Lecture 5: Basic Differential Equations

June 8, 2020

1 Simple Practice:

1.1 a) Is $y = \sqrt{10 - x}$ a solution to:

$$\frac{dy}{dx} = -\frac{x}{y} \tag{1}$$

1.2 b) Is $f(x) = 2(ln(x))^3$ a solution to:

$$f'(x) = \frac{3f(x)}{xln(x)} \tag{2}$$

1.3 c) Is $y = -\frac{x}{4} + \frac{3}{4}$ a solution to:

$$\frac{dy}{dx} = x + 4y - 2 \tag{3}$$

1.4 d) Is $y = -\frac{2}{3}x + \frac{1}{9}$ a solution to:

$$y' = 2x + 3y - 1 \tag{4}$$

2 More advanced differential equations:

2.1

Which one of the following families of functions solves the differential equation:

$$y' = \frac{3y}{x} \tag{5}$$

for all values of the constant C?

1. $y = Cx^2$
2. $y = Cx^3$
3. $y = Clnx$
4. $y = x^3 + C$

2.2

Which one of the following families of functions solves the differential equation:

$$x^3y' - e^y = 0 \tag{6}$$

for all values of the constant C?

1. $y = -ln(\frac{1}{2x^2} + C)$
2. $y = -ln(\frac{1}{2x^2}) + C$

Use your answer above to find the value of C that satisfies the condition $y(1) = -ln(3)$

2.3

An unknown function f solves the differential equation

$$f'(x) = f(x) + 3x \tag{7}$$

and satisfies the condition f(1) = 0. Use this information to complete the statements below:

1. $f'(1) =?$
2. $f'(1)$ is positive, zero, or negative?

3. $f''(1) = ?$

4. $f''(1)$ is positive, zero, or negative?

5. At the point 1 the function f is

   - (increasing or decreasing) and (concave up or concave down) ?
   - at a local maximum ?
   - at a local minimum ?

2.4

The differential equation

$$y' = 2x + 3y + 1 \tag{8}$$

has a solution in the form $y = mx + b$.

What are the values of m and b for this solution?