
II.8.2 (a)

$$\begin{aligned} q(x, y, z) &= 2x^2 + 5y^2 + 2z^2 + 2xz \\ &= \begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \end{aligned}$$

(b)

$$\begin{aligned} |A_1| &= \begin{vmatrix} 2 \end{vmatrix} = 2 > 0 \\ |A_2| &= \begin{vmatrix} 2 & 0 \\ 0 & 5 \end{vmatrix} = 10 > 0 \\ |A_3| &= \begin{vmatrix} 2 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 2 \end{vmatrix} = 15 > 0 \end{aligned}$$

Thus by theorem 8.8, q is positive definite.

(c)

$$\begin{aligned} 0 &= \begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 5-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{vmatrix} \\ &= (5-\lambda)((2-\lambda)(2-\lambda)-1) \\ &= (5-\lambda)\left(3-4\lambda+\lambda^2\right) \\ &= (5-\lambda)(3-\lambda)(1-\lambda) \end{aligned}$$

Eigenvalues, $1, 3, 5$, are all positive, so q is positive definite.
