

Homework 6

15. (a)

$$a + 3b - c + 2d = 0 \quad (1)$$

$$2a + 2b - c + d = 0 \quad (2)$$

$$-a + b + d = 0 \quad (1 - 2)$$

$$a = 2; b = 1; d = 1 \quad (\text{arbitrary choice})$$

$$c = 7$$

$$w + 3x - y + 2z = 0 \quad (3)$$

$$2w + 2x - y + z = 0 \quad (4)$$

$$-w + x + z = 0 \quad (5: 3 - 4)$$

$$aw + bx + cy + dz = 0$$

$$2w + x + 7y + z = 0 \quad (6)$$

$$3w + 7y = 0 \quad (6 - 5)$$

$$w = 7; y = -3 \quad (\text{arbitrary choice})$$

$$x - 8y = 0 \quad (4 - 6)$$

$$x = -24$$

$$z = 31$$

$(2, 1, 7, 1)$ and $(7, -24, -3, 31)$ form a basis for V^\perp .

(b) $\ker(\mathbf{A}) = \{\mathbf{v} \in V | \mathbf{A}\mathbf{v} = \mathbf{0}\}$, so $\mathbf{A} = \begin{bmatrix} 2 & 1 & 7 & 1 \\ 7 & -24 & -3 & 31 \end{bmatrix}$.

(c)

$$\begin{bmatrix} 2 & 1 & 7 & 1 \\ 7 & -24 & -3 & 31 \end{bmatrix} \left(\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ -1 \\ 5 \end{bmatrix} \right) = \mathbf{0}$$