CONVOLUTIONAL NEURAL NETWORKS:

DISCRETE CONVOLUTIONS

Convolution operations first published by D’Alembert in 1754

Discrete convolutions are matrix operations that can be used to apply **filters** to a matrix or array

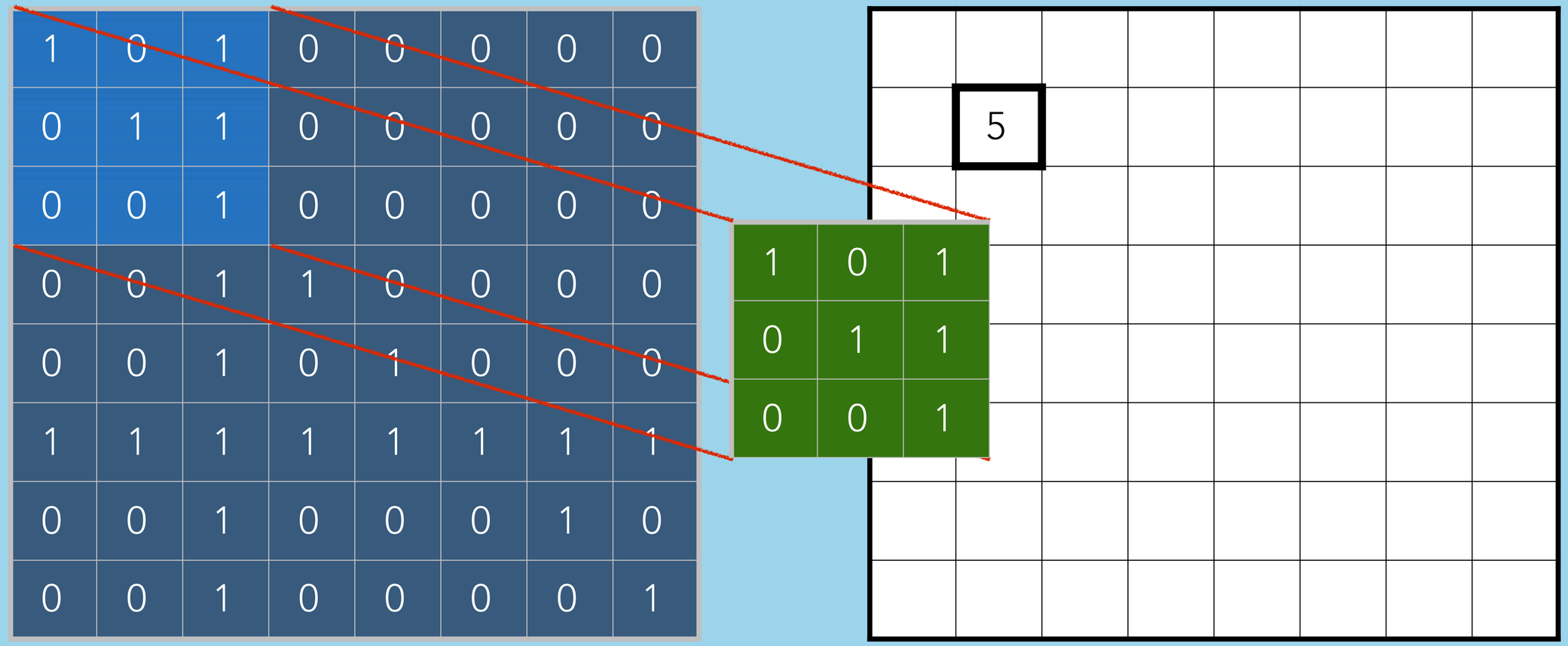
The convolutional neural network architecture was first described by Kunihiko Fukushima in 1980

pre-defined filters

Discrete convolutions are matrix operations that can be used to apply **filters** to a matrix or array

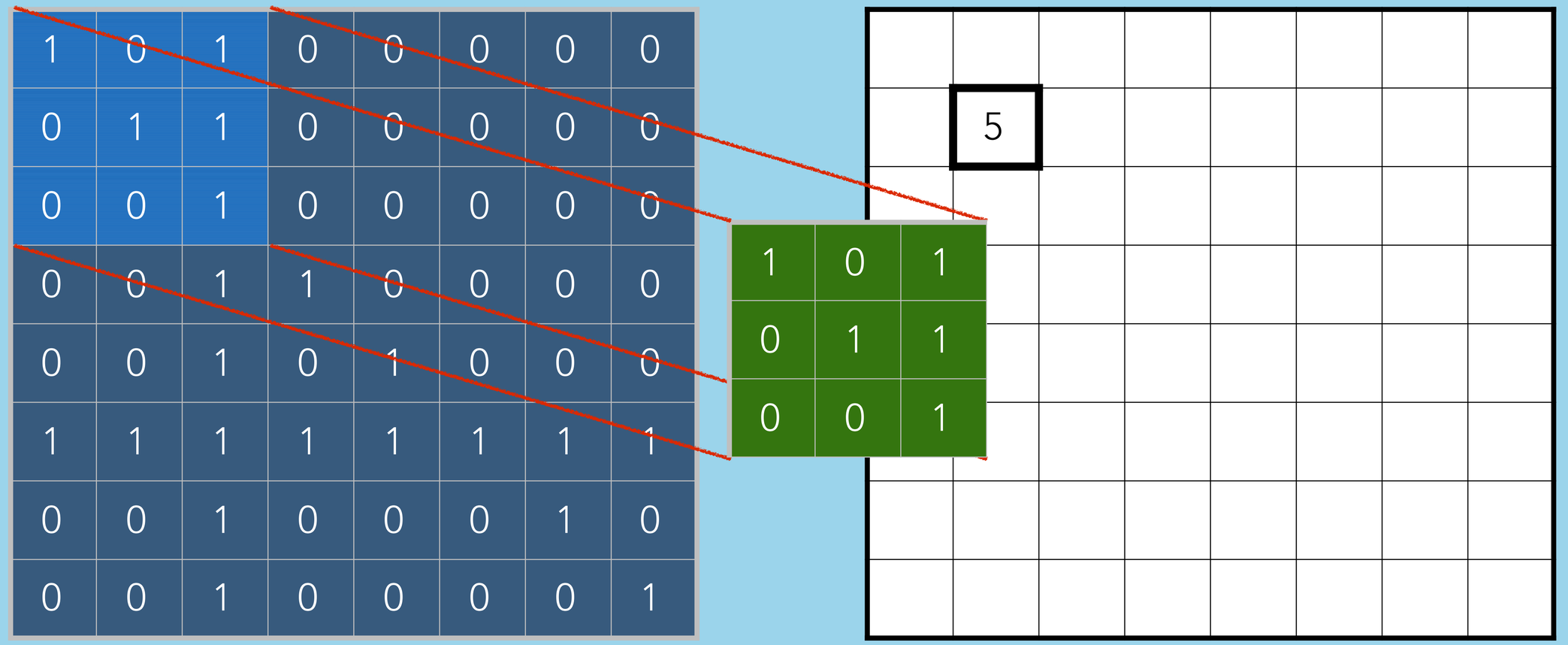
Discrete Convolutions $C = A \circledast h$

$$C[m,n] = \sum_{j=-\omega}^{\omega} \sum_{i=-\omega}^{\omega} h[i+\omega, j+\omega] * A[m+i, n+j]$$



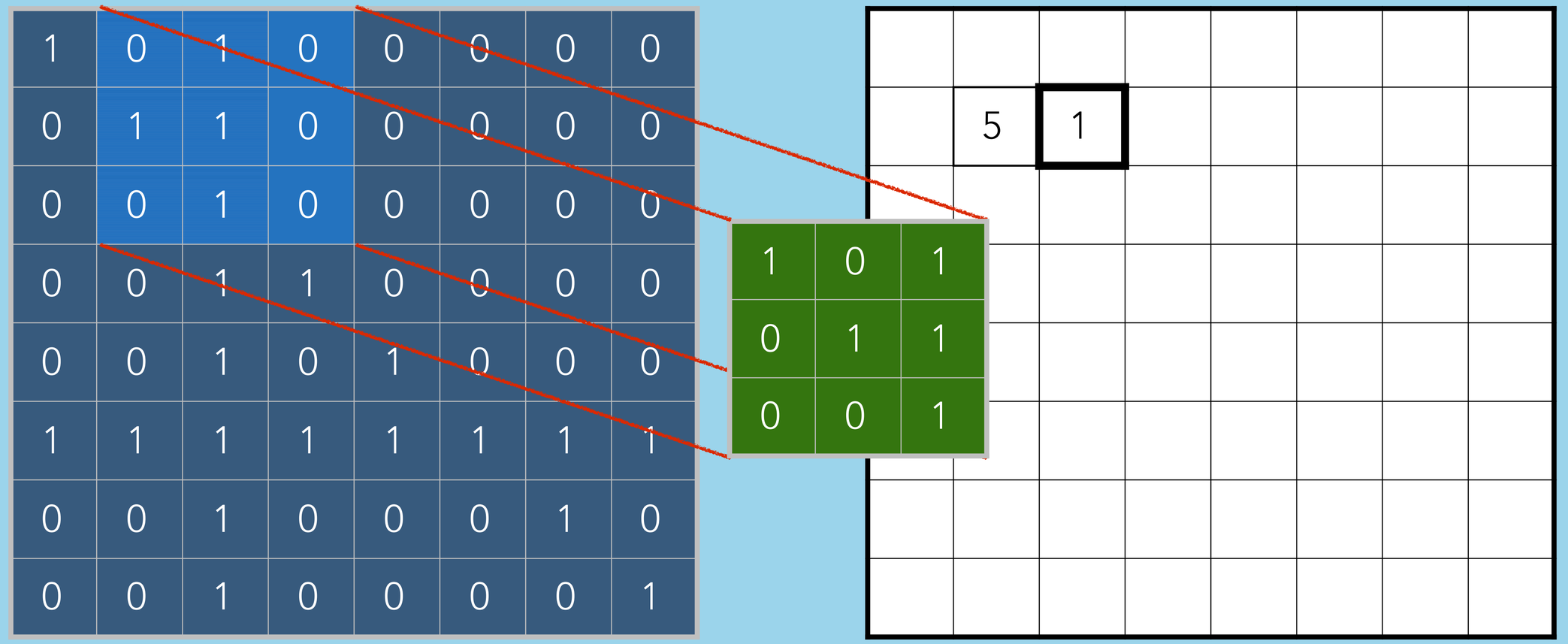
Discrete Convolutions $C = A \circledast h$

Filter Stride Edges



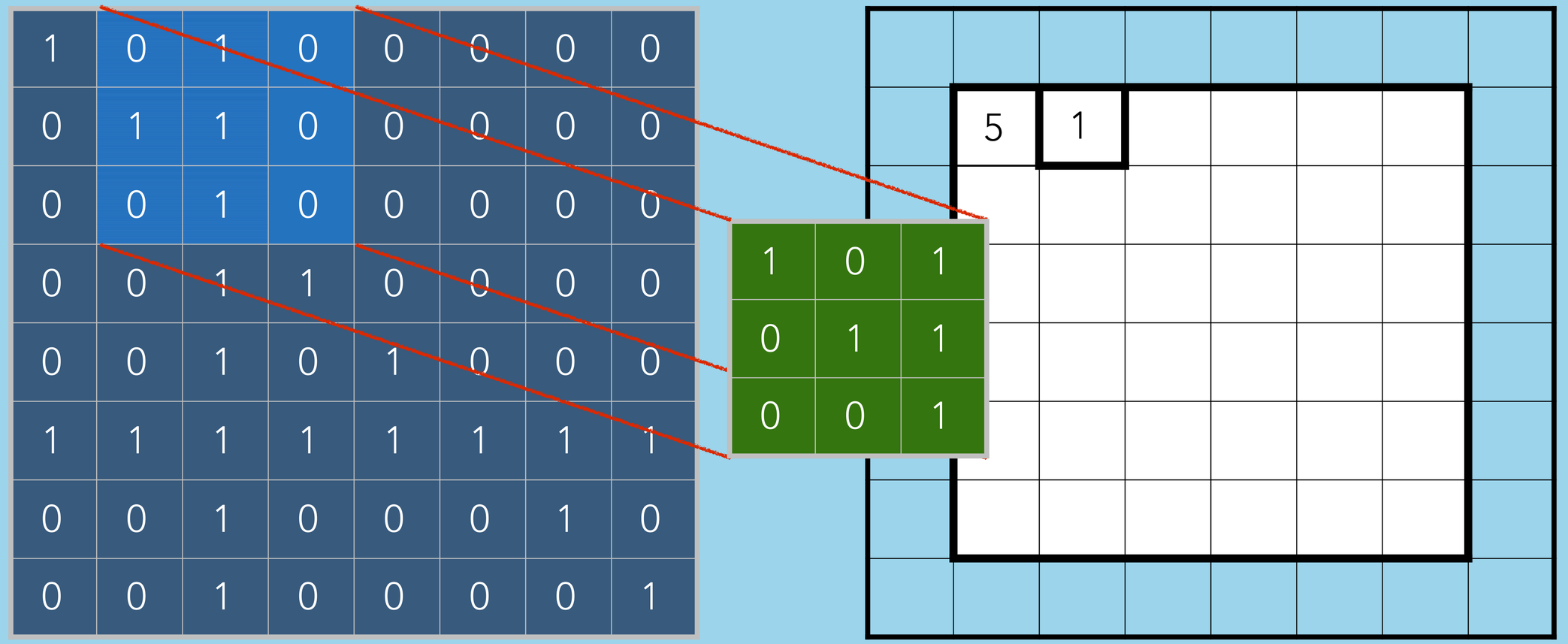
Discrete Convolutions $C = A \circledast h$

Filter Stride Edges



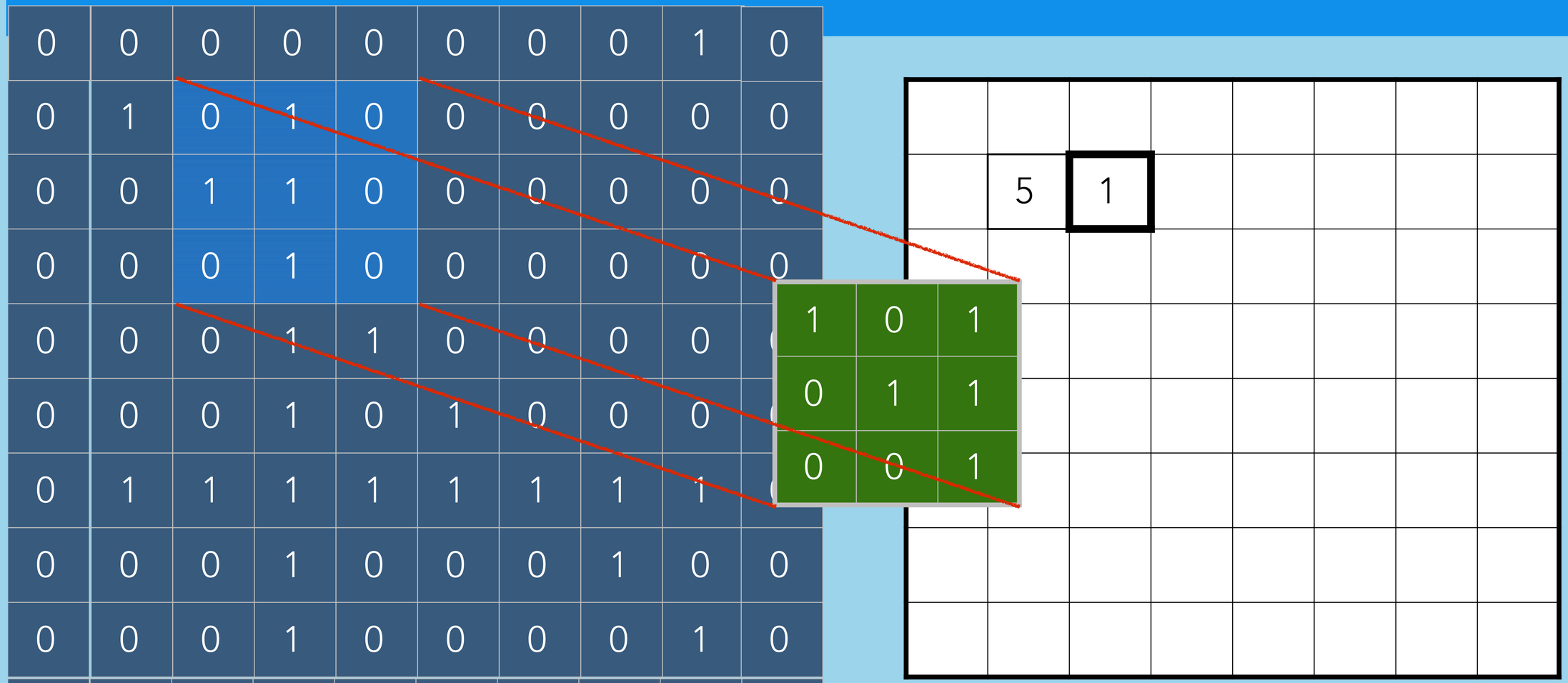
Discrete Convolutions $C = A \circledast h$

Filter Stride Edges



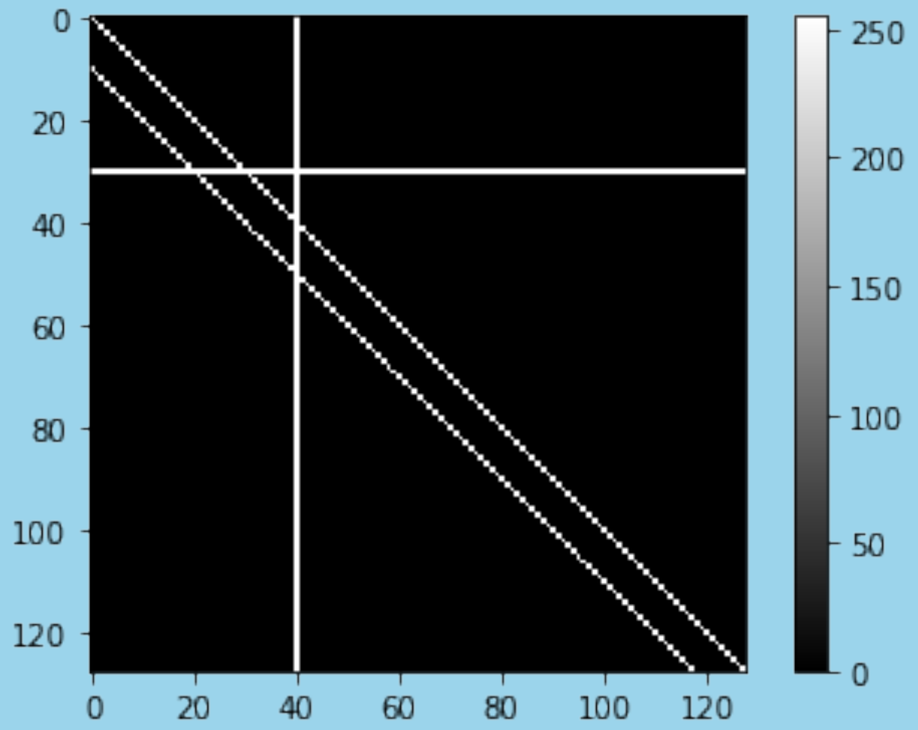
Discrete Convolutions $C = A \circledast h$

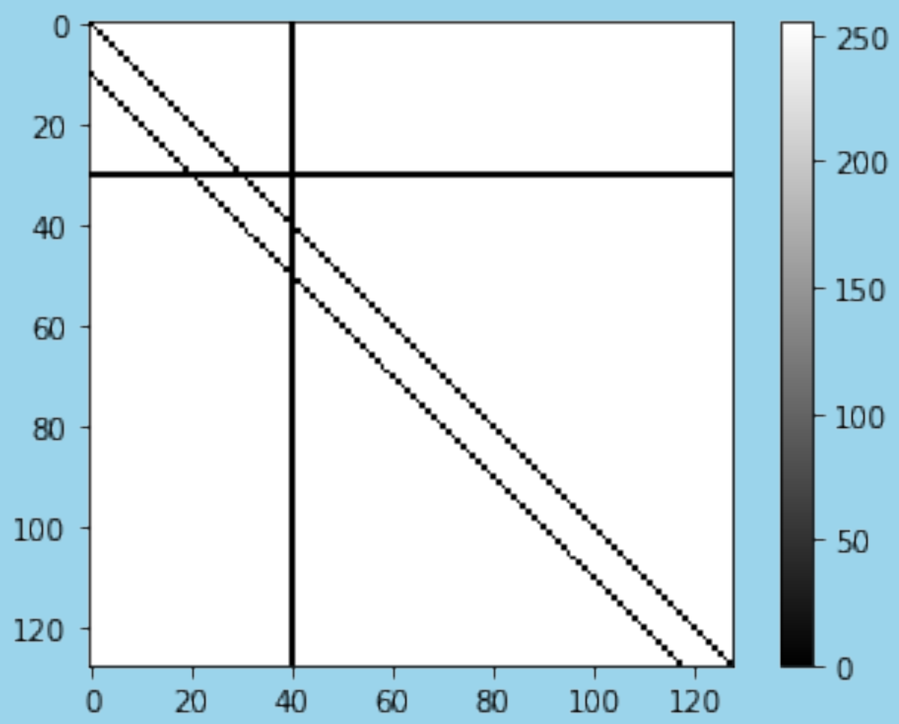
Filter Stride Edges



Discrete Convolutions $C = A \circledast h$

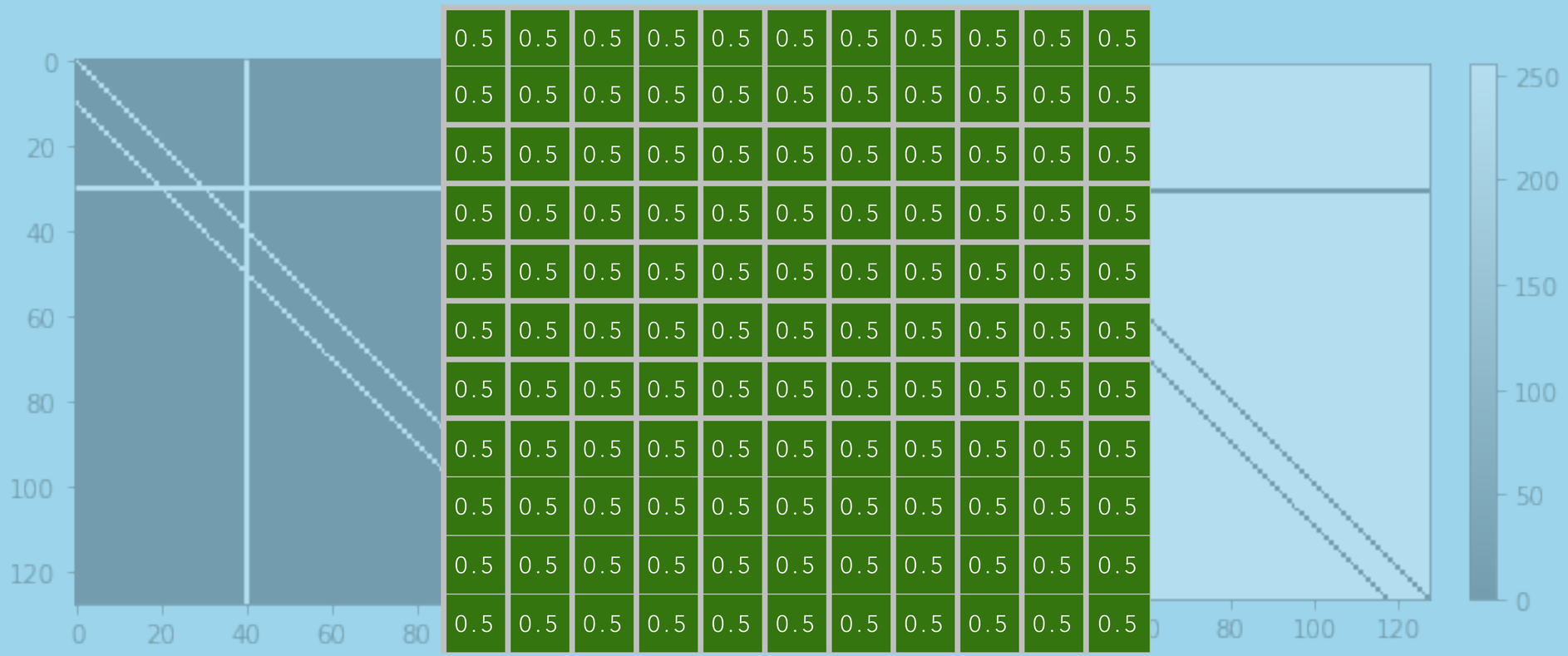
Filter	Stride	Edges
(11x11)	1	None





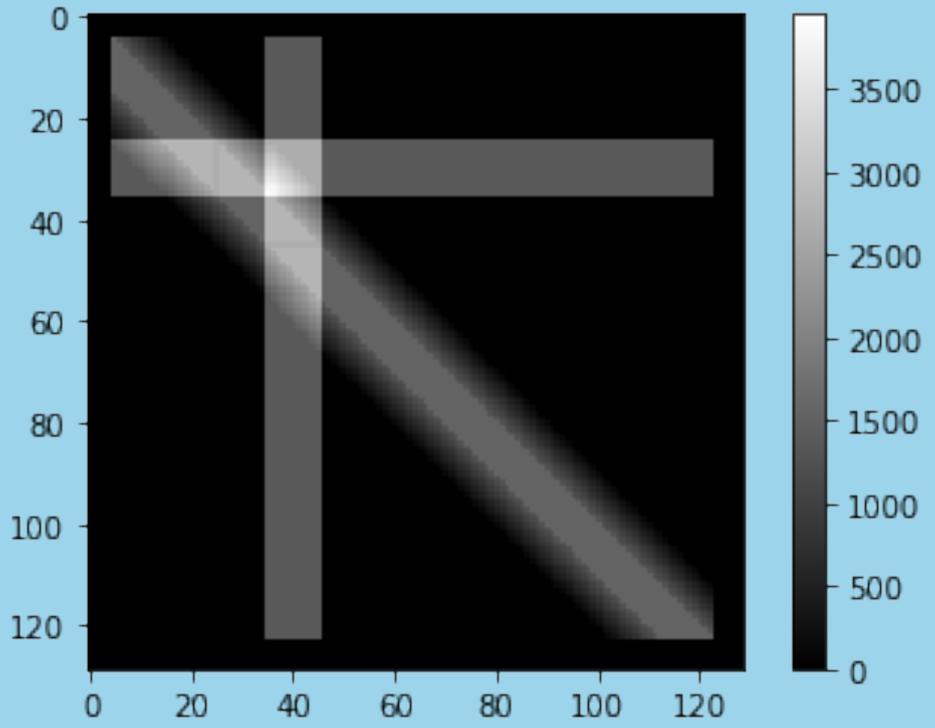
Discrete Convolutions $C = A \circledast h$

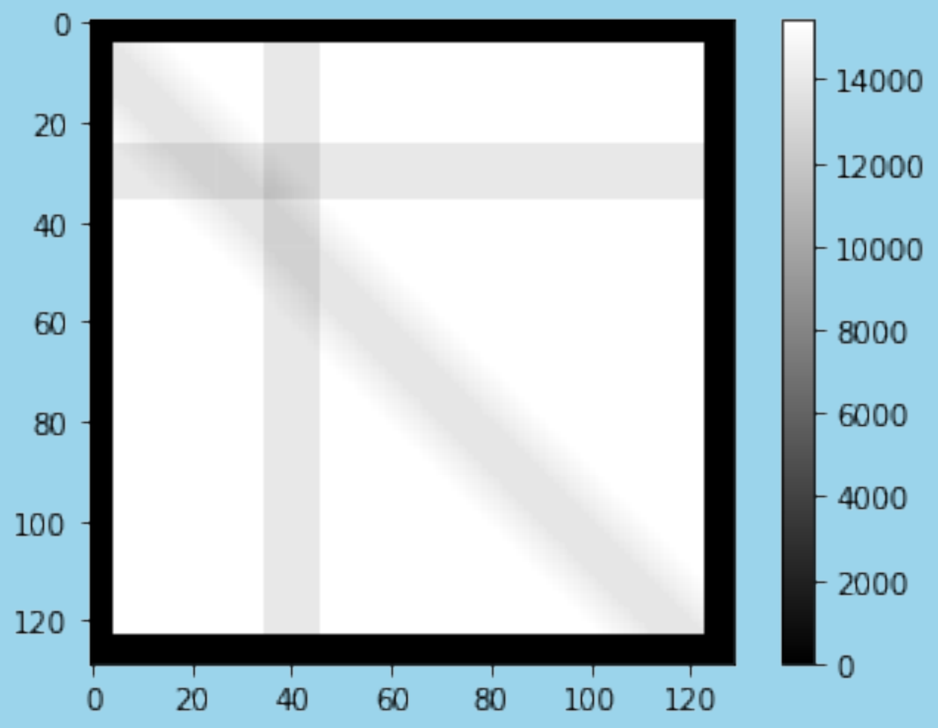
Filter	Stride	Edges
(11x11)	1	None



Discrete Convolutions $C = A \circledast h$

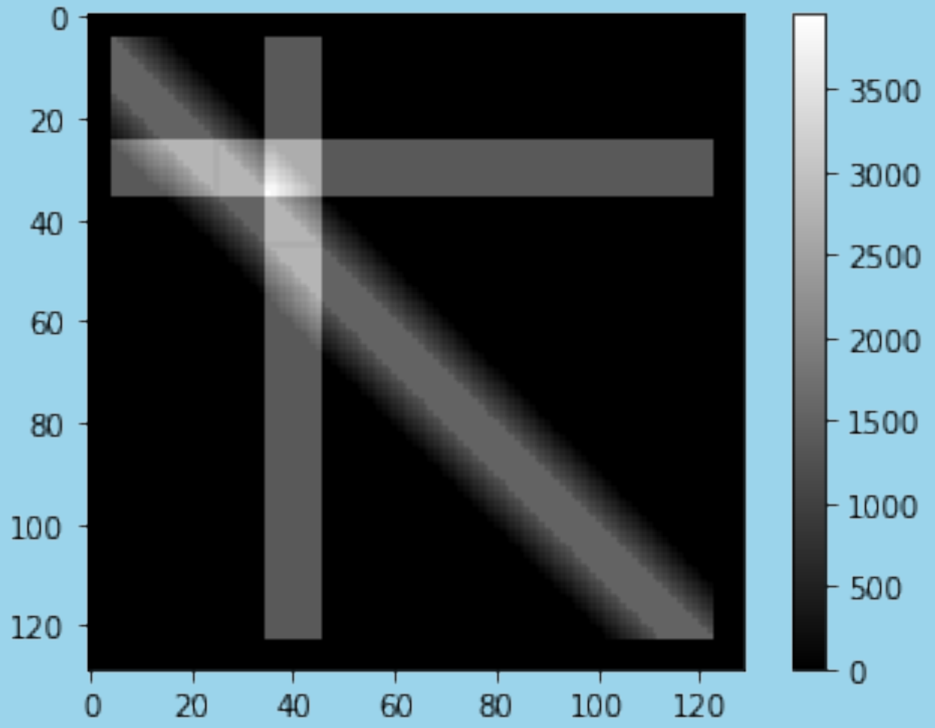
Filter	Stride	Edges
(11x11)	1	None

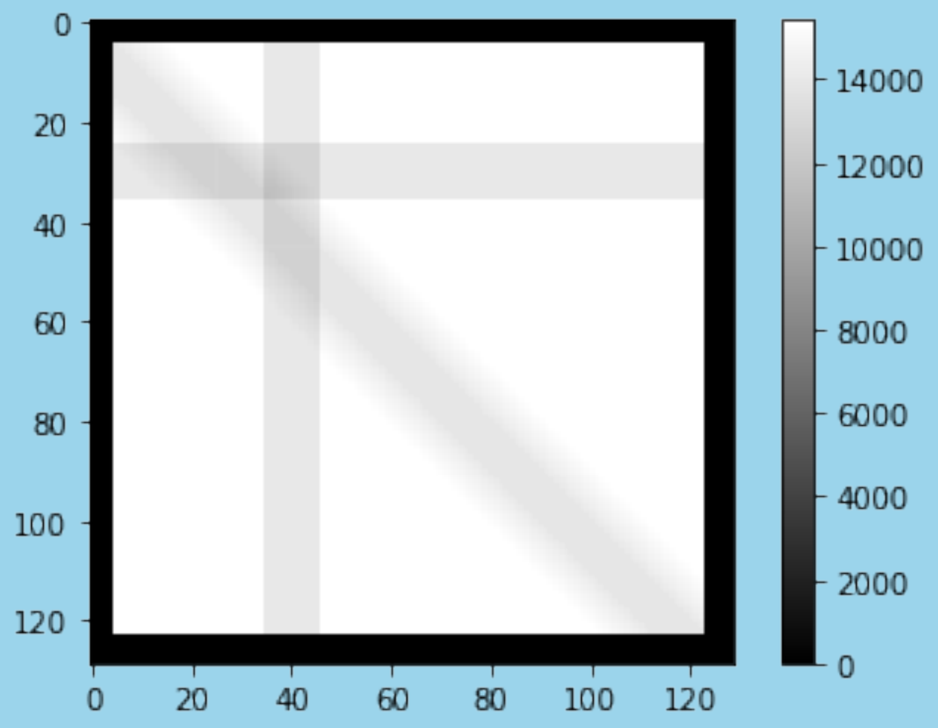




Discrete Convolutions $C = A \circledast h$

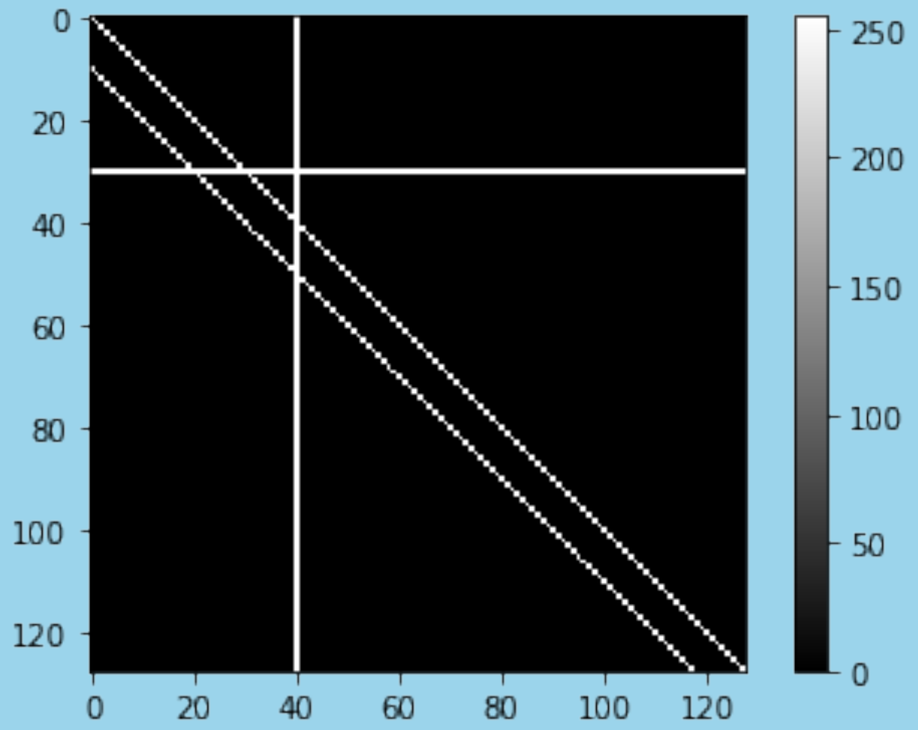
Blurring Filter

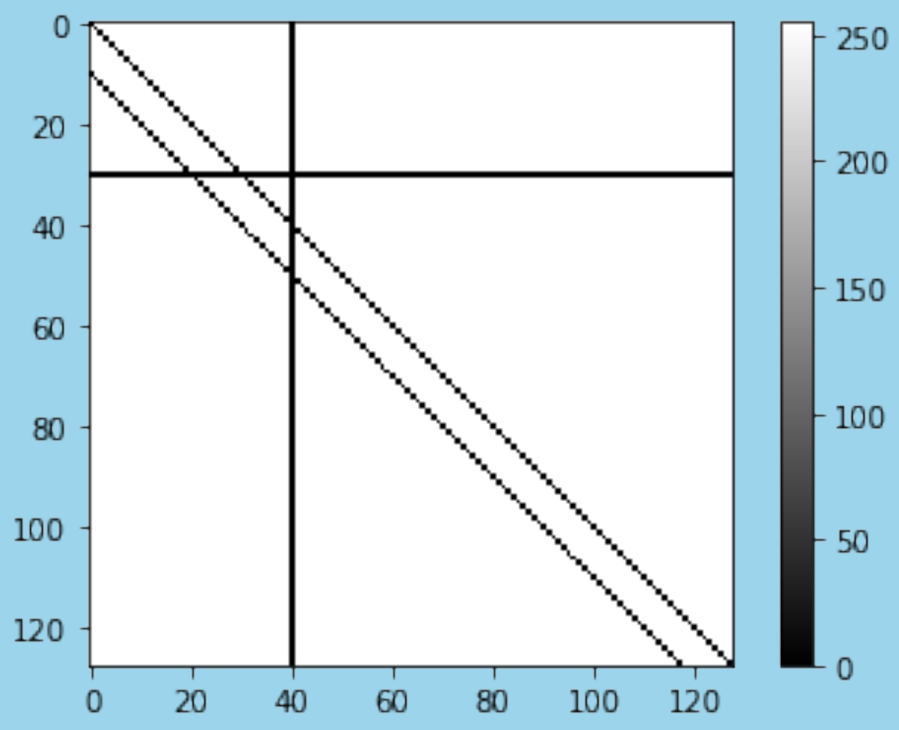




Discrete Convolutions $C = A \circledast h$

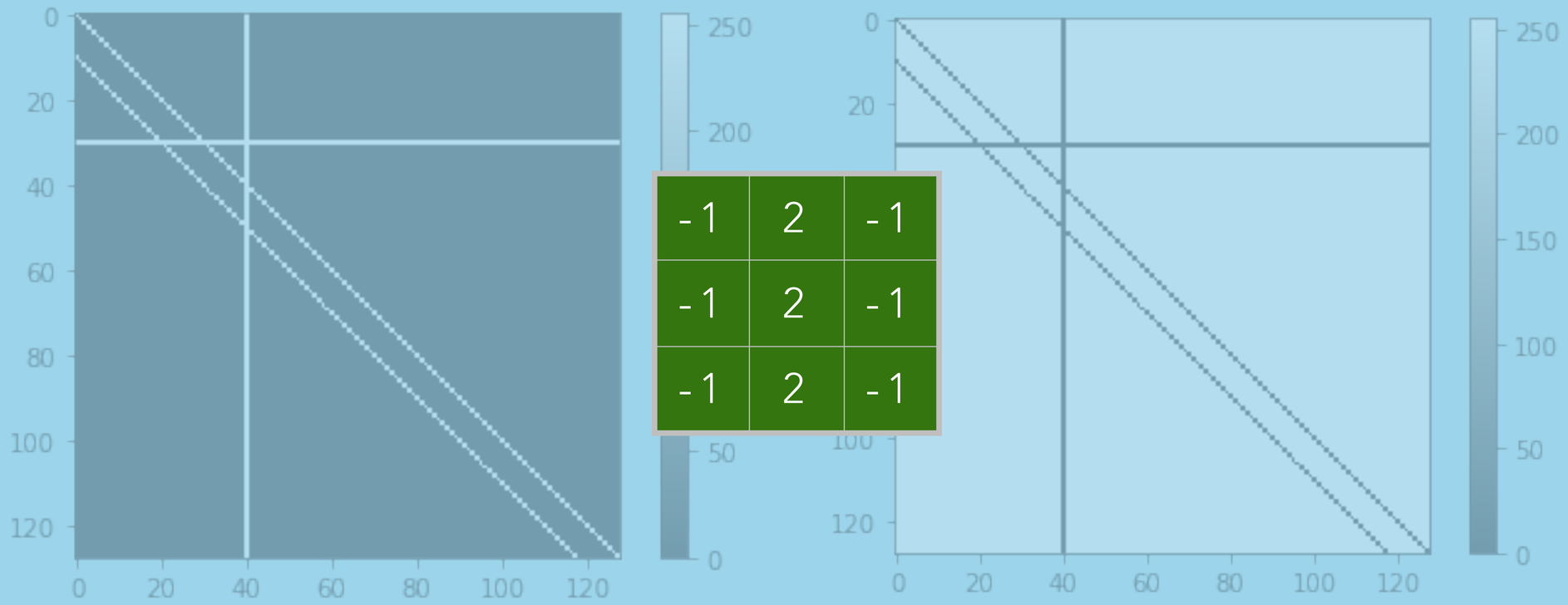
Filter	Stride	Edges
(3x3)	1	None





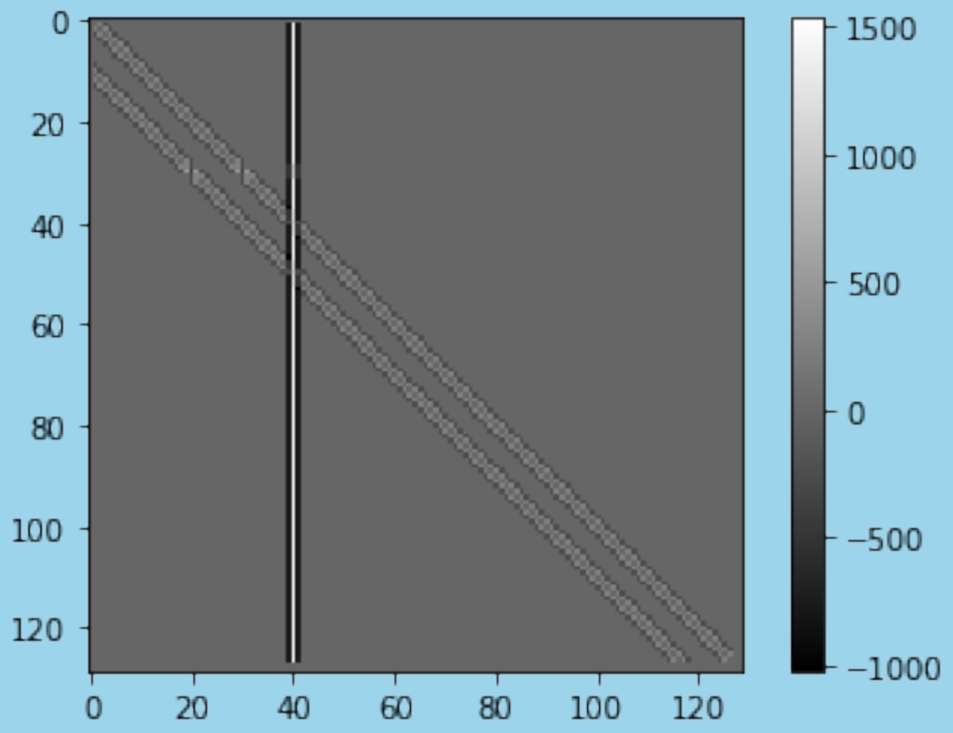
Discrete Convolutions $C = A \circledast h$

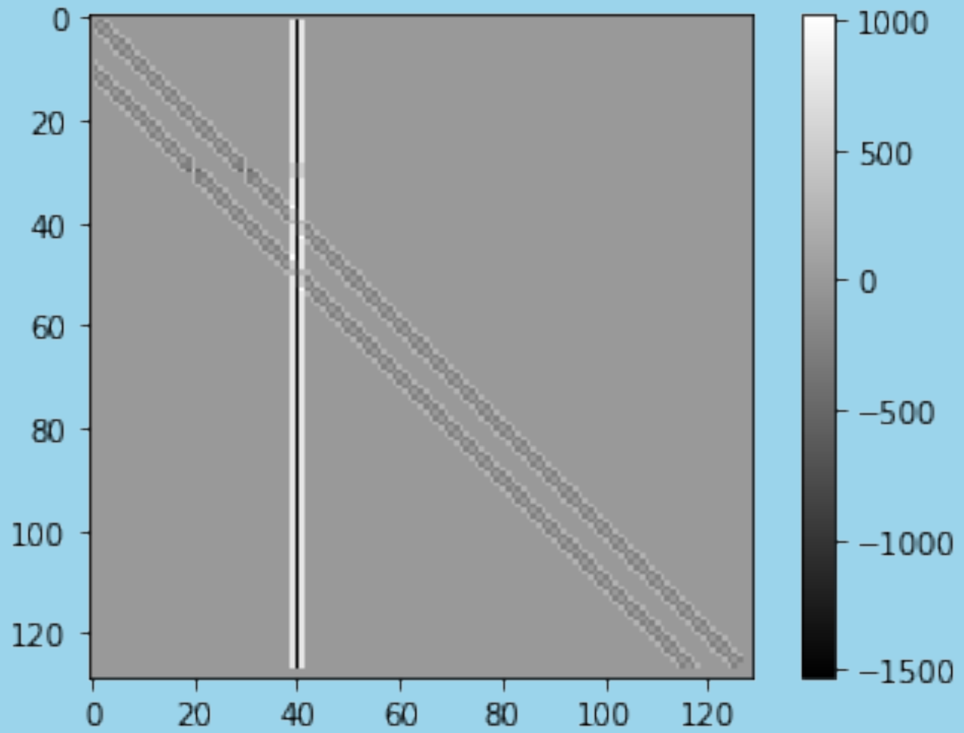
Filter	Stride	Edges
(3x3)	1	None



Discrete Convolutions $C = A \circledast h$

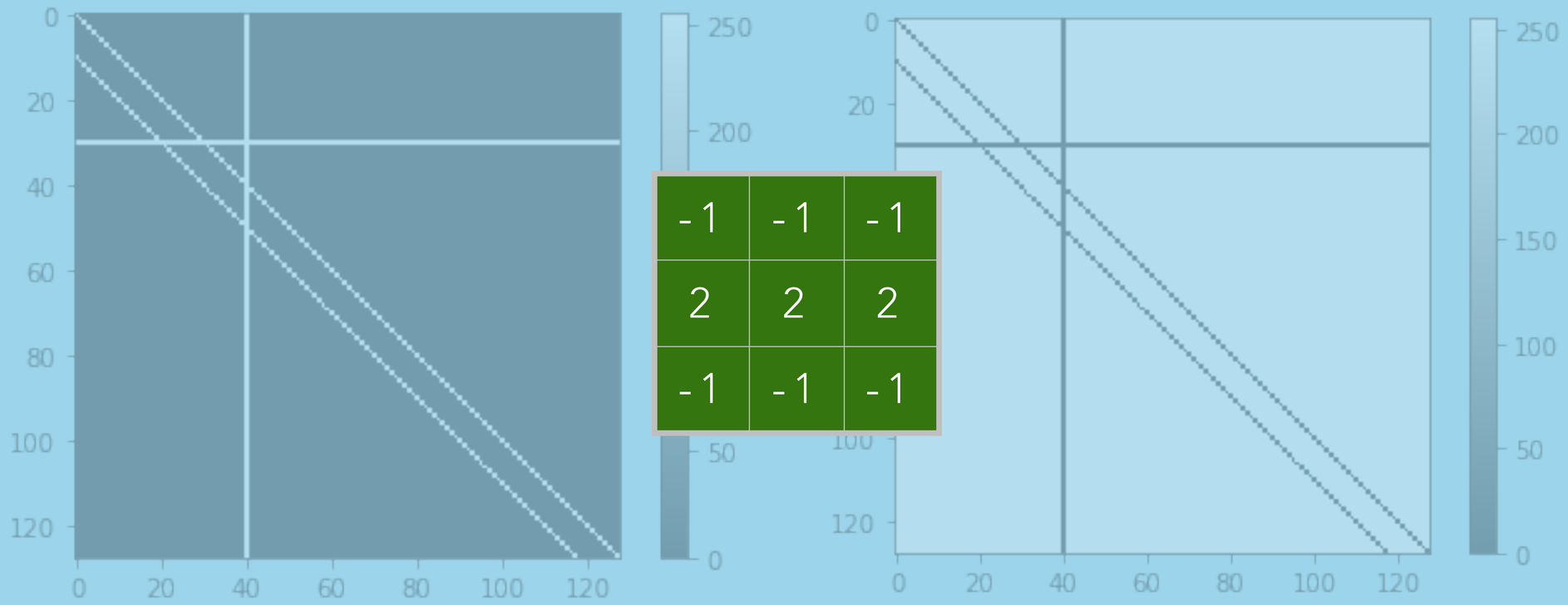
Filter	Stride	Edges
(3x3)	1	None





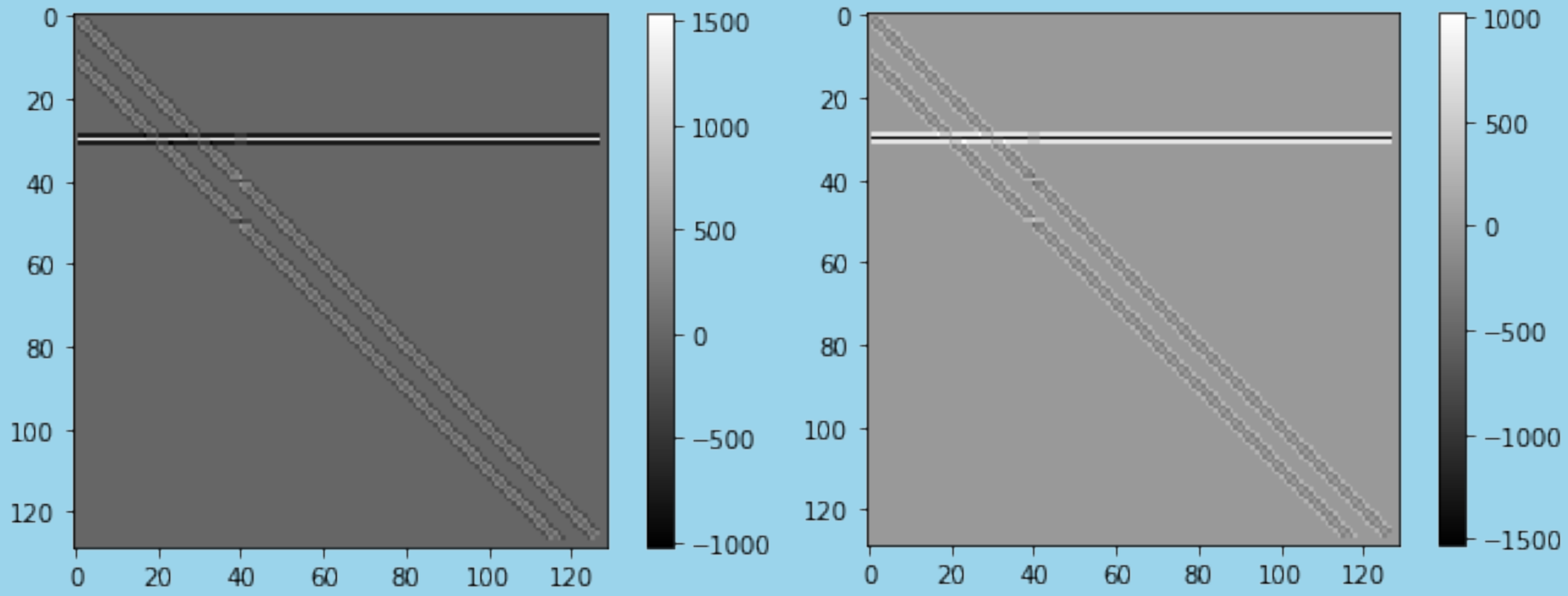
Discrete Convolutions $C = A \circledast h$

Filter	Stride	Edges
(3x3)	1	None



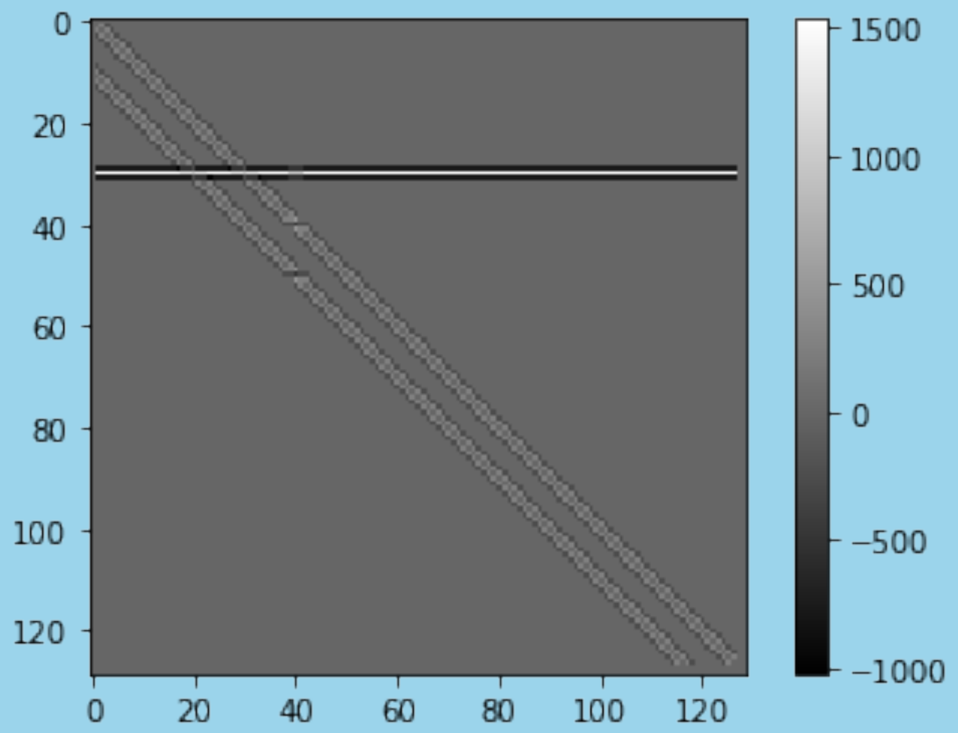
Discrete Convolutions $C = A \circledast h$

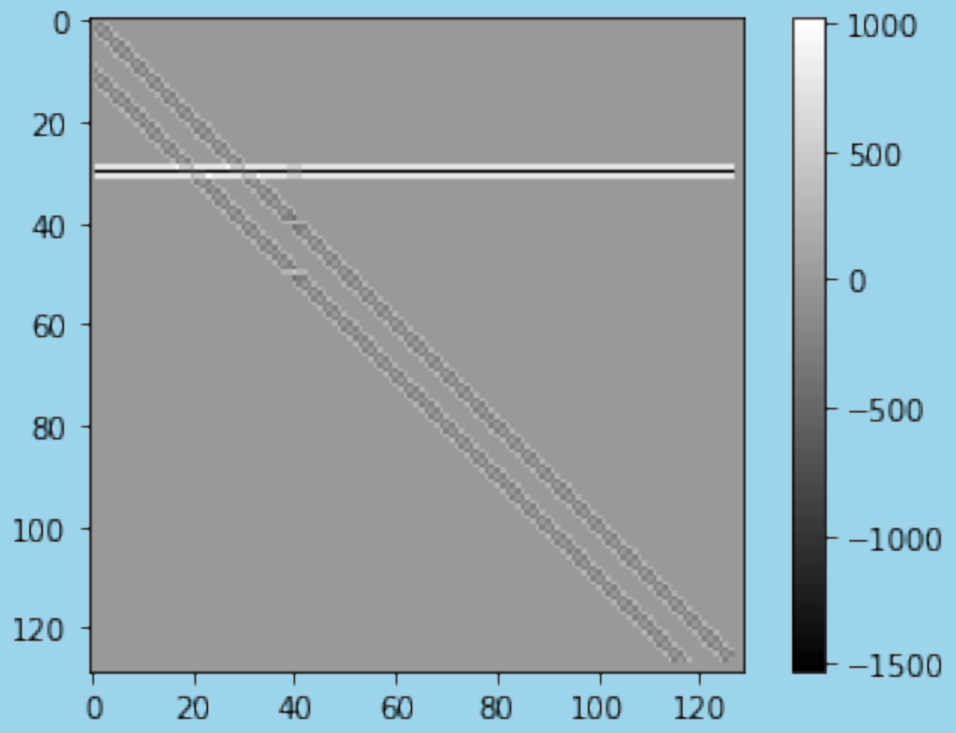
Filter	Stride	Edges
(3x3)	1	None



Discrete Convolutions $C = A \circledast h$

Line Detection / Extraction





Discrete Convolutions $C = A \circledast h$

Photograph

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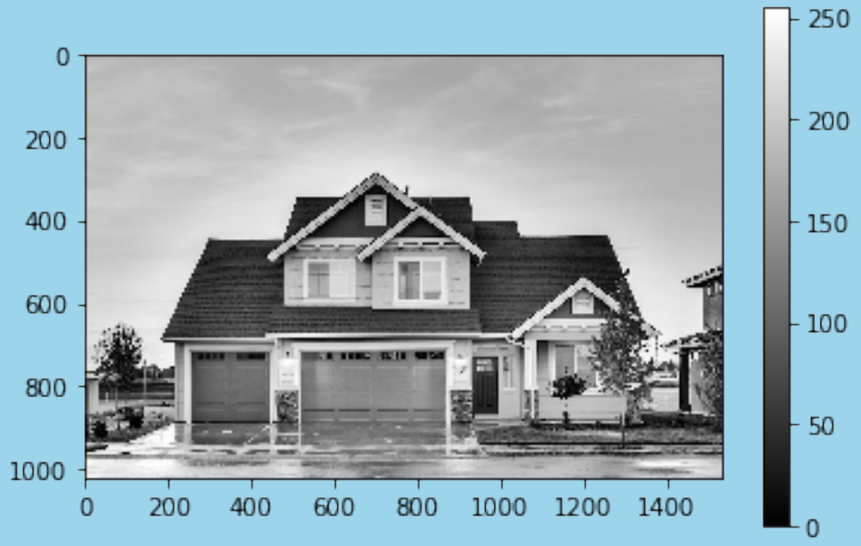
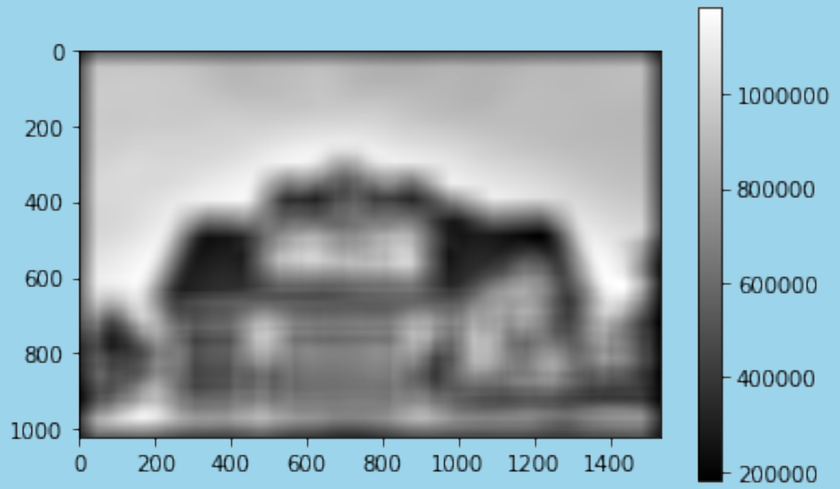
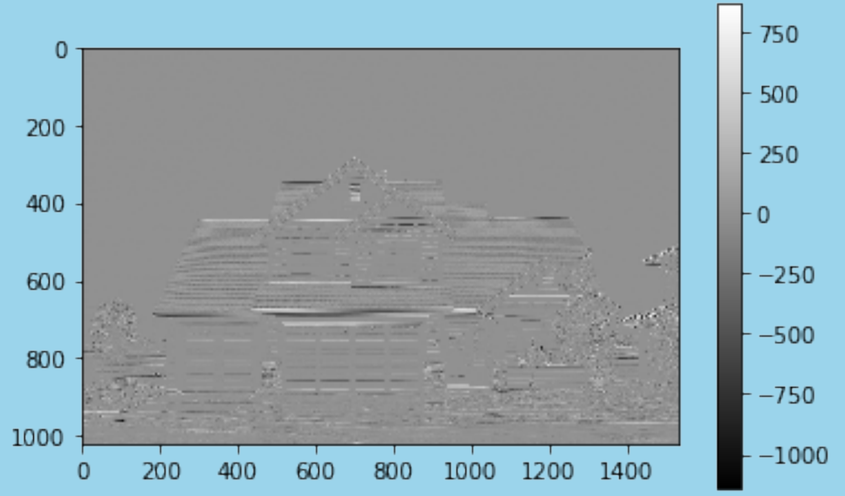
Photograph

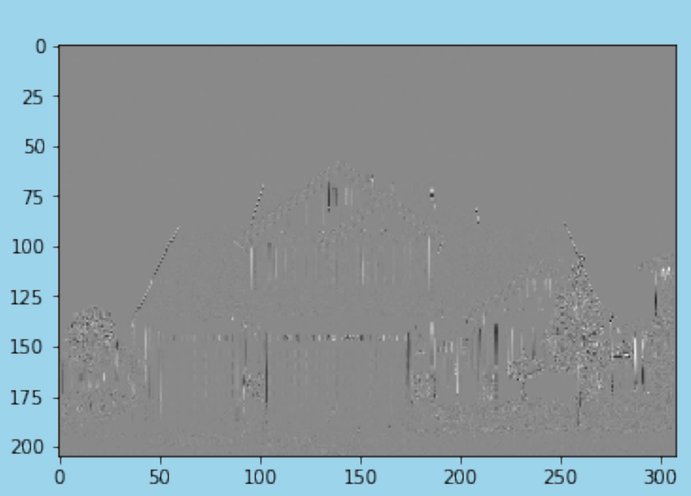
Discrete Convolutions $C = A \circledast h$



Filter (101x101)

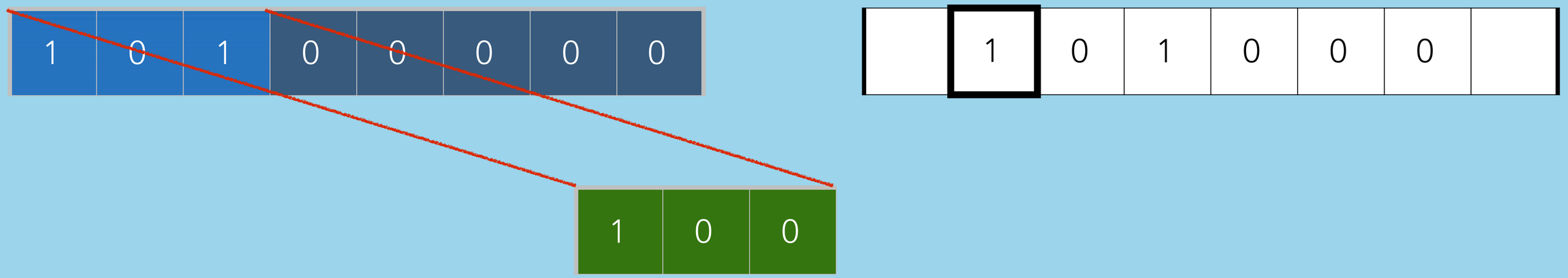






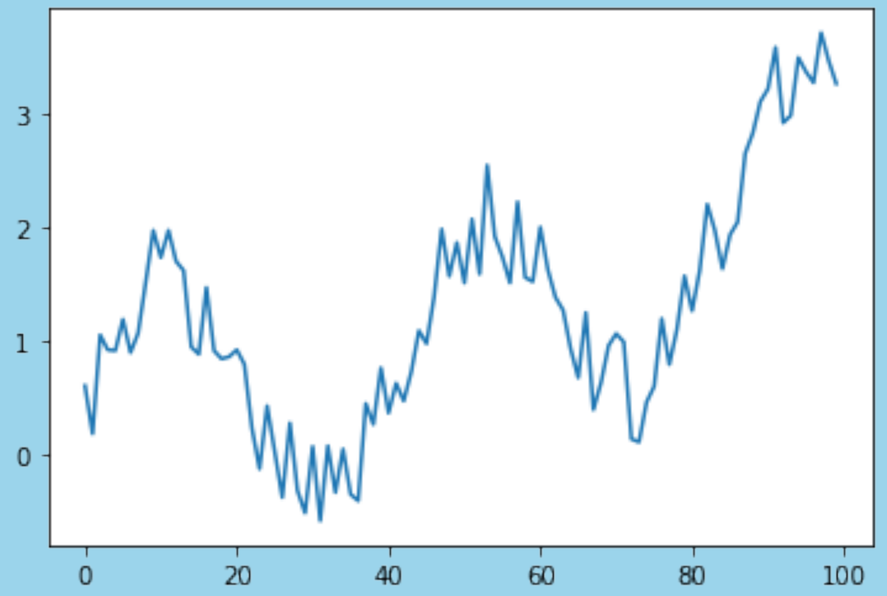
N-D Discrete Convolutions

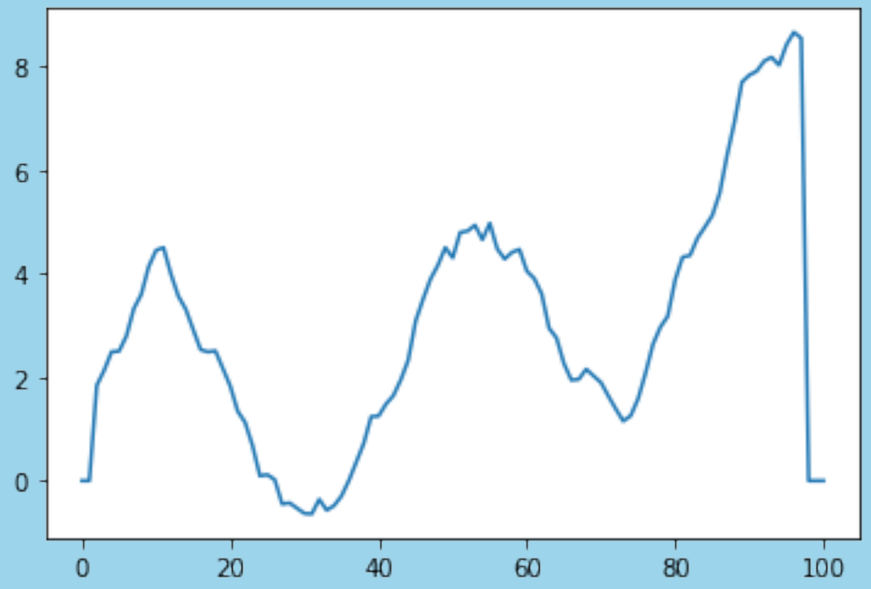
$$C[m,n] = \sum_{i=-\omega}^{\omega} h[i+\omega] * A[m+i]$$



N-D Discrete Convolutions

Filter	Stride	Edges
(1x5)	1	None





0.5	0.5	0.5	0.5	0.5

Discrete Convolutions

Discrete convolutions apply to **array** or **matrix**-like data

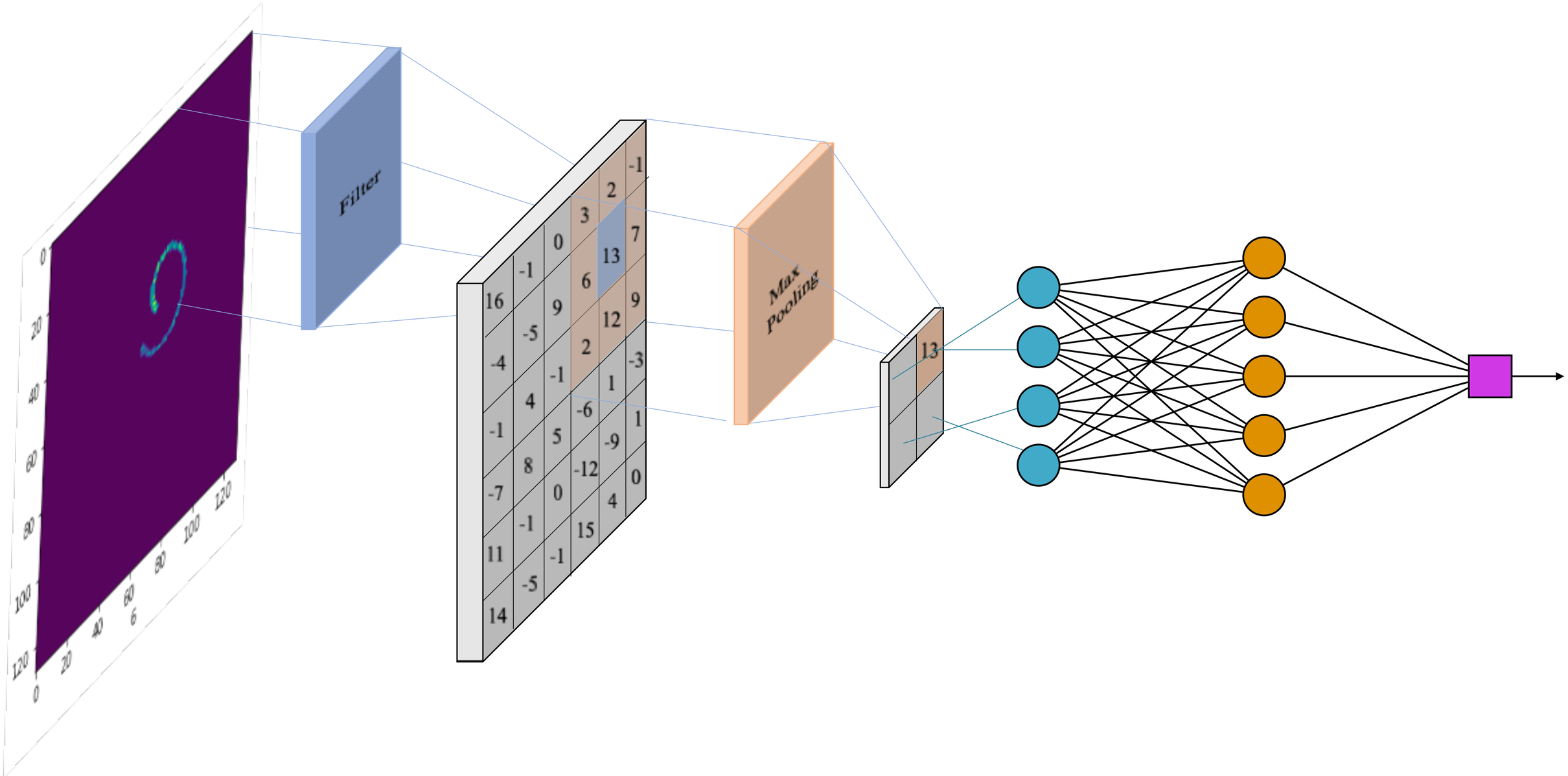
Discrete convolutions are matrix operations that can be used to apply **filters** to a matrix or array

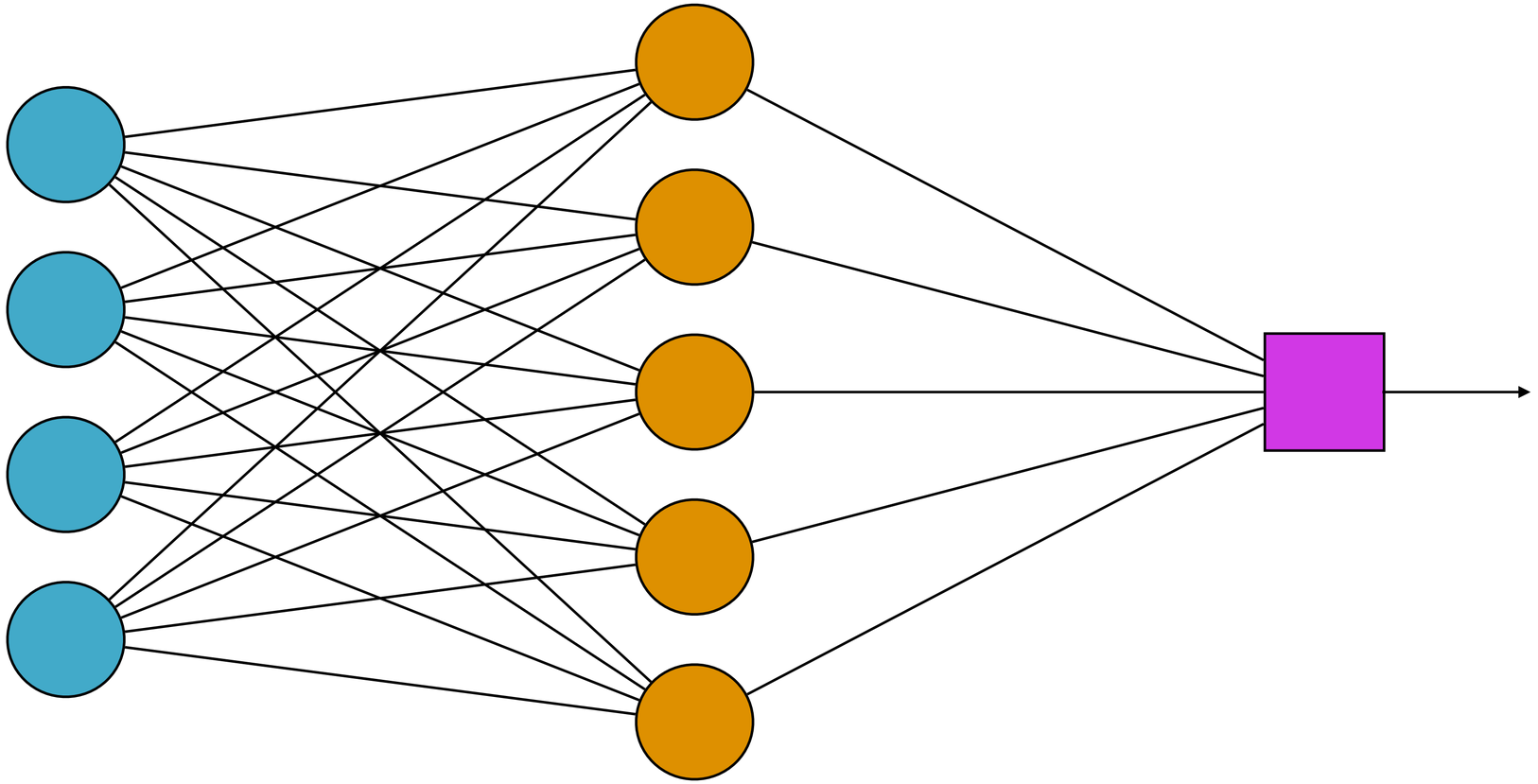
These filters can **extract features** or transform the input

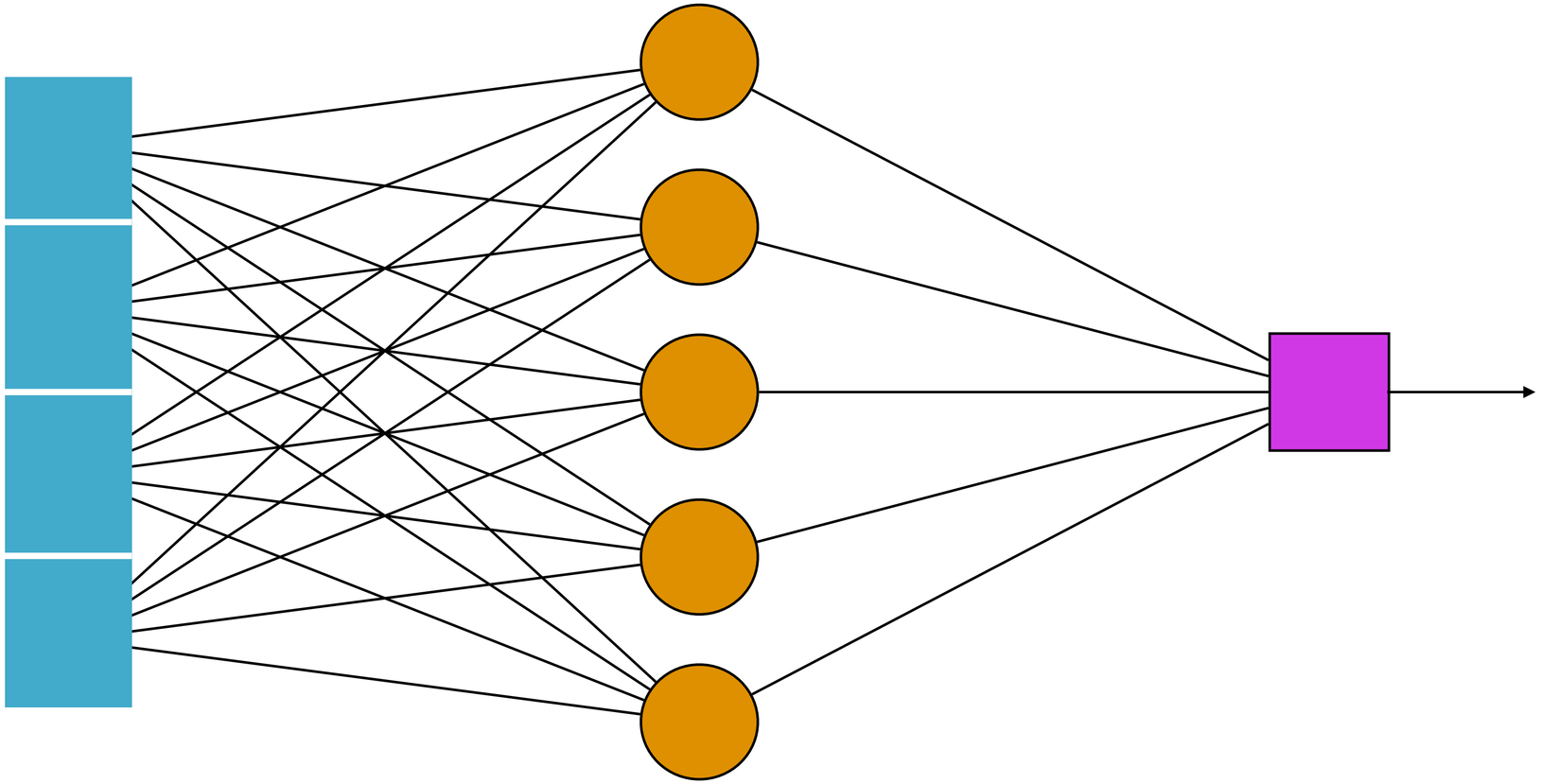
CONVOLUTIONAL NEURAL NETWORKS:

ARCHITECTURE AND TRAINING

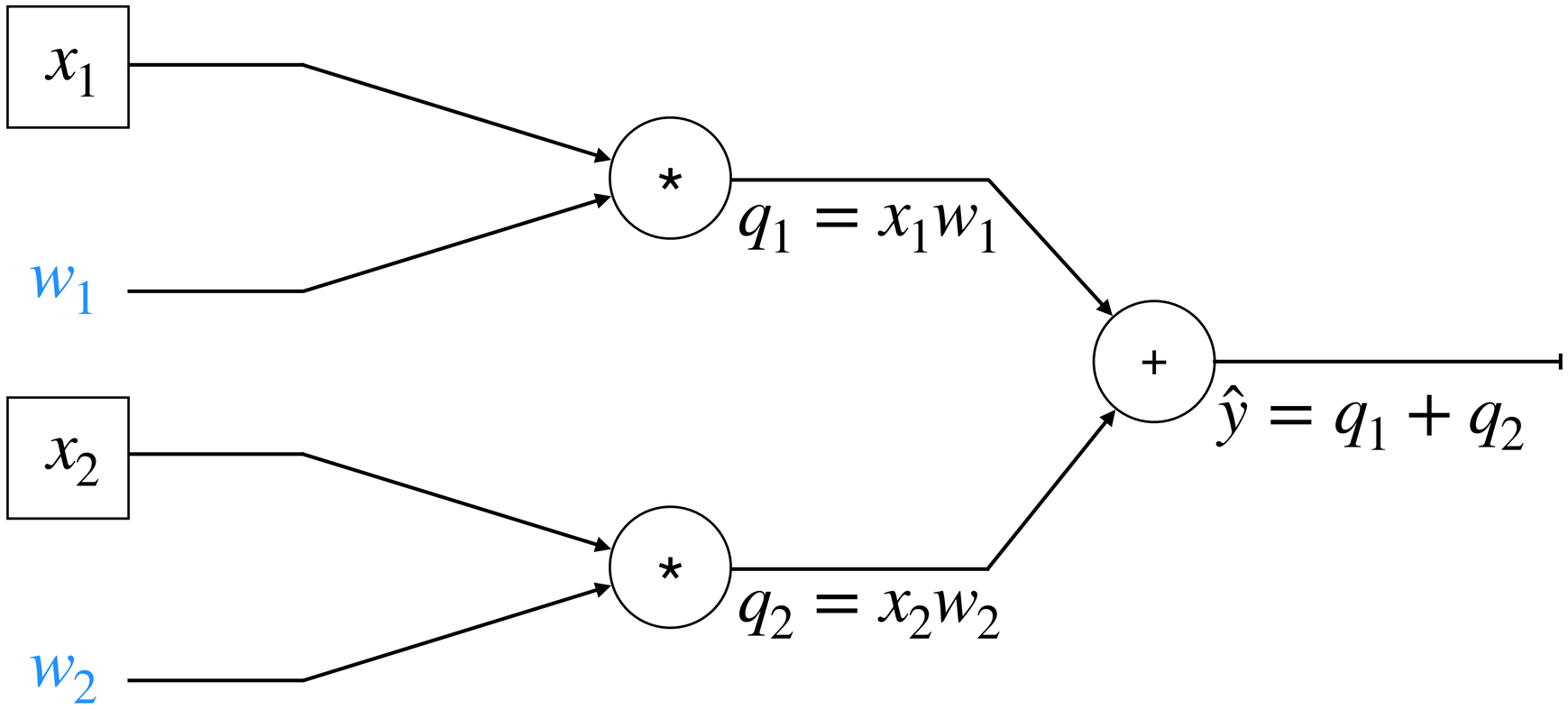
CONVOLUTIONAL NEURAL NETWORKS







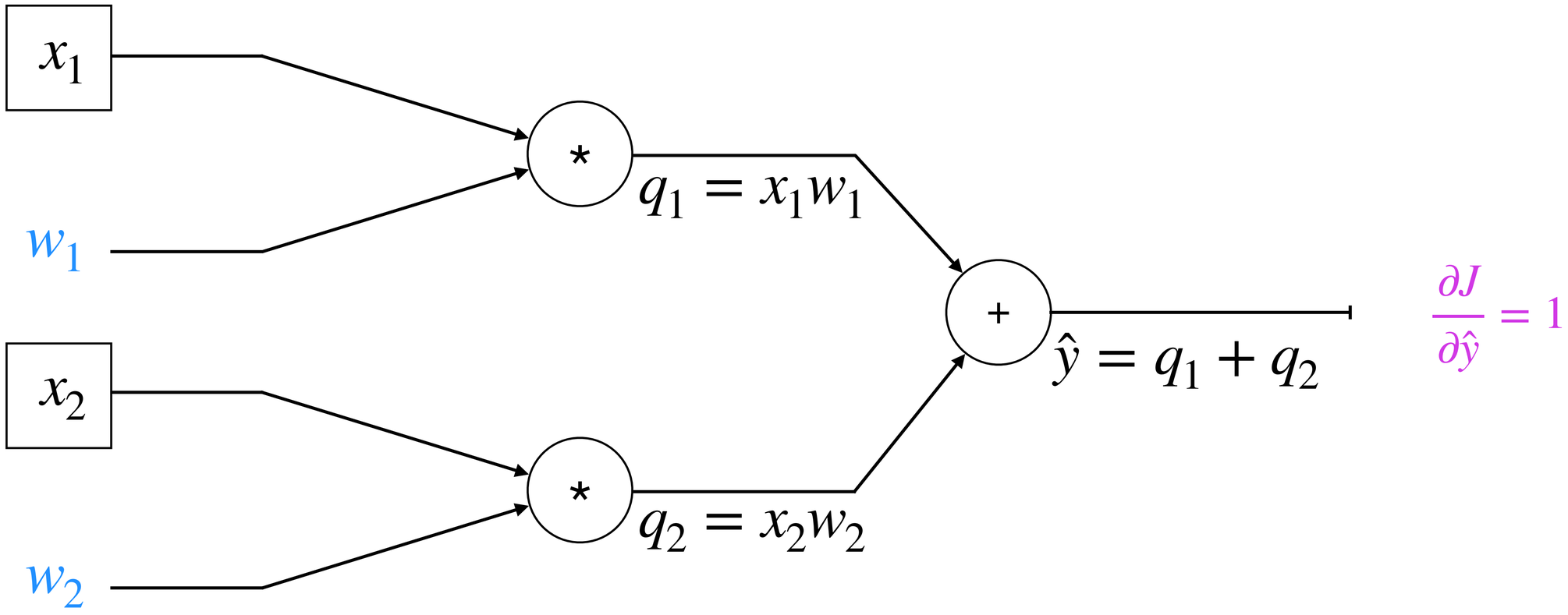
BACKPROPAGATION



Loss function

$$J(w) = \hat{y} - y$$

BACKPROPAGATION



Loss function

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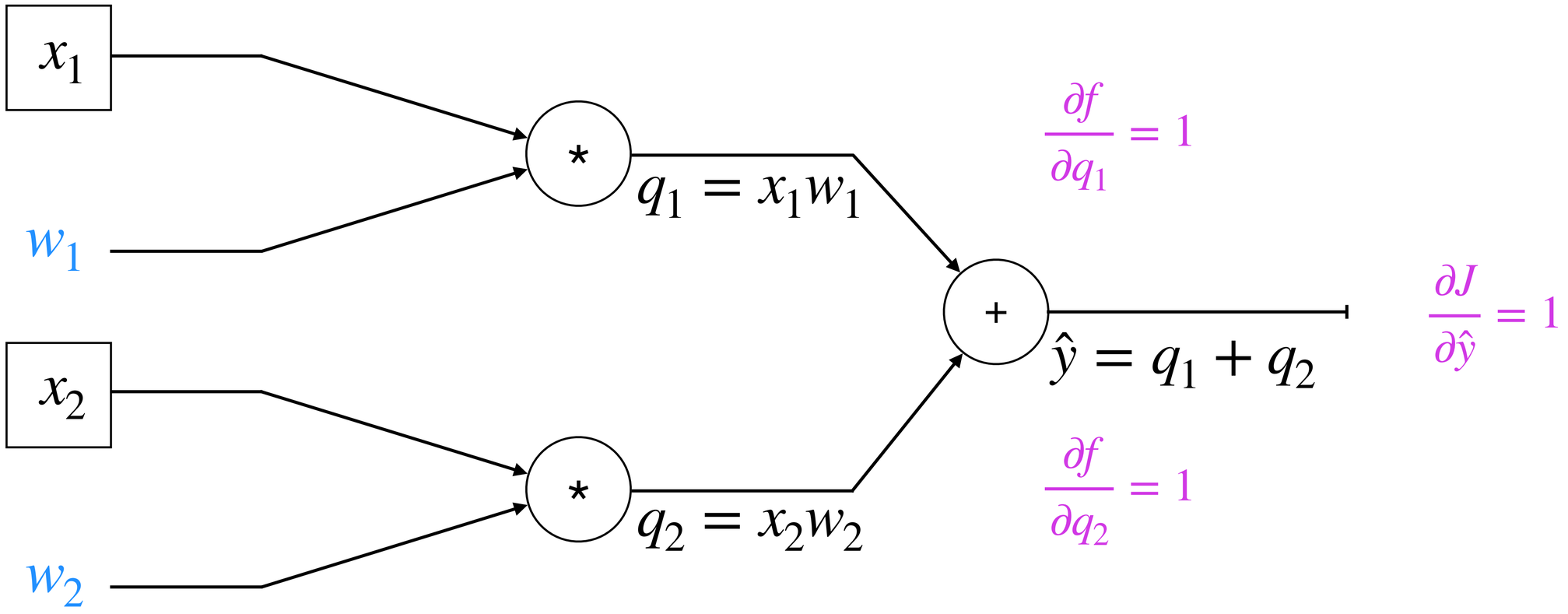
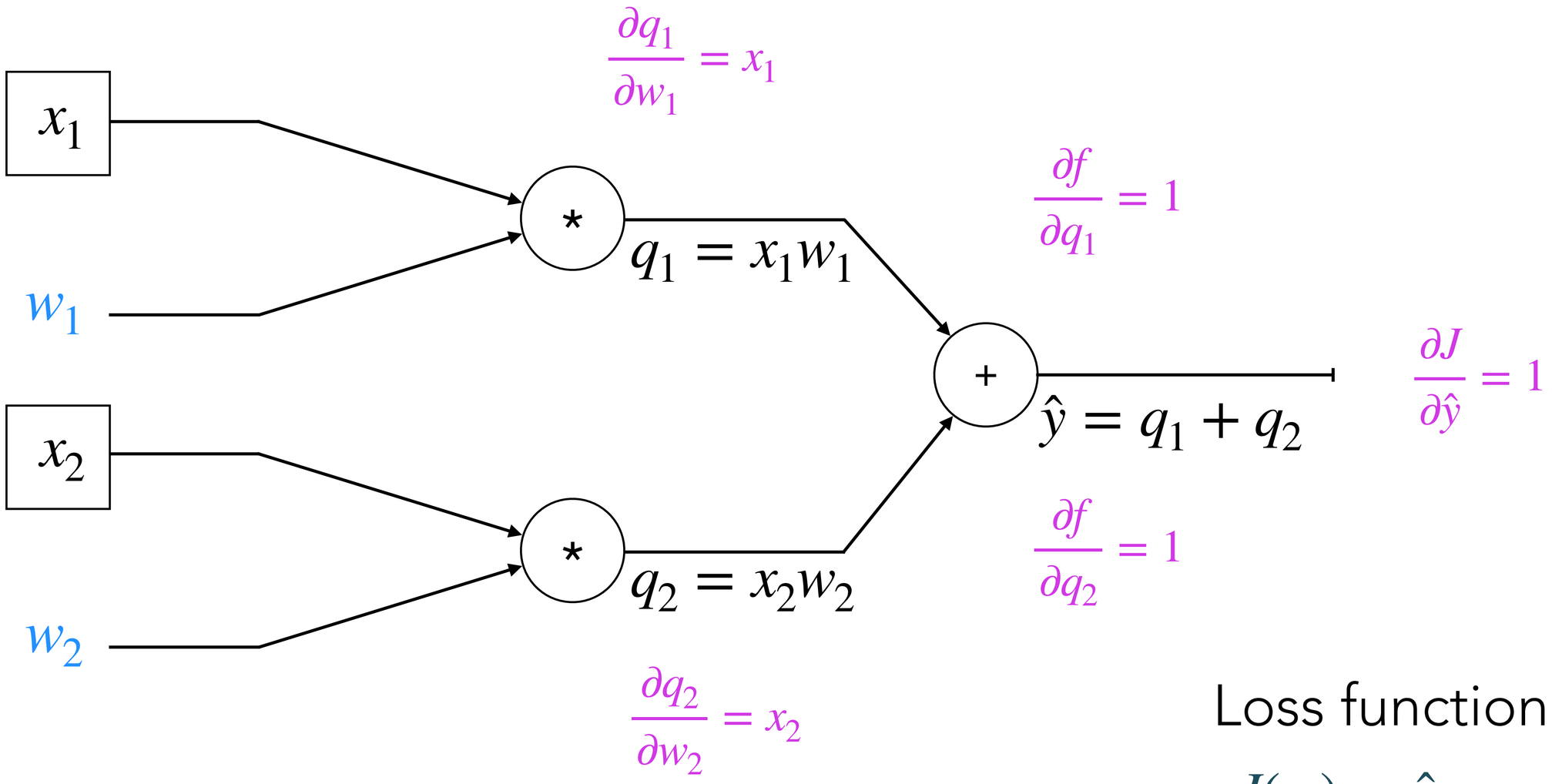
BACKPROPAGATION



Loss function

$$J(w) = \hat{y} - y$$

BACKPROPAGATION

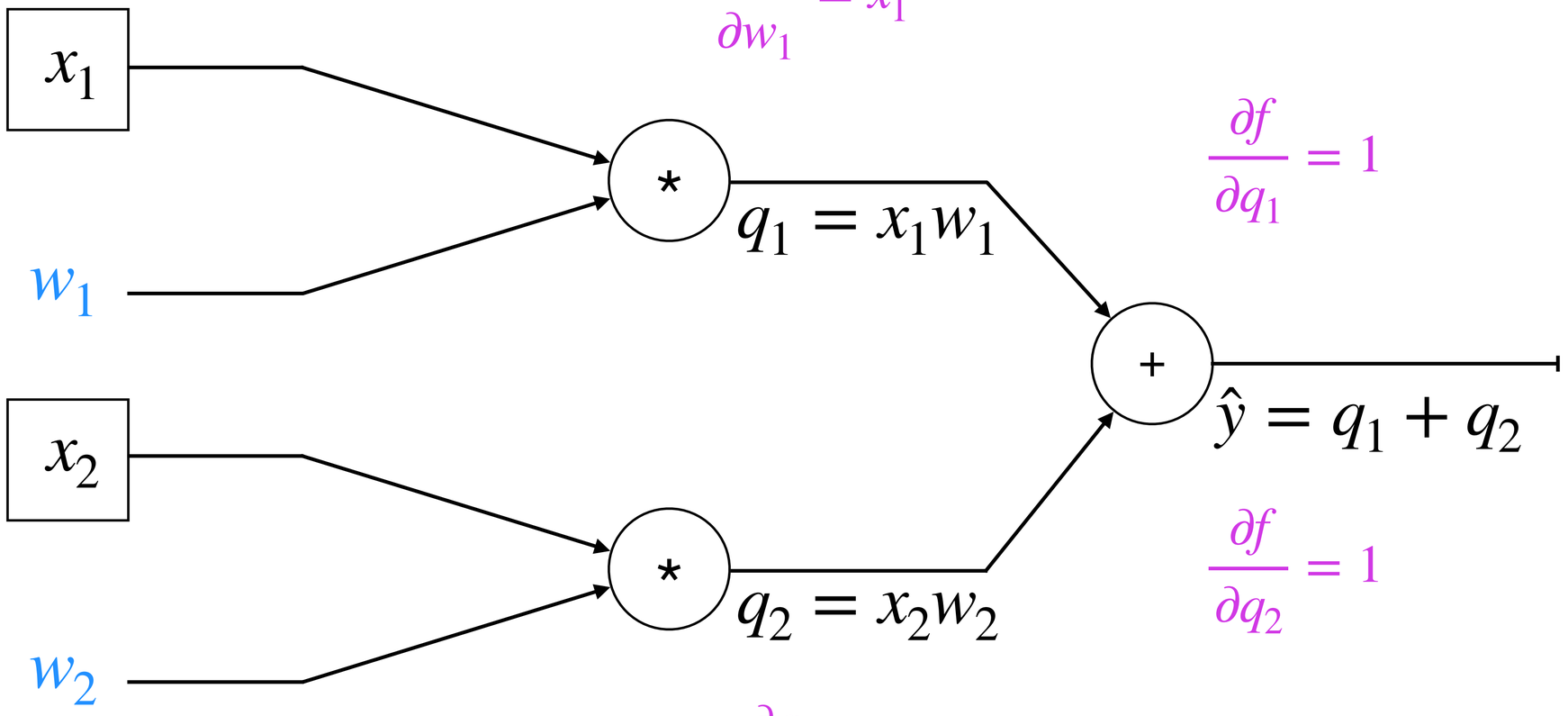


Loss function

$$J(w) = \hat{y} - y$$

BACKPROPAGATION

$$w_1 = w_1 + \eta * \frac{\partial J}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial q_1} \frac{\partial q_1}{\partial w_1}$$



$$\frac{\partial q_1}{\partial w_1} = x_1$$

$$\frac{\partial f}{\partial q_1} = 1$$

$$\frac{\partial J}{\partial \hat{y}} = 1$$

$$\frac{\partial f}{\partial q_2} = 1$$

$$\frac{\partial q_2}{\partial w_2} = x_2$$

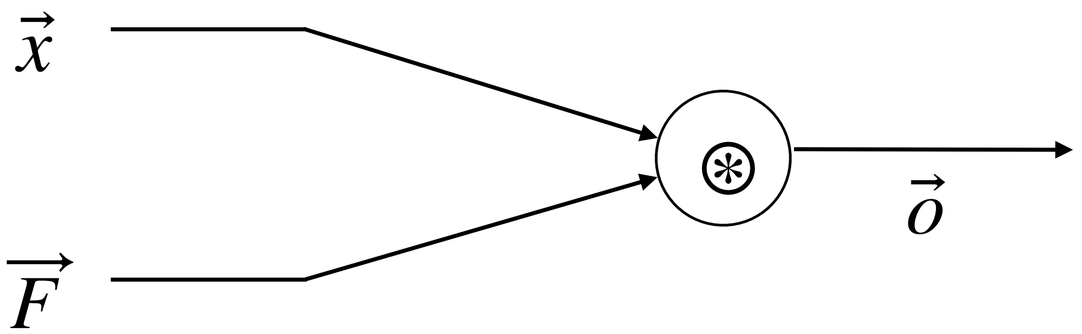
$$w_2 = w_2 + \eta * \frac{\partial J}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial q_1} \frac{\partial q_1}{\partial w_2}$$

Loss function

$$J(w) = \hat{y} - y$$

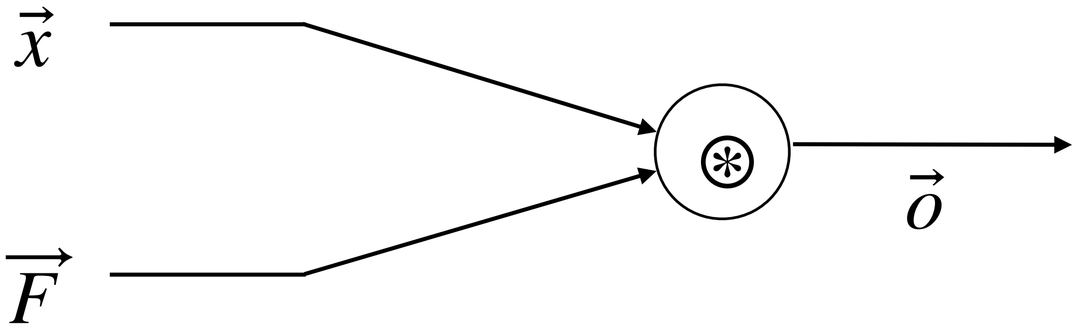
BACKPROPAGATION

$$\vec{o} = \vec{x} \circledast \overrightarrow{F}$$

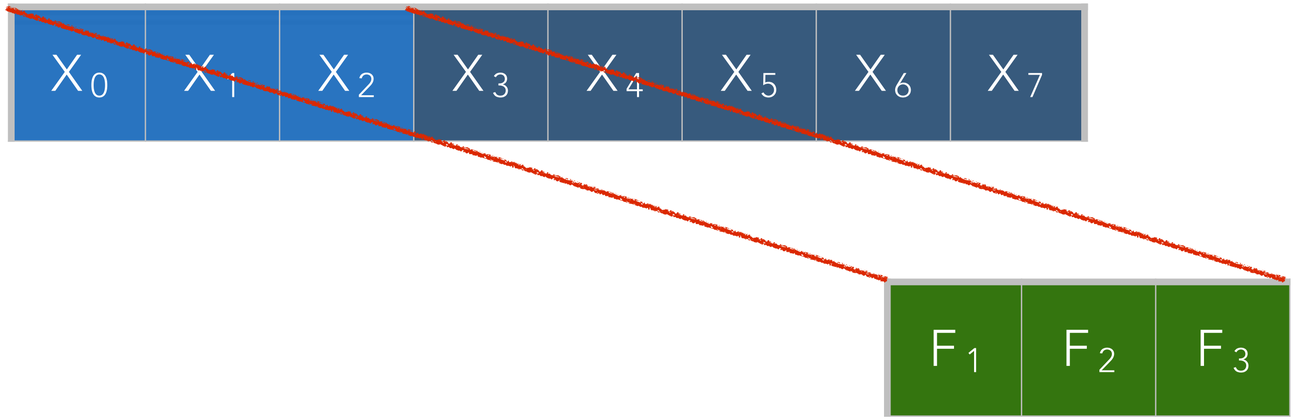


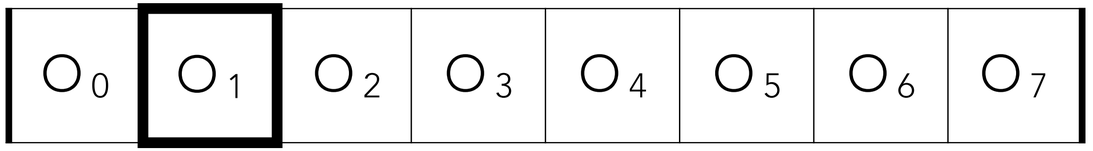
BACKPROPAGATION

$$\vec{o} = \vec{x} \circledast \vec{F}$$



$$o[n] = (x \circledast F)[n] = \sum_{i=-\omega}^{\omega} x[i+n+\omega] * F[i+\omega]$$



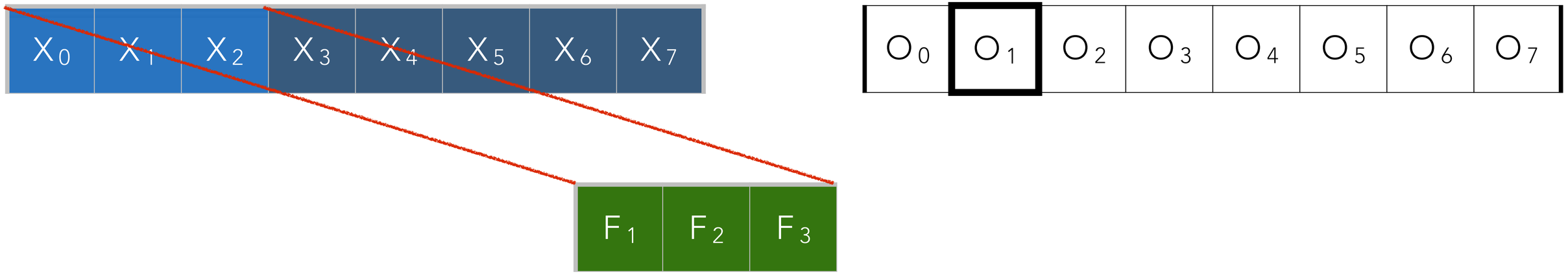


BACKPROPAGATION

$$F_i \leftarrow F_i - \eta \frac{\partial J}{\partial F_i}$$

$$\frac{\partial J}{\partial F_i} = \sum_{k=1}^{M} \frac{\partial J}{\partial o_k} \frac{\partial o_k}{\partial F_i}$$

$$o[n] = (x \circledast F)[n] = \sum_{i=-\omega}^{\omega} x[i+n+\omega] * F[i+\omega]$$



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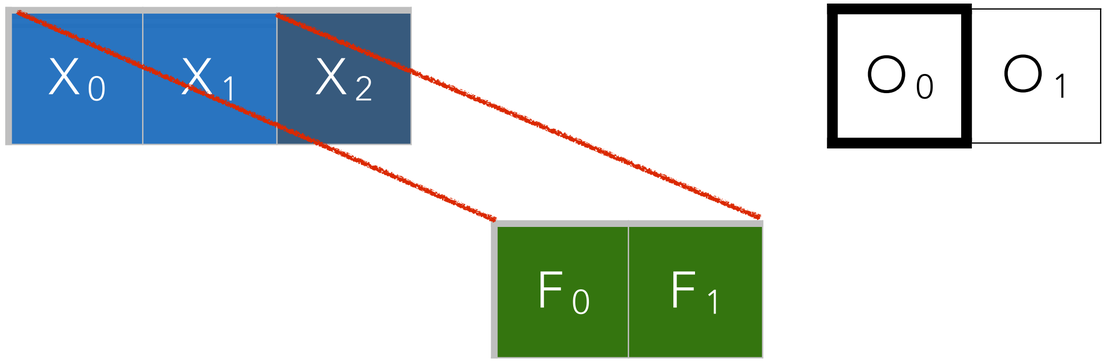


$$o_0 = F_0 x_0 + F_1 x_1$$

$$o_1 = F_0 x_1 + F_1 x_2$$

BACKPROPAGATION

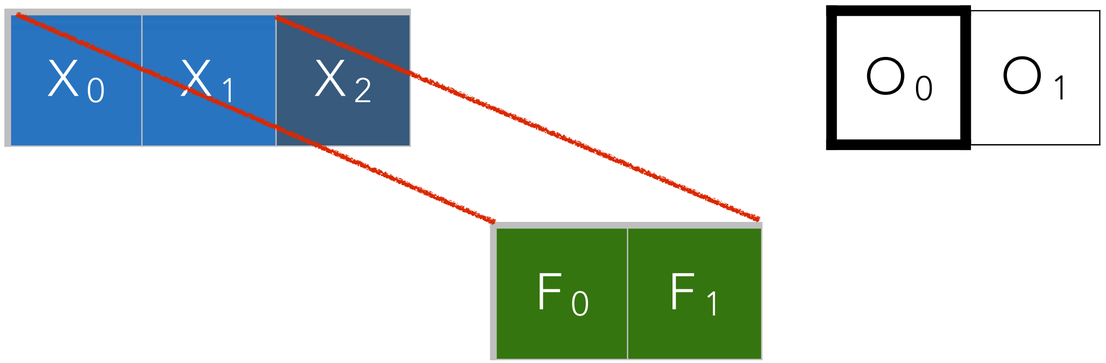
$$\frac{\partial J}{\partial F_1} = \frac{\partial J}{\partial o_0}\frac{\partial o_0}{\partial F_1} + \frac{\partial J}{\partial o_1}\frac{\partial o_1}{\partial F_1}$$

$$o_0 = F_0 x_0 + F_1 x_1$$
$$o_1 = F_0 x_1 + F_1 x_2$$

$$F_i \leftarrow F_i - \eta \frac{\partial J}{\partial F_i}$$

$$\frac{\partial J}{\partial F_i} = \sum_{k=1}^{M} \frac{\partial J}{\partial o_k}\frac{\partial o_k}{\partial F_i}$$

$$o[n] = (x \circledast F)[n] = \sum_{i=-\omega}^{\omega} x[i+n+\omega] * F[i+\omega]$$



BACKPROPAGATION

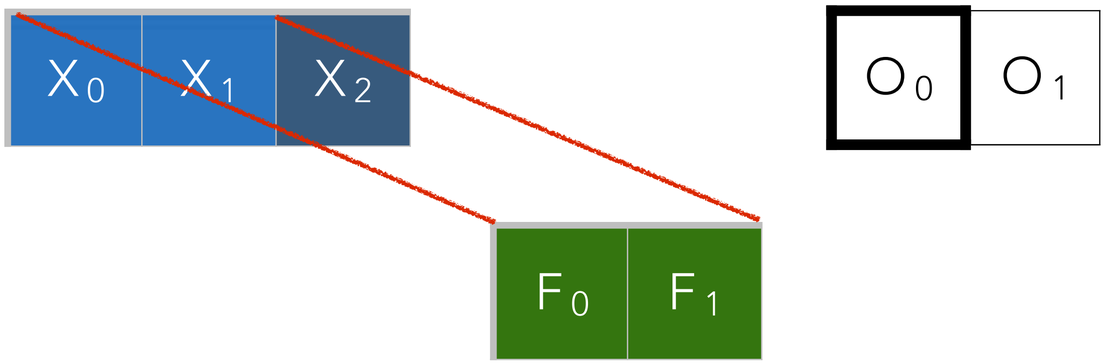
$$\frac{\partial J}{\partial F_1} = \frac{\partial J}{\partial o_0}\frac{\partial o_0}{\partial F_1} + \frac{\partial J}{\partial o_1}\frac{\partial o_1}{\partial F_1}$$

$$o_0 = F_0 x_0 + F_1 x_1$$
$$o_1 = F_0 x_1 + F_1 x_2$$

$$F_i \leftarrow F_i - \eta \frac{\partial J}{\partial F_i}$$

$$\frac{\partial J}{\partial F_i} = \sum_{k=1}^{M} \frac{\partial J}{\partial o_k}\frac{\partial o_k}{\partial F_i}$$

$$o[n] = (x \circledast F)[n] = \sum_{i=-\omega}^{\omega} x[i+n+\omega] * F[i+\omega]$$



BACKPROPAGATION

$$\frac{\partial J}{\partial F_1} = \frac{\partial J}{\partial o_0}\frac{\partial o_0}{\partial F_1} + \frac{\partial J}{\partial o_1}\frac{\partial o_1}{\partial F_1}$$

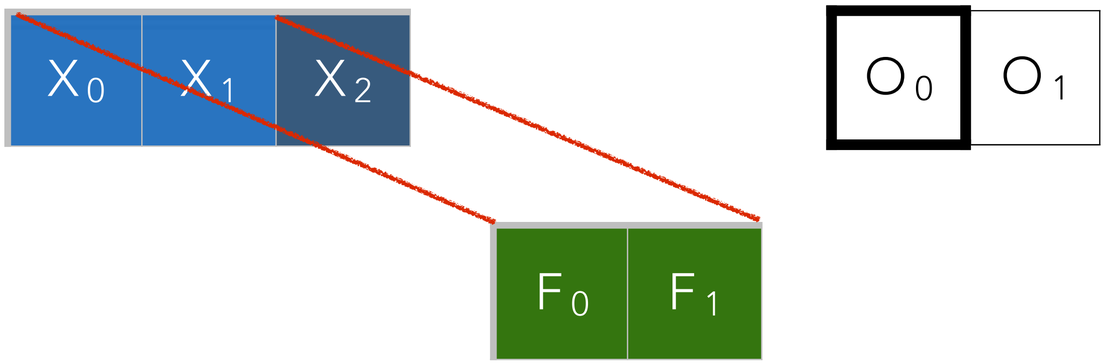
$$o_0 = F_0 x_0 + F_1 x_1$$

$$o_1 = F_0 x_1 + F_1 x_2$$

$$F_i \leftarrow F_i - \eta \frac{\partial J}{\partial F_i}$$

$$\frac{\partial J}{\partial F_i} = \sum_{k=1}^{M} \frac{\partial J}{\partial o_k}\frac{\partial o_k}{\partial F_i}$$

$$o[n] = (x \circledast F)[n] = \sum_{i=-\omega}^{\omega} x[i+n+\omega] * F[i+\omega]$$



BACKPROPAGATION

$$\frac{\partial J}{\partial F_1} = \frac{\partial J}{\partial o_0}\frac{\partial o_0}{\partial F_1} + \frac{\partial J}{\partial o_1}\frac{\partial o_1}{\partial F_1}$$

$$F_i \leftarrow F_i - \eta \frac{\partial J}{\partial F_i}$$

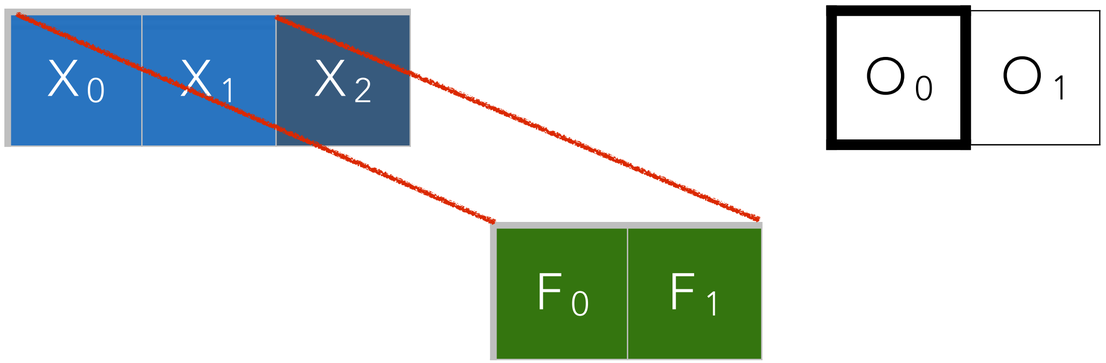
$$\frac{\partial J}{\partial F_i} = \sum_{k=1}^{M} \frac{\partial J}{\partial o_k}\frac{\partial o_k}{\partial F_i}$$

$$o_0 = F_0 x_0 + F_1 x_1$$
$$o_1 = F_0 x_1 + F_1 x_2$$

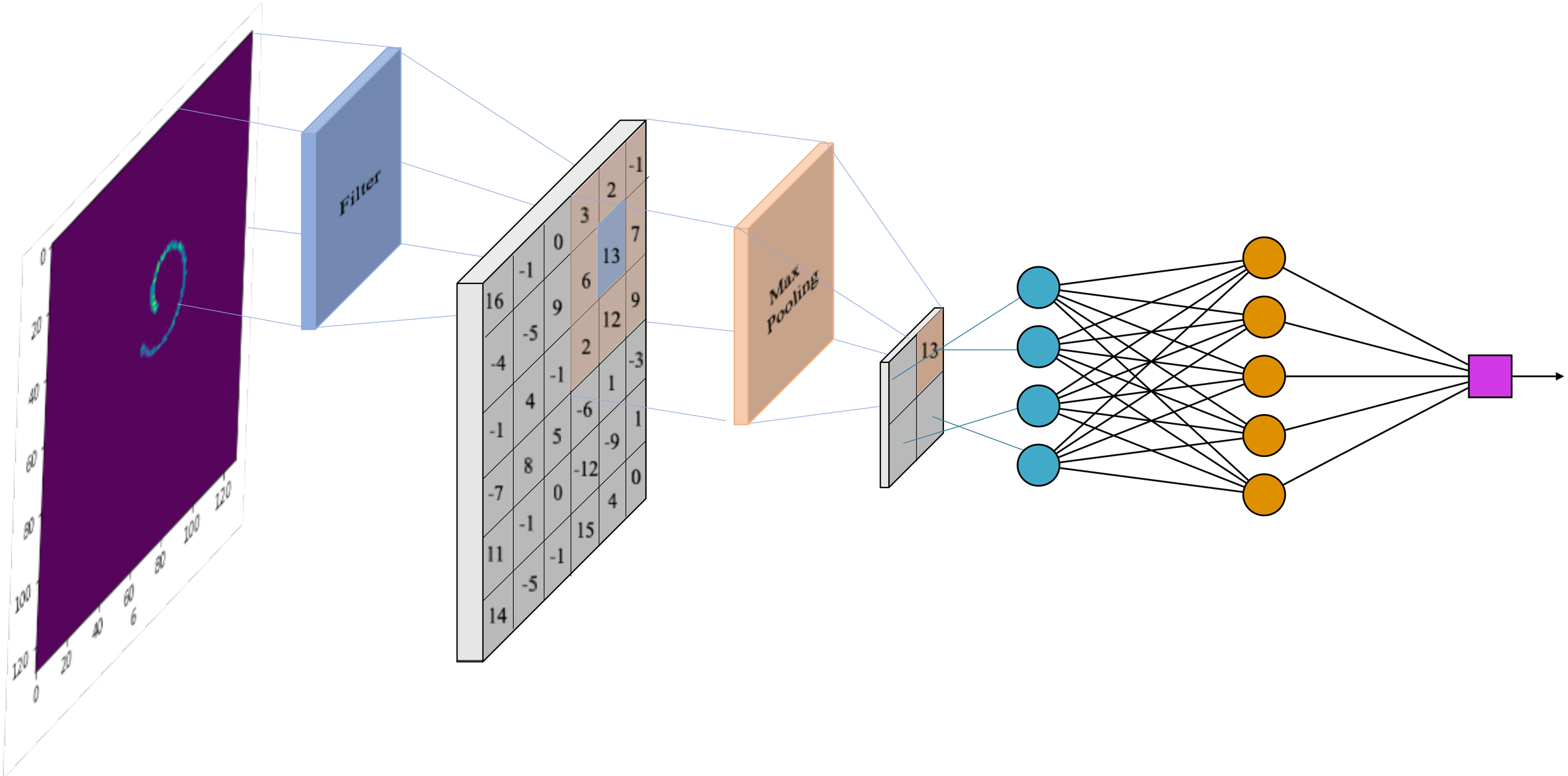
$$o[n] = (x \circledast F)[n] = \sum_{i=-\omega}^{\omega} x[i+n+\omega] * F[i+\omega]$$

$$\frac{\partial o_0}{\partial F_1} = x_1$$

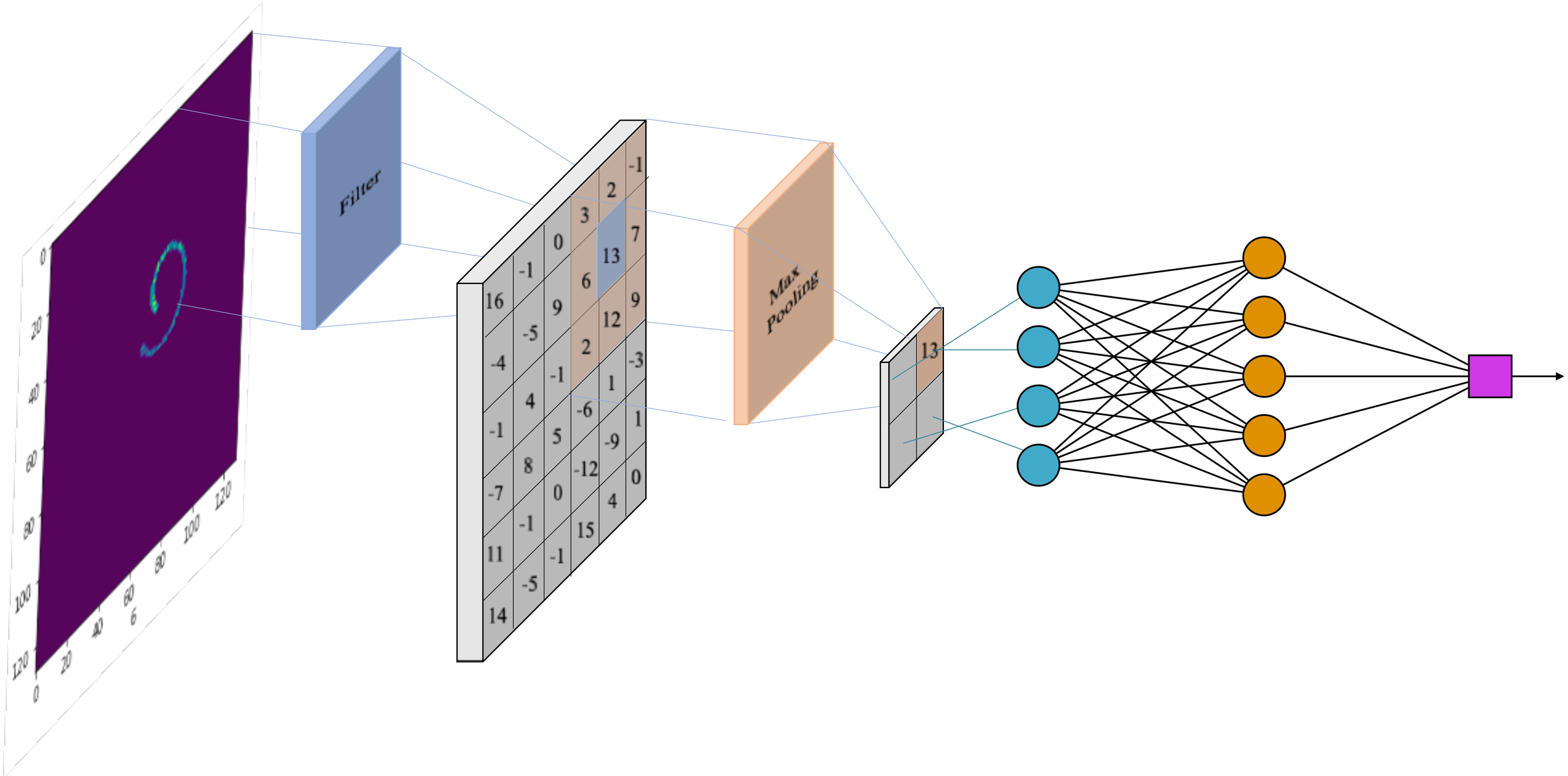
$$\frac{\partial o_1}{\partial F_1} = x_2$$



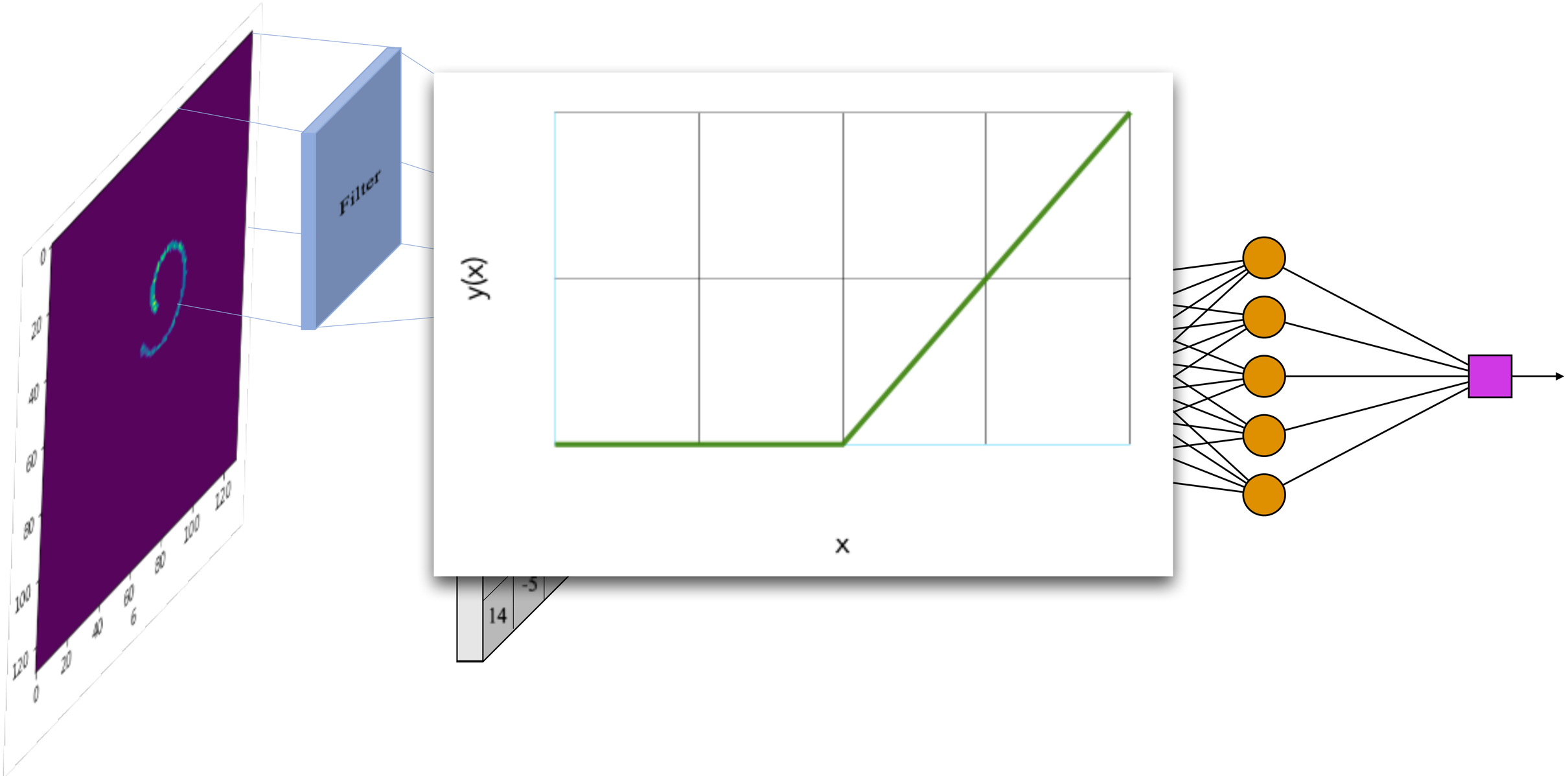
CONVOLUTIONAL NEURAL NETWORKS



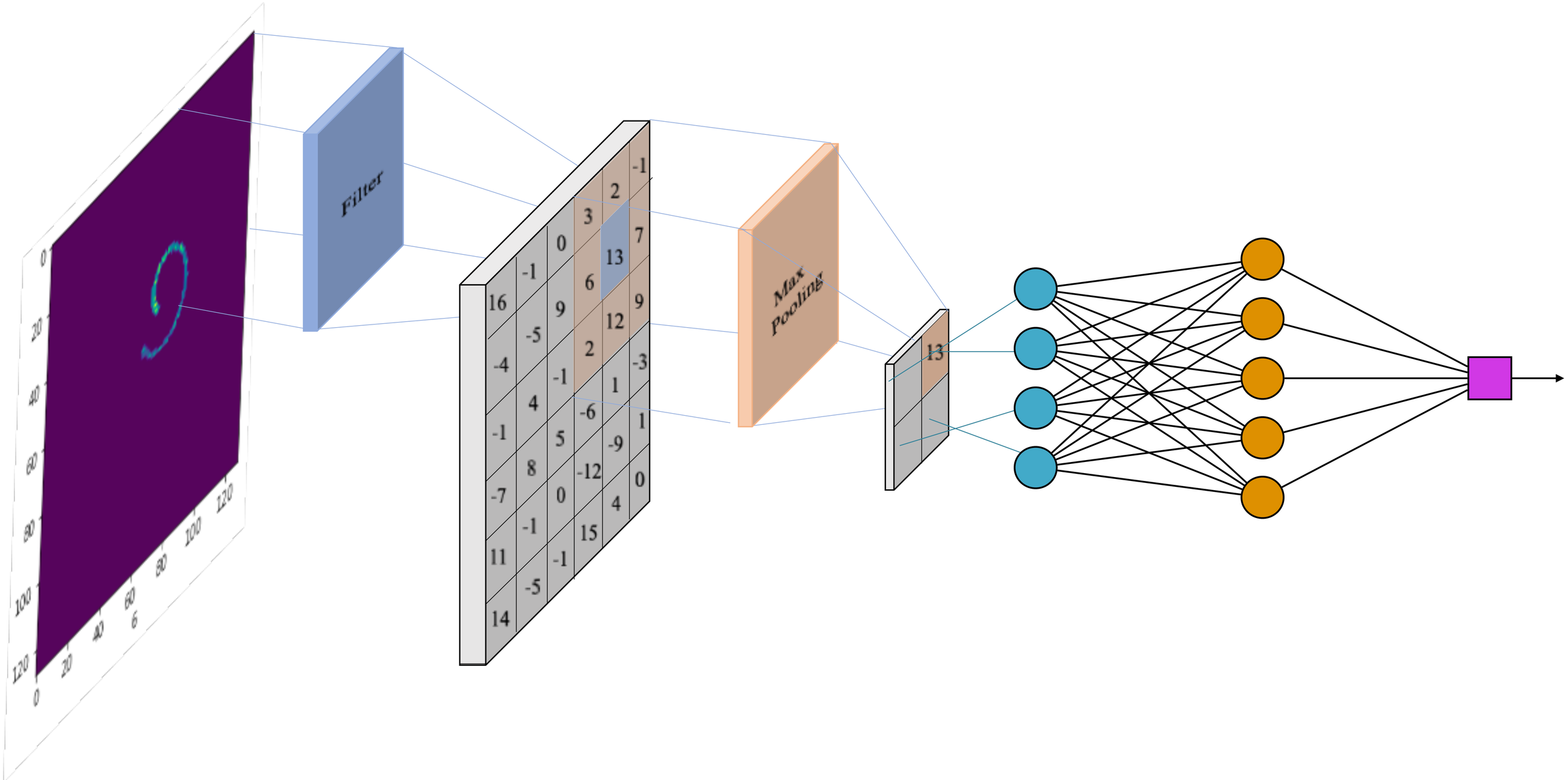
CONVOLUTIONAL NEURAL NETWORKS



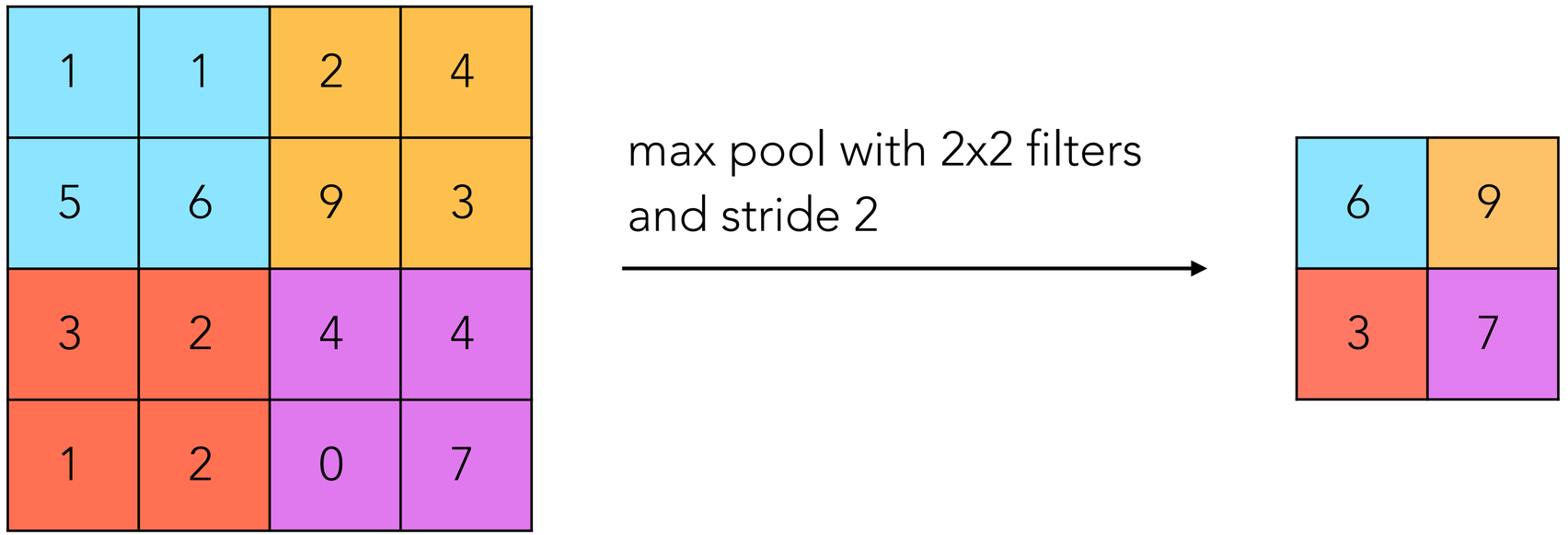
CONVOLUTIONAL NEURAL NETWORKS



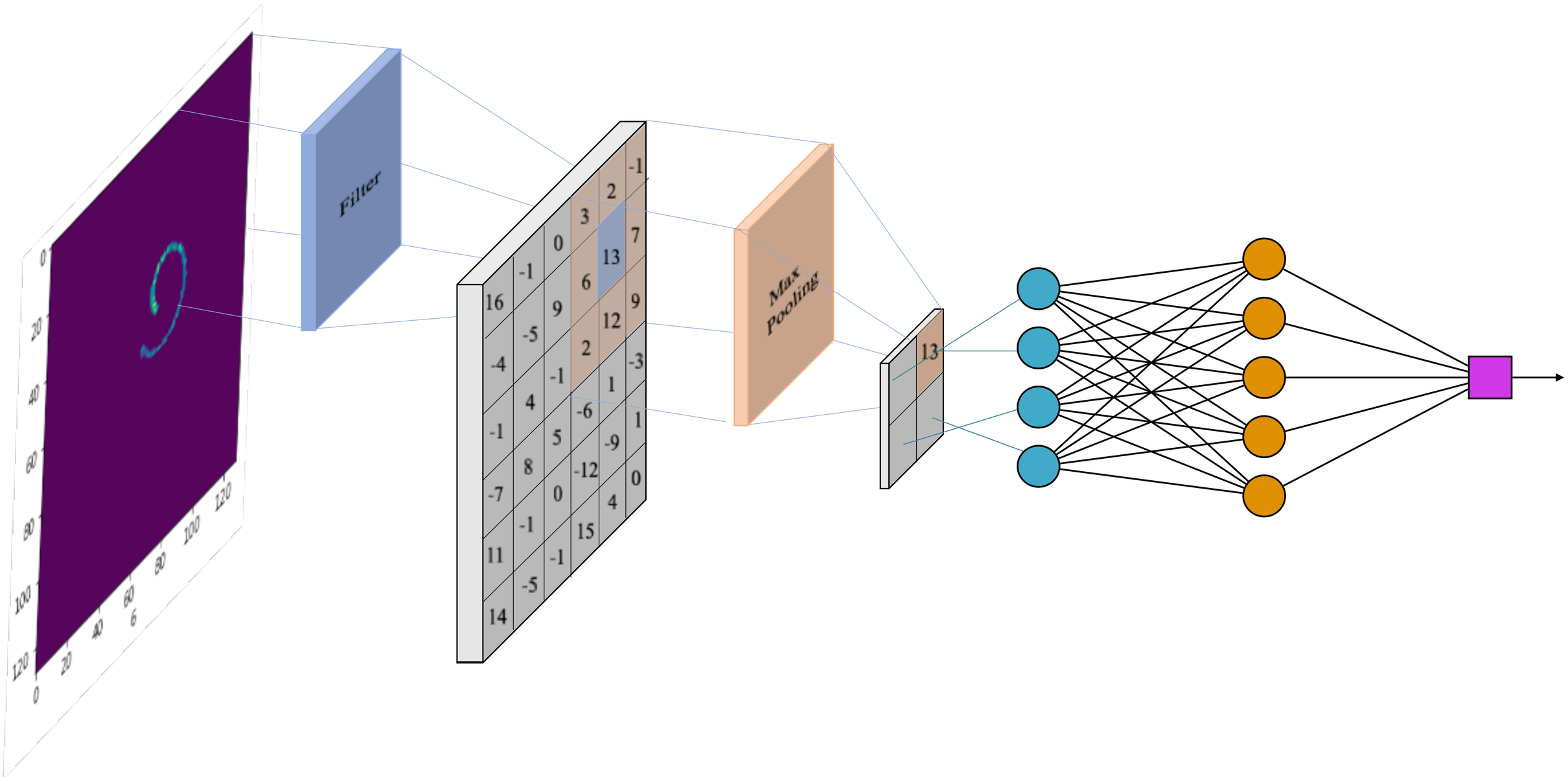
CONVOLUTIONAL NEURAL NETWORKS



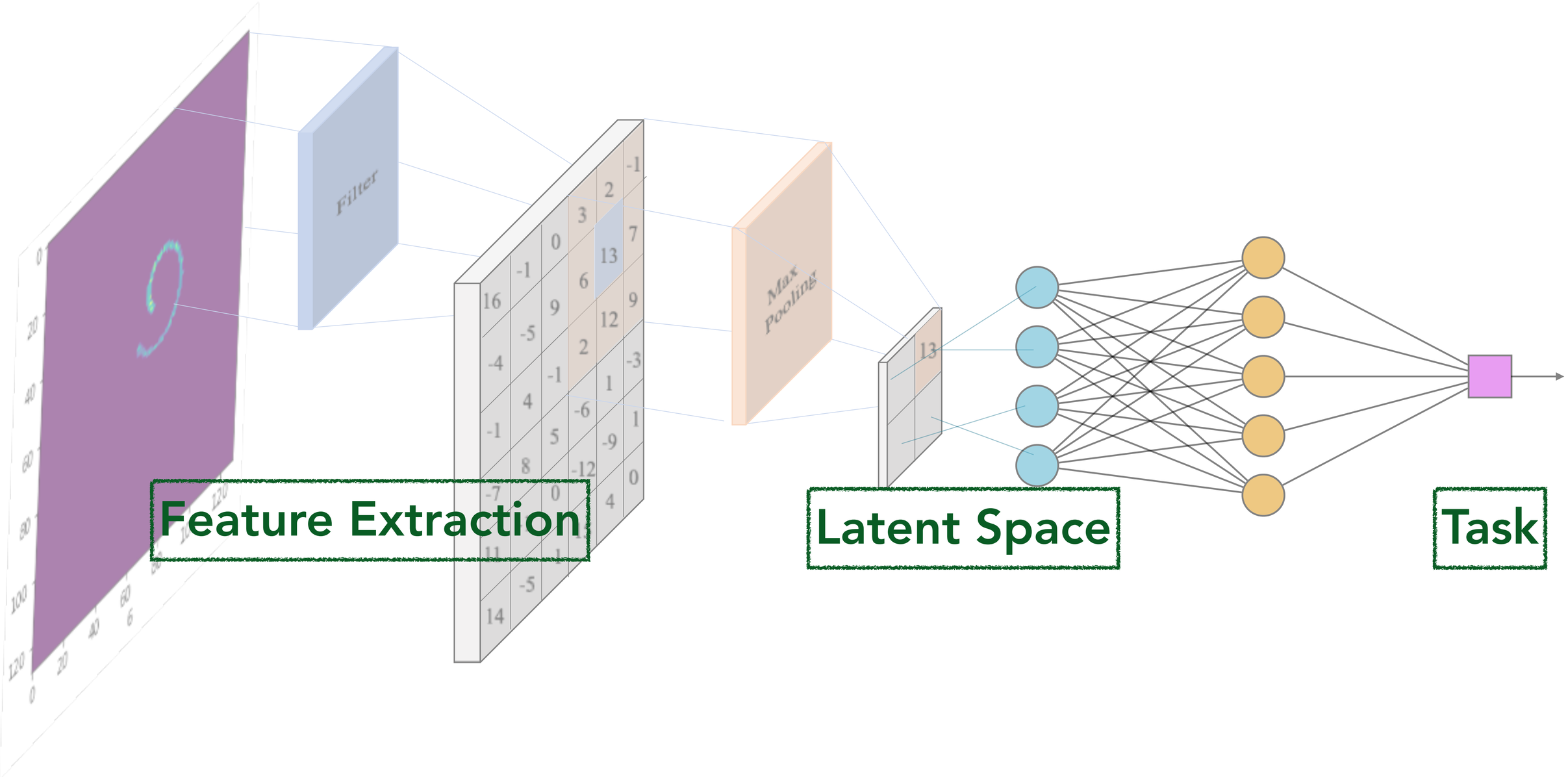
POOLING


1 1 2 4
5 6 9 3
3 2 4 4
1 2 0 7
max pool with 2x2 filters
and stride 2
6 9
3 7

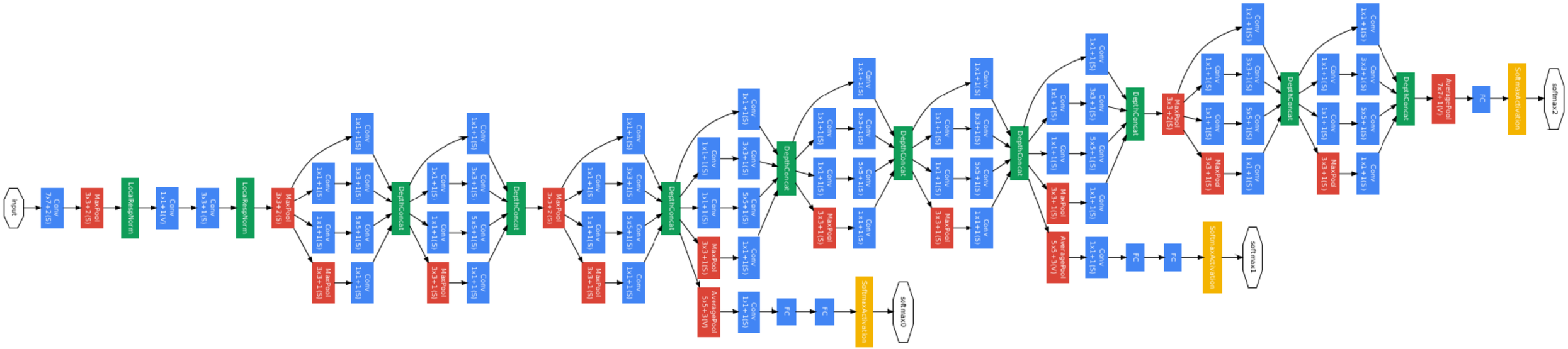
BACKPROPOGATION



CONVOLUTIONAL NEURAL NETWORKS



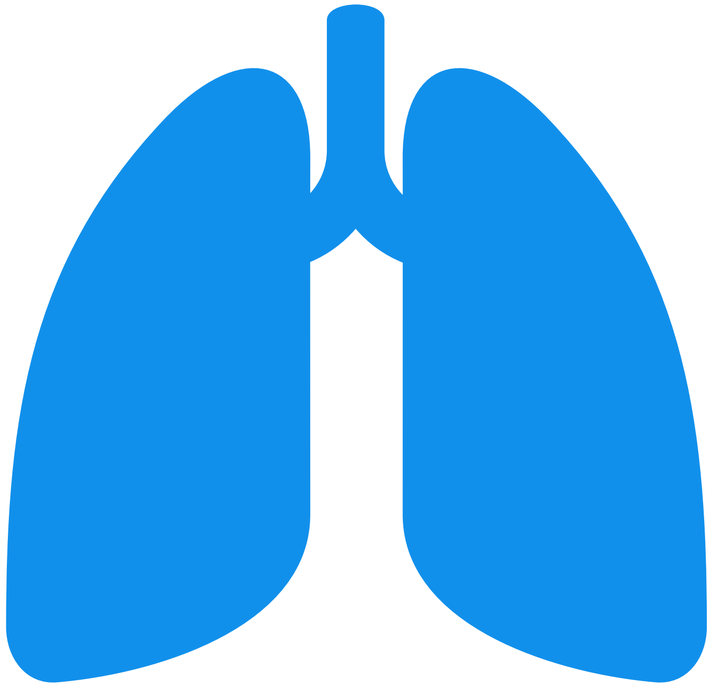
CONVOLUTIONAL NEURAL NETWORKS



CHRISTIAN SZEGEDY ET. AL. GOING DEEPER WITH CONVOLUTIONS.

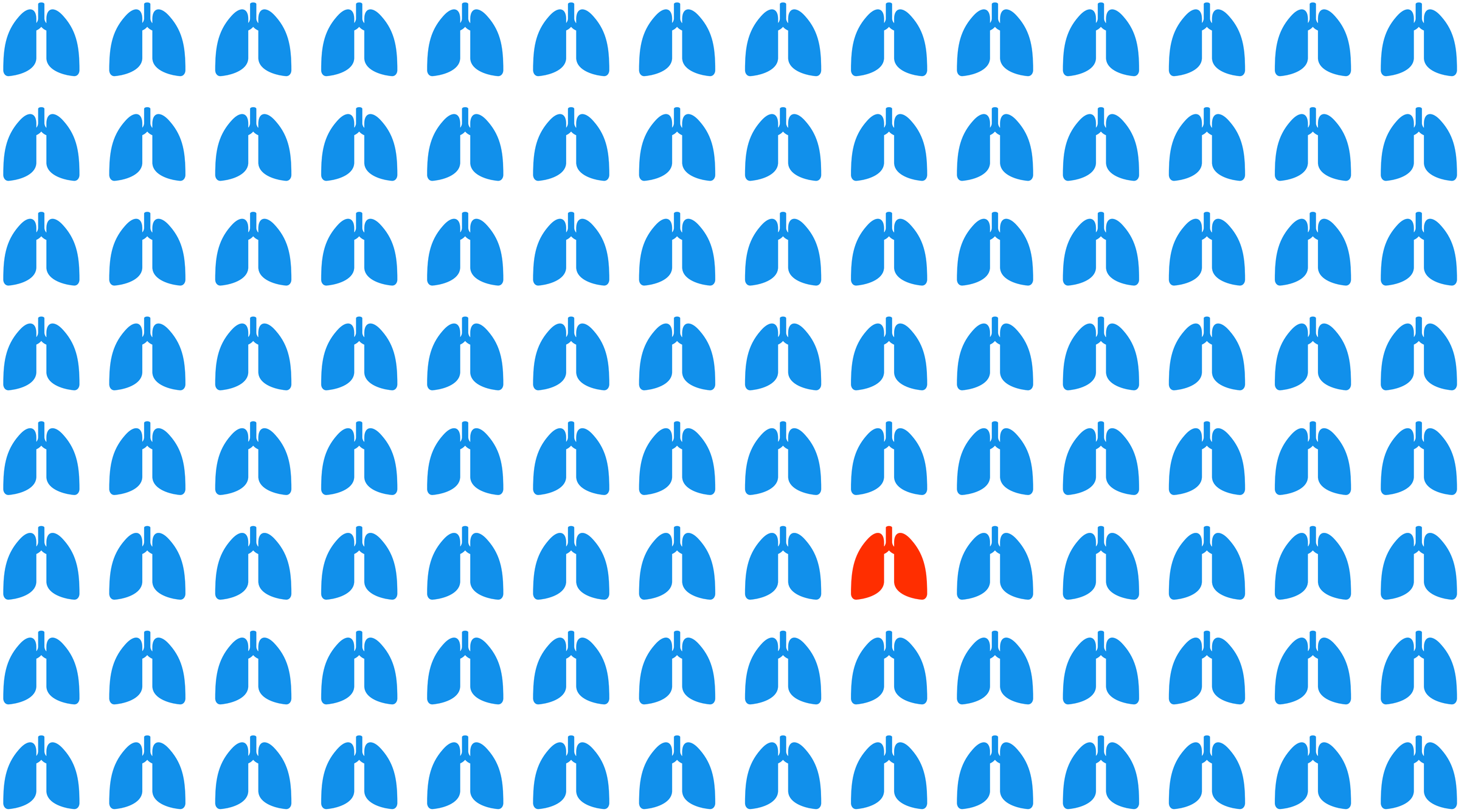
Application 2: Can we use machine learning to **accurately** classify events in detectors?

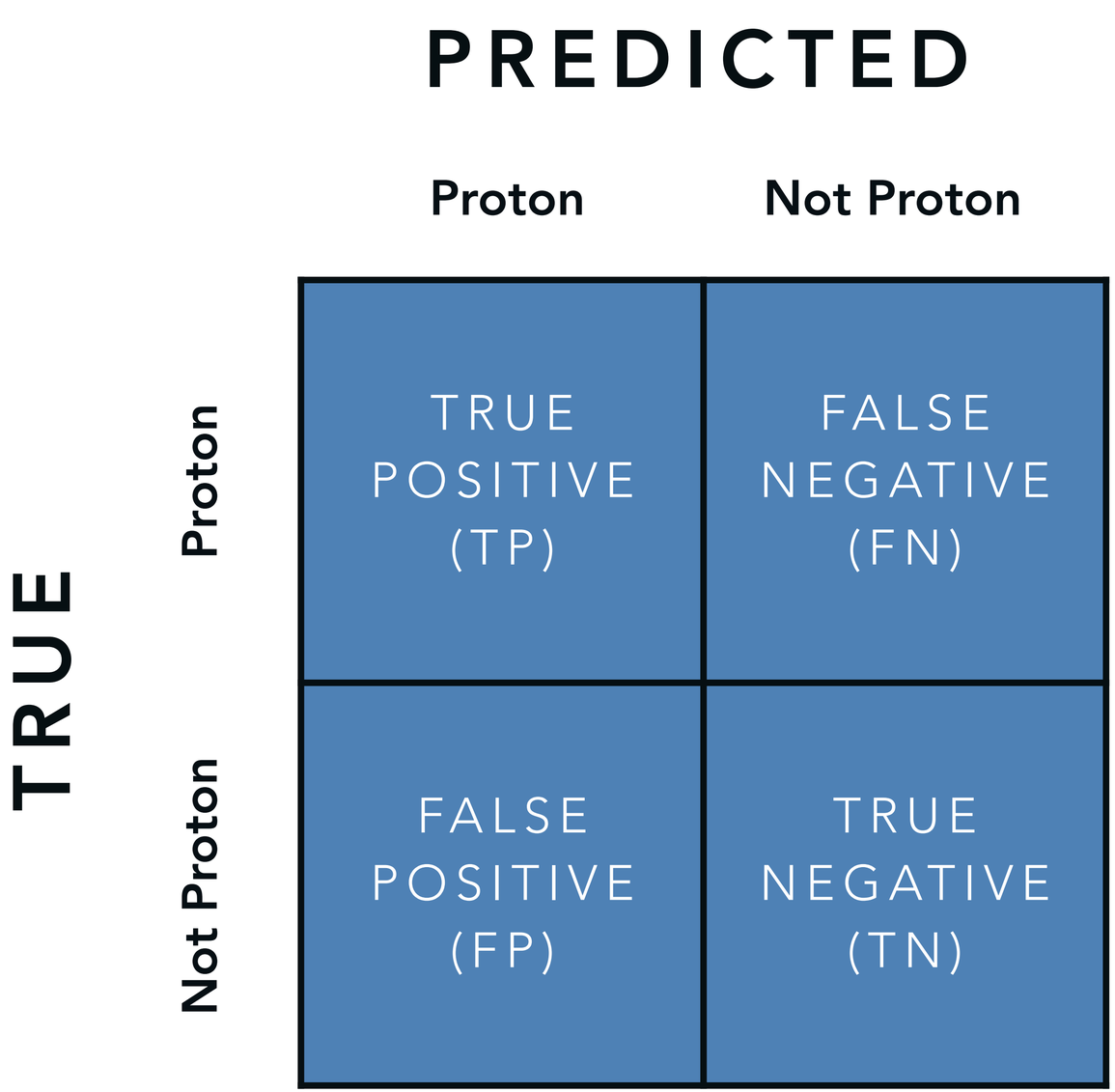
Metrics

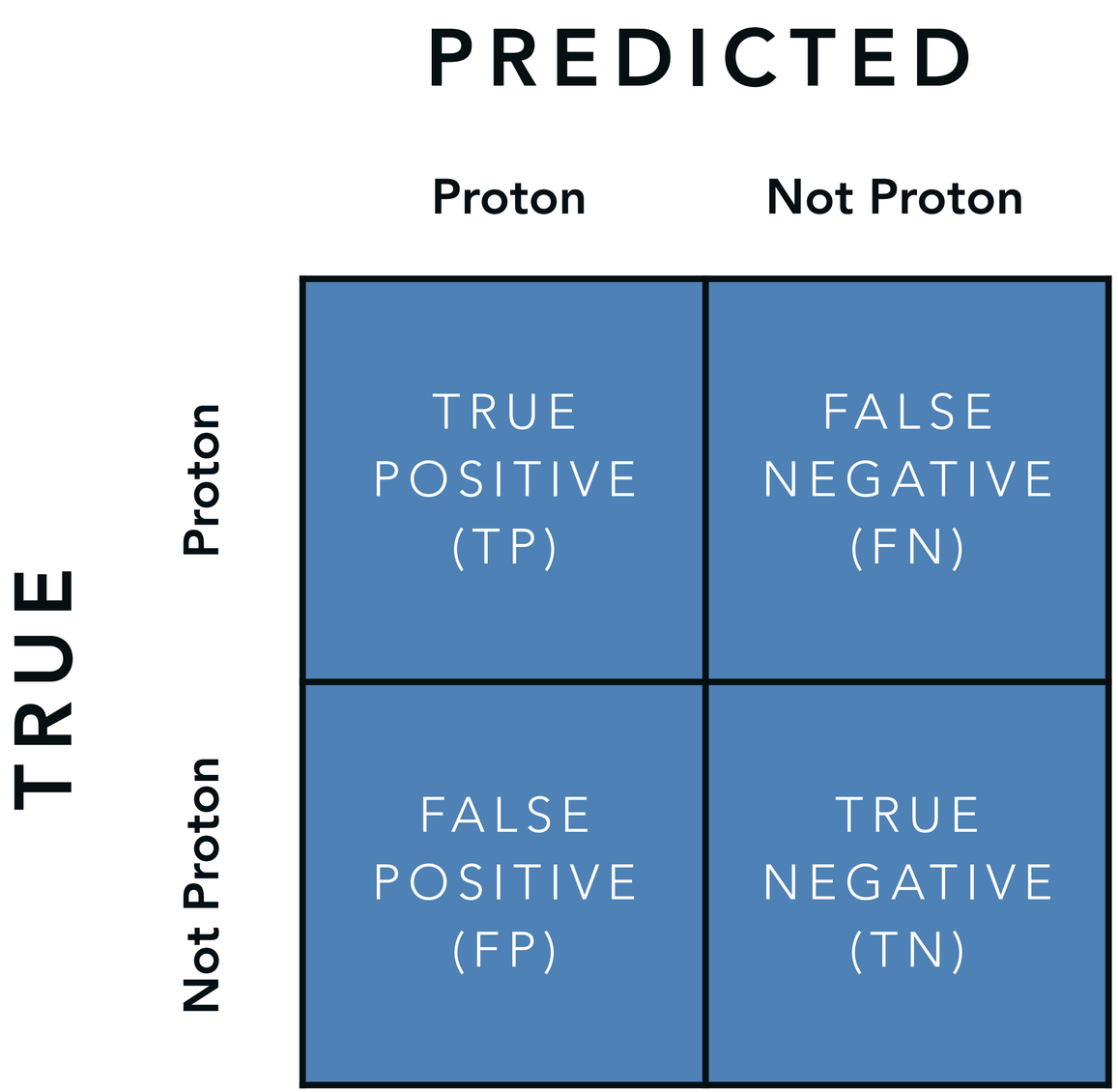


Detect Lung Cancer

99% Accuracy




PREDICTED
Proton
Not Proton
TRUE
Proton
Not Proton
TRUE POSITIVE (TP)
FALSE NEGATIVE (FN)
FALSE POSITIVE (FP)
TRUE NEGATIVE (TN)

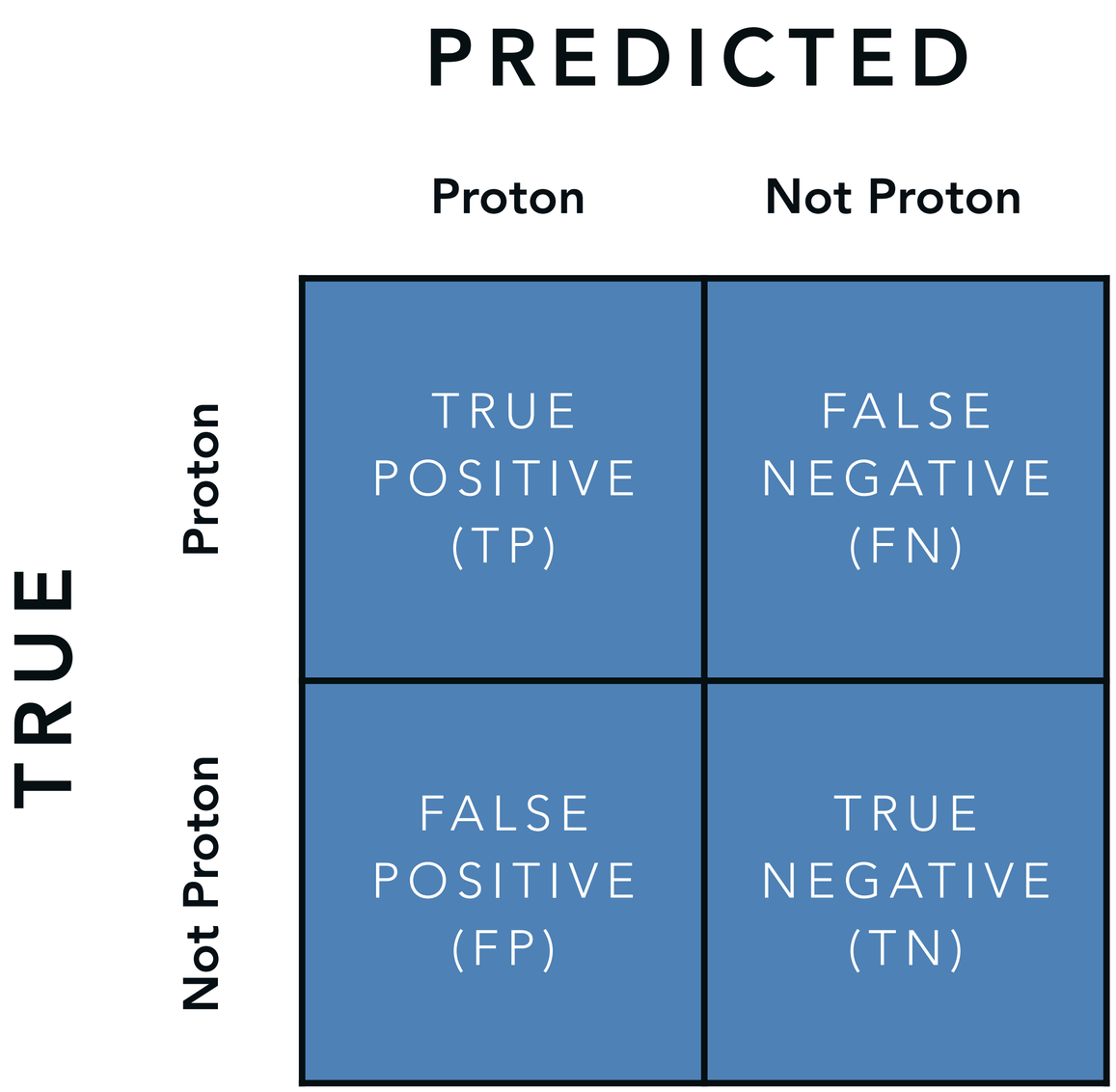

PREDICTED
Proton
Not Proton
TRUE
Proton
Not Proton
TRUE POSITIVE (TP)
FALSE NEGATIVE (FN)
FALSE POSITIVE (FP)
TRUE NEGATIVE (TN)

PREDICTED

TRUE

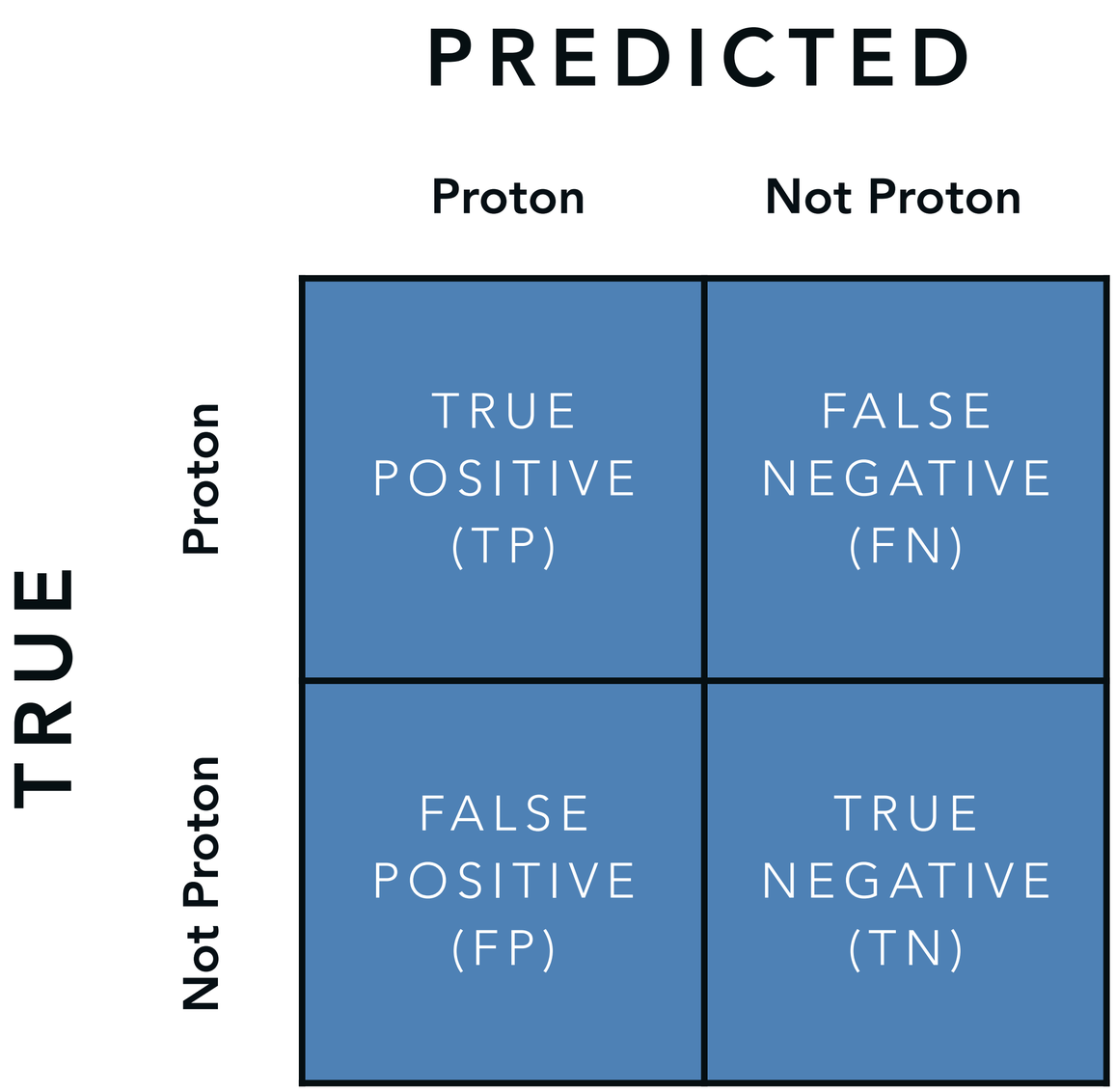
	Proton	Not Proton
Proton	TRUE POSITIVE (TP)	FALSE NEGATIVE (FN)
Not Proton	FALSE POSITIVE (FP)	TRUE NEGATIVE (TN)

$$\text{accuracy} = \frac{TP + TN}{TP + FN + FP + TN}$$



$$\text{accuracy} = \frac{TP + TN}{TP + FN + FP + TN}$$

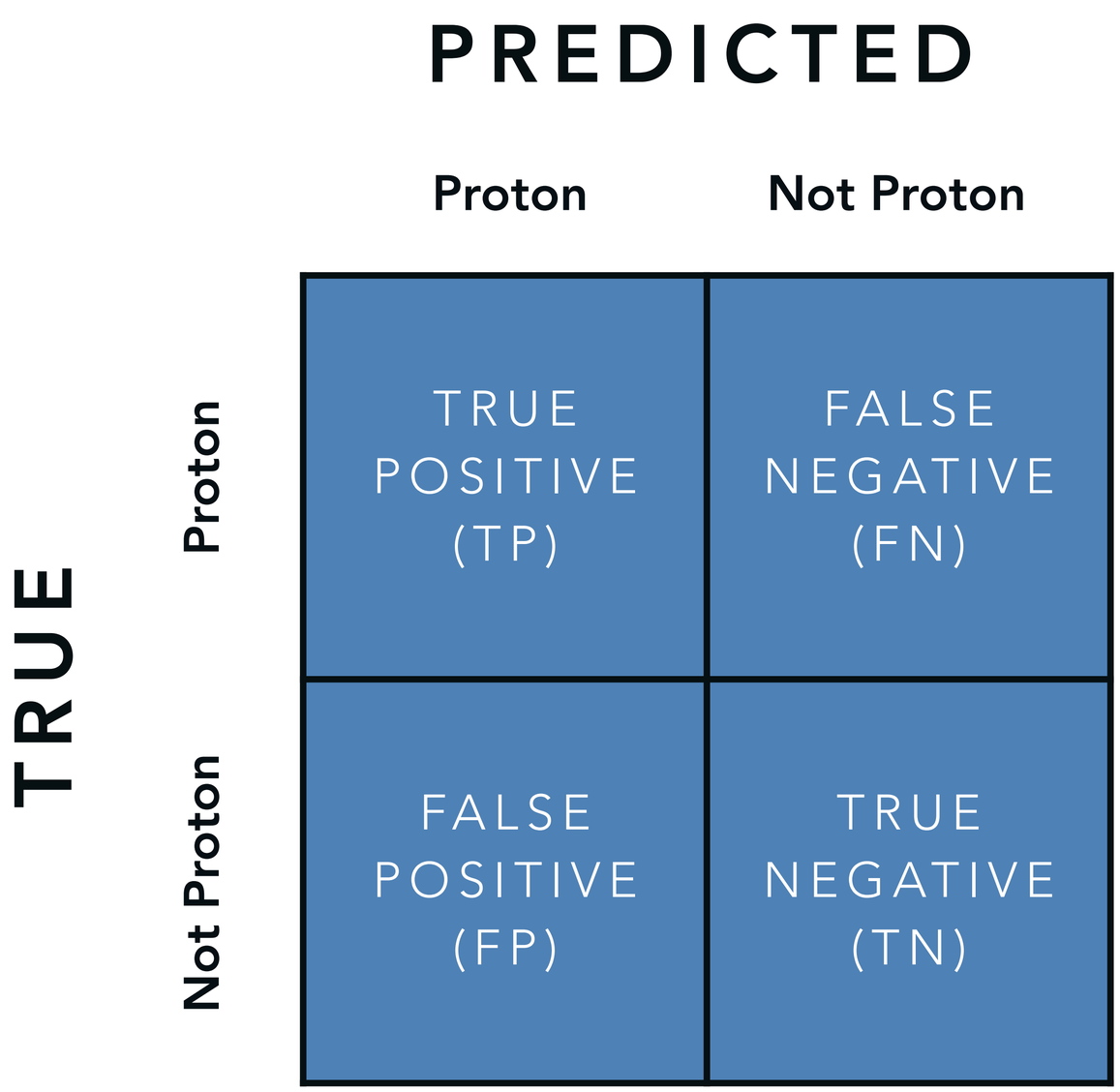
$$\text{precision} = \frac{TP}{TP + FP}$$



$$\text{accuracy} = \frac{TP + TN}{TP + FN + FP + TN}$$

$$\text{precision} = \frac{TP}{TP + FP}$$

$$\text{recall} = \frac{TP}{TP + FN}$$

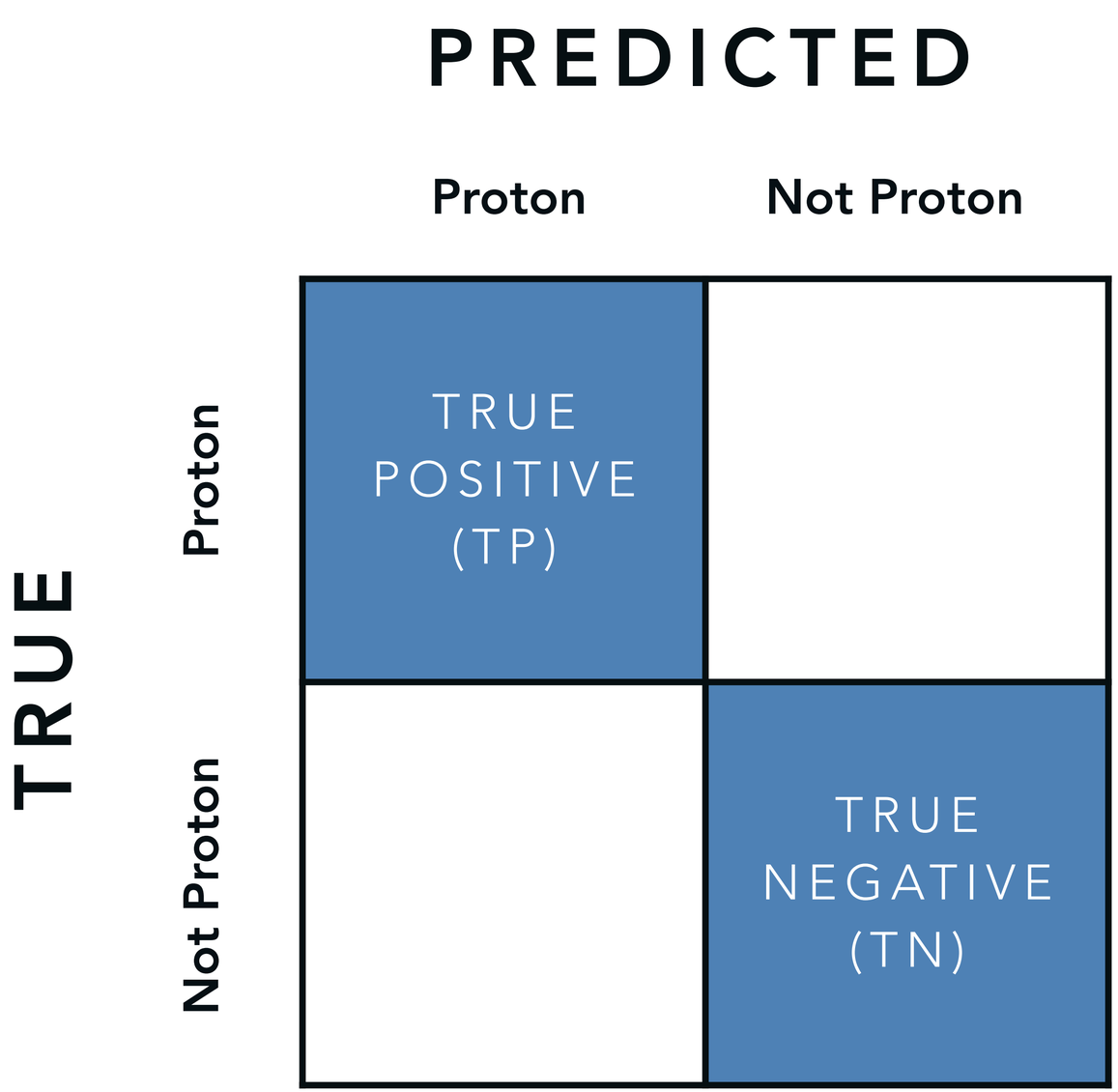


$$\text{accuracy} = \frac{TP + TN}{TP + FN + FP + TN}$$

$$\text{precision} = \frac{TP}{TP + FP}$$

$$\text{recall} = \frac{TP}{TP + FN}$$

$$F1 = \frac{2 \cdot \text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

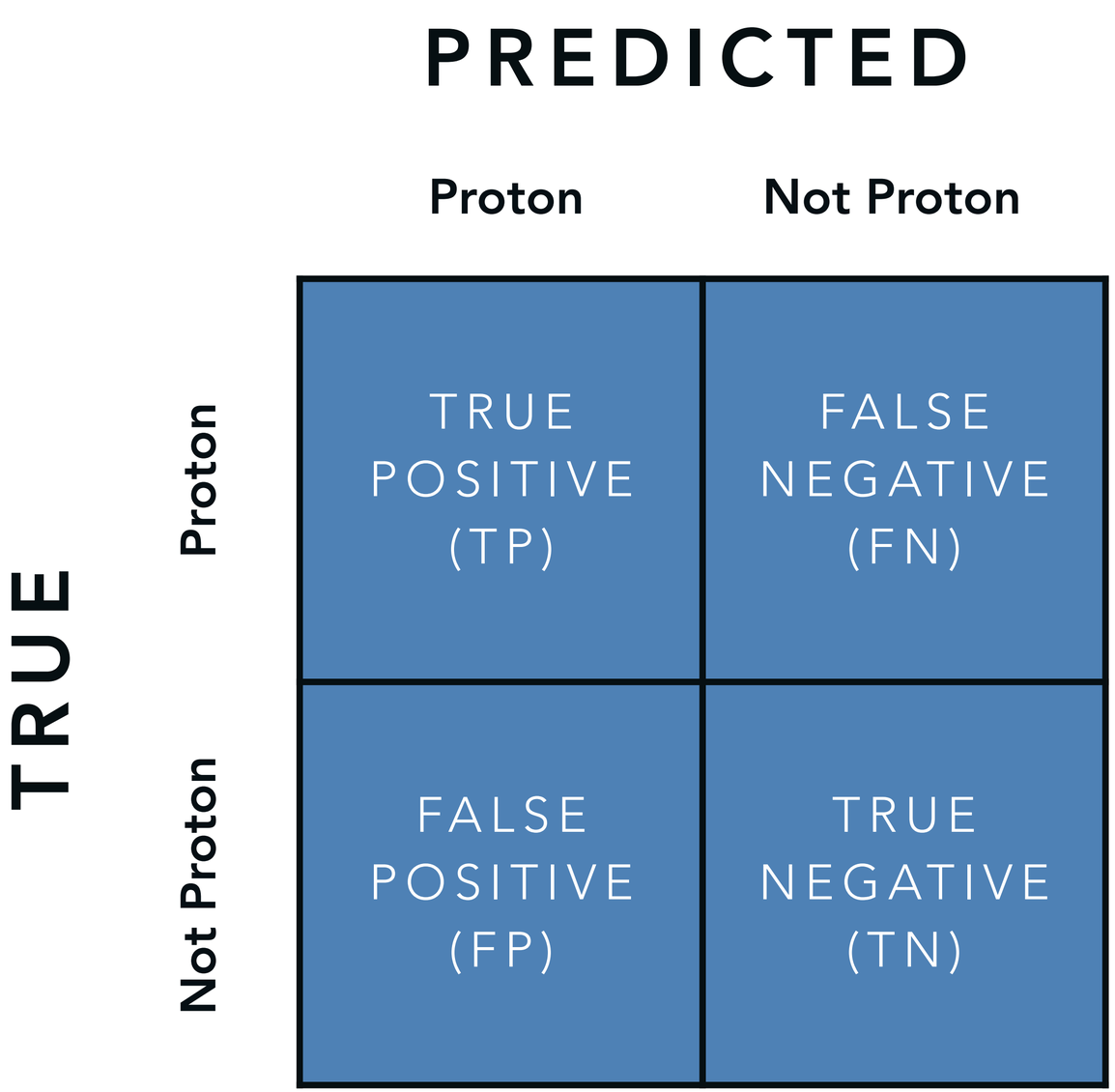

PREDICTED
Proton
Not Proton
TRUE
Proton
Not Proton
TRUE POSITIVE (TP)
TRUE NEGATIVE (TN)

PREDICTED

TRUE

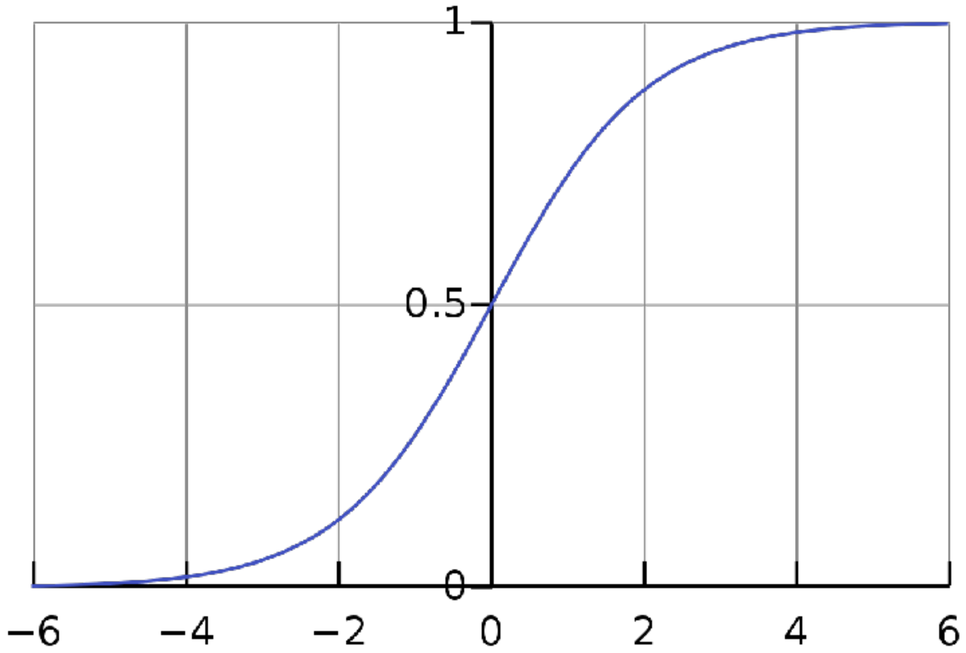
	Proton	Not Proton
Proton	TRUE POSITIVE (TP)	
Not Proton		TRUE NEGATIVE (TN)

PERFECT MODEL



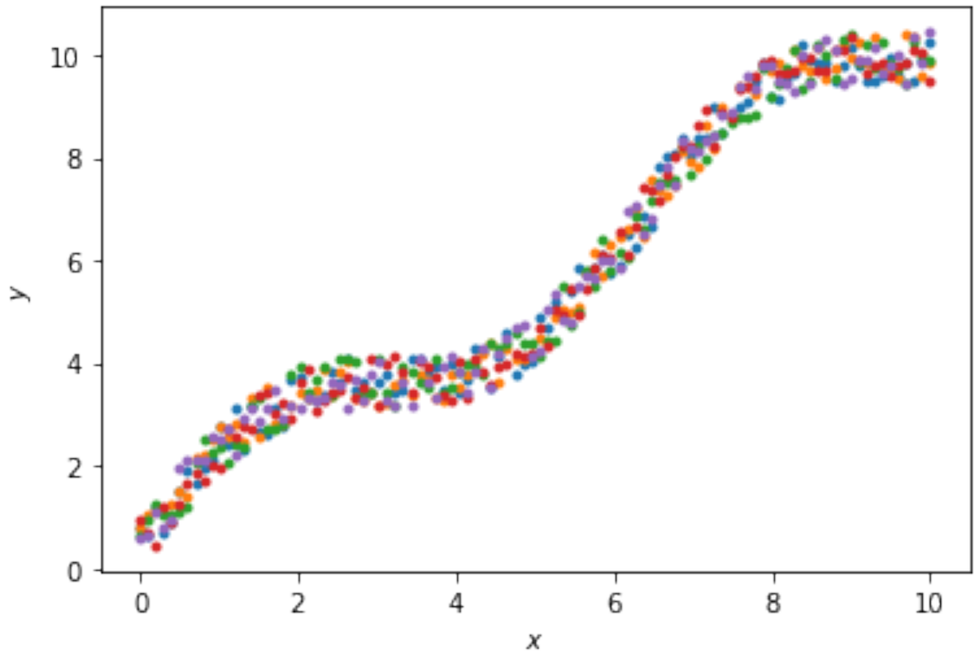
What does this matrix look like for multi-class classification?

MULTI-CLASS CLASSIFICATION



$$\text{Softmax}(\mathbf{x})_i = \frac{e^{x_i}}{\sum_{j=1}^{C} e^{x_j}}$$

LOSS FUNCTIONS



Loss function

Mean squared error (MSE)

$$J(w) = \frac{1}{N}\sum_{i=0}^{N}(\hat{y}_i - y_i)^2$$

Mean absolute error (MAE)

$$J(w) = \frac{1}{N}\sum_{i=0}^{N}|\hat{y}_i - y_i|$$

Cross entropy (CE)

$$\mathscr{L}(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_{i=1}^{N}\sum_{j=1}^{C} y_{ij} \cdot \log(\hat{y}_{ij})$$

CHOOSING AN ARCHITECTURE

HOW MANY LAYERS?

HOW MANY NODES PER LAYER?

LEARNING RATE

DROPOUT?

WHAT ACTIVATION FUNCTION(S)?

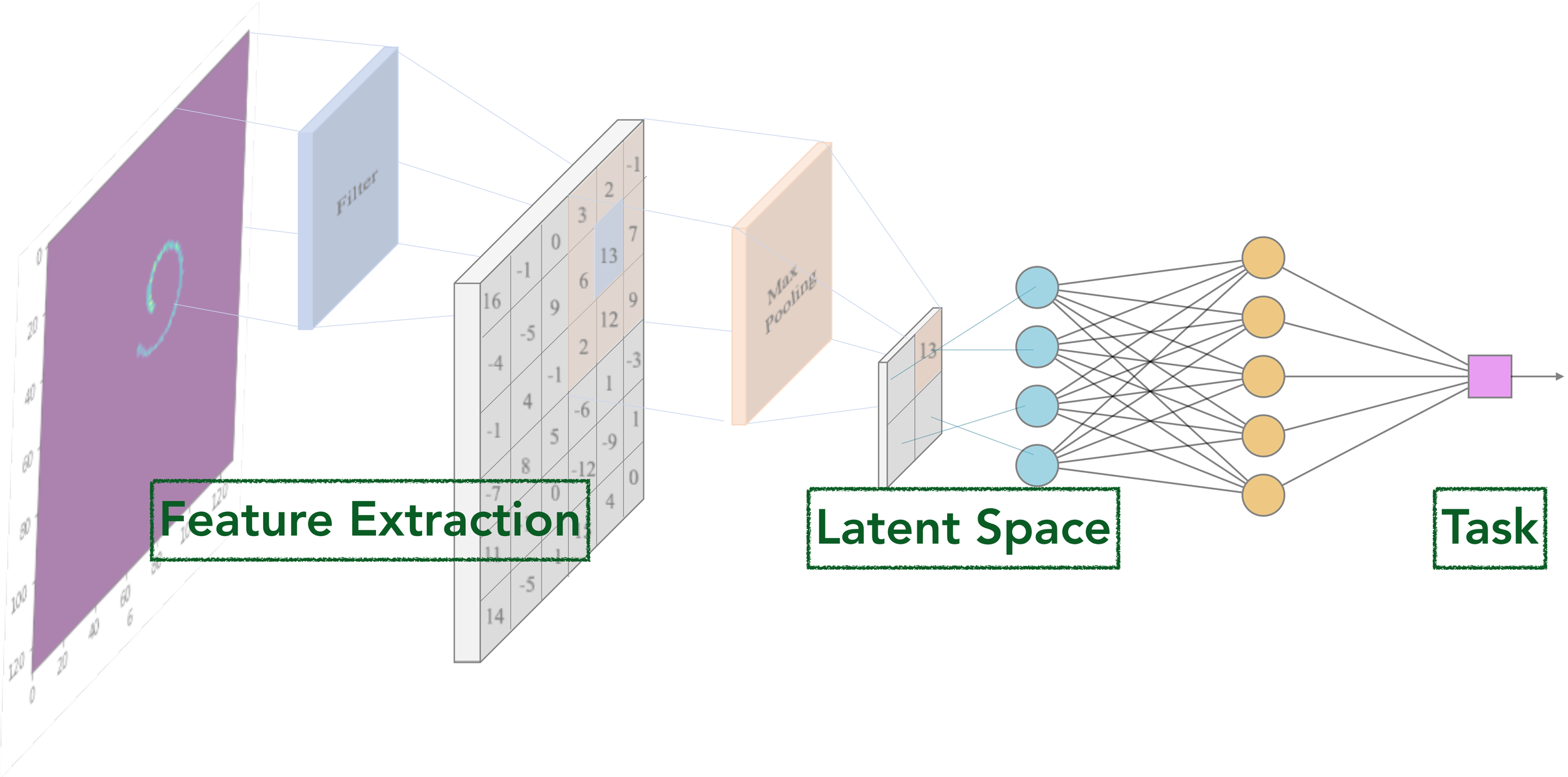
HOW MANY CONVOLUTION LAYERS?

FILTER SIZE?

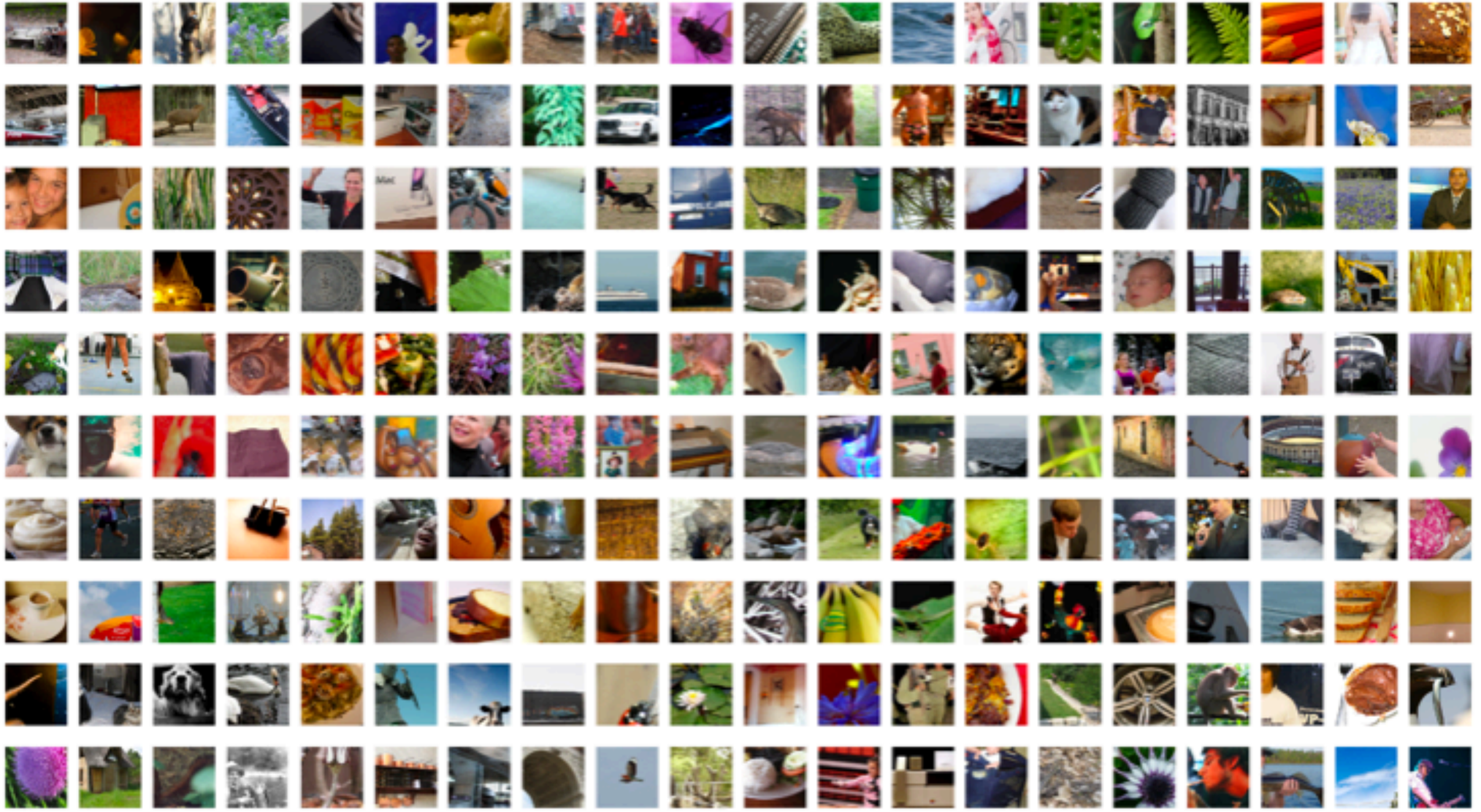
STRIDE?

POOLING?

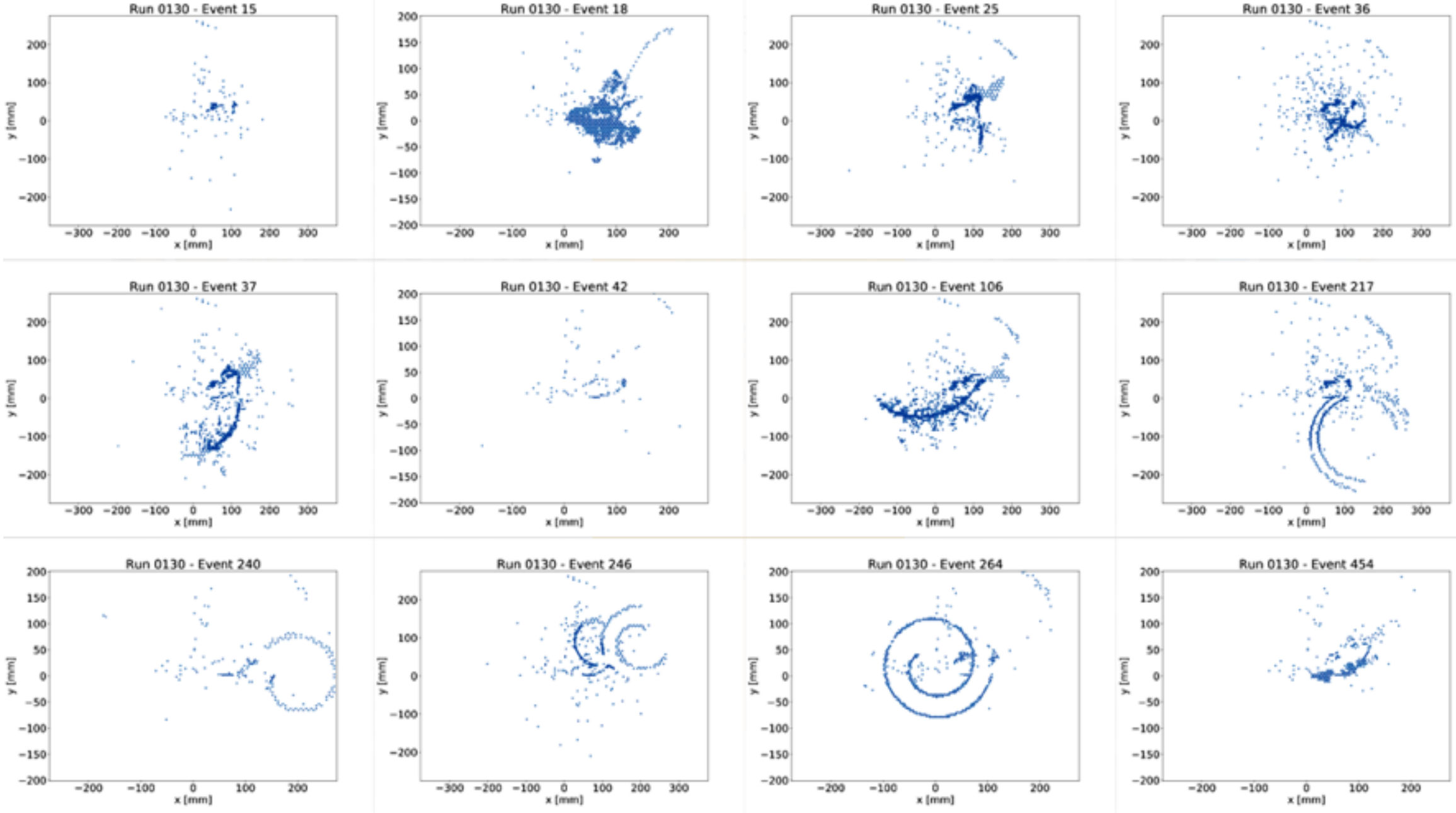
PRE-TRAINED MODELS

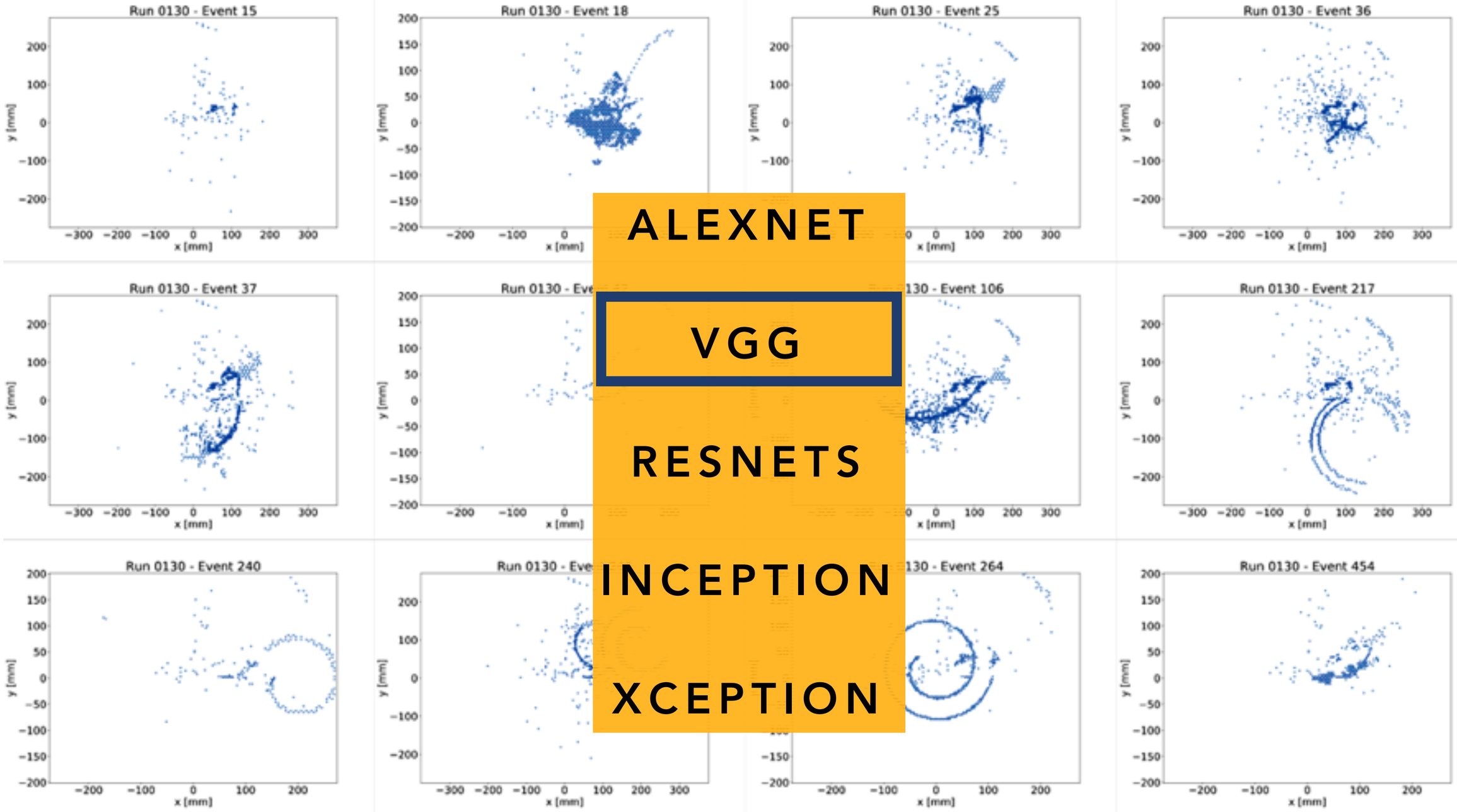


PRE-TRAINED MODELS



PRETRAINED MODELS


Run 0130 - Event 15
Run 0130 - Event 18
Run 0130 - Event 25
Run 0130 - Event 36
Run 0130 - Event 37
Run 0130 - Event 42
Run 0130 - Event 106
Run 0130 - Event 217
Run 0130 - Event 240
Run 0130 - Event 246
Run 0130 - Event 264
Run 0130 - Event 454
x [mm]
y [mm]



ALEXNET

VGG

RESNETS

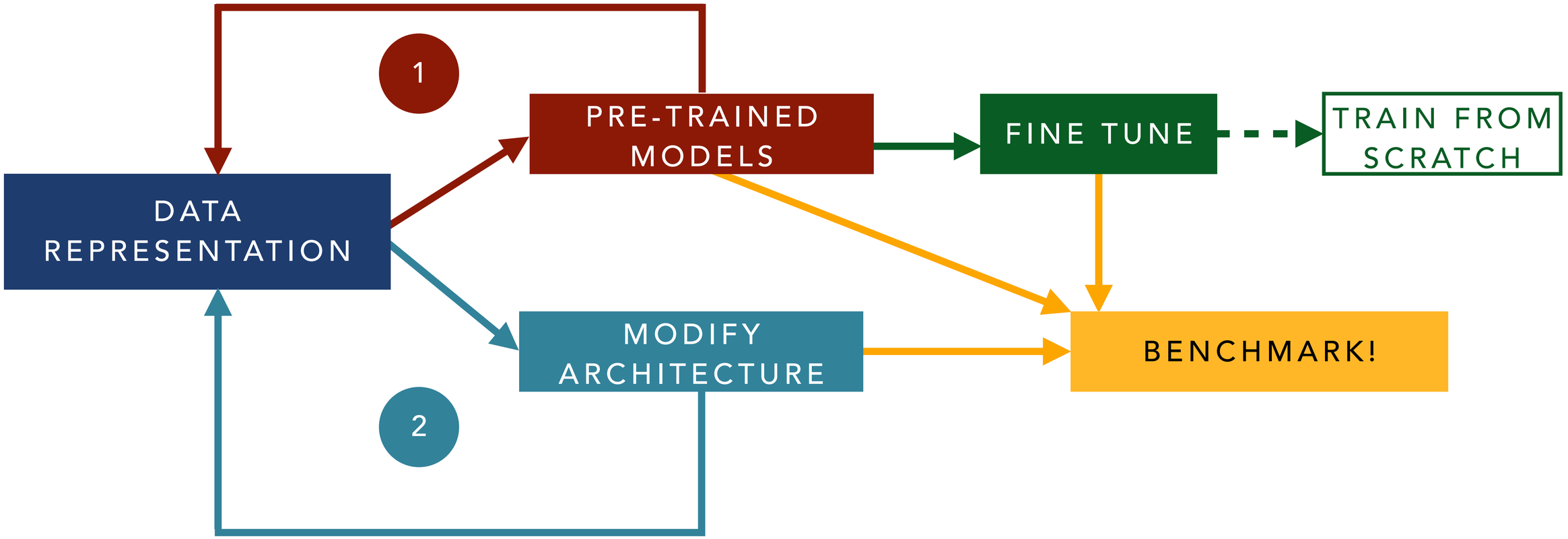
INCEPTION

XCEPTION

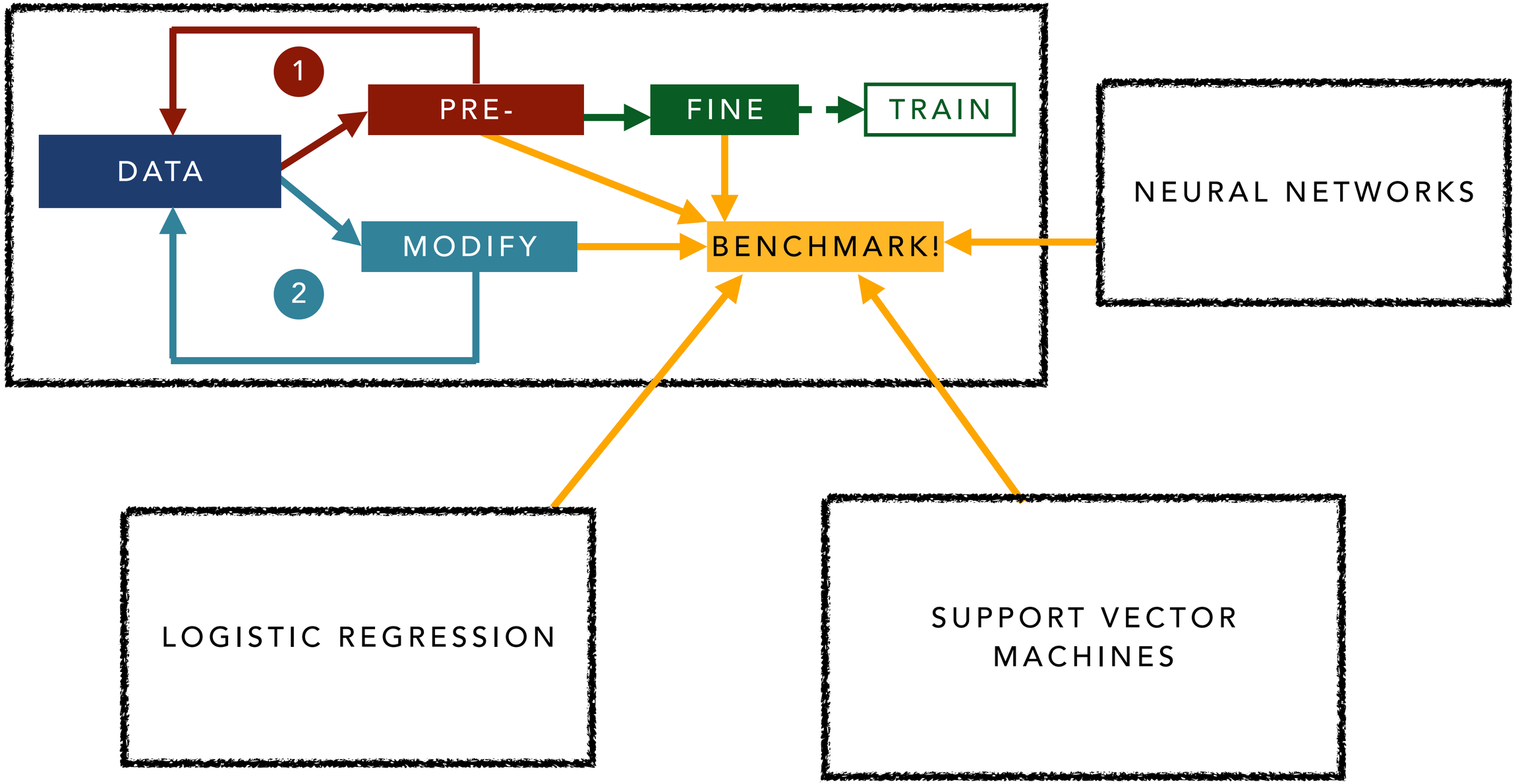
J. Z. TAYLOR, HONOR'S THESIS, DAVIDSON COLLEGE

M.P. KUCHERA, ET. AL. NUCLEAR INSTRUMENTATION AND METHODS A (2019)

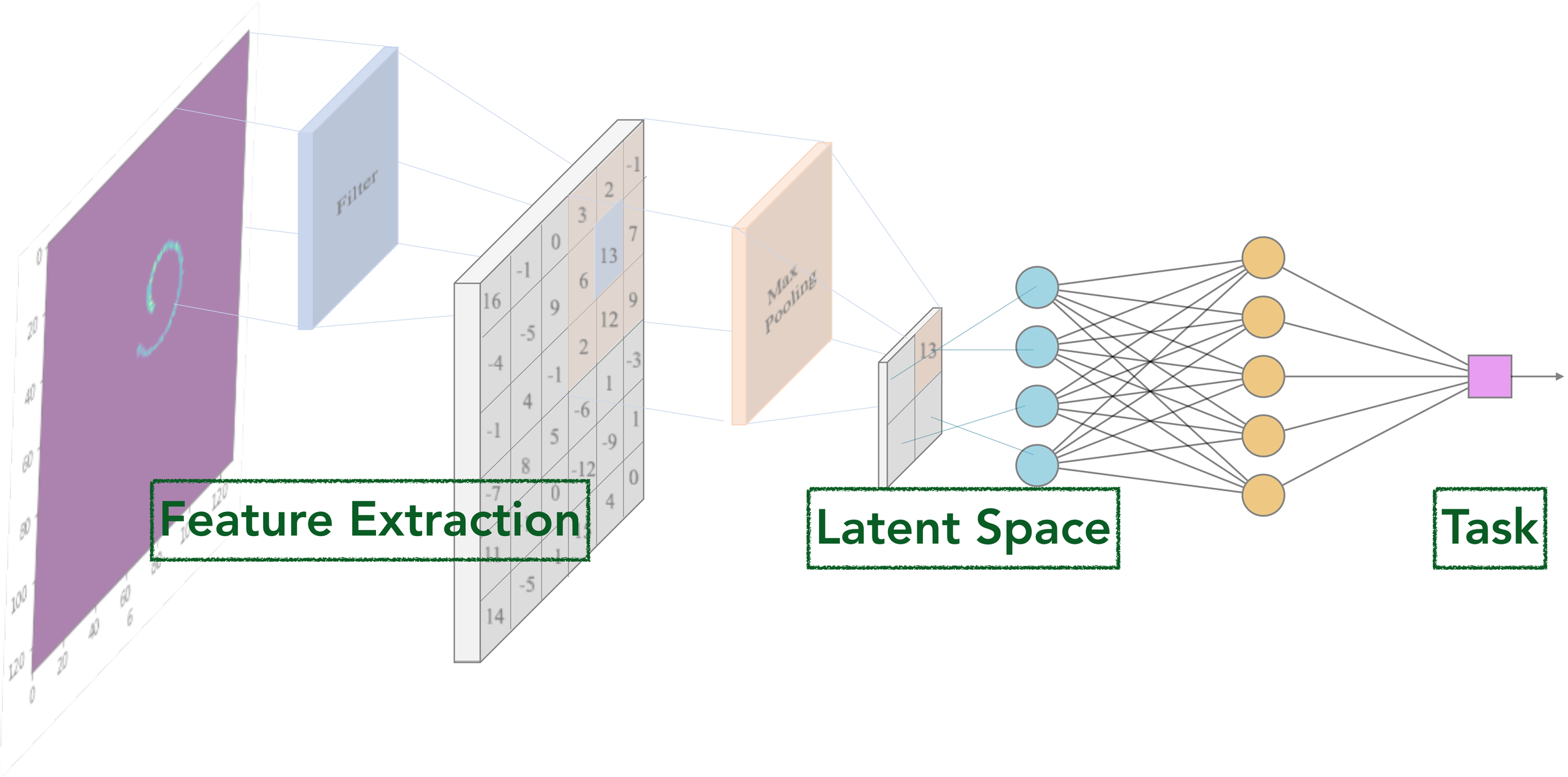
EXAMPLE WORKFLOW



EXAMPLE WORKFLOW



CONVOLUTIONAL NEURAL NETWORKS



OTHER NETWORK MECHANISMS:

-POINT CONVOLUTIONS

-ATTENTION