

18. Find tangent planes to the surface $z = Ax^2 + By^2$.

$$G(x, y) = (x, y, Ax^2 + By^2)$$

$$G'(x, y) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2Ax & 2By \end{bmatrix}$$

$$G'(x, y)^\perp = s \begin{bmatrix} 2Ax \\ 2By \\ -1 \end{bmatrix}$$

So by plugging in x and y to $G'(x, y)^\perp$, the tangent planes at $\mathbf{p}_1 = (0, 0, 0)$, $\mathbf{p}_2 = (a, 0, Aa^2)$, $\mathbf{p}_3 = (0, b, Bb^2)$ are:

$$P_1 : \quad [0 \quad 0 \quad -1] \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) = [0]$$

$$P_2 : \quad [2Aa \quad 0 \quad -1] \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} a \\ 0 \\ Aa^2 \end{bmatrix} \right) = [0]$$

$$P_3 : \quad [0 \quad 2Bb \quad -1] \left(\begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 0 \\ b \\ Bb^2 \end{bmatrix} \right) = [0]$$