
Homework 3

8. (a) The limit points are the closed ball of radius r about the origin.
   (b) The only limit point is 0.
   (c) There are no limit points.
   (d) The limit points are $\mathbb{R}^n$.

9. (a) True.
   (b) True.
   (c) True.
   (d) True.
   (e) True.
   (f) True.
   (g) False.

10. (a) $\mathbf{a}$ must be a limit point: $B_r(\mathbf{a})$ must contain at least one point, distinct from $\mathbf{a}$, in D for every $r > 0$.
    (b) $\forall \varepsilon > 0, \exists \delta > 0$ such that $0 < |\mathbf{x} - \mathbf{a}| < \delta$ implies $|f(\mathbf{x}) - L| < \varepsilon$.
    (c) $\forall \varepsilon > 0, \exists \delta > 0$ such that $\mathbf{x} \in B_\delta(\mathbf{a}), \mathbf{x} \neq \mathbf{a}$ implies $f(\mathbf{x}) \in B_\varepsilon(\mathbf{L})$.

11. (a) Closed.
    (b) Both open and closed.
    (c) Neither open nor closed.
    (d) Open.
    (e) Closed.
    (f) Neither open nor closed.
    (g) Open.
    (h) Closed.
    (i) Closed.

12. Compact sets are: b, e, h, i

13. (a) $|\mathbf{x}_2 - \mathbf{x}_1|$, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
    (b) $|\mathbf{x}_2 - \mathbf{x}_1|$, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
    In $\mathbb{R}^n$: $|\mathbf{x}_2 - \mathbf{x}_1|$, $\sqrt{\sum_{i=1}^n (x_{2,i} - x_{1,i})^2}$

14. $f = s \circ (p \circ (\pi_1, \pi_1), p \circ (2, \pi_1, \pi_3, \pi_3)$
    Since f is composed of continuous functions, f is continuous.

I.7.6 Let $\mathbf{a}$ be a boundary point of $D \subset \mathbb{R}^n$. Then either $\mathbf{a} \in D$ or $\mathbf{a} \notin D$.
Suppose $\mathbf{a} \in D$. Then we are done.
Suppose $\mathbf{a} \notin D$. Then since $\mathbf{a}$ is a boundary point, for every $r > 0$, $B_r(\mathbf{a})$ contains at least one point of D. So $\mathbf{a}$ is a limit point.
Thus every boundary point of D is either in D, or a limit point of D.

I.8.4 Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$, and the inverse image $f^{-1}(U) = \{\mathbf{x} \in \mathbb{R}^n | f(\mathbf{x}) \in U\}$ is open for any open set $U \subset \mathbb{R}^m$.