
Homework 10

26. (a)

$$\begin{aligned} Q(x,y) &= \mathbf{x}^\top \mathbf{A}\mathbf{x} \\ &= \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= \begin{bmatrix} ax + cy & bx + dy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= ax^2 + cxy + bxy + dy^2 \\ &= ax^2 + (b+c)xy + dy^2 \end{aligned}$$

Thus $Q(x,y)$ 0^{th}- nor 1^{st}-degree terms.

(b) $Q(x,y) = y^2$ is neither positive definite, nor negative definite, nor nondefinite, so taking its coefficients and the result from (a), one corresponding matrix is $\mathbf{A} = \left[\begin{smallmatrix} 0 & 2 \\ -2 & 1 \end{smallmatrix}\right]$.

27. (a)

$$\begin{aligned} f(x,y) &= ax^2 + 2bxy + cy^2 \\ f_x &= 2ax + 2by \\ f_{xx} &= 2a \\ f_{xy} &= 2b \\ f_y &= 2bx + 2cy \\ f_{yy} &= 2c \\ \mathbf{H}_f(x,y) &= \begin{bmatrix} 2a & 2b \\ 2b & 2c \end{bmatrix} \end{aligned}$$

(b)

$$\begin{aligned} q(x,y) &= \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 2a & 2b \\ 2b & 2c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= \begin{bmatrix} 2ax + 2by & 2bx + 2cy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= 2ax^2 + 2bxy + 2bxy + 2cy^2 \\ &= 2ax^2 + 4bxy + 2cy^2 \end{aligned}$$

ACoSV II

7.3
