**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Work will begin after 10 min so average time will be 55 min.**

**Standardizing a normally distributed variable:**

**Z= (X - μ) / σ**

**Z= (60-55)/8**

**Z=5/8**

**Z = 0.625**

**1 – pnorm(0.625)**

**= 0.030396**

**The probability that the service time exceeds 60mins is approximately 0.2659 or about 26.59% . This is the probability that the service manager can not meet to his commitment to have the car ready with in 1 hr from drop off.**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**False**

**Z= (X - μ) / σ**

**Z= (44-38)/6**

**Z= 6/6**

**Z = 1**

**Norm.cdf(1)= 0.8413**

**People above 44 age = 100-84.13**

**=15.87%**

**63 out of 400.**

**Z= (X - μ) / σ**

**Z=(38-38)/6**

**Z=0/6**

**Z = 0**

**Norm.cdf(0)= 0.5**

**People between 38 and 44 age = 84.13 – 50**

**34.13%**

**137 out of 400.**

**More employee at the processing center are older than 44 than between 34 and 44 is FALSE.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**True**

**Z= (X - μ) / σ**

**Z= (30-38)/6**

**Z=-8/6**

**Z = - 1.333**

**norm.cdf(-1.33) = 0.091759135602**

**= 9.15%**

**36 out of 400**

**A training program for the employees under age of 30 at the center would be expected to attract about 36 employees is TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**X1 ~ N(μ, a^2), where X1 follows normal distribution with the mean μ and variance a^2**

**X2 ~ N(μ,a^2), where X2 follows normal distribution with the mean μ and variance a^2**

**X1 and X2 are independent and identically distributed (iid) random variable.**

**Difference between 2X1 and X1 + X2:**

1. **Distribution of 2X1:**

**If X1 follows a normal distribution N(μ, a^2), then the random variable 2X1 will also follow a normal distribution. The mean of 2X1 will be 2 μ and the variance will be 4a^2.**

**So 2X1~N(2** **μ ,4a^2).**

1. **Distribution of X1 + X2:**

**Since X1 and X2 are independent and identically distributed, the sum of two independent normal random variable is itself a normal random variable, The mean of X1 + X2 will be 2** **μ and the variance of X1 + X2 will be** **2a^2.**

**So X1 + X2 ~ N(2 μ, 2a^2)**

1. **Comparison:**

**Both 2X1 and X1 + X2 are normal random variable but they have different distributions and parameters.**

**For 2X1 Distribution is : N(2** **μ, 4a^2)**

**Mean : 2 μ**

**Variance: 4a^2**

**For X1 + X2 :**

**Distribution : N(2** **μ 2a^2)**

**Mean : 2** **μ**

**Variance: 2 a^2**

**Mean is same varianc and distribution is different. Distribution 2x1 has higher variance.**

**The variance and the distribution shapes differ between the twos.**

**This difference in variance is due to the scaling effect of multiplication in 2X1 lading to a wider spread of values.**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**We have 0.5th and 99.5th percentile Z scores**

**Z(0.5) = -2.576**

**Z(99.5) = 2.576**

**Z = (x – 100)/ 20**

**X = 20Z +100**

**A = -(20 \* 2.576) + 100**

**= 48.5**

**B = (20 \* 2.576) + 100**

**= 151.5**

**Two values symmetric about mean for given standard normal distribution ,**

**Hence answer is 48.5 & 151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

**Company’s profit: P~N(5+7,32+42)**

**N(12,52)**

**95% Of the probability lies between1.96 S.D of the mean**

**=(12-1.96\*5, 12+1.96x5)**

**=(12-9.8, 12+9.8)**

**=(2.2, 21.8)**

**=(2.2\*45),(21.8\*45)**

**=99rs, 981rs.**

1. Specify the 5th percentile of profit (in Rupees) for the company

**ANS:**

**P = 0.05**

**From P value of X square table**

**p-12 = -1.644**

**5**

**P = 12- 8.82**

**P = 3.78**

**(3.78\*45 =170.1)**

**5th percentile of profit is Rs 170.1 Million.**

1. Which of the two divisions has a larger probability of making a loss in a given year?

**The first division of company have larger probability of making loss in year.**