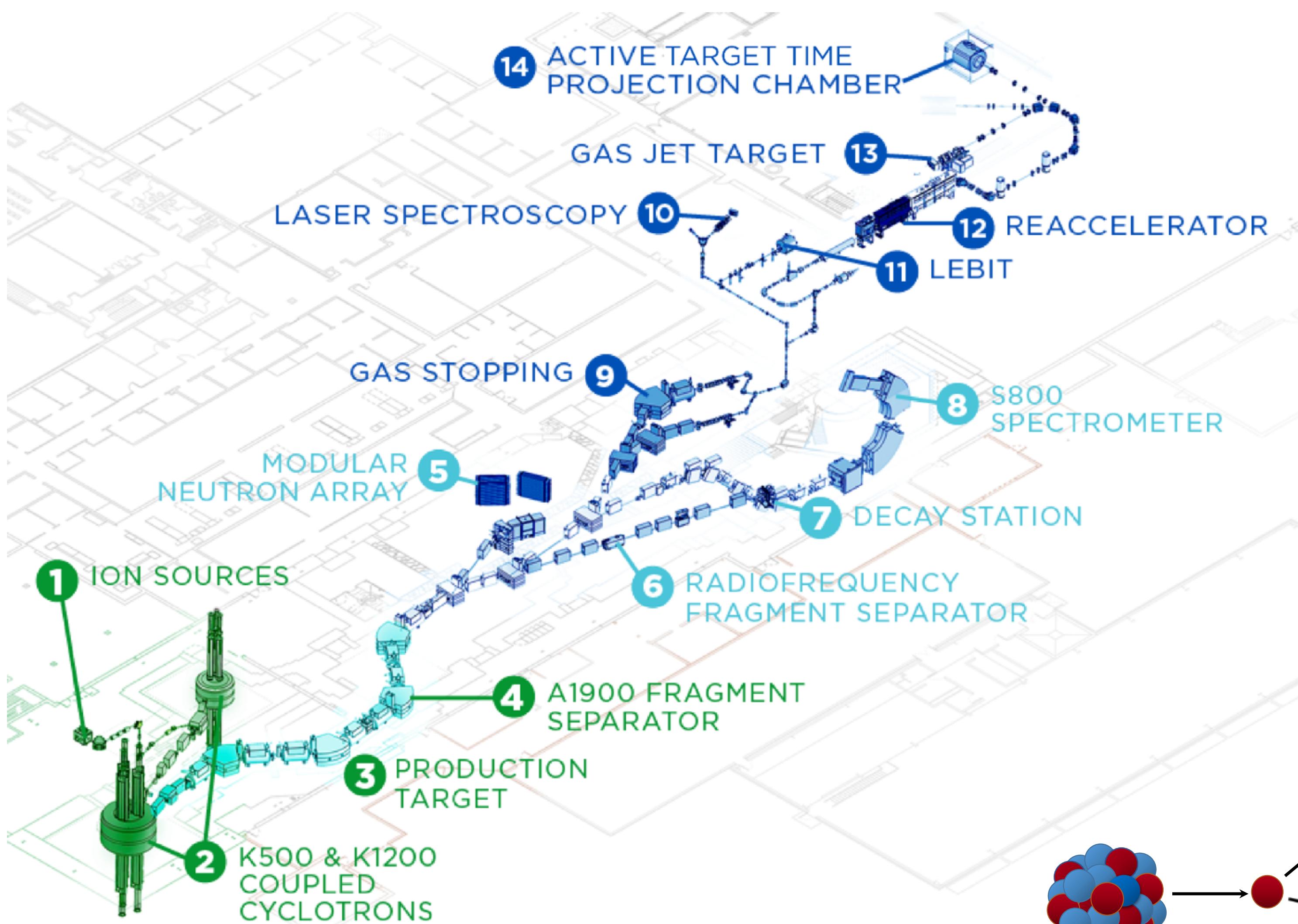
CLAS 12



CMS

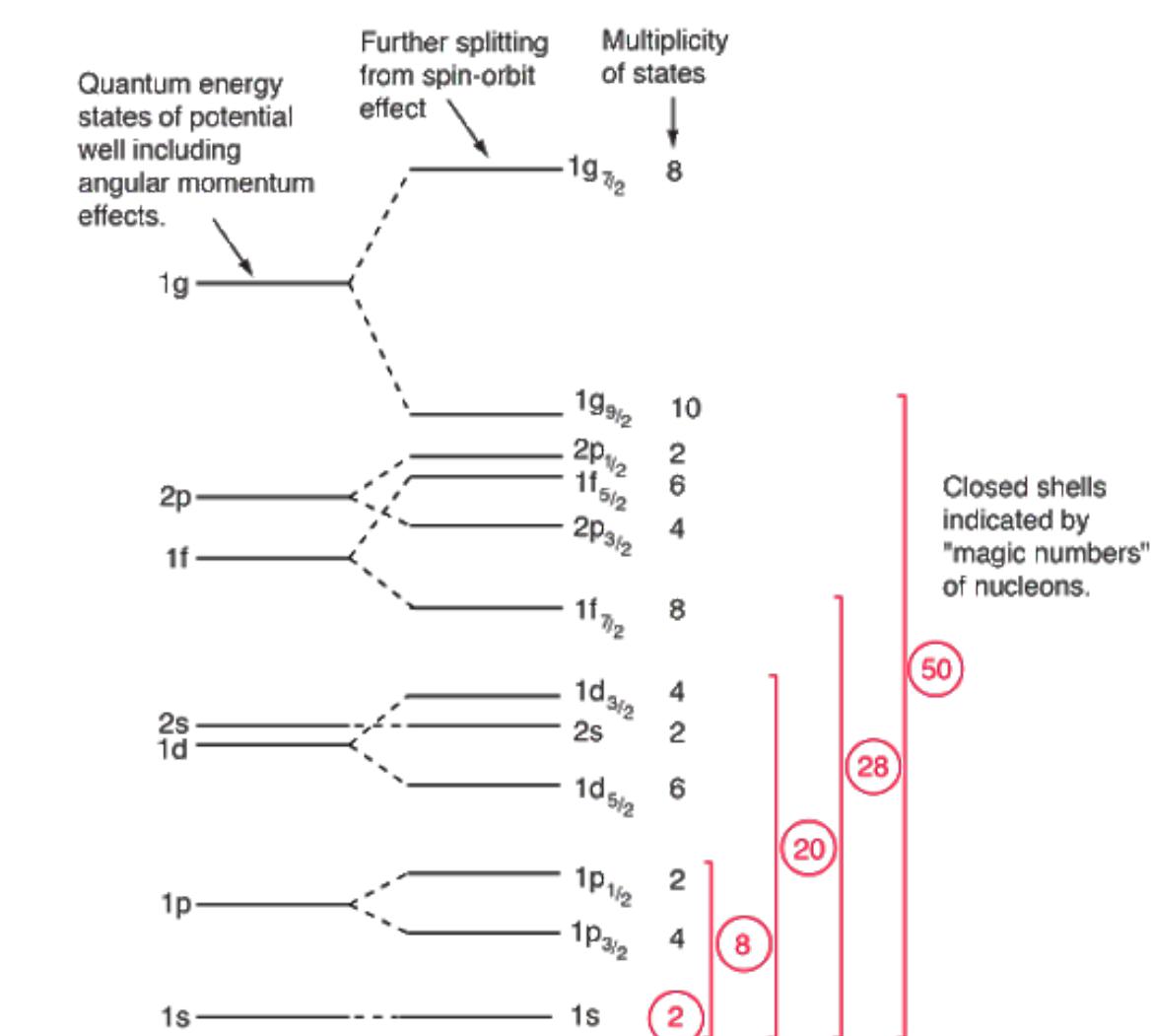






# ALPhA

DAVIDSON COLLEGE

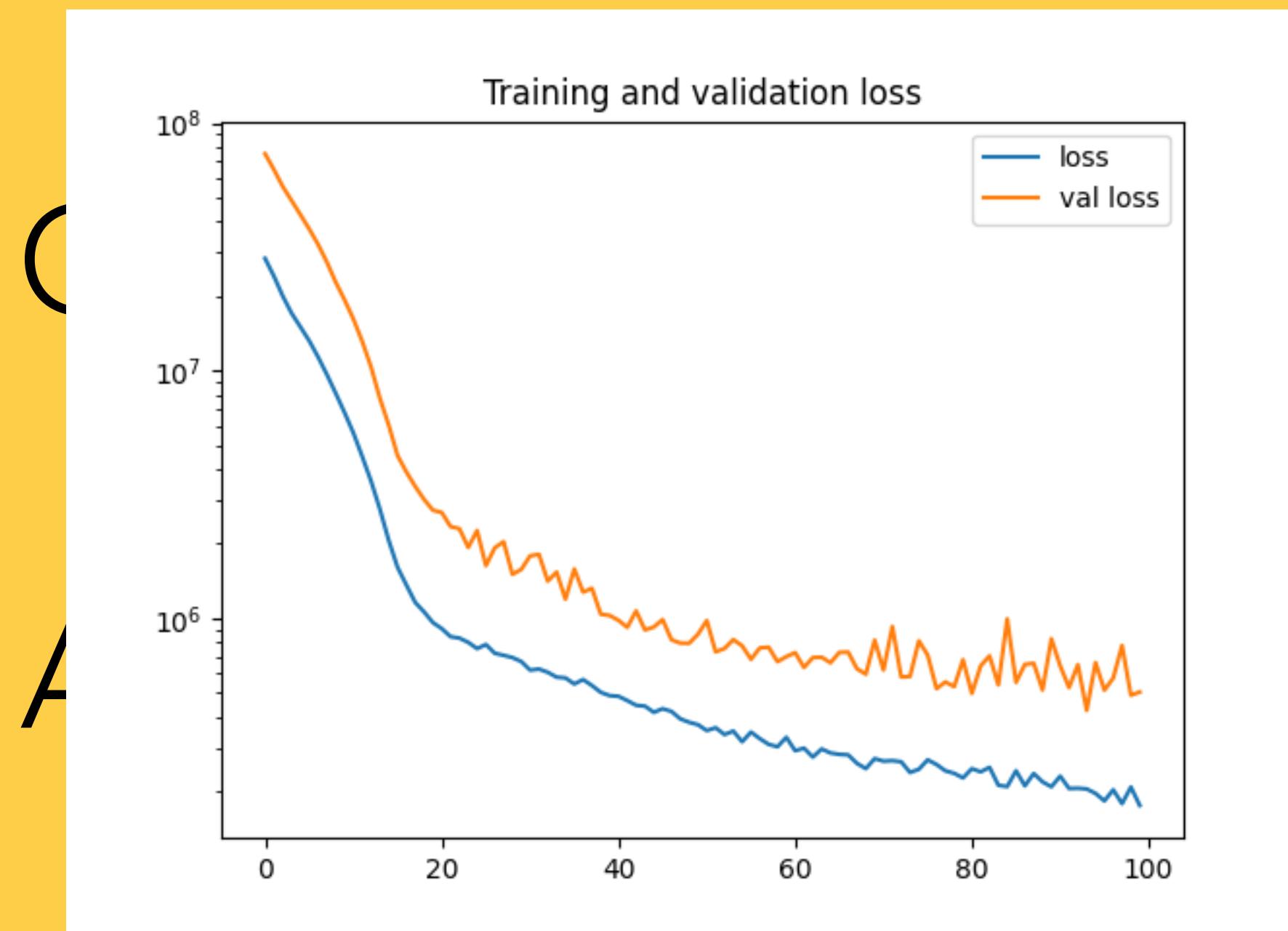
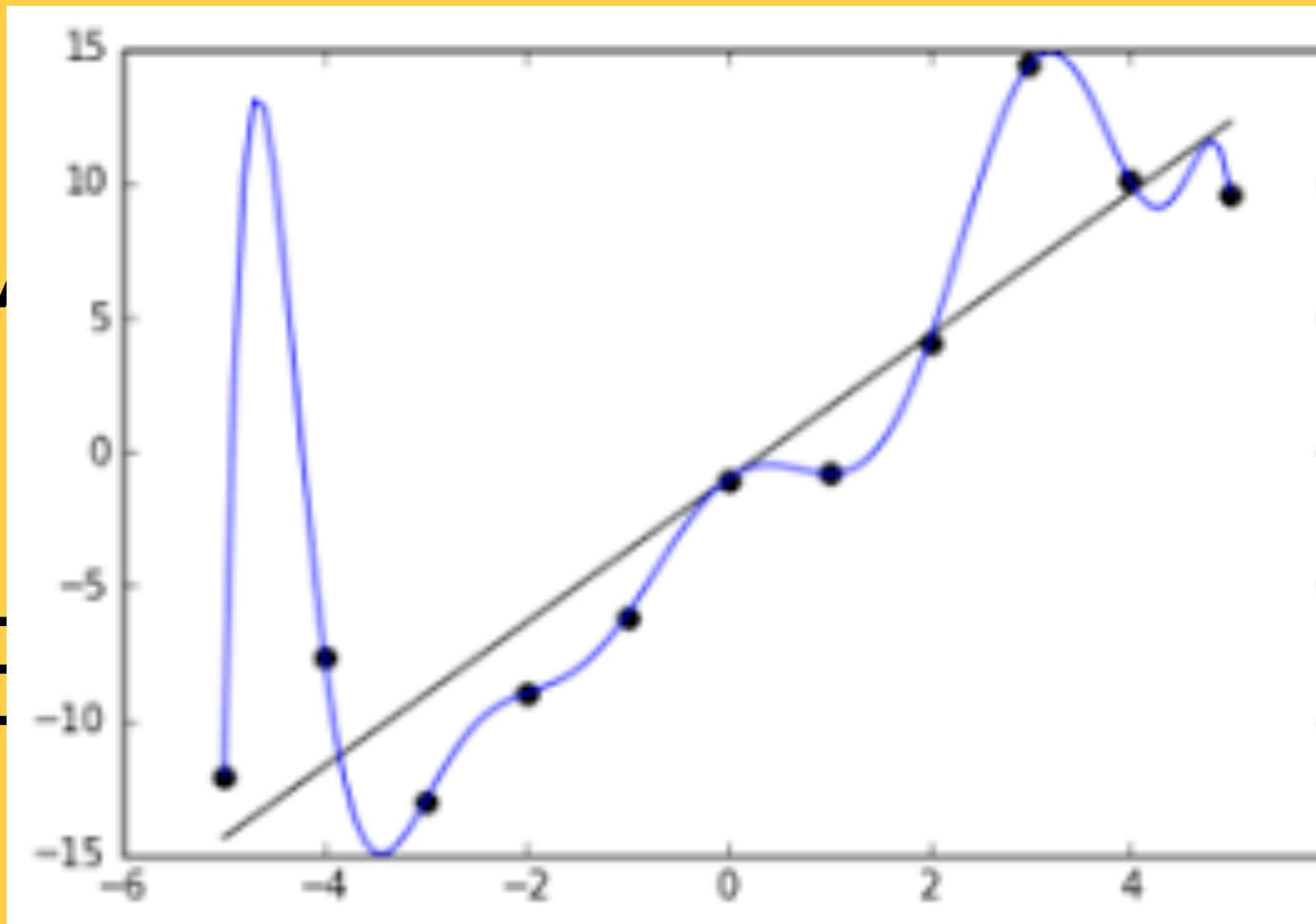


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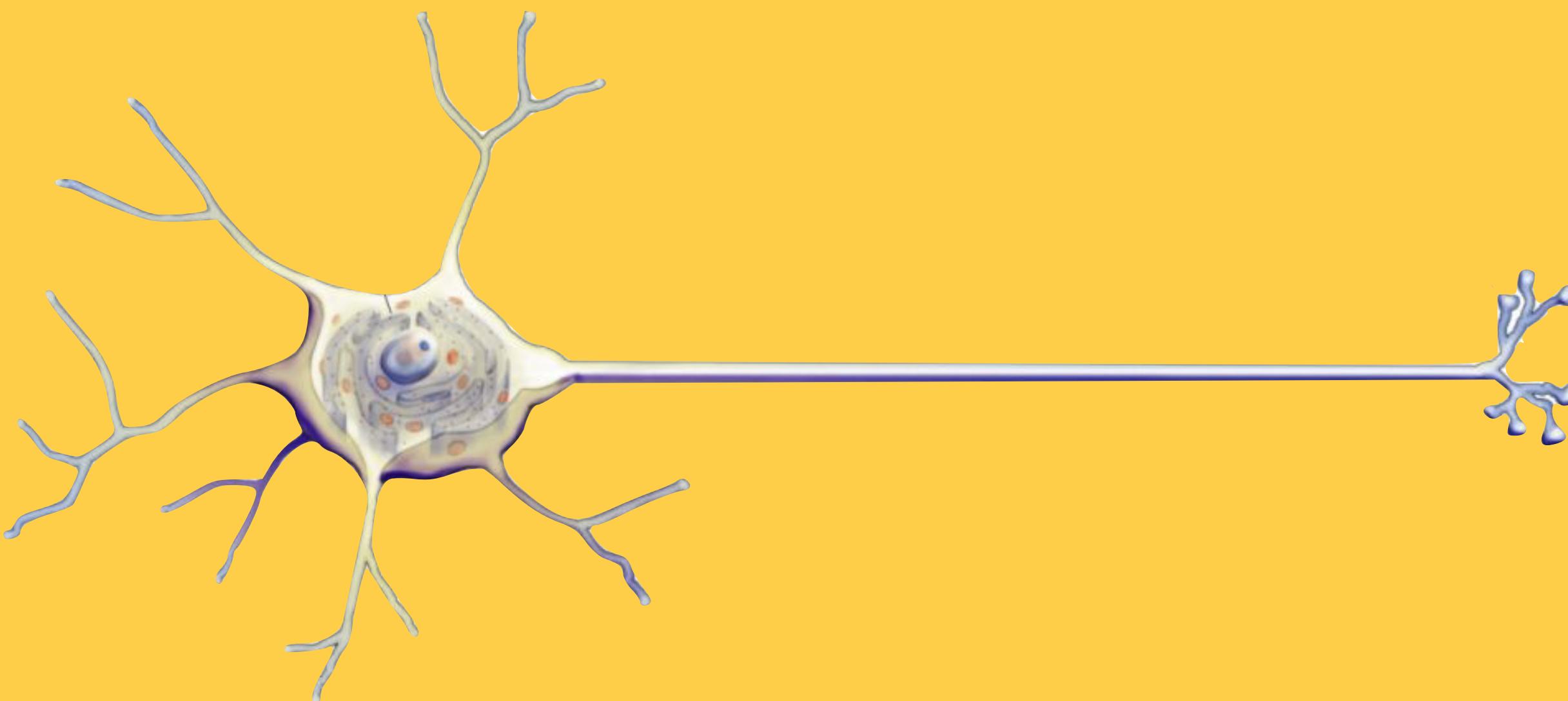
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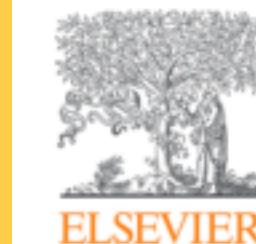
DA



# NEURON

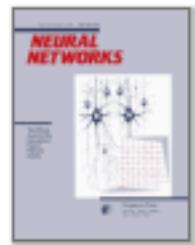


# MATHEMATICS



Neural Networks

Volume 4, Issue 2, 1991, Pages 251-257



## Approximation capabilities of multilayer feedforward networks

Kurt Hornik 

Show more 

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[https://doi.org/10.1016/0893-6080\(91\)90009-T](https://doi.org/10.1016/0893-6080(91)90009-T)

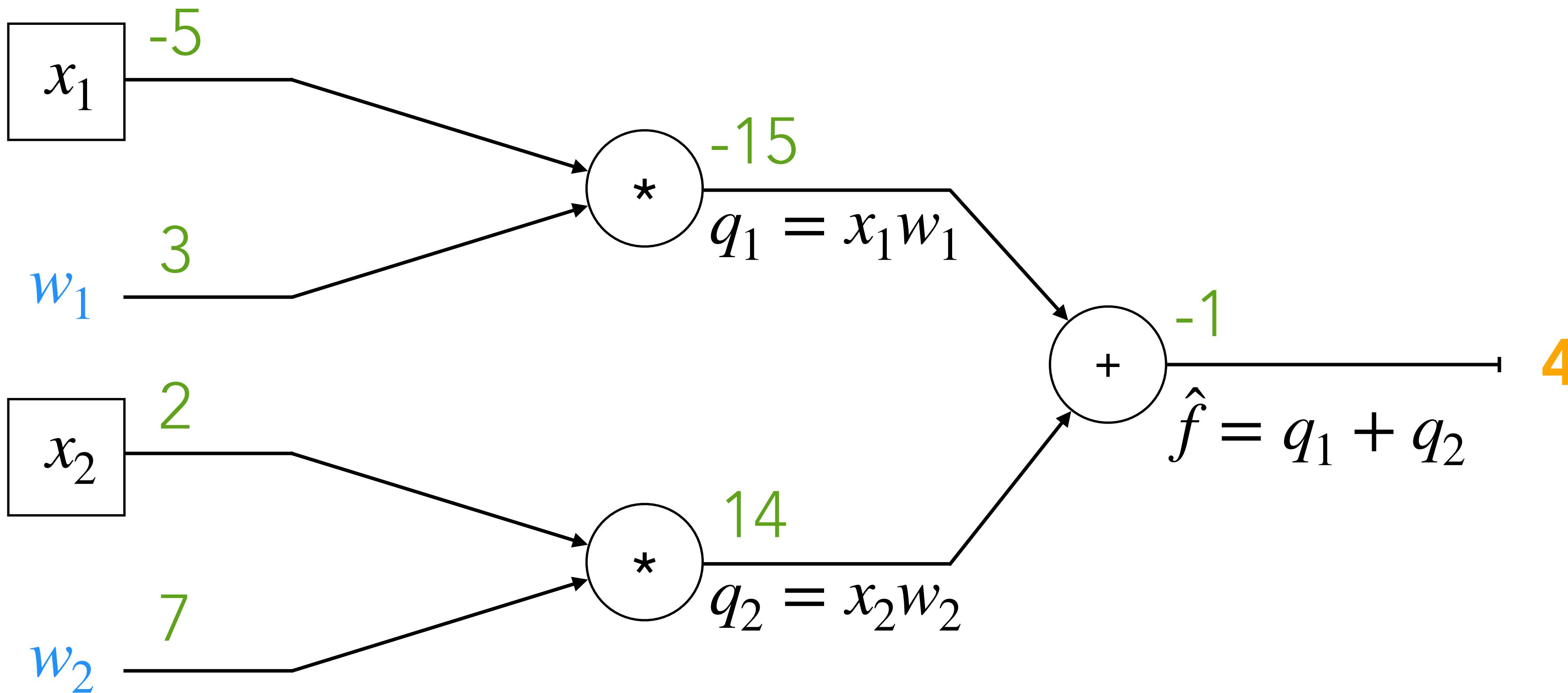
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### Abstract

We show that standard multilayer feedforward networks with as few as a single hidden layer and arbitrary bounded and nonconstant activation function are universal approximators with respect to  $L^p(\mu)$  performance criteria, for arbitrary finite input environment measures  $\mu$ , provided only that sufficiently many hidden units are available. If the activation function is continuous, bounded and nonconstant, then continuous mappings can be learned uniformly over compact input sets. We also give very general conditions ensuring that networks with sufficiently smooth activation functions are capable of arbitrarily accurate approximation to a function and its derivatives.

# MATHEMATICS

# COMPUTATIONAL GRAPH

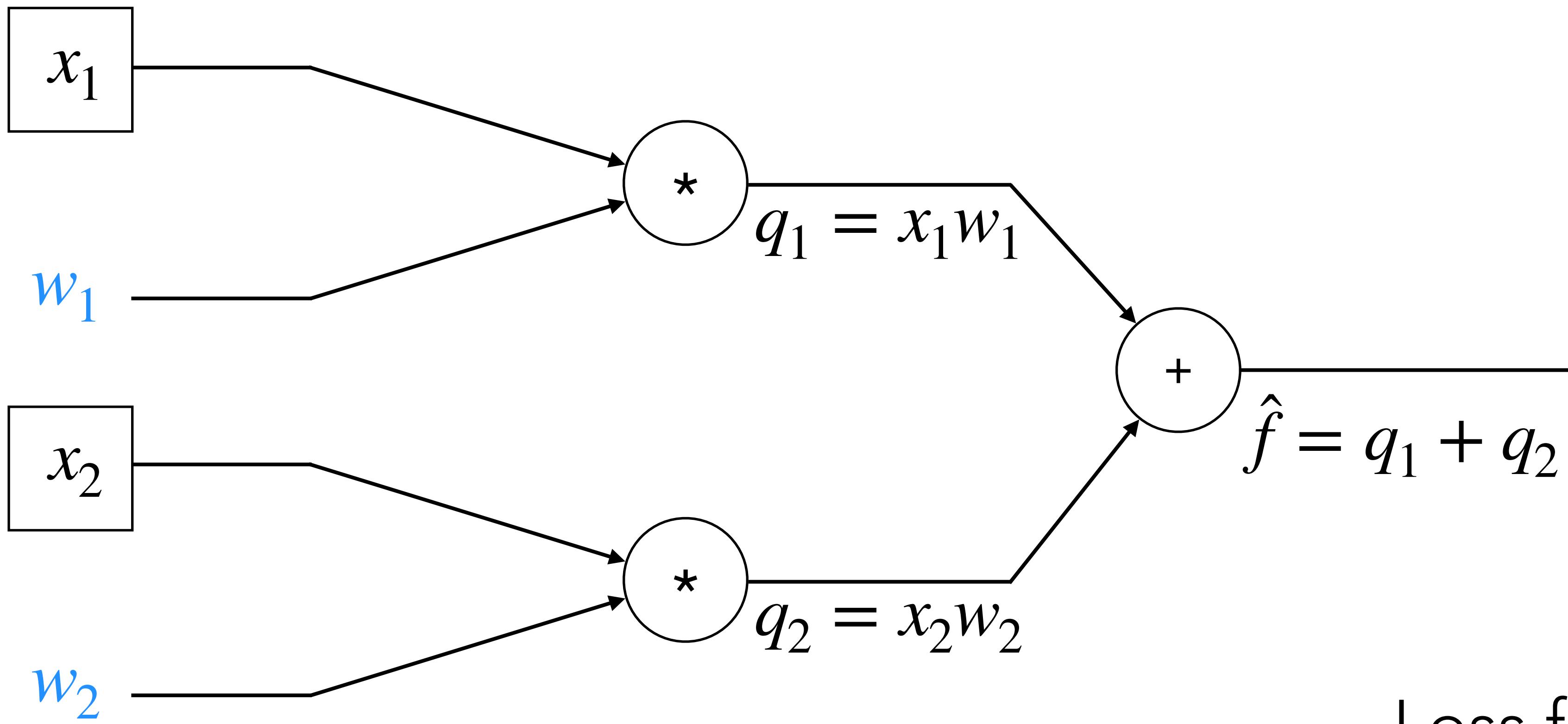


$$\hat{f} = x_1 w_1 + x_2 w_2$$

# MACHINE LEARNING

## SUPERVISED LEARNING

# REGRESSION

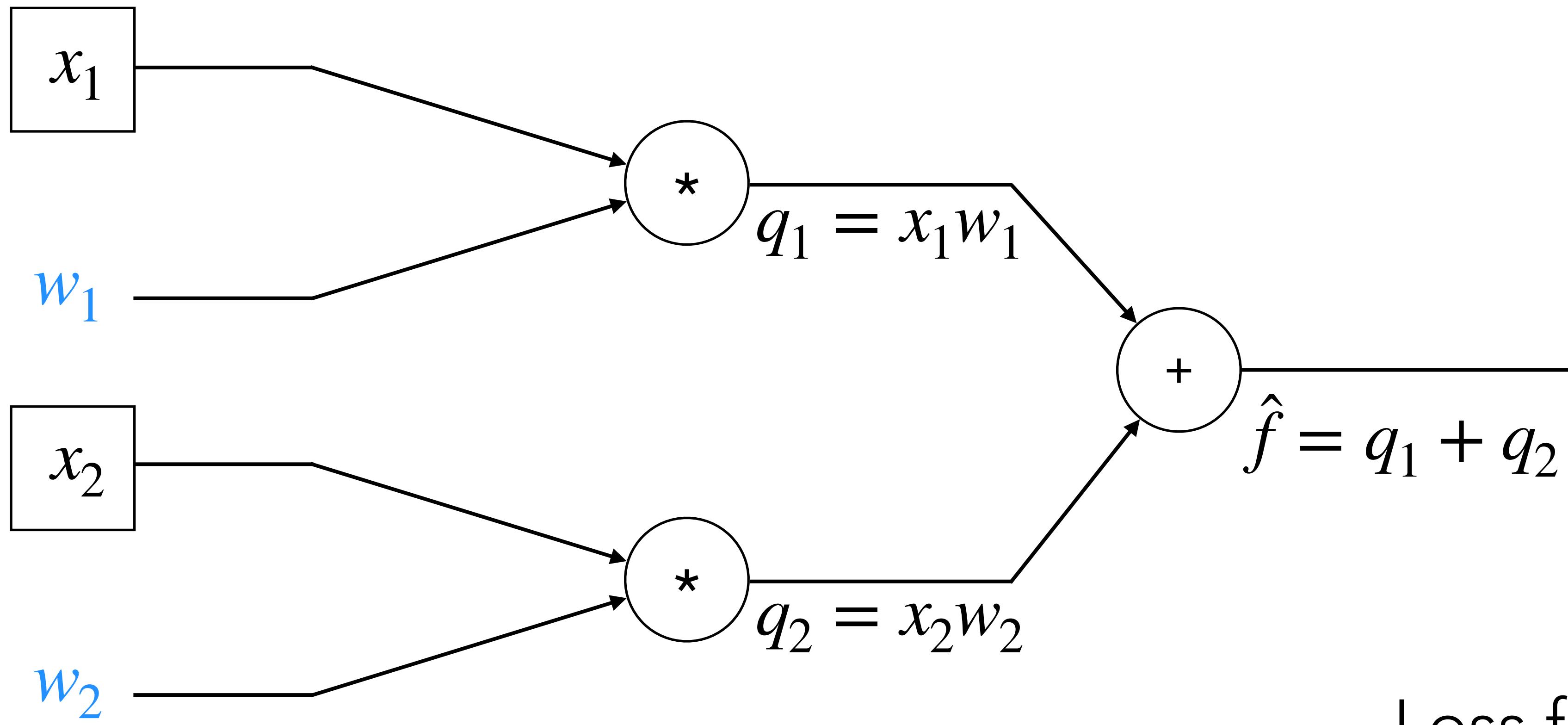


Loss function

$$\hat{f} = x_1 w_1 + x_2 w_2$$

$$J(w) = \hat{f} - f$$

# SUPERVISED LEARNING

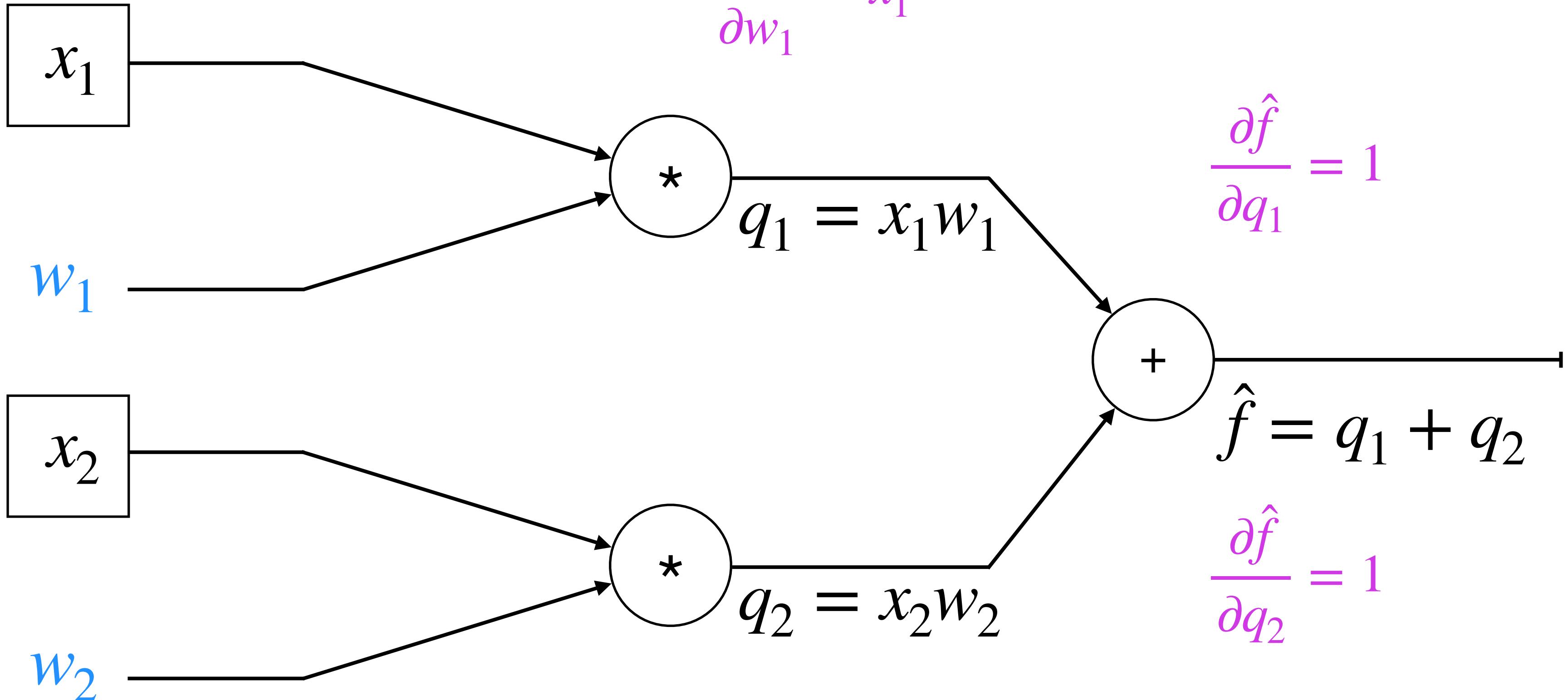


$$\hat{f} = x_1 w_1 + x_2 w_2$$

$$J(w) = \hat{f} - f$$

# BACKPROPAGATION

$$w_1 = w_1 - \eta * \frac{\partial J}{\partial \hat{f}} \frac{\partial \hat{f}}{\partial q_1} \frac{\partial q_1}{\partial w_1}$$



$$w_2 = w_2 - \eta * \frac{\partial J}{\partial \hat{f}} \frac{\partial \hat{f}}{\partial q_2} \frac{\partial q_2}{\partial w_2}$$

$$\frac{\partial q_1}{\partial w_1} = x_1$$

$$\frac{\partial \hat{f}}{\partial q_1} = 1$$

$$\frac{\partial J}{\partial \hat{f}} = 1$$

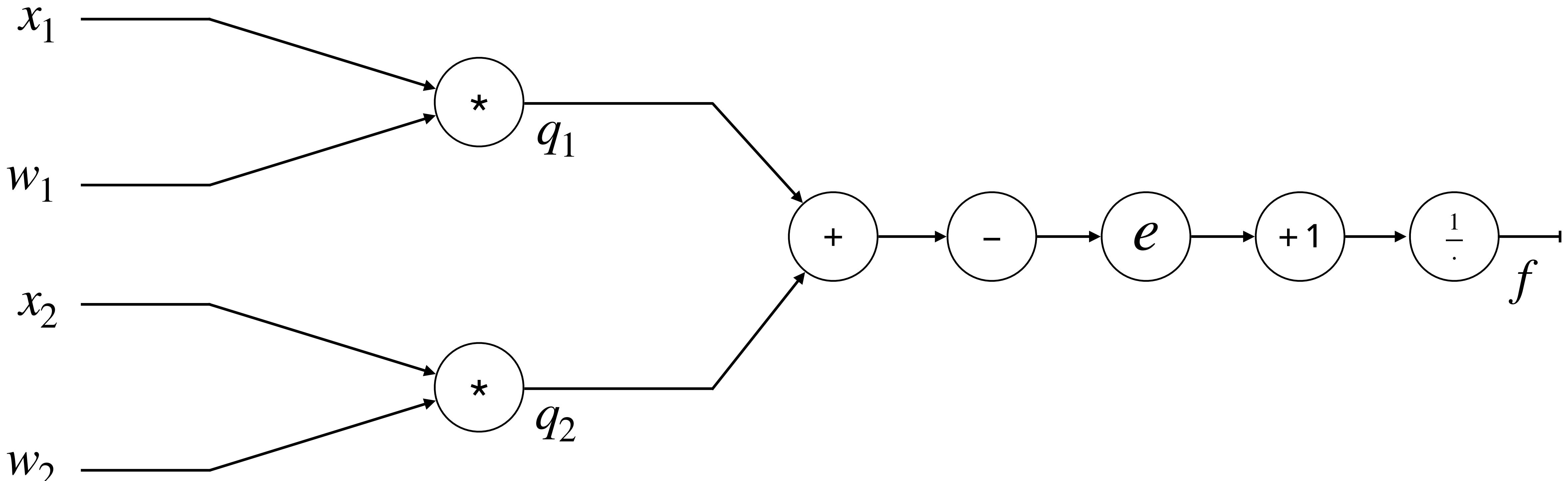
$$\frac{\partial \hat{f}}{\partial q_2} = 1$$

Loss function

$$J(w) = \hat{f} - f$$

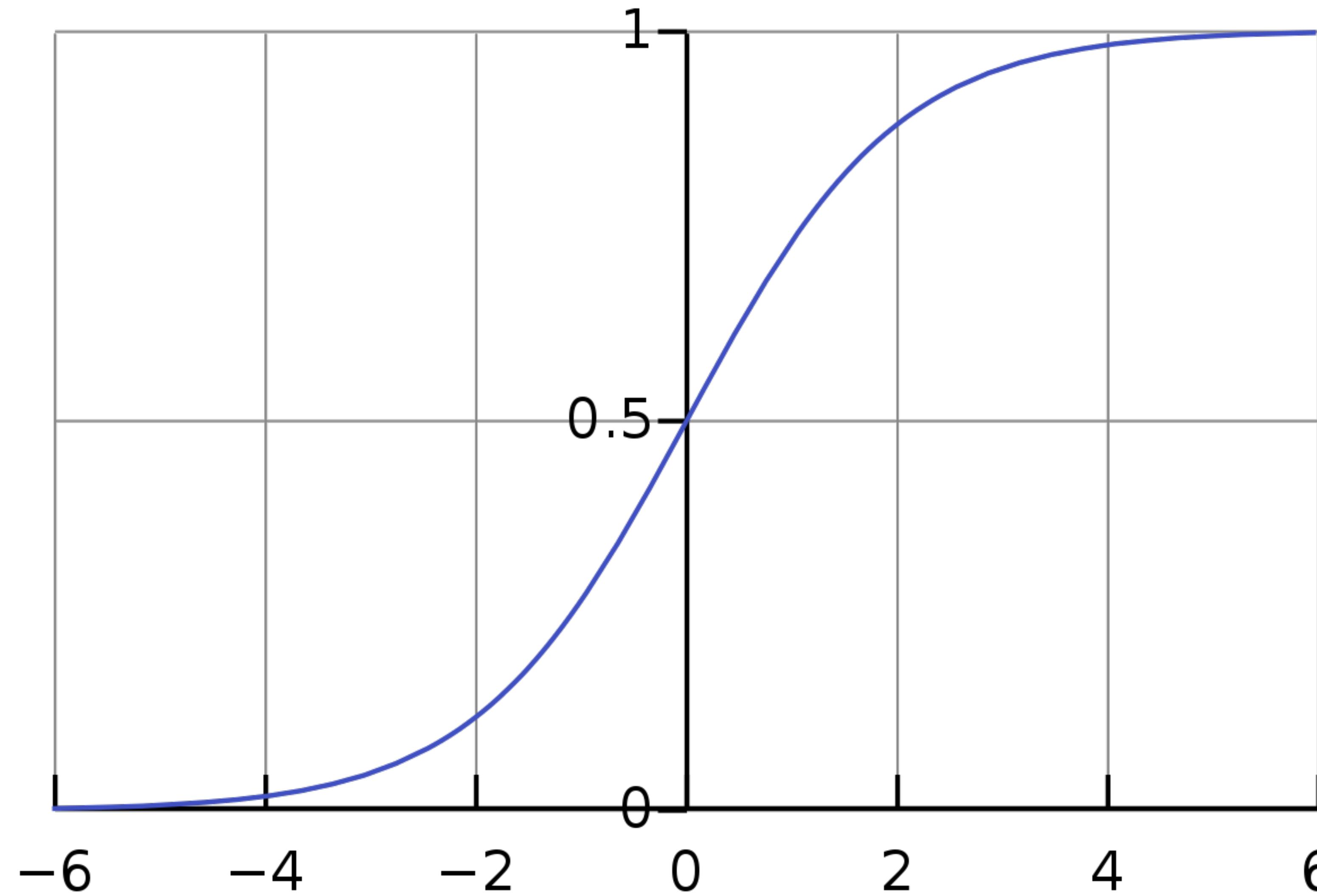
$$\frac{\partial q_2}{\partial w_2} = x_2$$

# LOGISTIC REGRESSION

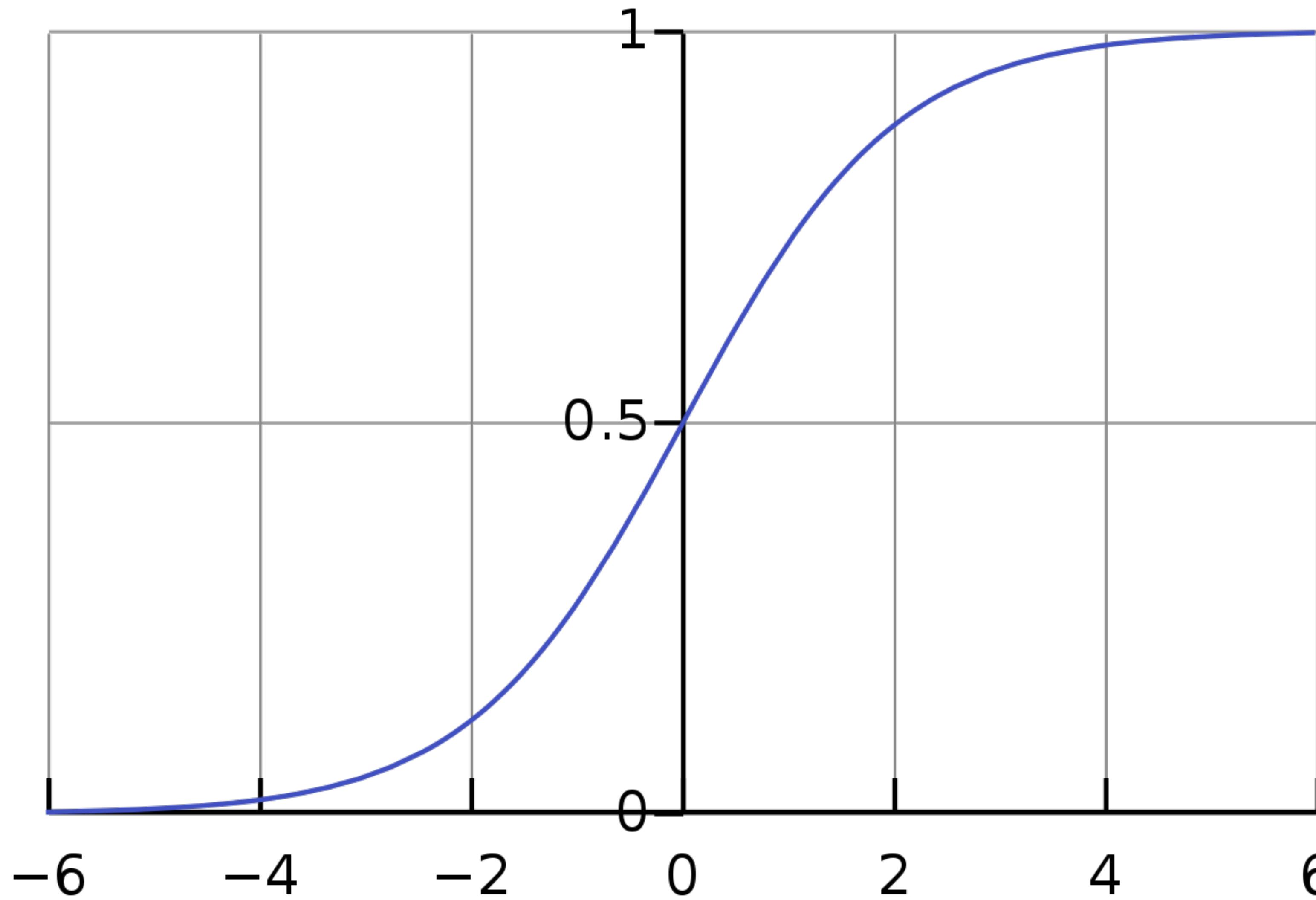


$$f = \frac{1}{1 + e^{-(x_1 w_1 + x_2 w_2)}}$$

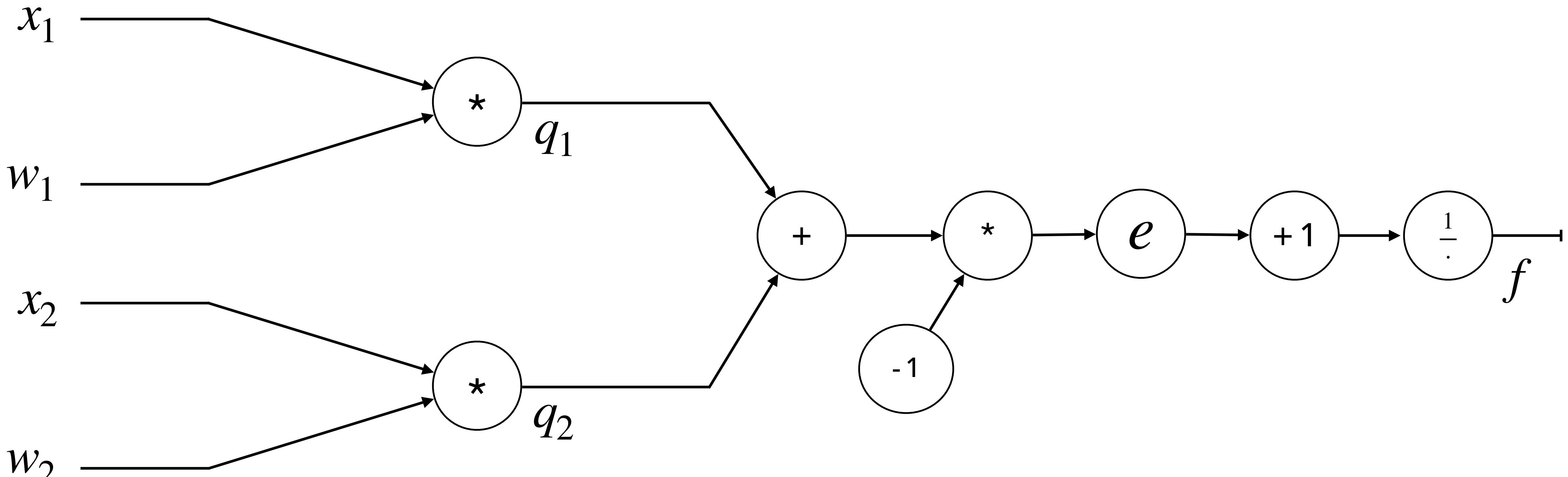
# LOGISTIC REGRESSION



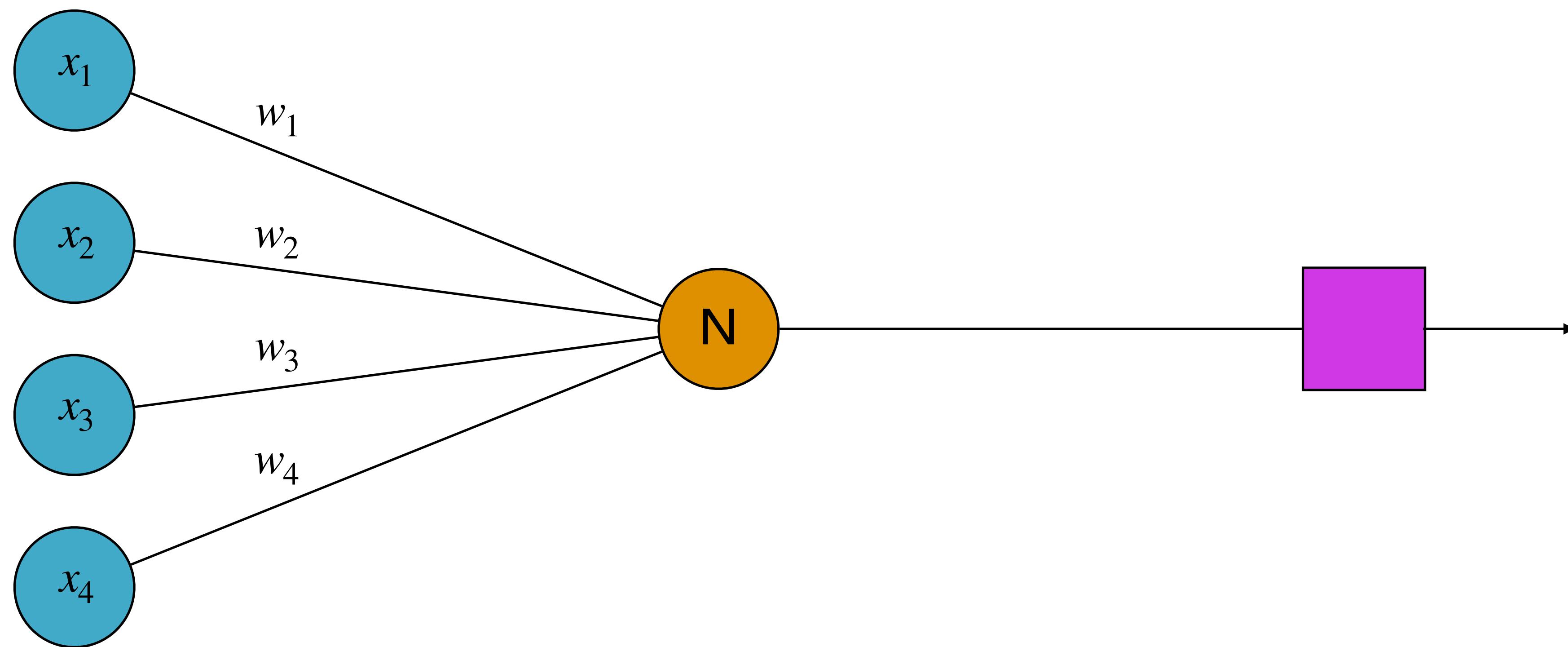
# BINARY CLASSIFICATION



# LOGISTIC REGRESSION



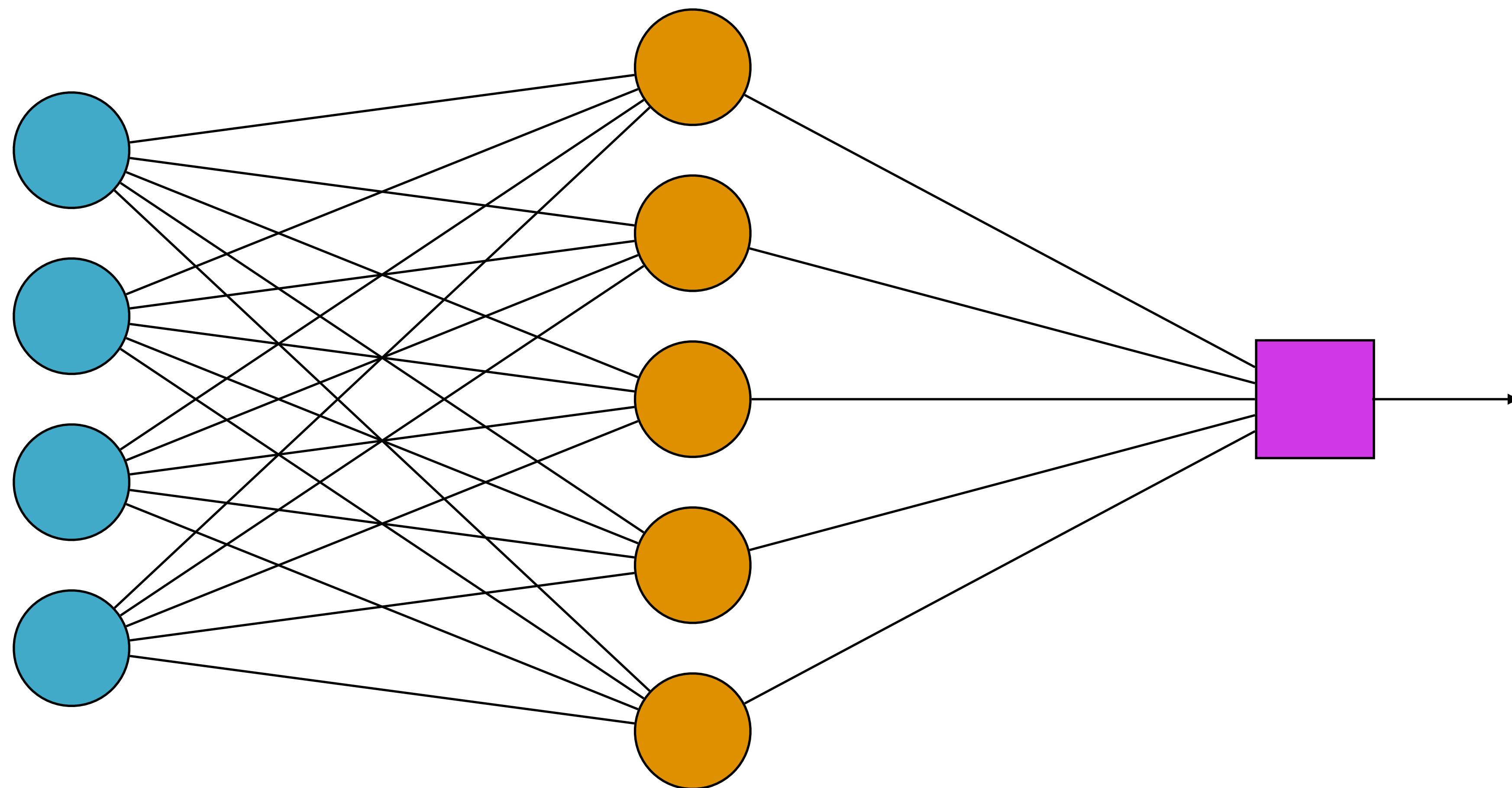
$$f = \frac{1}{1 + e^{-(x_1 w_1 + x_2 w_2)}}$$



Features

Summation  
+ Nonlinearity

Output



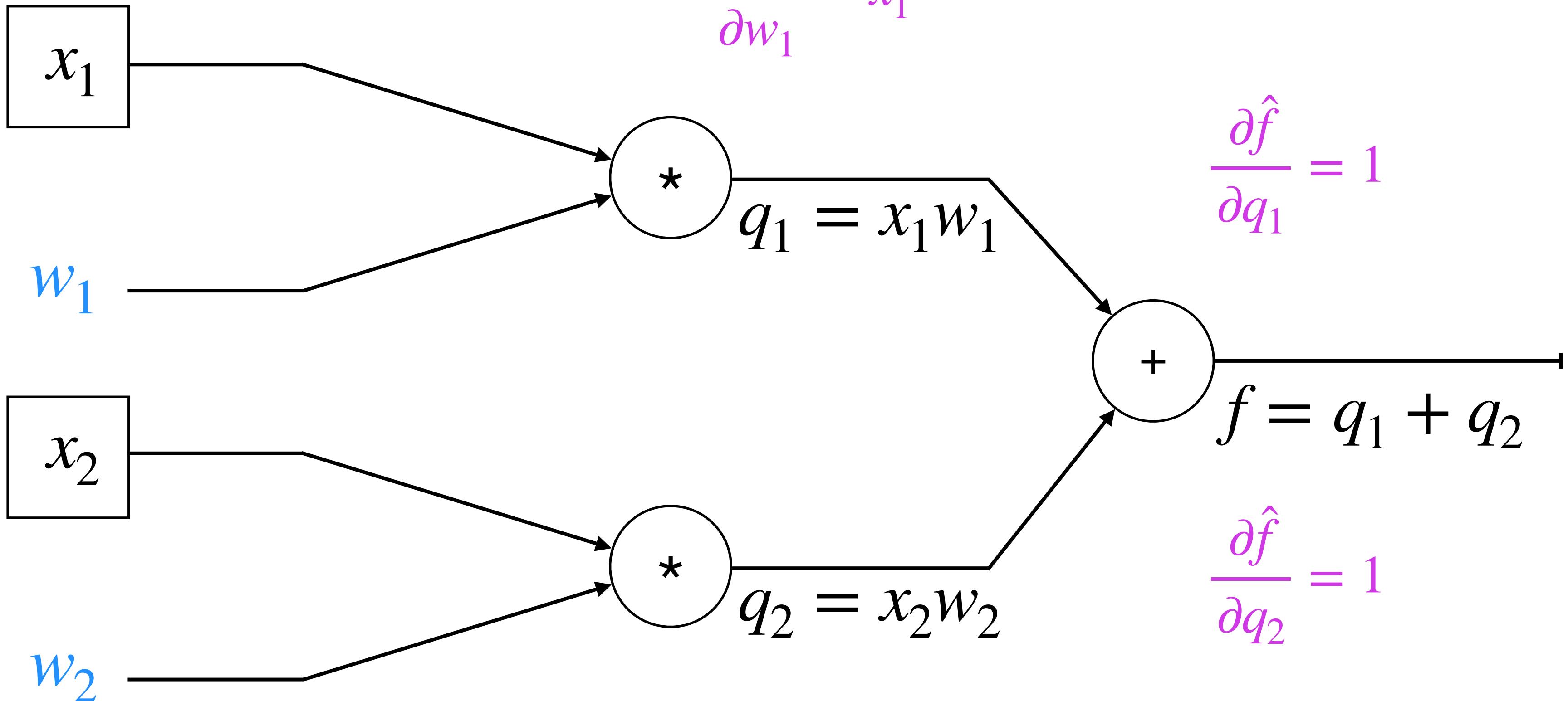
Features

Hidden Layer

Output

# BACKPROPAGATION

$$w_1 = w_1 + \eta * \frac{\partial J}{\partial \hat{f}} \frac{\partial \hat{f}}{\partial q_1} \frac{\partial q_1}{\partial w_1}$$



$$w_2 = w_2 + \eta * \frac{\partial J}{\partial \hat{f}} \frac{\partial \hat{f}}{\partial q_2} \frac{\partial q_2}{\partial w_2}$$

$$\frac{\partial q_1}{\partial w_1} = x_1$$

$$\frac{\partial \hat{f}}{\partial q_1} = 1$$

$$\frac{\partial J}{\partial \hat{f}} = 1$$

$$\frac{\partial \hat{f}}{\partial q_2} = 1$$

Loss function

$$J(w) = \hat{f} - f$$

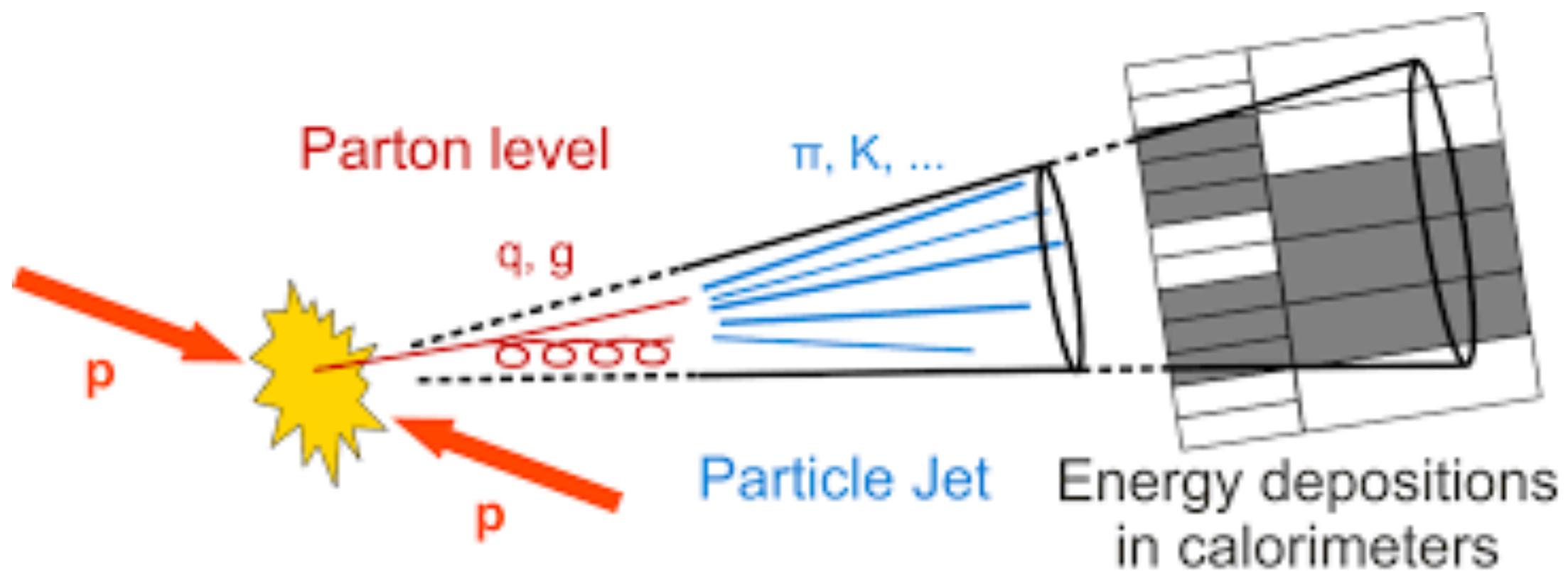
$$\frac{\partial q_2}{\partial w_2} = x_2$$

# AUTOMATIC DIFFERENTIATION



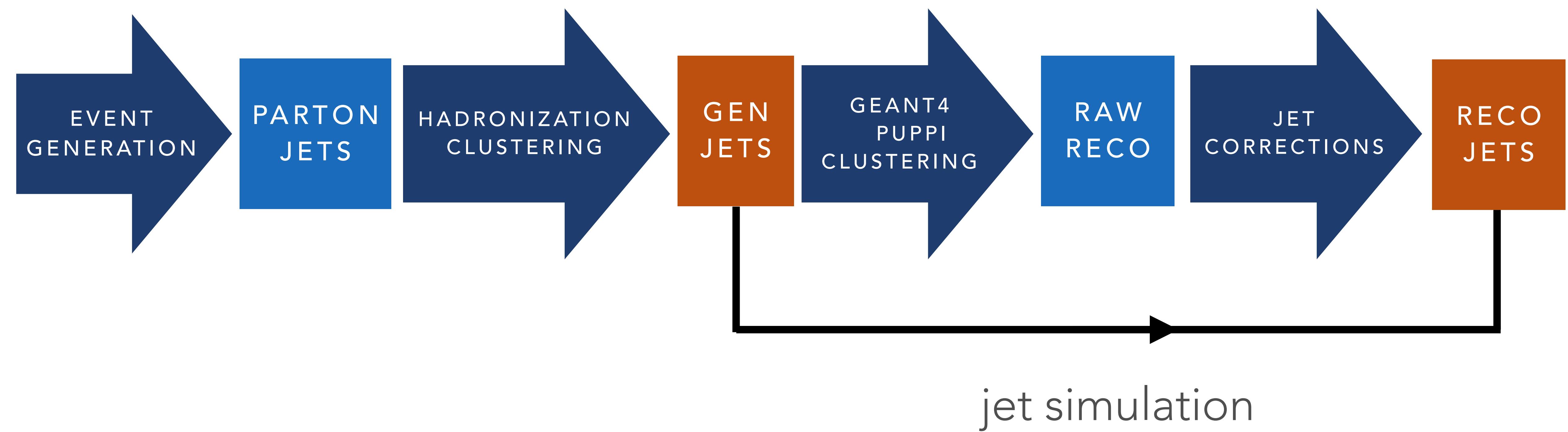
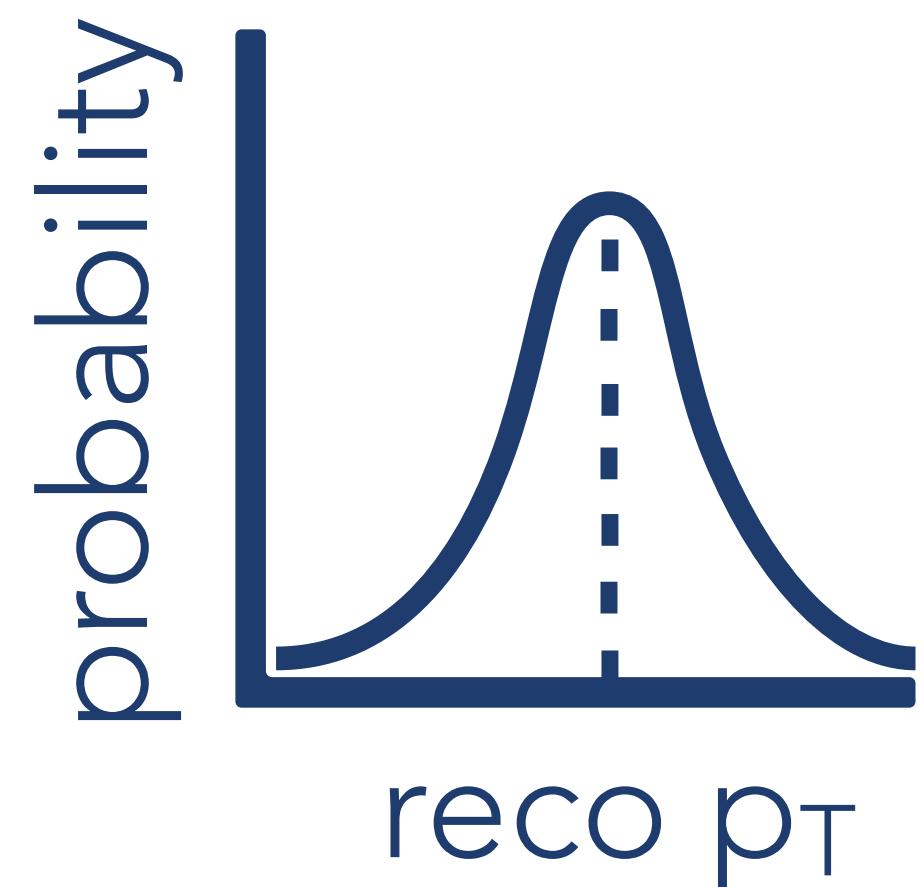
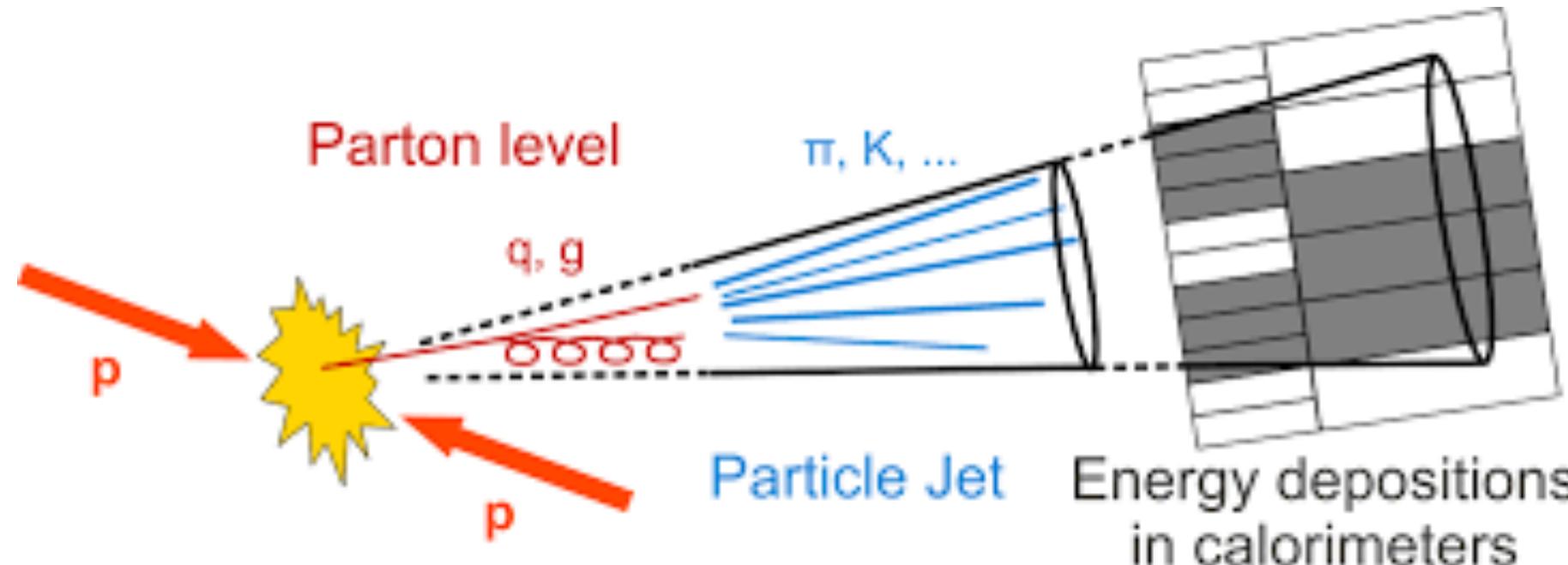
**Application 1:** Can we model stochastic processes?

# JET SIMULATION AND CORRECTION

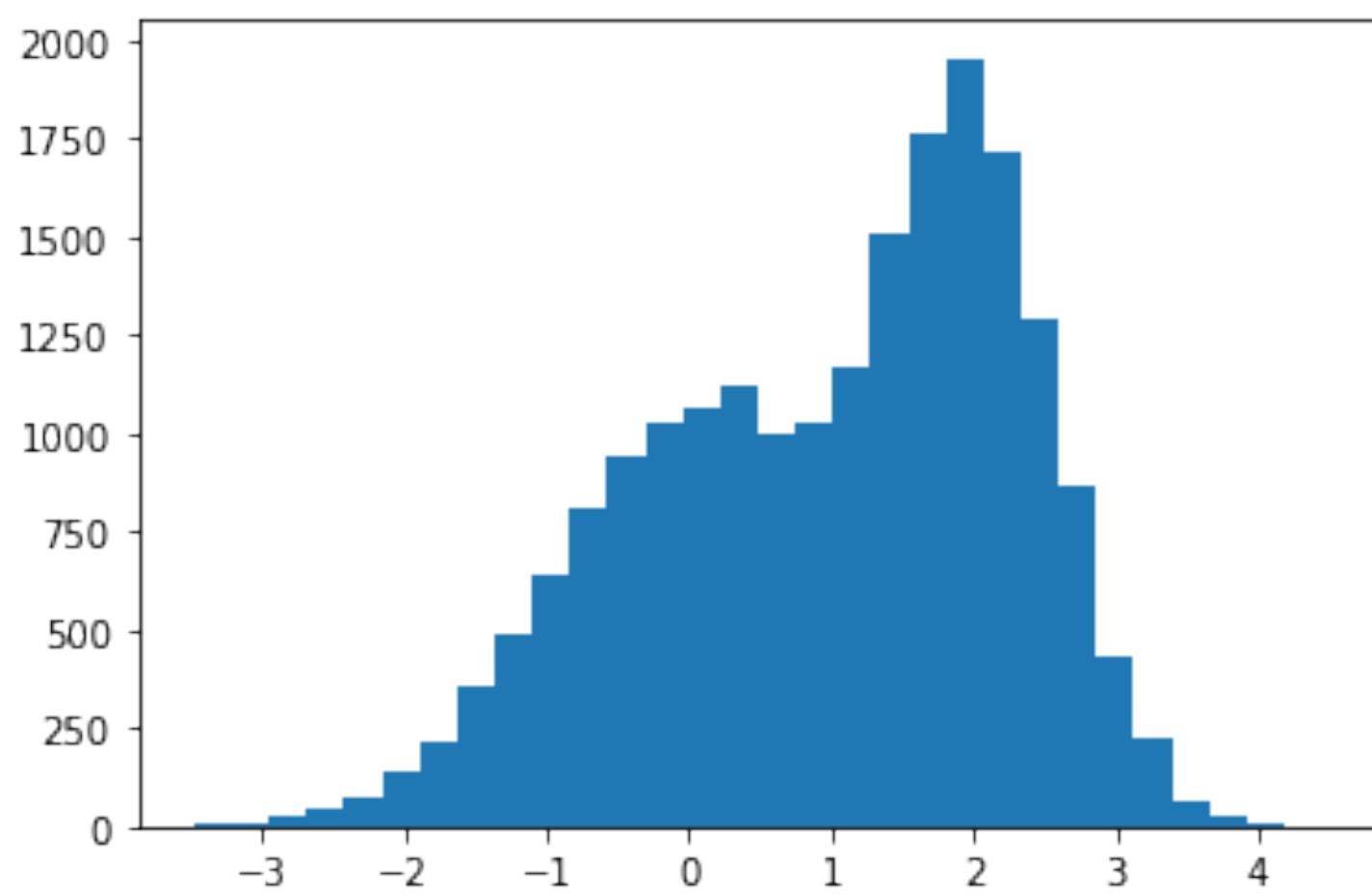


**DATASET:** Generated via Pythia8 + Delphes + anti- $K_T$  + FastJet

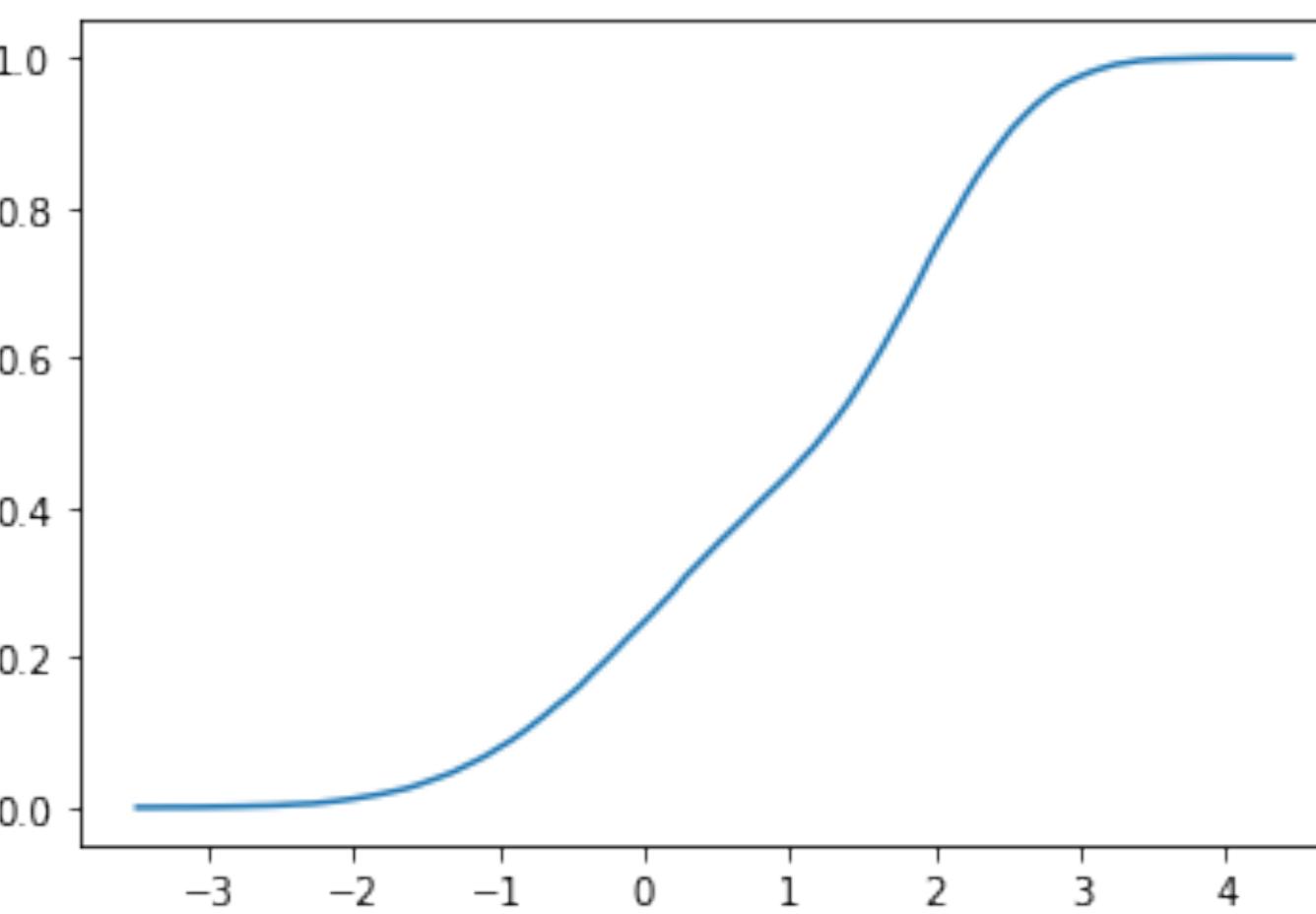
# NEED FOR DISTRIBUTION PREDICTIONS



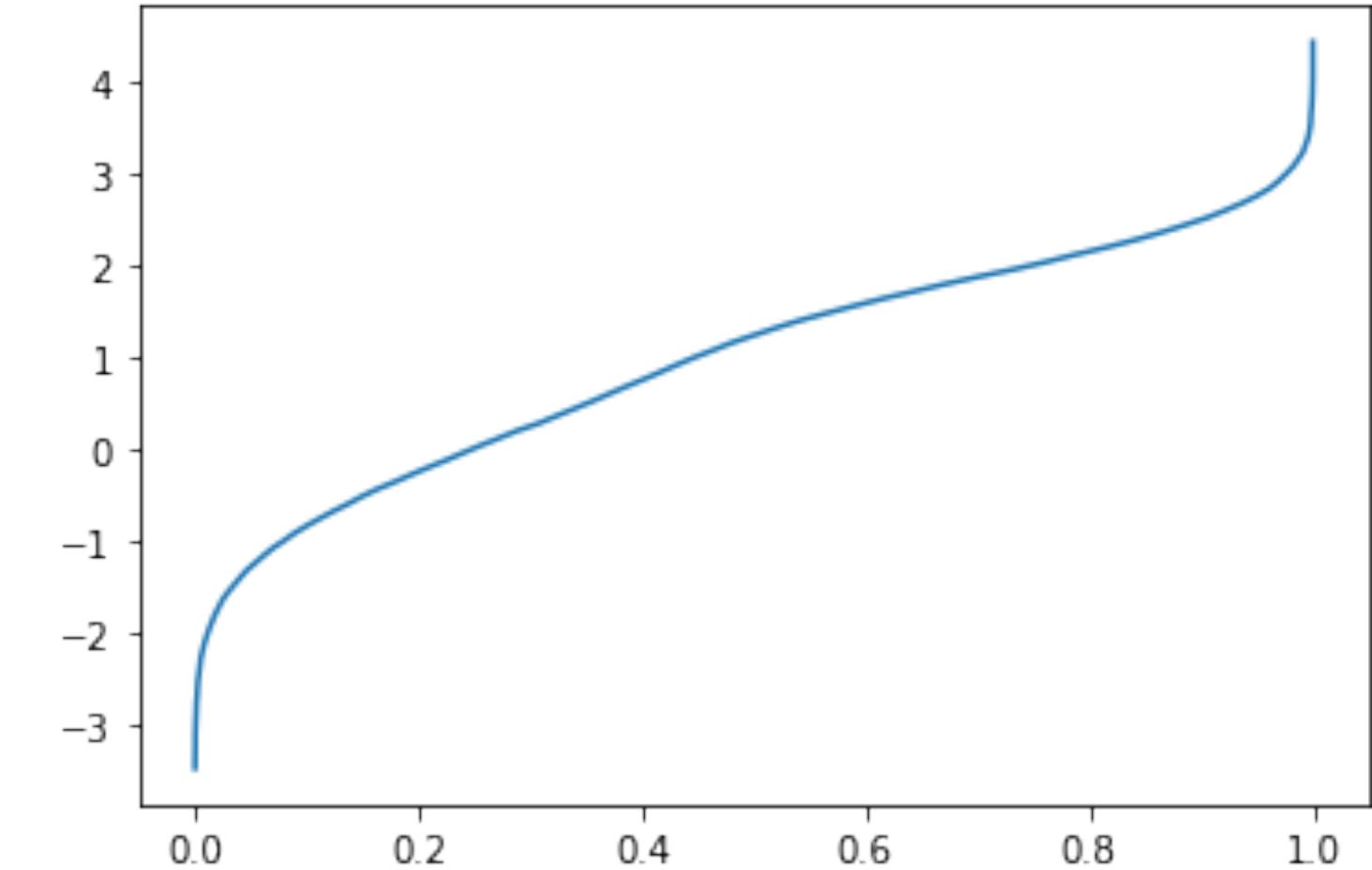
# IMPLICIT QUANTILE NETWORKS STATS REVIEW



data

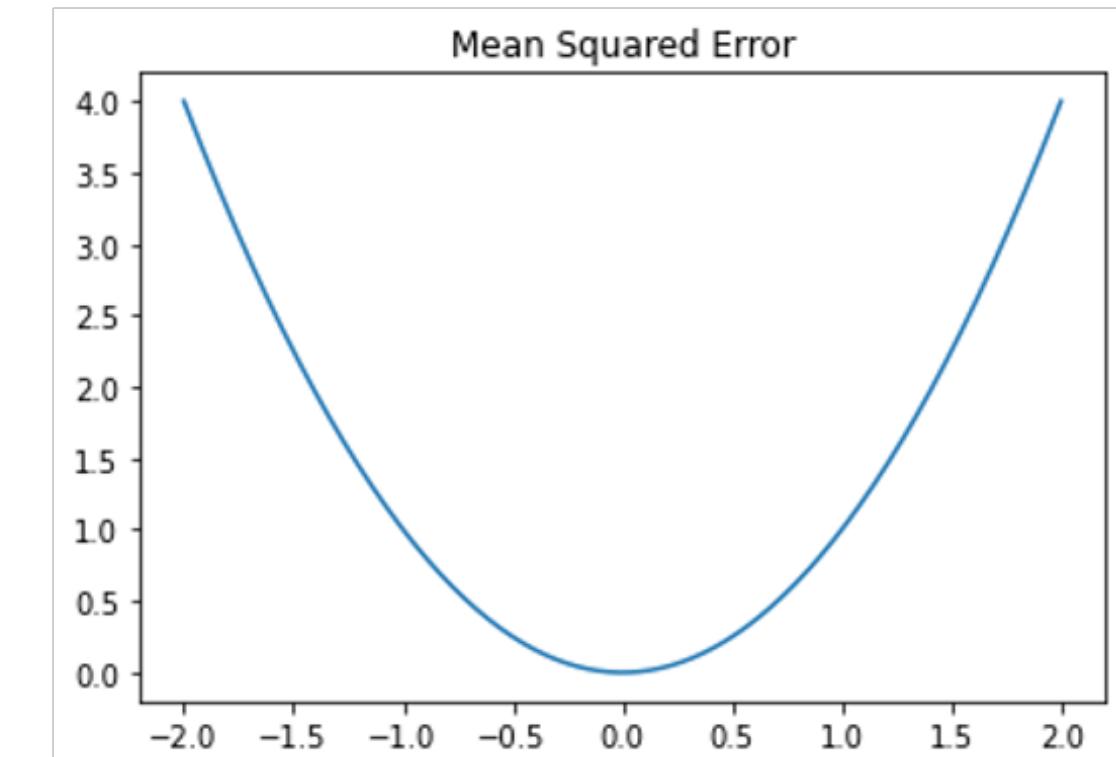
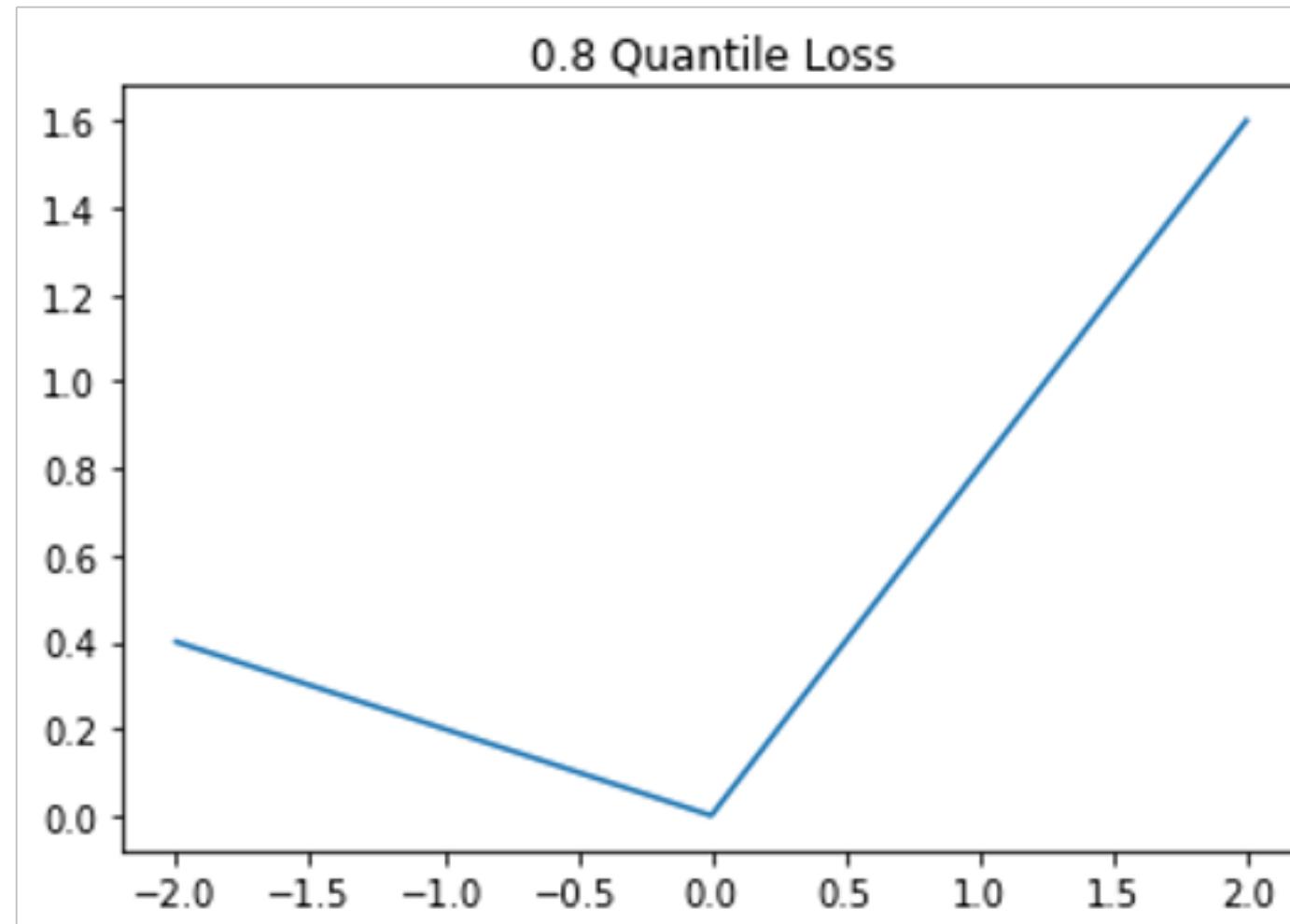
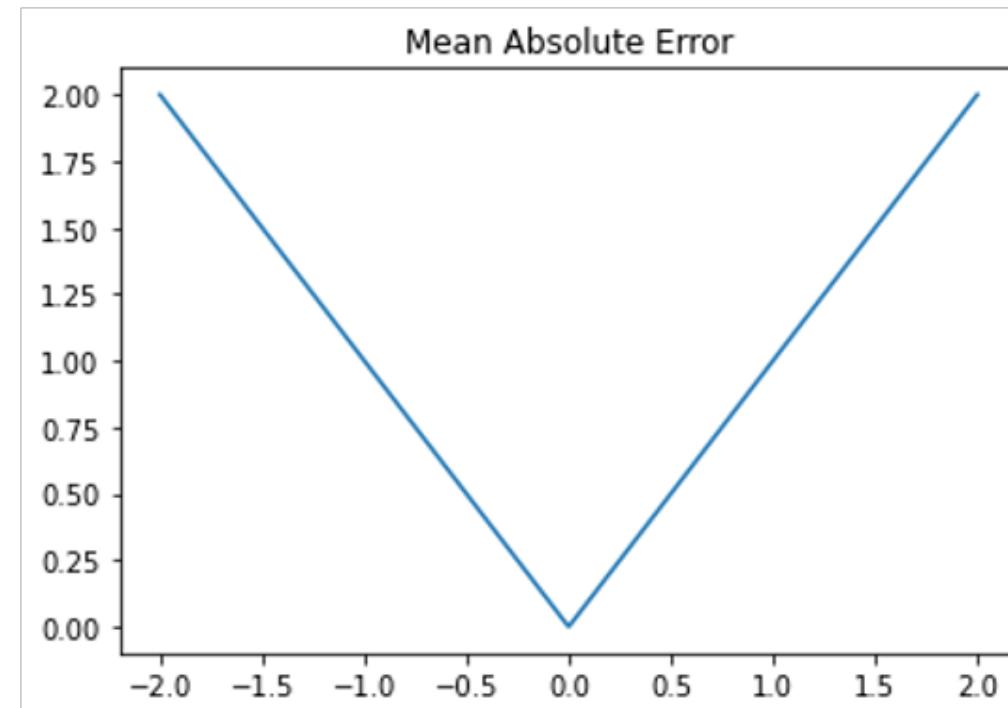


CDF



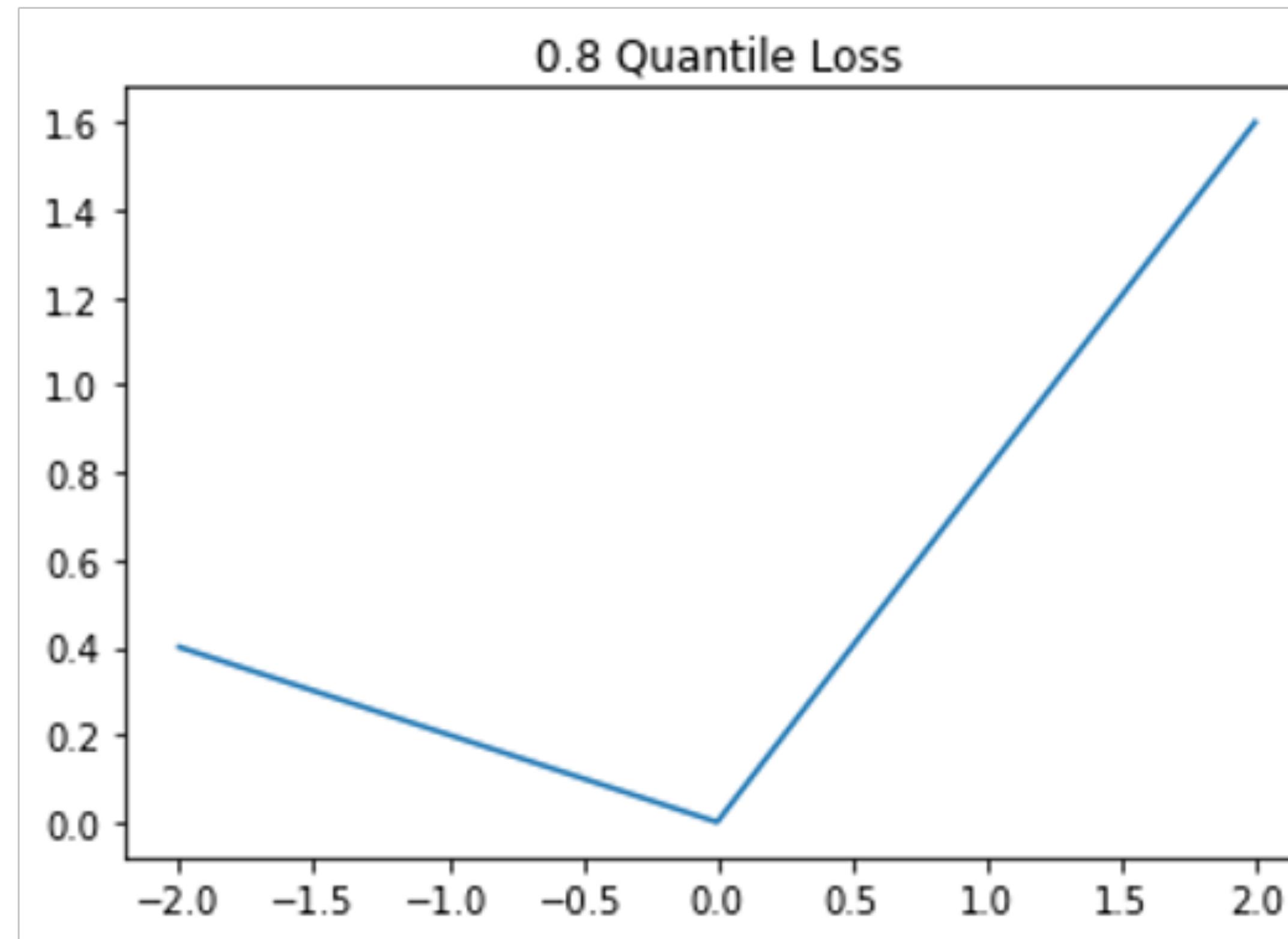
quantile function

# IMPLICIT QUANTILE NETWORKS LOSS FUNCTION



$$\mathcal{L}(f, x, y, \tau) = \begin{cases} \tau(y - f(x, \tau)) & y \geq f(x, \tau) \\ (\tau - 1)(y - f(x, \tau)) & y < f(x, \tau) \end{cases}$$

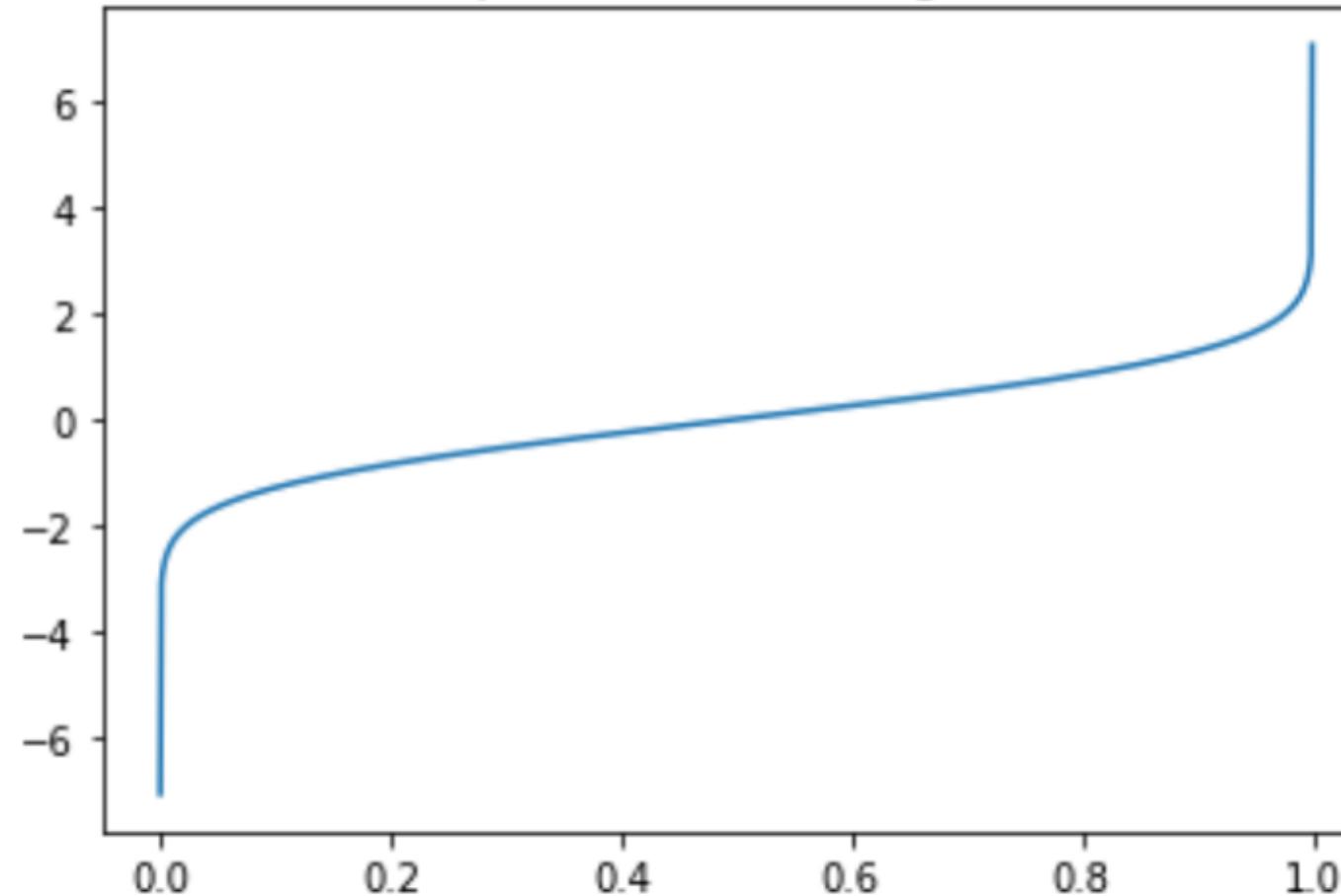
# IMPLICIT QUANTILE NETWORKS LOSS FUNCTION



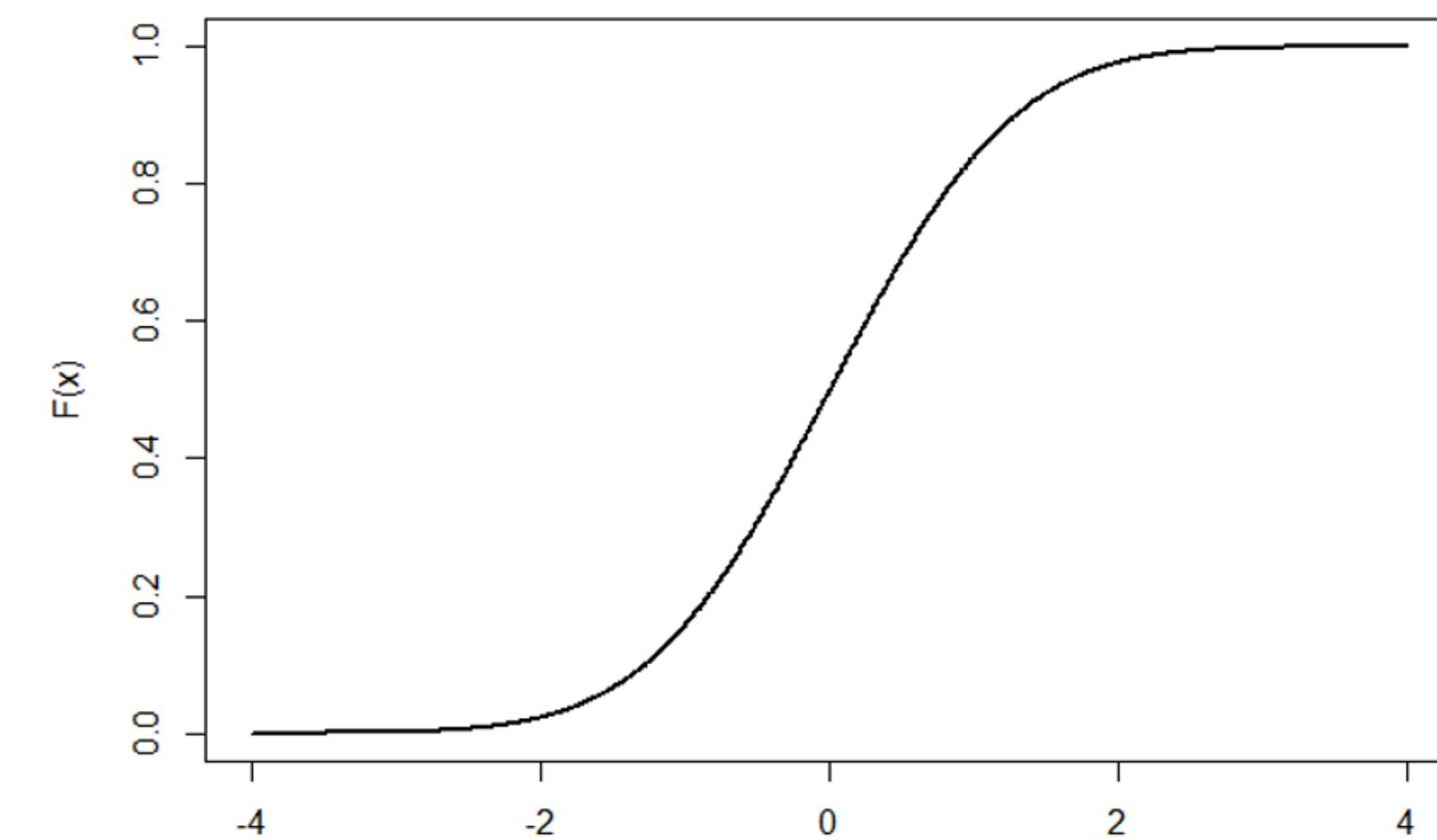
$$\mathcal{L}(f, x, y, \tau) = \begin{cases} \tau(y - f(x, \tau)) & y \geq f(x, \tau) \\ (\tau - 1)(y - f(x, \tau)) & y < f(x, \tau) \end{cases}$$

# IMPLICIT QUANTILE NETWORKS LOSS FUNCTION

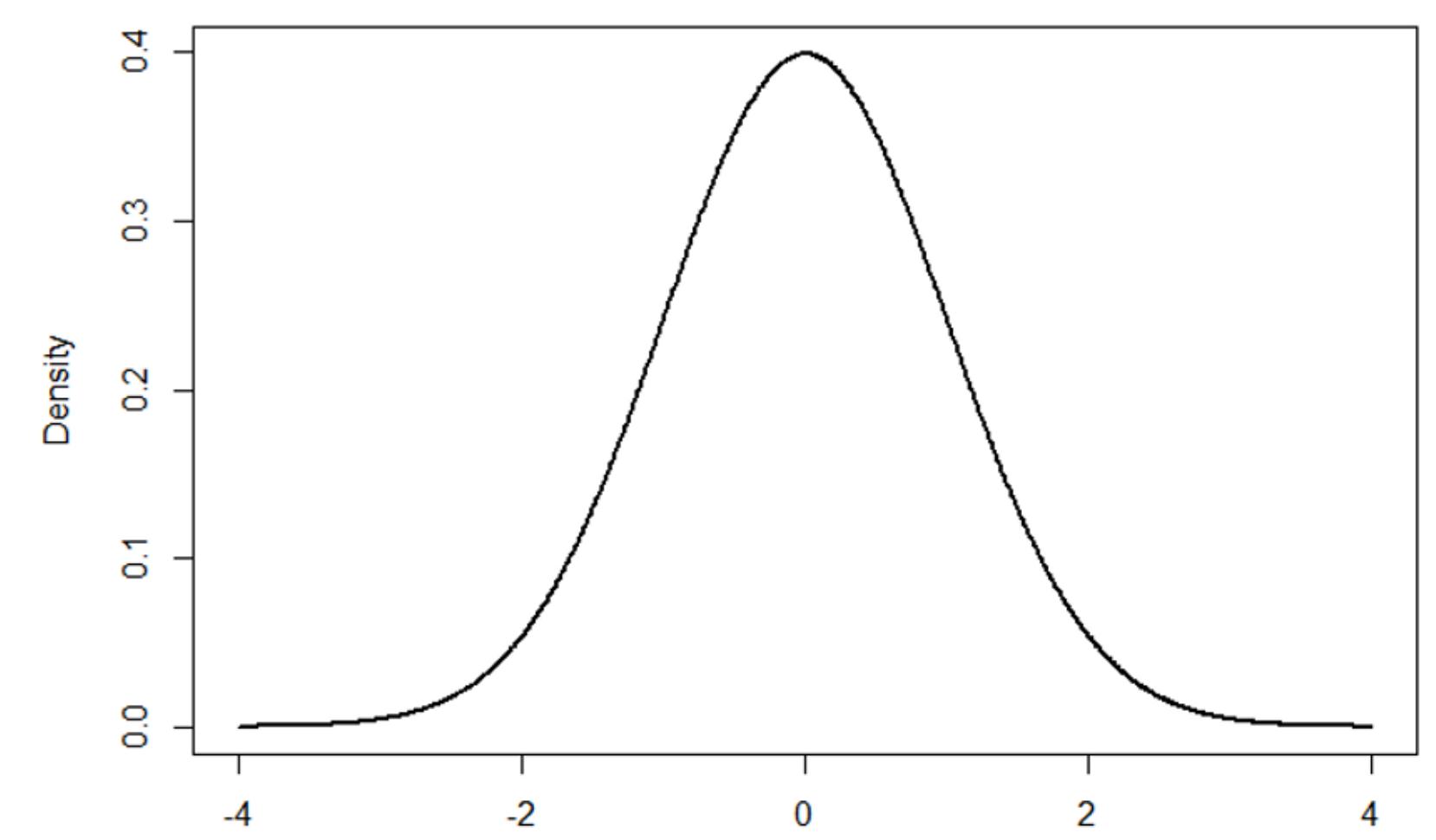
raw quantile function, sigma = 1



Cumulative distribution function (CDF)

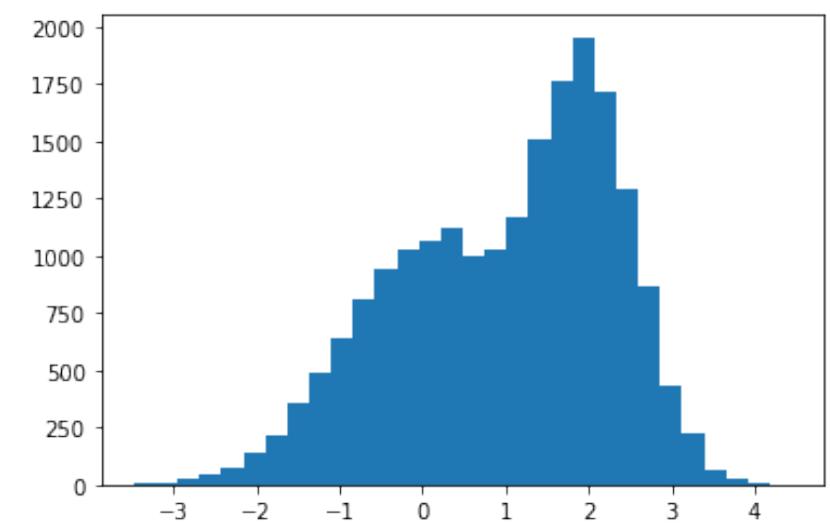


Probability density function (PDF)

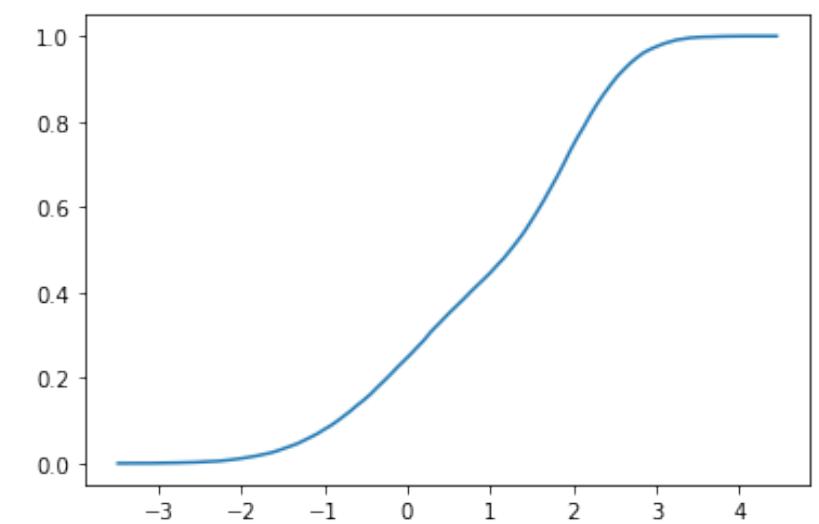


$$\mathcal{L}(f, x, y, \tau) = \begin{cases} \tau(y - f(x, \tau)) & y \geq f(x, \tau) \\ (\tau - 1)(y - f(x, \tau)) & y < f(x, \tau) \end{cases}$$

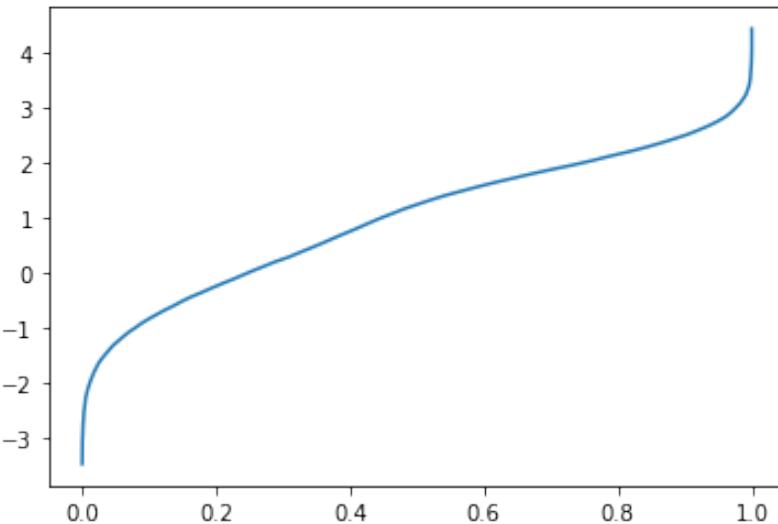
# IMPLICIT QUANTILE NETWORKS STATS REVIEW



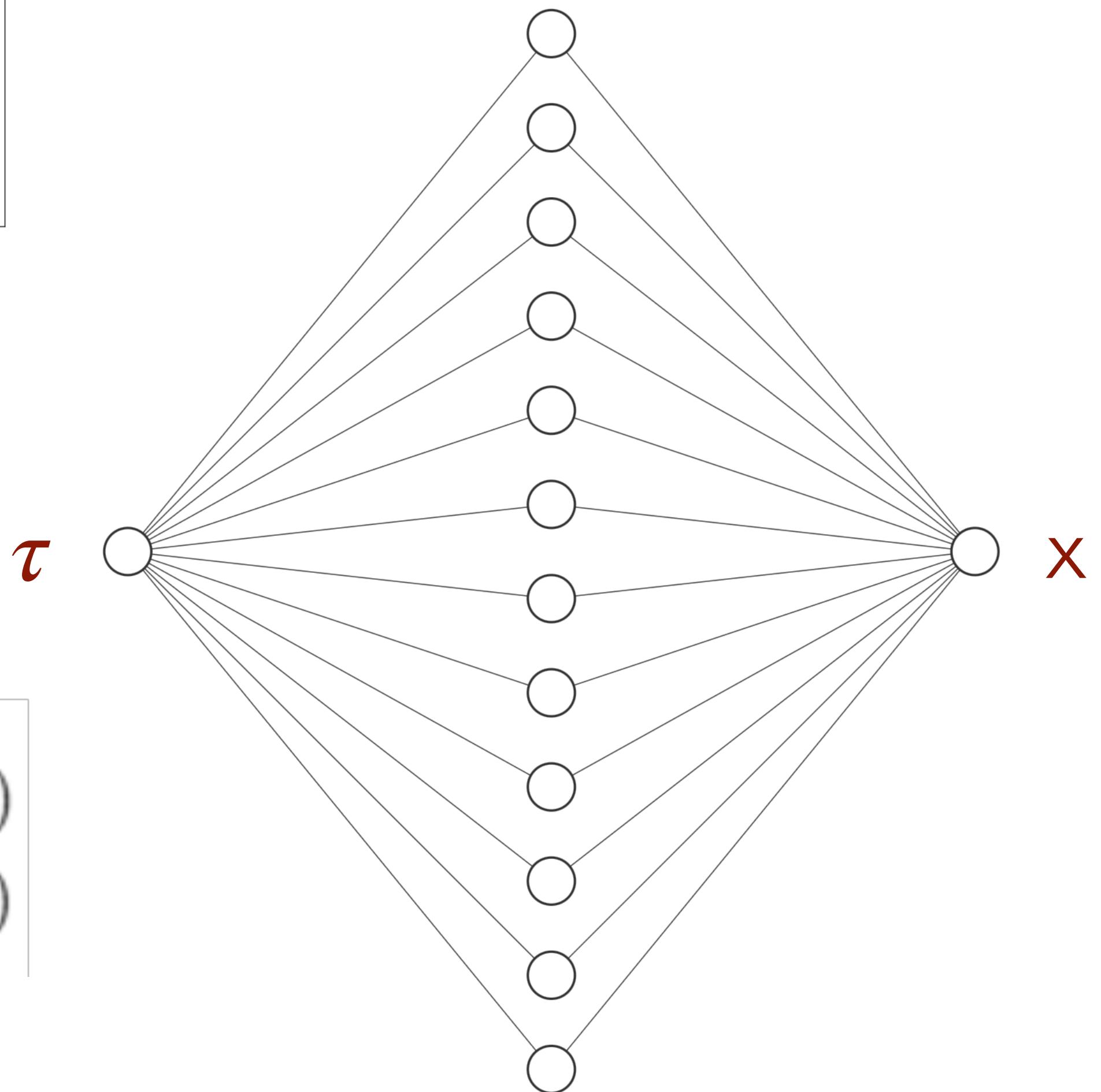
data ( $x$ )



CDF



quantile function ( $\tau$ )



$$\mathcal{L}(f, x, y, \tau) = \begin{cases} \tau(y - f(x, \tau)) & y \geq f(x, \tau) \\ (\tau - 1)(y - f(x, \tau)) & y < f(x, \tau) \end{cases}$$

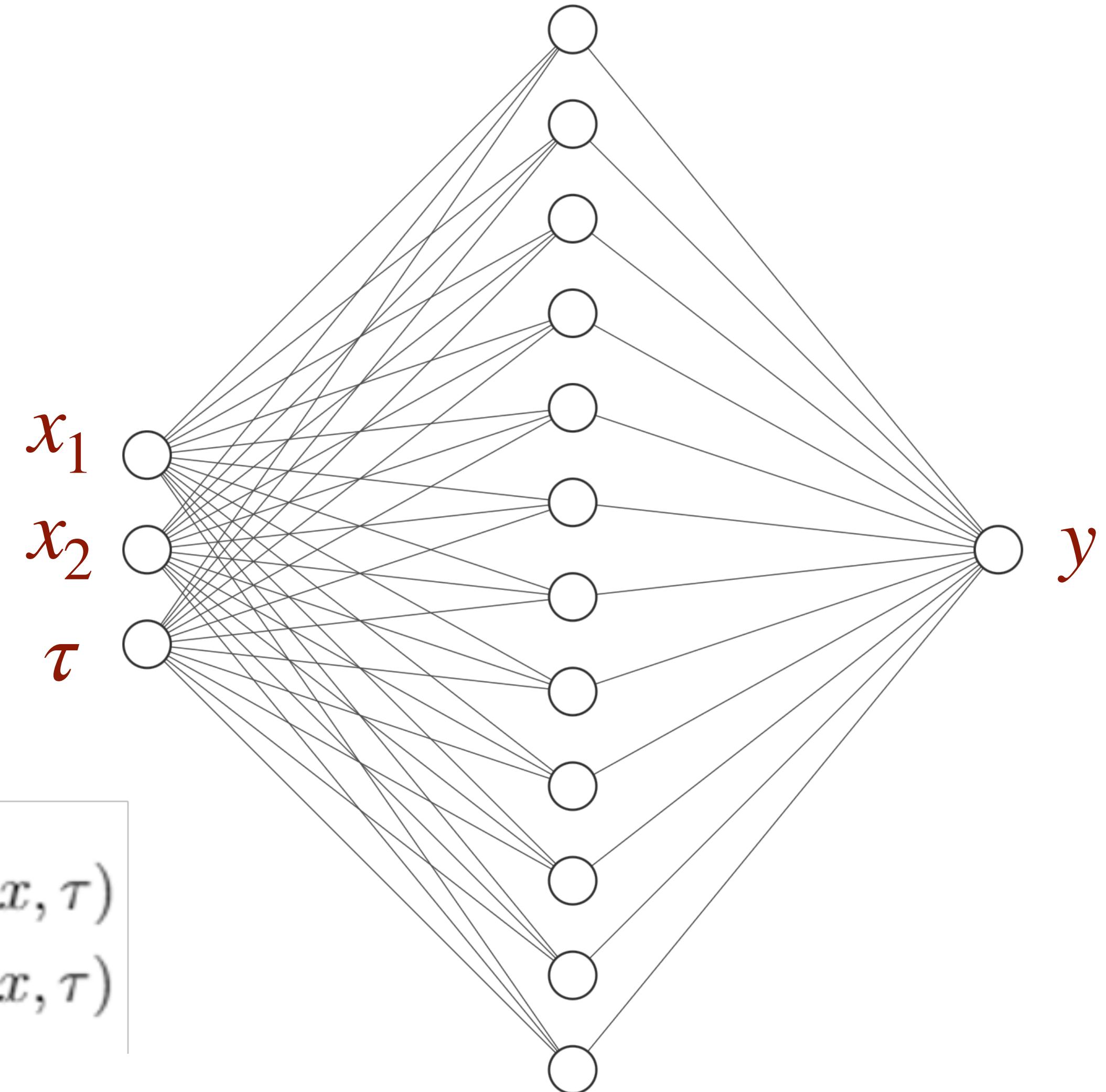
# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$$p(y | x_1, x_2)$$

$$(x_1, x_2) \rightarrow y$$

---

$$\mathcal{L}(f, x, y, \tau) = \begin{cases} \tau(y - f(x, \tau)) & y \geq f(x, \tau) \\ (\tau - 1)(y - f(x, \tau)) & y < f(x, \tau) \end{cases}$$



# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$$p(p'_T,\eta',\phi',m' \mid p_T,\eta,\phi,m)$$

$$(p_T,\eta,\phi,m) \rightarrow (p'_T,\eta',\phi',m')$$

$$p(y^{(1)},y^{(2)},\ldots,y^{(n)}\,|\,\mathbf{x})=p(y^{(1)}\,|\,\mathbf{x})p(y^{(2)}\,|\,\mathbf{x},y^{(1)})\prod_{i=3}^np(y^{(i)}\,|\,\mathbf{x},y^{(1)},\ldots,y^{(i-1)})$$

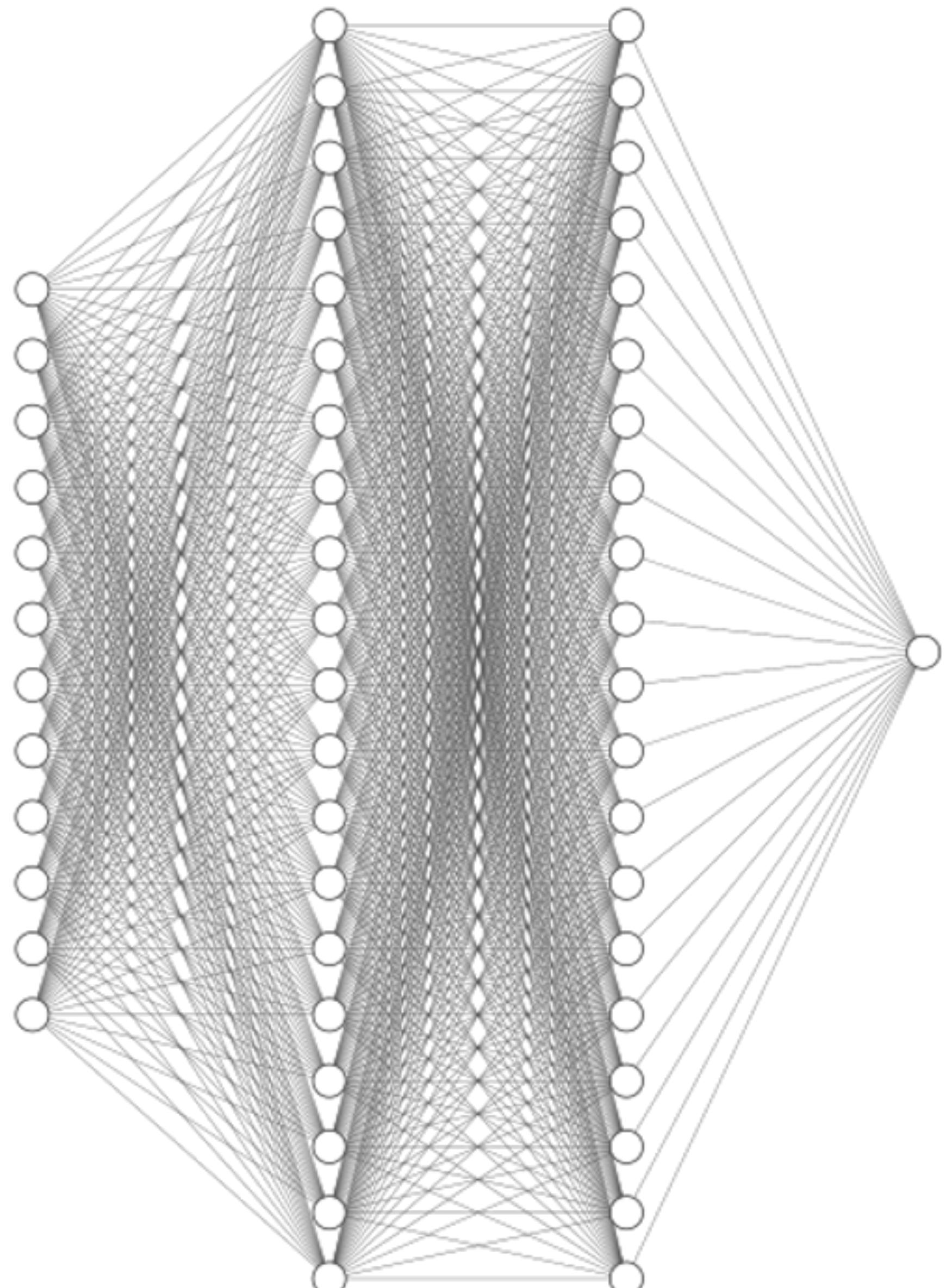
# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$$p(p'_T, \eta', \phi', m' | p_T, \eta, \phi, m)$$

$$(p_T, \eta, \phi, m) \rightarrow (p'_T, \eta', \phi', m')$$

$$p(y^{(1)}, y^{(2)}, \dots, y^{(n)} | \mathbf{x}) = \boxed{p(y^{(1)} | \mathbf{x})} \boxed{p(y^{(2)} | \mathbf{x}, y^{(1)})} \prod_{i=3}^n \boxed{p(y^{(i)} | \mathbf{x}, y^{(1)}, \dots, y^{(i-1)})}$$

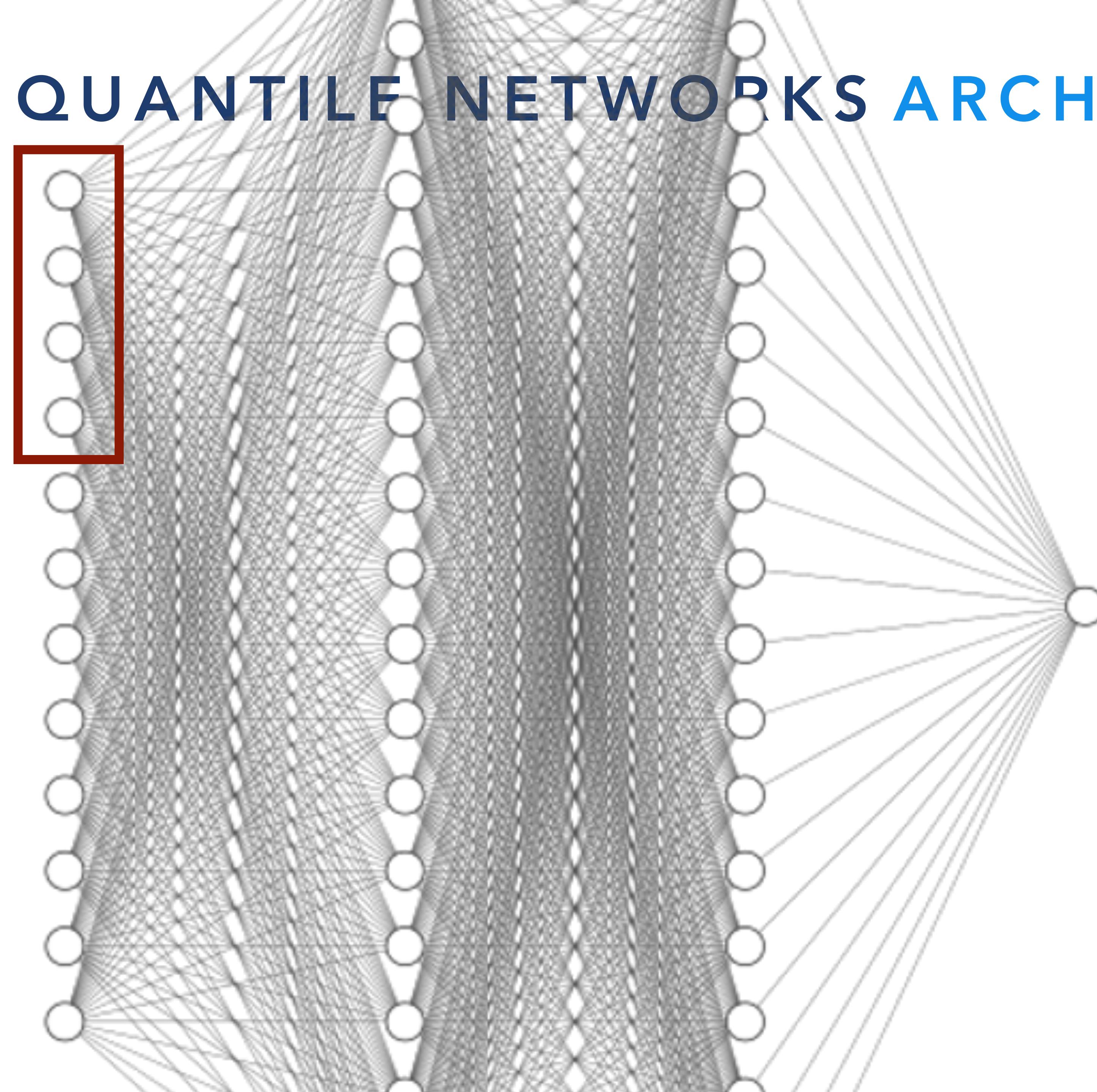
# IMPLICIT QUANTILE NETWORKS ARCHITECTURE



$$(p_T, \eta, \phi, m) \rightarrow (p'_T, \eta', \phi', m')$$

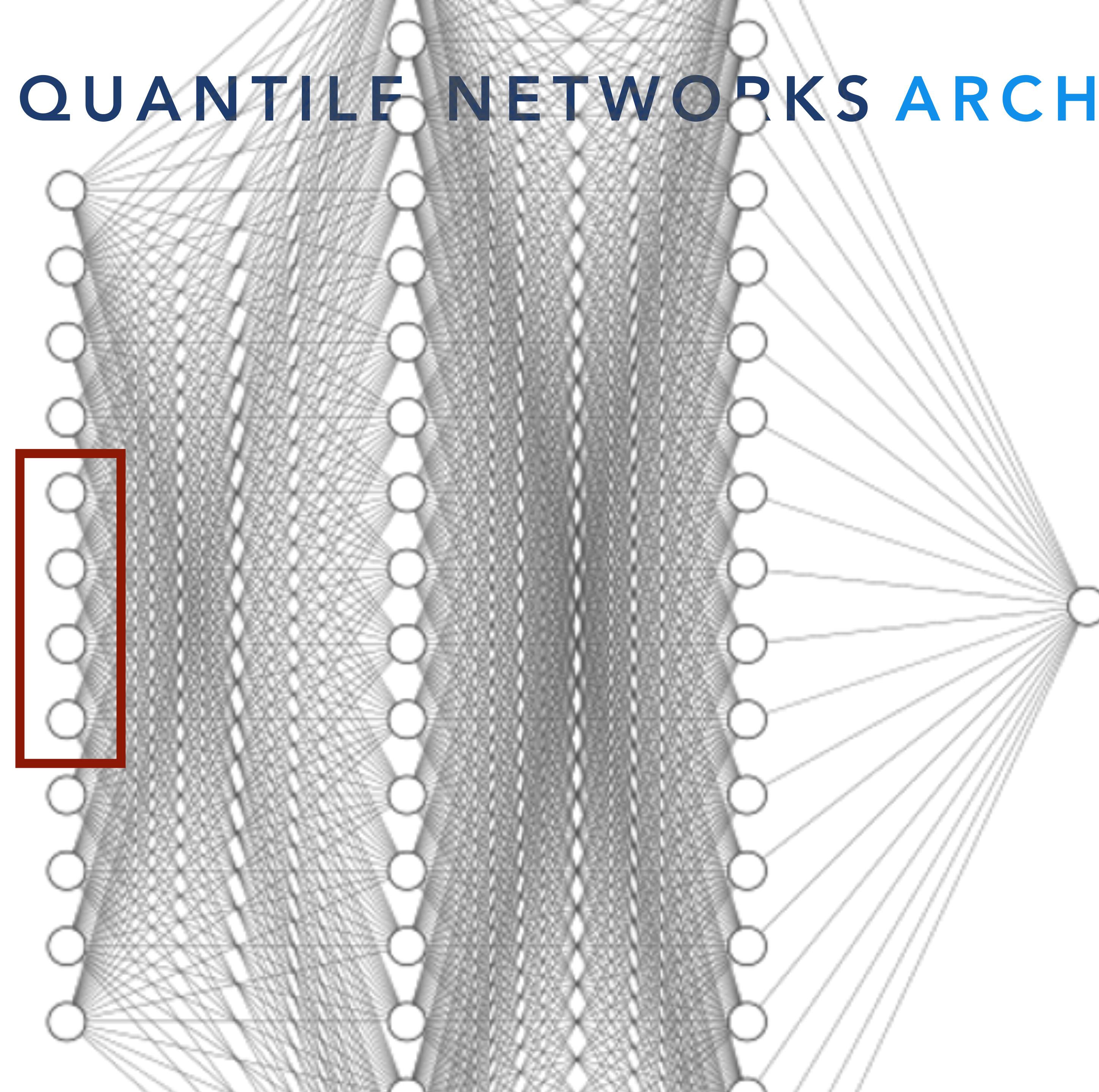
# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$(p_T, \eta, \phi, m)$



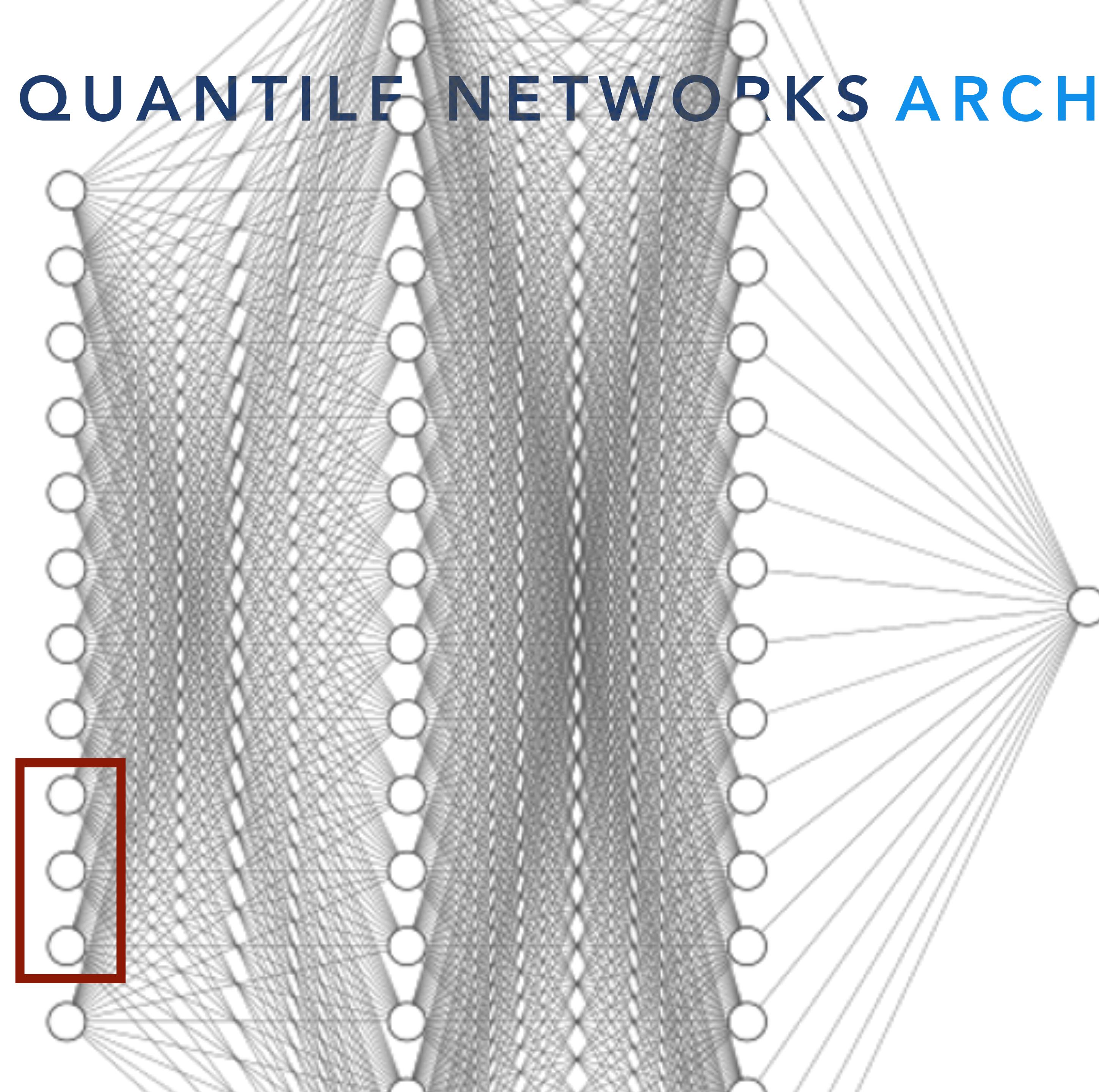
# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$(p'_T, \eta', \phi', m')$   
[0,0,1,0]



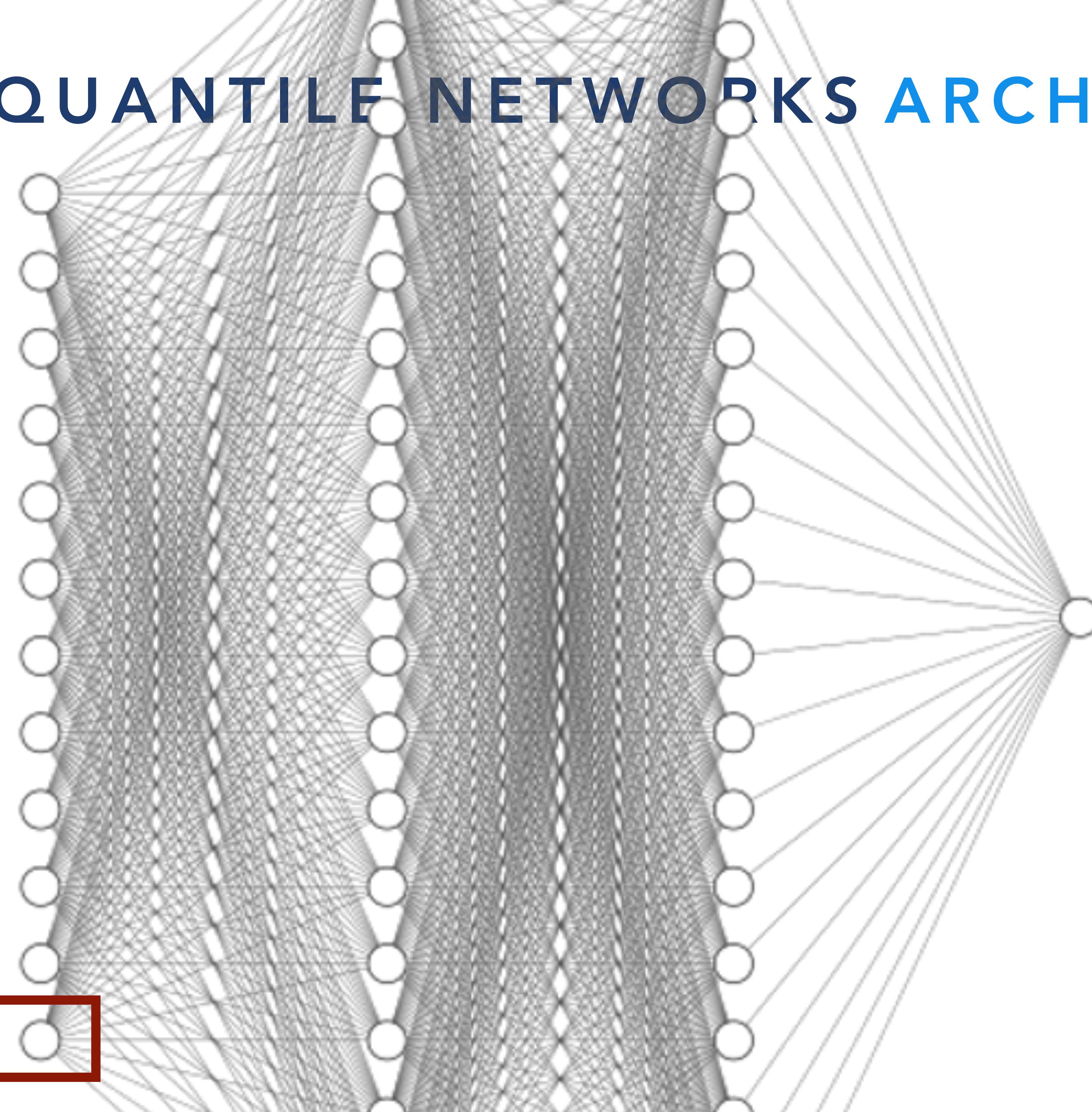
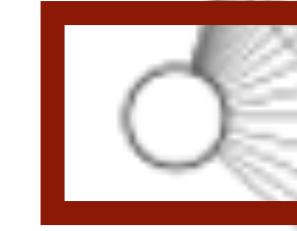
# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$(p'_T, \eta', \phi')$



# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$\tau \sim U(0,1)$



# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

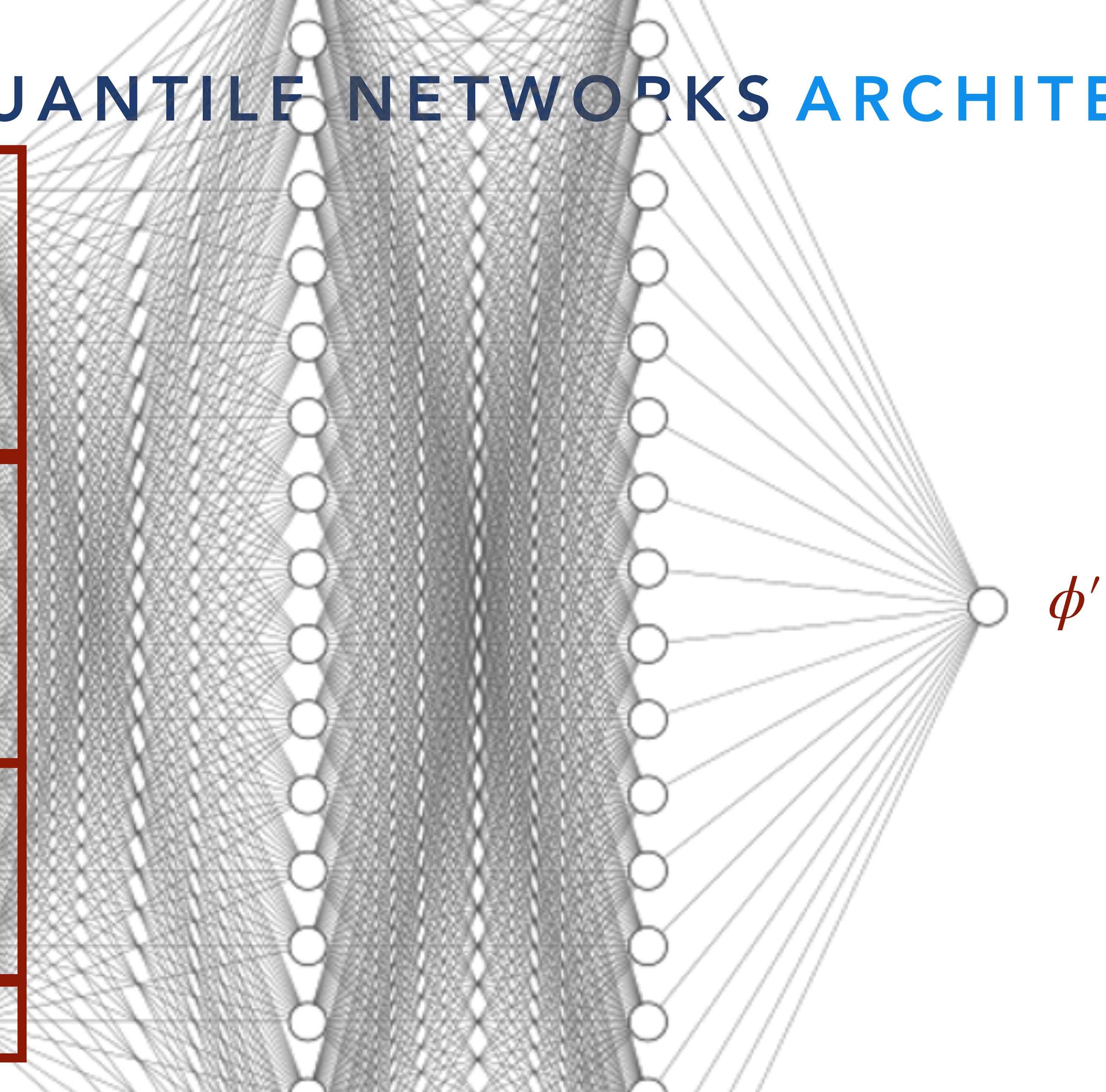
$(p_T, \eta, \phi, m)$

$(p_T, \eta, \phi, m)$

$[0,0,1,0]$

$(p'_T, \eta', \phi')$

$\tau \sim U(0,1)$



# IMPLICIT QUANTILE NETWORKS ARCHITECTURE

$(p_T, \eta, \phi, m)$

$(p_T, \eta, \phi, m)$

$[0,0,1,0]$

$(p'_T, \eta', \phi')$

$\tau \sim U(0,1)$

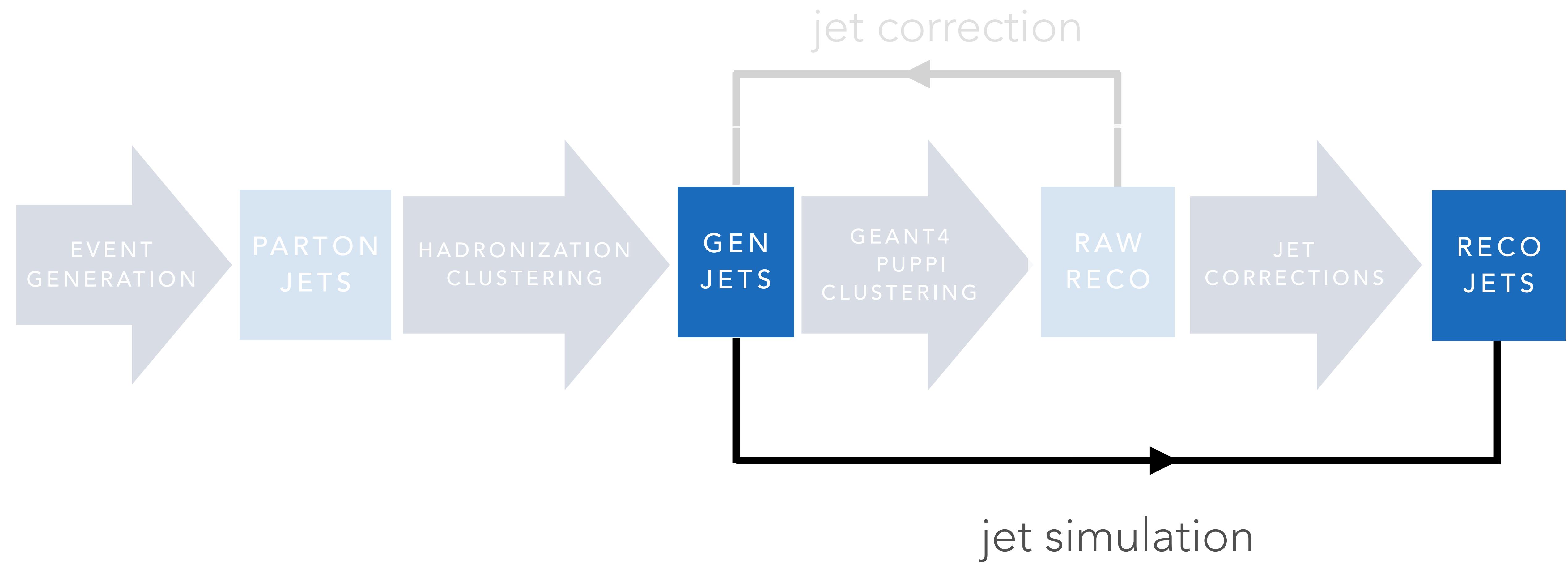


$(p_T, \eta, \phi, m, 1, 0, 0, 0, 0, 0) \rightarrow (p'_T),$   
 $(p_T, \eta, \phi, m, 0, 1, 0, 0, p'_T, 0, 0) \rightarrow (\eta'),$   
 $(p_T, \eta, \phi, m, 0, 0, 1, 0, p'_T, \eta', 0) \rightarrow (\phi'),$   
 $(p_T, \eta, \phi, m, 0, 0, 0, 1, p'_T, \eta', \phi') \rightarrow (m'),$

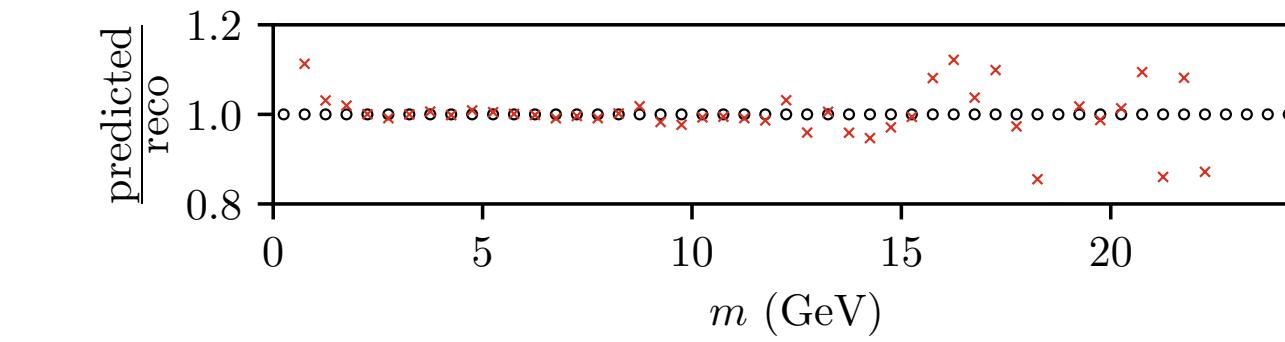
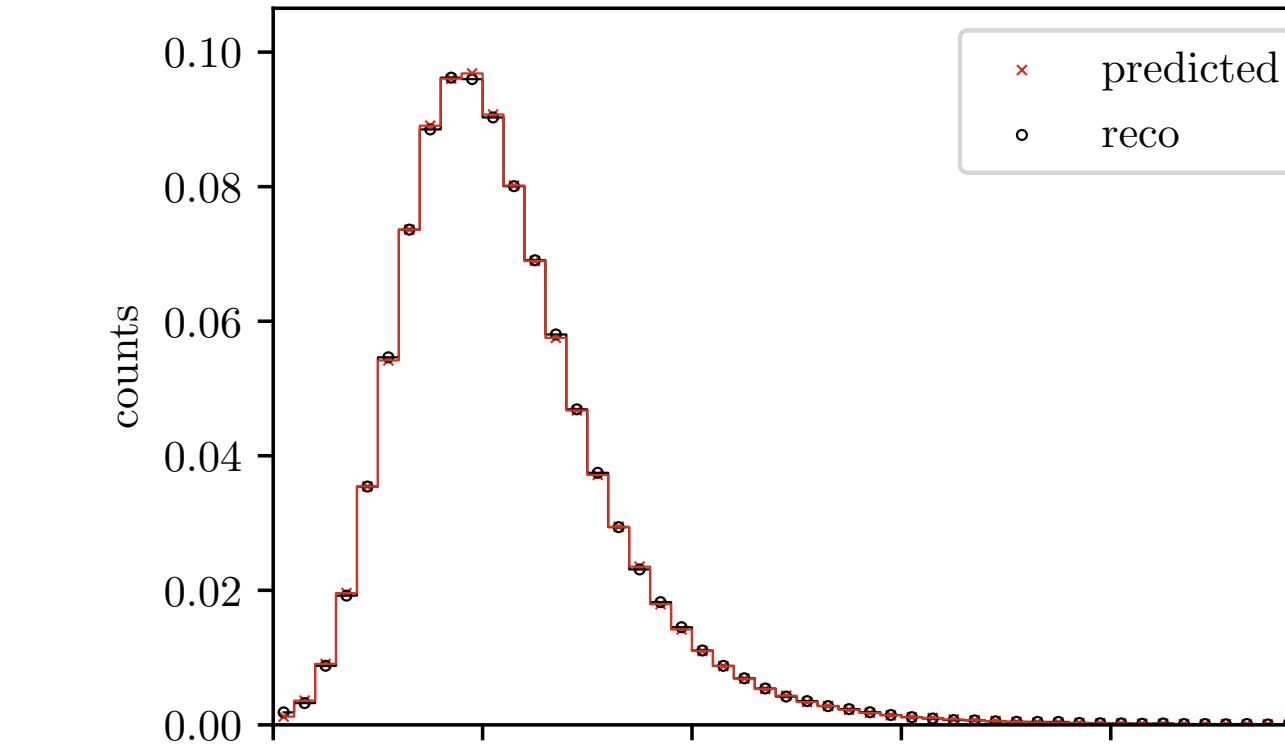
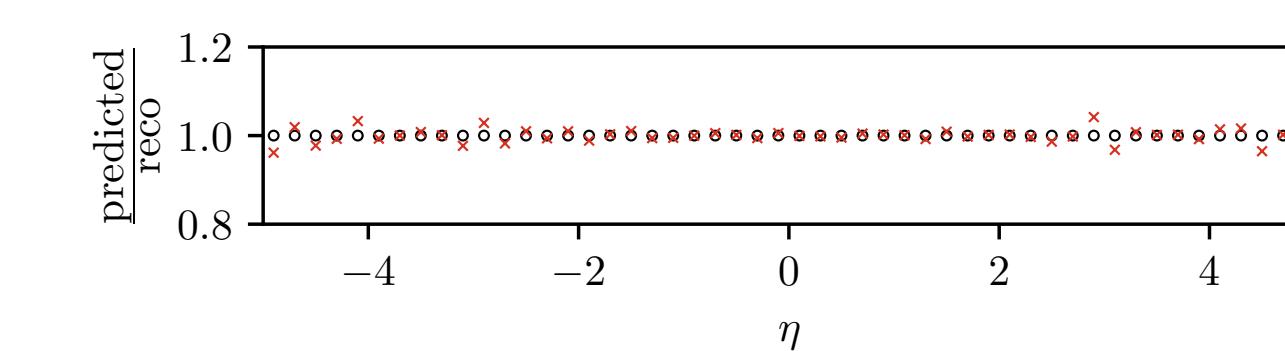
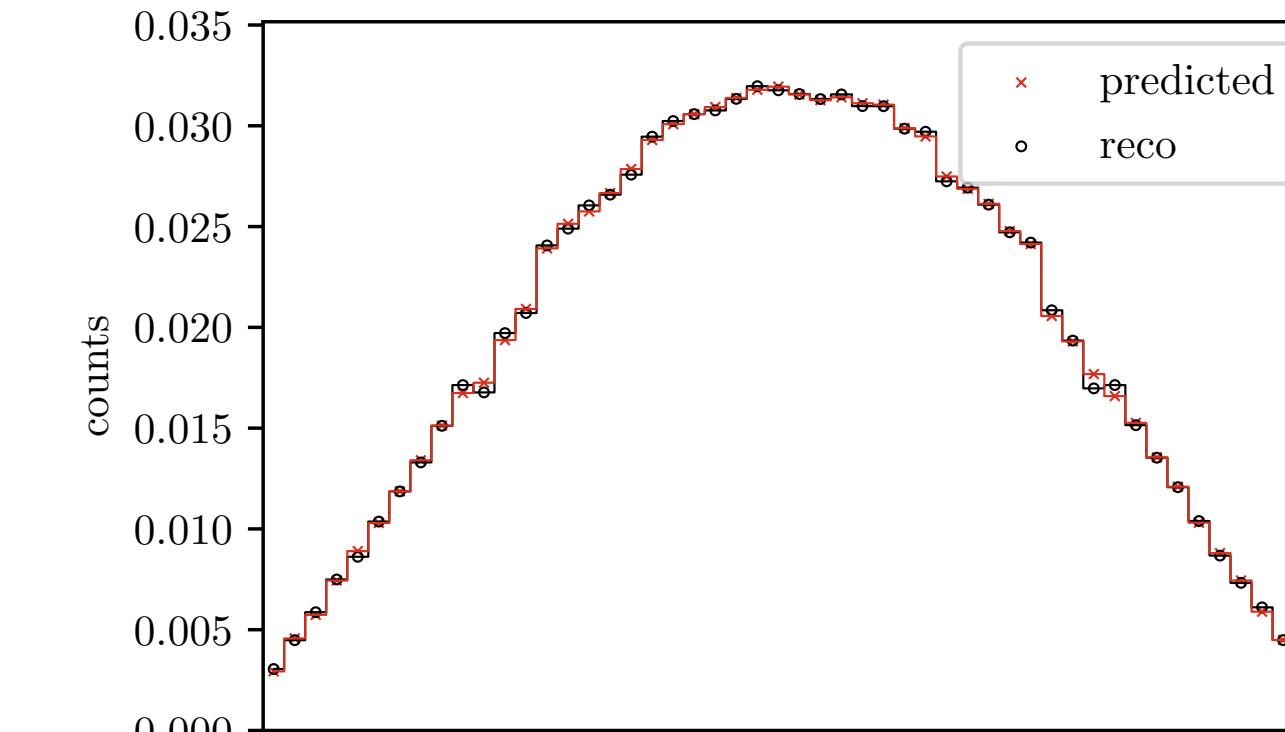
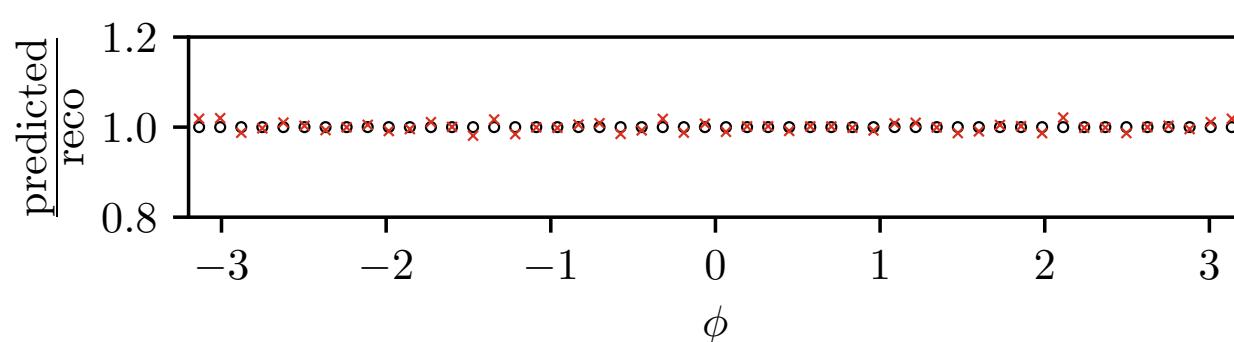
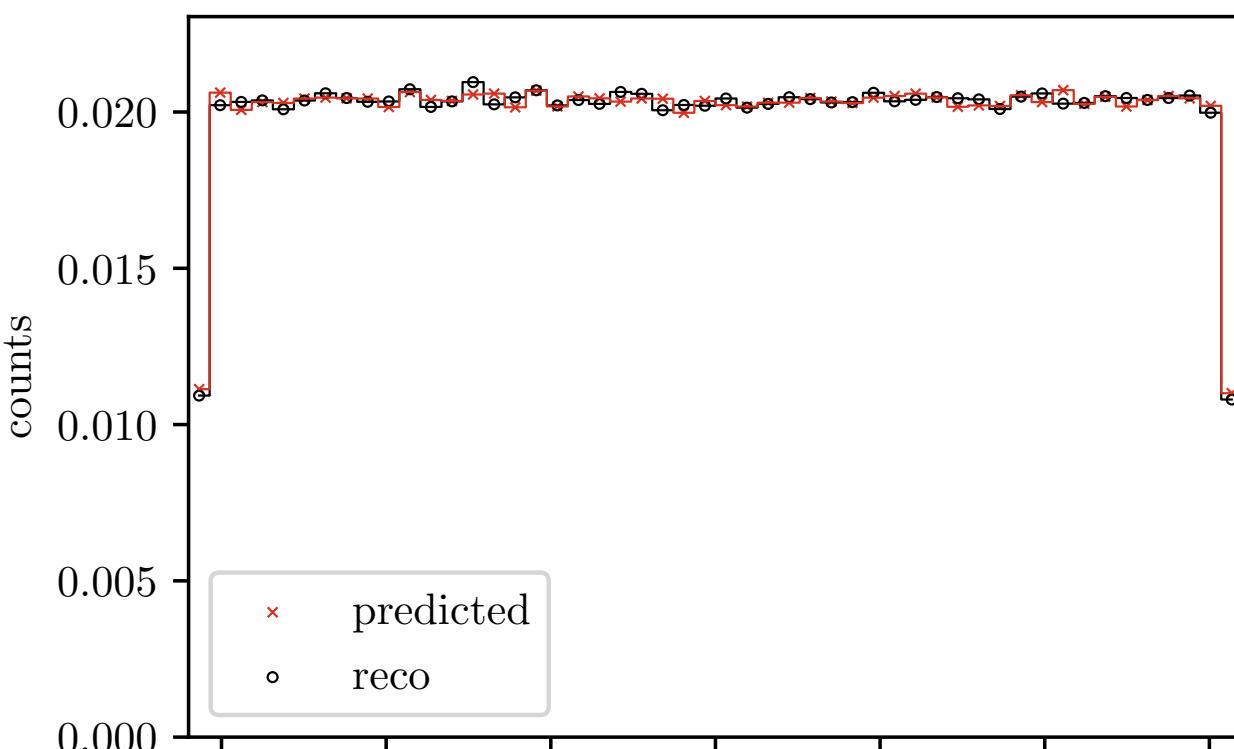
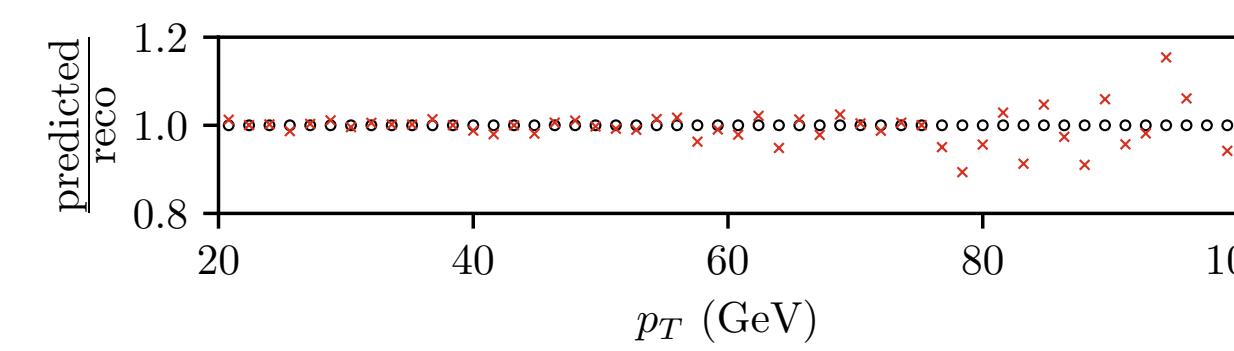
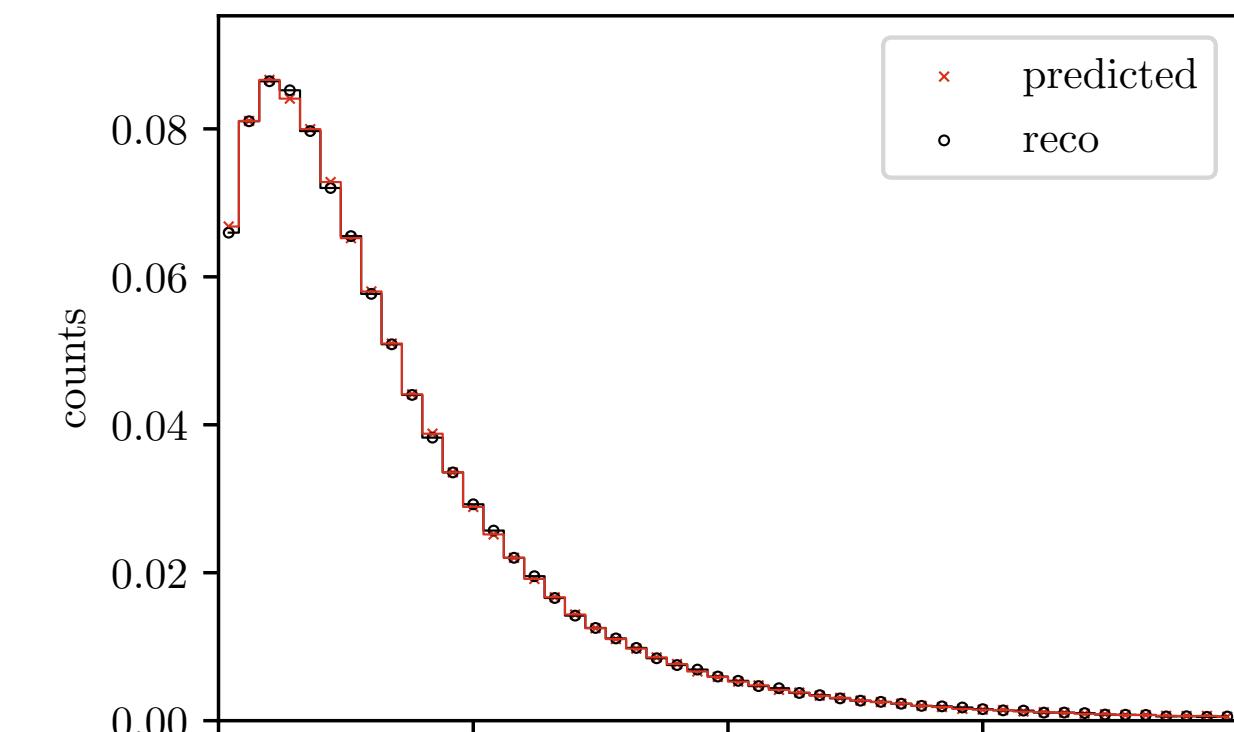
$\phi'$

$$p(A, B, C, D) = p(A | D)p(B | A, D)p(C | A, B, D)$$

# RESULTS JET SIMULATION

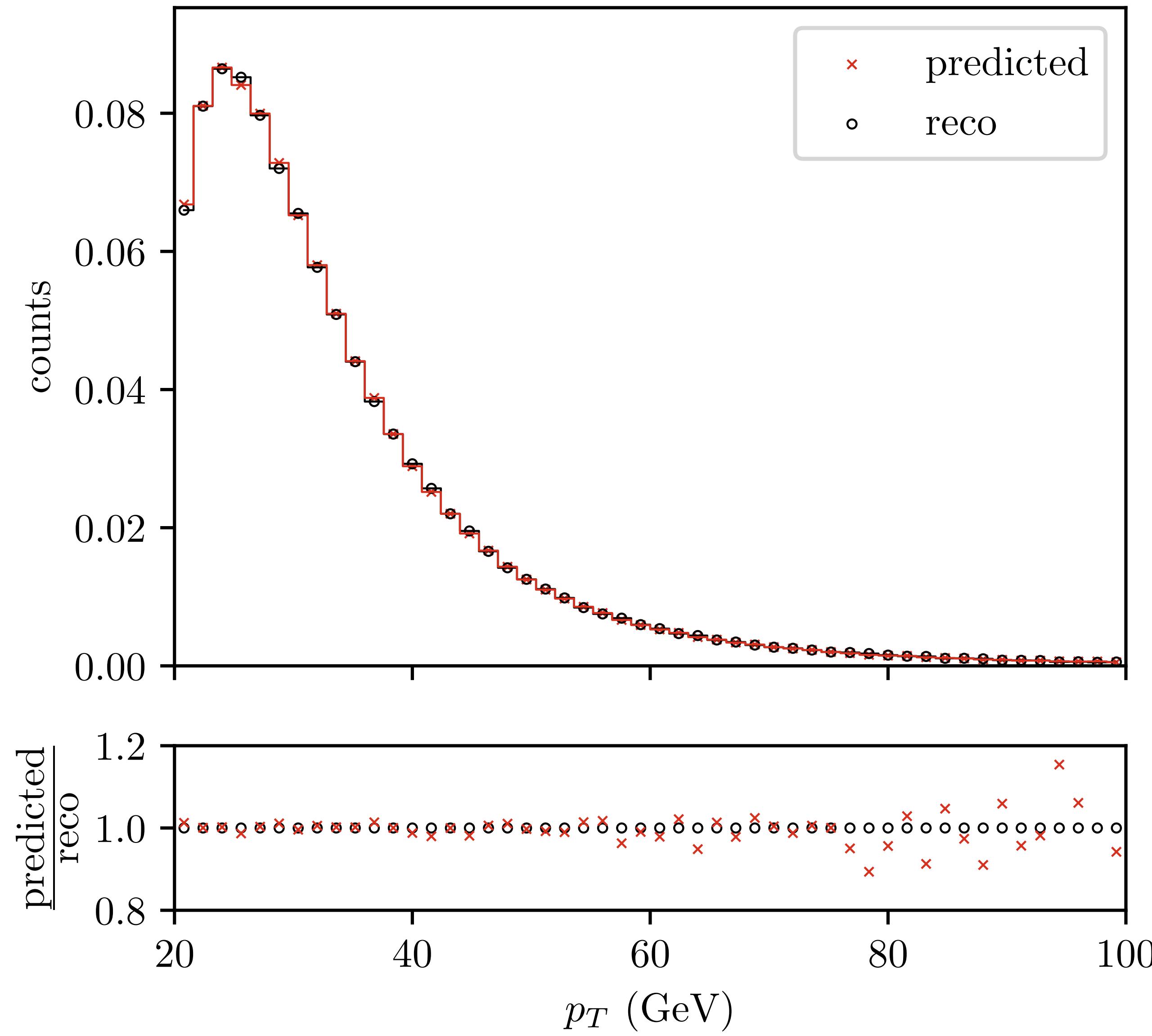


# RESULTS JET SIMULATION



[A single black vertical bar is located here.]

# RESULTS JET SIMULATION



# RESULTS JET SIMULATION: SUBSPACE



## Implicit Quantile Neural Networks for Jet Simulation and Correction

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