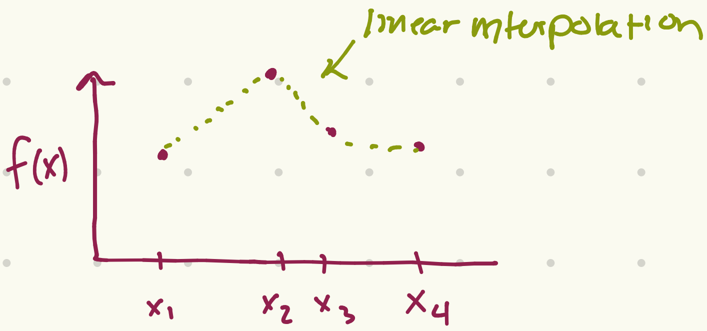
Interpolation:

$x_1, \dots, x_n \Rightarrow$ not of our choice

$f_1, \dots, f_n$



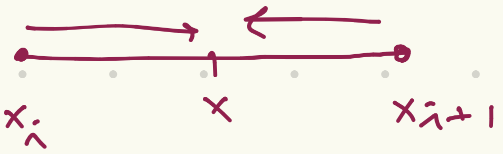
Linear Interpolation

$x_i \le x \le x_{i+1}$ $\qquad f(x) = f_i + \frac{x - x_i}{x_{i+1} - x_i}(f_{i+1} - f_i)$ $\qquad f_i \equiv f(x_i)$

truncation error scales $\sim f'' |x_{i+1} - x_i|^2$

Lagrang Interpolation:

$$f(x) = \frac{x - x_{i+1}}{x_i - x_{i+1}} f_i + \frac{x - x_i}{x_{i+1} - x_i} f_{i+1}$$



$$= \sum_{k=i}^{i+1} f_k P_k^{(1)}(x) \qquad P_k^{(1)}(x) = \frac{x - x_j}{x_k - x_j}$$

build a higher order interpolater:

$$f(x) = \sum_{k=1}^{N} f_k P_k^{(N)}(x), \quad P_k^{(N)}(x) = \prod_{j \ne k}^{N} \frac{x - x_j}{x_k - x_j} \qquad (j \ne k)$$

Cubic Splines:

linear scheme: $f(x) = A f_i + B f_{i+1}$ $\quad A = \frac{x_{i+1} - x}{x_{i+1} - x_i}$ $\quad B = 1 - A$

want: a cubic polynomial fc w/

- fc = 0 @ boundaries
- fc consist of linearly interpolating 2nd order derivatives: f_i''

$$f = \underbrace{A f_i + B f_{i+1}}_{\text{linear}} + \underbrace{C f_i'' + D f_{i+1}''}_{\text{cubic}}$$

impose: $C(x_i) = C(x_{i+1}) = D(x_i) = D(x_{i+1}) = 0$

and: $C''(x_i) = 1, \; D''(x_i) = 0$
$C''(x_{i+1}) = 0, \; D''(x_{i+1}) = 1$ } reproduce proper f''

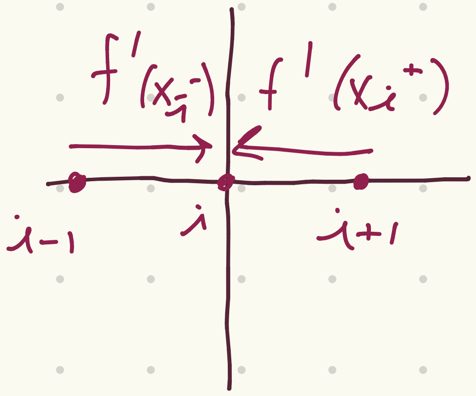
$$\Rightarrow \quad C = \frac{1}{6} A (A^2 - 1)(x_{i+1} - x_i)^2$$

$$D = \frac{1}{6} B (B^2 - 1)(x_{i+1} - x_i)^2$$

} $C'' = A$, $D'' = B$

Need f_i''s: we'll use f' across boundaries needs to be continuous

$$f'(x_i^+) = f'(x_i^-)$$



gives $(N-2)$ conditions of f_i''

need to supply others:

<u>Natural Spline:</u>

$$f_1'' = f_N'' = 0$$