

# Translating from Recurrence Relations to an Agent-Based Representation

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## Abstract

*This dissertation investigates a process to increase the acceptance of agent-based models by translation from recurrence relation models. The work presented in this dissertation is designed to combat two main aspects that hinder the up take in agent-based models: a lack of a model-wide formalism, and a lack of familiarity to those unaccustomed to the technique.*

*The importance of this work relates in particular to the field of economics, where agent-based models are yet to become widely accepted. This lack of acceptance leads to an absence of work based on agent-based models in top economics journals. This creates a problem for modelling complex systems and emergent behaviour, as agent-based models are in many cases a natural modelling technique for these systems, due in part to their message passing behaviour.*

*The process presented in this dissertation is to create an agent-based model equivalent to a recurrence relation model. Recurrence relation models are both widely accepted and have a model-wide formalisation. Creating an equivalence between these two models allows for the recurrence relation model to give the agent-based model both a formalisation and a familiar representation, for economists.*

*This work comprises the creation of a bespoke recurrence relation language, the design of step-by-step translation between the models, and a numeric testing of the equivalence of these models.*

*The bespoke recurrence relation language presented in this work allows for an experiment involving complex and interacting systems to be laid out in a clear manner, using recurrence relations.*

*A design for an eight-step translation that turns an experiment written in the bespoke recurrence relation language into an agent-based model is given.*

*The agent-based model created in this translation is shown to be numerically equivalent to the initial recurrence relation model, for a number of starting values. Each step in the translation process is also shown to be numerically equivalent, for these same values.*

*The work presented in this dissertation contributes to the field by adding a method for both formalising and expressing an agent-based model using a recurrence relation model. This work gives a method, which can allow for agent-based models to become more widely accepted within the field of economics, giving broader visibility to a powerful tool for investigating complex financial markets.*

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# 1 Introduction

Complex systems have been an area of interest for a long time in many fields such as physics, biology and chemistry [1]. More recently this is true in the fields of economics and finance [2], especially as the financial markets have become increasingly automated and interconnected. Of particular interest is emergent behaviour and emergence in complex systems, these have been defined as “a property of a system is emergent; if it is not a property of any fundamental element, and emergence is the appearance of emergent properties and structures on a higher level of organization or complexity” by Fromm as well as others [3–5].

Due to its nature emergent behaviour can often be damaging to the system in which it occurs. As such an area of research within the study of complex systems in economics is focussed on regulation [6], with the aim of being able to introduce regulations to stop future occurrences of damaging emergence, such as possible flash crashes. A flash crash is a sudden drop and then recovery in the price of securities or commodities, this maybe the result automated trading [7]. A prominent example of this is the flash crash of 2010, which is thought to have, at least in part, been caused by high-frequency traders [8, 9].

Emergent behaviour in markets can occur in a large number of ways, but more often the behaviour of interest is created over a time period. In general there are two ways of viewing systems in time, these are either continuous (where the system progress smoothly through time) or discrete (where the system takes quantised steps through time, with each step advancing the system).

For automated traders, and high-frequency traders in particular, the latter approach is best used to describe them. This is due to the fact that computers trade in discrete time, and because of the speeds involved in most of these trades this quantisation of time becomes very important [10].

For discrete time systems two particular modelling techniques that can be used are recurrence relations and agent-based models. These two techniques model systems in different ways, with recurrence relations being analytic and agent-based models being dynamic. Recurrence relations take a more mathematical approach modelling a system as a set of equations that interact through function calls between different equations. Agent-based models on the other hand model a system as a set of agents that interact through messages sent between them.

Both of these techniques have been used extensively in a large range of fields for modelling complex systems [11], however within the field of finance use of agent-based models is predominantly missing from top economics journals [12, 13].<sup>1</sup> This absence of agent-based models can be explained by two factors: the lack of a formal definition for the model as a whole, and the technique itself being unfamiliar and not resonating with many researchers in economics [13].

The lack of widespread acceptance of agent-based models within economics can cause an issue for the development of work in this field on complex interacting systems and emergent behaviour. Agent-based models are well known for their ability to model these kinds of systems, often being considered the natural way for describing systems composed of independent entities [14]. They allow these systems to be modelled in such a way that facilitates the exploration of aggregate properties, interaction dynamics, and emergent behaviour. The modularity of their design allows for complexity of agents to be easily tuned, as well as for the models to be easily expanded [14]. Agent-based models in many fields are considered very powerful and well suited to computational modelling of complex systems, with a number of advantages over other modelling techniques [12, 15–18].

The other popular modelling technique for complex systems, recurrence relation models, has a system-wide formalisation, and is widely accepted and well used within economics [19–21]. This motivates this dissertations investigation of a correctness preserving transformation from a

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<sup>1</sup>This is discussed further in Section 2.2.

recurrence relation model to an equivalent agent-based model.

The recurrence relation model gives the agent-based model a system-wide formalism and the translation seeks to make agent-based models more familiar to those experienced with recurrence relations.

This translation is from a recurrence relation model to an agent-based model, as opposed to the reverse. This trajectory is taken as the aim is to familiarise those already accustomed to the former with the latter, hence it makes sense to start with the recognised model. Facilitating them in taking advantage of agent-based models ability to investigate message passing, something, which is difficult in recurrence relation models.<sup>2</sup>

This translation has a number of fundamental challenges associated with it, which will be discussed in Section 3.3, but include:

- The two paradigms of the models are substantially different.
- How are function calls related to message passing?
- How can the idea of agents having an infinite list of their values through out time be derived from the recurrence relations?
- How can the idea of private and public information be introduced, and what data should fall into each category?
- How can time limited messages be introduced?
- How can output messages containing message information be introduced?
- How can the recurrence relations be split in a way that does not splinter the model?

The translation process in this dissertation takes a step-by-step approach, with each step gradually translating the model. This allows for these issues to be tackled in a more modular fashion. This also makes the equivalence of the two models more apparent, if each small step is equivalent then so too must be the models, assuming the transitivity of the equivalence.

The objectives of the project this dissertation describes are as follows:

- The creation bespoke recurrence relation language and formal definition of its syntax.
- The design of a step-by-step translation process, turning a model written in the bespoke recurrence relation language into an agent-based model.
- The validation and testing of this step-by-step translation.

This dissertation presents both a bespoke recurrence relation language and the design of the steps needed to translate a system expressed in this language into an agent-based model. Though not within the initial project scope, also presented is the first three steps of implementing this translation process into an automated program. These steps take the form of a lexer, parser, and first translation step.

The agent-based model translated to is a reduced form of the InterDyne simulator. InterDyne was chosen in particular due to it being designed to investigate emergent behaviour within the financial markets, this software was also a convenient choice due to access to the source code and local expertise.

The creation of this translation process contributes to the field by giving a method whereby economists can become familiar with agent-based models, as well as giving a formalisation to the agent-based models. This has the potential to increase the uptake in use of agent-based models and to increase the acceptance of work already done using agent-based models.

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<sup>2</sup>Translation in the opposite direction is discussed in Section 7.1.

The benefits of this work are timely, matching with the increased interest in agent-based models within finance, and with the interest by regulatory bodies in interaction and emergent phenomenon [22–24].

The dissertation layout is as follows. In Section 2 a background is given, covering: emergent behaviour generally and in the financial markets, a short assessment of the literature and description of both recurrence relations and agent-based models, and an in-depth description of InterDyne. In Section 3 a description and analysis of the problem is given, covering: a detailed description of the problem this dissertation addresses, a review of similar work in tackling this problem, and a description of the challenges with the translation approach taken. In Section 4 the bespoke recurrence relation is given, along with its syntax and presentation. In Section 5 the design of the translation is given, with an example system translated from the recurrence relations to an agent-based model. In Section 6 the translation is discussed, with: comparisons to the desired agent-based model given, numerical testing of the equivalence of the steps given, and additional work outside the scope of the project presenting the first stages in implementing this translation given. In Section 7 the dissertation is concluded and summarised with notes for further work given.



## 2 Background

This section covers a description of emergent behaviour, feedback loops as a type of this behaviour, and their appearance in the financial markets. Recurrence relations and agent-based models are introduced as a method for modelling emergent behaviour, and are described. InterDyne is introduced as an example of an agent-based model designed for the financial markets. Its applicability to modelling aspects of the financial market will be discussed and a description of its design will also be given.

### 2.1 Emergent Behaviour

Emergent behaviour is a term used to describe a macro-level behaviour of a system that is not obvious from analysis of the micro-level behaviour of the system; more formally this is a behaviour that can not be predicted through analysis of any single component of a system [25].

A misunderstanding of emergence can lead to the fallacy of division, this is that a property of the system as a whole must also be a property of an individual component of the system; water for example has a number of properties including being able to be cooled down to become ice and heated to become steam, saying the same must also be true of a molecule of water however is incorrect. This concept continues into economics, being called the fallacy of composition, where what is true for the whole economy may not hold for an individual and vice versa [22].

A simple way to demonstrate emergence is in the Game of Life [26], which is an example of a cellular automaton; this game takes place on an infinite two-dimensional grid in which, cells can either be ‘alive’, coloured for example green, or ‘dead’, a different colour usually black. Whether a cell is ‘alive’ or ‘dead’ is based on a set of simple rules:

1. ‘Alive’ cells will transition to be ‘dead’ cells in the next time step if they have fewer than two ‘alive’ neighbours.
2. ‘Alive’ cells with two or three ‘alive’ neighbours remain ‘alive’ at the next time step.
3. ‘Alive’ cells will transition to be ‘dead’ cells in the next time step if they have more than three ‘alive’ neighbours.
4. ‘Dead’ cells with exactly three ‘alive’ neighbours will transition to ‘alive’ at the next time step.

With this simple set up very complex patterns evolving through time can be created, these patterns can be seen as emergence, with an individual cell not being able to encapsulate this behaviour. A natural phenomena similar to this is the formation of symmetries and patterns within snowflakes.

Emergent behaviour can be seen occurring naturally in many other cases, with physics offering a number of well explored examples. For instance the n-body problem [27], this historically is explained as n planets interacting in such a way as to produce complex behaviour, despite each individual body following Newtonian laws. An interesting aspect of the n-body problem is that it can be reduced down to three bodies and still exhibit complex emergent behaviour. This example shows that a system need not be overly complex or large to display emergent behaviour.

The emergent behaviour within the n-body problem is caused by interaction dynamics, this is the communication between different elements of a system. The interactions here takes the form of gravitational pulls, if these pulls were not present then the system as a whole, and every individual present would maintain a constant velocity, unless they physically collided [28].

Though this example deals with a physical phenomenon, interaction based emergence is present in many different systems. For example interactions can take the form of verbal and visual communication in social systems, with negative emergent behaviour in this case being the break down of social cooperation [29]. The financial markets can be thought of as a complex system,

with interaction dynamics, hence one can posit that the markets would exhibit emergence. This is true and the financial markets have been seen to exhibit a large selection of emergent behaviour, such as the formation of patterns, bubbles and crashes [3]. These particular examples derive from feedback loops with in the markets, in a similar process to that of the interaction between short range and long range feedback loops in chemical reactions [30].

Feedback loops are a prominent type of emergent behaviour that can occur from interaction dynamics. Feedback loops occur where the input information to an entity is in some way dependent on the output information of that same entity, normally from a previous moment in time. In their simplest form this can just be a single entity supplying an input to its self, shown in Fig. 1.

In finance this could be seen as a simple trader who decides how much to sell based solely on their inventory at the previous time step.

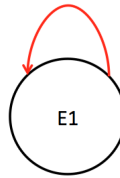


Figure 1: *Simple feedback loop with an entity supplying its input from its output.*

Feedback loops can encompass multiple entities, in Fig. 2 a feedback loop is shown that encloses two different entities. An output from  $E2$  is passed to  $E3$  which in turn creates a new input for  $E2$ ; though  $E2$  input is not directly its own output, it does depend upon it.

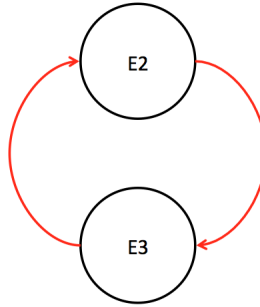


Figure 2: *Feedback loop between two entities, with each output being transformed by the other entity before becoming an input.*

A loop such as the one in Fig. 2, is only considered a feedback loop if information is passed through out the entire loop. If  $E3$  produced a constant output, or an output that does not depend upon its input from  $E2$ , then this would not be a feedback loop as the new input to  $E2$  does not depend on its output.

These examples are very simple, feedback loops can be much more complex, encompassing any number of entities, each of whom can have very complex algorithms for transforming their inputs. Feedback loops can operate across time, meaning that an event in the past can eventually feedback to a present decision. For a feedback loop containing a large number of entities the time scale on which the feedback occurs can be come significantly large.

Though feedback loops are often assumed to be a negative property, some can be stabilising due to a benign effect [31].

Feedback loops can be present in a system in two ways, either they can be a constant fixture, called a static feedback loop, or they can form and change, called a dynamic feedback loop. A static feedback loop is present in the system from the start whether this is intentional and known, or unintentional and unknown to the members of the system. Dynamic feedback loops may not be present at the start and can form and change over time, with new entities joining or leaving them, allowing them to increase or decrease in size or effect, to split or merge, or to disappear.

Due to the potential complexity of feedback loops both in construction and in time, they can be difficult to detect, therefore methods are usually used to expose them. For static loops, forms of static analysis can be used such as analysing initial setup; this is possible since the loops do not change through out time. Dynamic loops can be much harder to observe and analyse, an important aspect to detecting these loops is the interactions, messages sent between different entities within the system. Since the loops can evolve over time being able to track and analyse these messages over a time series is vitally important for the analysis of these loops; this time dependent analyse is called dynamic analyse.

Since feedback loops can be destabilising and damaging to the system in which they occur, there is interest in studying this emergence in the aim to prevent monetary loss and damage to the economy.

Flash crashes are a notable form of emergence behaviour, due to feedback loops, that are considered damaging. These cause the price of a security or commodity to drop very rapidly, becoming disconnected from its fundamental, before then recovering [32]. A particularly famous flash crash is that of 2010, in which the E-mini S&P 500 equity futures market dropped in price by more than 5%, before rebounding to close to its original price [8,32]. This whole process occurred very rapidly, lasting approximately thirty-six minutes and has been described as “one of the most turbulent periods in their history” for the US financial markets [33].

Research done by [9,34] describes how the crash may have unfolded due to a feedback loop between High-Frequency Traders (HFTs), known as “hot-potato” trading.

HFTs are a subset of algorithmic traders who normally participate in the market as arbitrageurs or market makers, they invest in ultra-high speed technology allowing them to detect, analyses and react to market condition in nanoseconds [35]. This means HFTs can trade huge quantities of assets in very short time frames, with some estimates stating that 10-40% of all trades where initiated by them during 2016 [36].

The feedback loop of “Hot Potato” trading, is when inventory imbalance is repeatedly passed between HFT market makers. A market maker is a trader who is required to have both a bid and an ask on the order book at all times, this means in theory that they are constantly buying and selling. A high frequency market maker as expected should be buying and selling very very often. Market makers make a profit from the spread and not from long positions, hence they want to keep inventories low to avoid the market moving against them. To achieve this market makers have strict inventory limits that if they pass will cause them to sell off an amount of inventory to return back into their normal trading region. This inventory now sold by the market maker can be bought by another market maker perhaps causing them to exceed their limits and sell, this process is “Hot Potato” trading and can in theory be cyclic and continue indefinitely [37].

This constant selling and buying of inventory can artificially inflate the trading volume of the market, changing how many traders operate and potentially leading to a flash crash.

Flash crashes have occurred on a number of occasions and in a large selection of markets, with a more recent example being a crash of the cryptocurrency Ethereum [38].

## 2.2 Methods for Modelling Emergent Behaviour in Finance

There are many different methods available for modelling systems that may exhibit emergent behaviour, such as: 1-period, 2-period and multi-period models, probabilistic models (e.g. hidden Markov models), ordinary differential equations and partial differential equations [39–42]. This dissertation focusses on two models that can be used for discrete non-equilibrium system: recurrence relations and agent-based models.

Both recurrence relations and agent-based models have been used in modelling the financial markets [20, 21, 43].

Recurrence relations have seen wide spread use in economics appearing in a large selection of journals, including numerous times in top journals [44–51]. This wide spread use of the technique implies the acceptance of recurrence relations within financial modelling.

Agent-based models are also used widely in economics and finance [52, 53]. However these papers are noticeably absent from top economics journals (with some exceptions, such as [54] and [55]) and tend to be published in the Journal of Economic Behavior & Organization and the Journal of Economic Development & Control [12, 13]. The lack of agent-based models present in top economic journals combined with the number of papers published in other journals shows the lack of wide spread acceptance for this technique [56–58].

Here these two modelling techniques will be described.

### 2.2.1 Recurrence Relations

Wimp [59] describes recurrence relations as follows, recurrence relations connect a discrete set of elements in a sequence, these elements are normally either numbers or functions, they can be used to define these sequences or produce the elements in them. They can be seen as equations that give the next term in a sequence based on the previous term or terms, hence defining said sequence. Recurrence relations are often used to define coefficients in series expansions, moments of weight functions, and members of families of special functions.

The most simplistic form of a recurrence relation is one where the next term depends only on the immediately preceding term. If the  $n^{th}$  element in the sequence is defined as  $x_n$ , then this recurrence relation can be written as,

$$x_{n+1} = f(x_n), \quad (1)$$

where  $f()$  is a function that calculates the next term based on the previous one. A recurrence relation does not just have to depend on its immediate previous term and can depend on any number of terms further back in the sequences. For example a recurrence relation depending on terms from two and three steps before can be written as,

$$x_{n+1} = f(x_{n-1}, x_{n-2}), \quad (2)$$

with  $f()$  now taking two inputs to produce the new term in the sequence [60].

Recurrence relations can also be used to define a sequence through time. In the simplest case, the enumerate  $n$  can be set to represent time  $t$ ; this is applicable to discrete time as it requires set steps between the different times. Just as in the previous examples, the simplest recurrence relation is,

$$x_{t+1} = f(x_t), \quad (3)$$

where  $x_t$  is the term at time  $t$  and  $f()$  gives the term at  $t + 1$  based on the term at  $t$ . Again this can be expanded to include terms from a number of previous time steps, allowing the memory of the sequence to be shown.

A recurrence relation for defining a sequence may also depend upon some parameter,  $\alpha$ , in its simplest case this can be written as,

$$x_{n+1} = f(x_n, \alpha). \quad (4)$$

The next term in the sequence may not only depend on previous terms within its own sequence and parameters, it can also be conditional on another sequence. For example one sequence through out time,  $x$ , may depend on another sequence through out time,  $y$ , a simple recurrence relation for this could be,

$$x_{t+1} = f(x_t, y_t), \quad (5)$$

with the sequence for  $y$  possibly depending on its own recurrence relation. The sequence for  $x$  may not even directly depend on its own sequence and could solely depend on  $y$ ,

$$x_{t+1} = f(y_t). \quad (6)$$

Though it could also indirectly depend on its self, if  $y$  was defined by a recurrence relation depending on  $x$ , such as,

$$y_{t+1} = f(x_t). \quad (7)$$

These cross sequence associations allow for complex interactions to be represented as time series defined by recurrence relations.

Recurrence relations have a number of benefits given by their construction, such as:

- A mathematical formalisation of the whole system being modelled is given when using recurrence relations due to three factors: the way in which they model the whole system as an entity, the static view point they give to the system, and the mathematical style which they take [61].
- Links within sets of recurrence relations are hard coded into the equations, making these relationships amenable to static analysis.
- This model lays out both the functionality of each equation and the relation between them in such a way to give a static representation of the system as a whole.

A downside to this modelling technique comes with complex equations. Equations such as those making function calls within conditional statements, can become increasingly less amenable to static analysis for dependencies. This is a problem similar to determining whether arguments will be evaluated, and makes tracking the flow of information, especially dynamically, very difficult [62]. This also leads to challenges in tracking the individual behaviour of recurrence relations or groups of recurrence relations [63].

### 2.2.2 Agent-Based Models

A modelling technique that takes a more dynamic approach is agent-based modelling. Agent-based modelling can be considered more of a mind set than a rigid methodology; this involves describing the system in question in terms of its components and then allowing these to interact. Agent-based models allow a system to be described naturally and are hence the canonical approach to modelling emergent phenomena. This method is a bottom up approach, allowing for each component of the system, agent of the model, to be created to a relevant degree of abstraction [64].

Agent-based models have been used to model a wide range of emergent behaviour including in the financial markets, examples of this are, noise traders [65], herding among traders [66], and fundamentalists [67].

Agent-based models are particularly suited to systems that, contain a number of autonomous components. Each component can be modelled independently and then allowed to

interact through messages sent between each other. This allows for an obvious visual design of the system, such as that shown in Fig. 3.

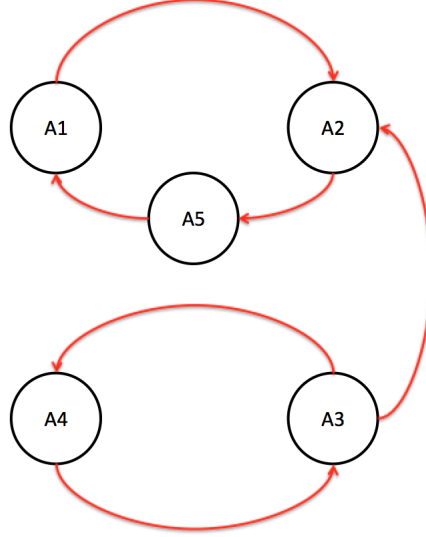


Figure 3: *Visualisation of the allowed interactions between five different agents, represented by black circles, with red lines representing the message paths.*

The main aspect of this model is the agents used. Though every system modelled can have drastically different agents, there are a few characteristics which should be in place for an agent to be considered an agent [68,69]:

- An agent is self contained, unique and identifiable, this requires an agent to have boundaries which can easily be used to determine what is and what is not part of the agent.
- An agent is autonomous and it can act independently during its interactions with other agents. Its has behaviour and decisions that can be associated with information acquired during communication with other agents.
- An agent's state varies over time, representing variables associated with its current position.
- An agent's behaviour is influenced by dynamic interactions with other agents.

This modelling technique has a number of benefits originating from its construction, these include:

- Being able to independently create each agent to varying degrees of complexity.
- Giving a clear visual representation of the system's interconnectedness.
- Giving a dynamic representation of the system.
- Easily expandable by adding more agents.

Since each agent in the system can be completely autonomous and independent from each other, save for message passing, this allows for them to be created individually. Hence each agent can describe its relevant component to a relevant degree of complexity, making the creation

of a simulation more intuitive and sectioned. This makes agent-based models in many cases the most natural way for describing systems composed of “behavioural” entities [14].

Due to the set up of this model a network representing the communication between different agents is easy to construct; this allows for topologies to be visualised and can aid in the creation of a simulation.

The use of messages and a bottom-up approach in this model, allows for a more dynamic view of a system to be achieved; messages can be more easily tracked through-out the system's evolution allowing for events to be more easily pinpointed and analysed.

The use of independent agents makes this technique very amenable to expansion, new agents can often be added with little or no change to previously existing agents. This allows these simulations to be confidently expanded to look at more complex systems, or to add new elements to an existing system.

These aspects of this approach make the modelling technique well qualified for modelling and analyse of emergent behaviour within systems of interacting components. However this approach does contain some draw backs including a one-to-many problem (where, though some high level behaviour may be noted in the system, this does not mean the only way of achieving this behaviour is the current system design) and the lack of a formal definition for the system as a whole [56].

## 2.3 InterDyne

An example of an agent-based model used in modelling the financial markets is InterDyne. InterDyne is bespoke simulator created by Clack and his research team at UCL [70], it is a general-purpose simulator for exploring emergent behaviour and interaction dynamics within complex non-equilibrium systems.

InterDyne's design is that of an agent-based model interacting via a harness. This creates a structure of individual autonomous agents who interact through messages sent through a harness to one another.

Similar to other agent-based models, InterDyne operates in discrete time rather than continuous time. These quantised time chunks, which move the simulation forward, can be left without proper definition (allowing operations to be defined in a number of time steps) or they can be equated to a real time (usually with the smallest time gap needed being a single time step and then all other timings being integer multiples of this). This discrete time is most important to message passing, with messages between agents only sent on an integer time step.

Messages in InterDyne are small packets of data, such as a series of numbers. An agent can send private messages that are only received by a single other agent (one-to-one messages), or it can send public broadcast messages received by any number of other agents, subscribed to a channel (one-to-many messages). To facilitate this a communication topology can be made for InterDyne, this is done in the form of a directed graph determining which agents can communicate with each other. Due to the directional nature of these messages this topology could allow an agent to send messages to another but not be able to receive messages from that same agent. Messages have a defined order to them, an agent will, unless otherwise instructed, always process messages in the order in which they arrive. To change the order in which messages arrive delays can be added to communication paths between agents, this can be a static delay which always applies to messages sent from one agent to another, meaning this will arrive a set number of time steps later. Or a more complex dynamic delay, which is achieved by using another agent to mediate the passing of these messages, delaying by an amount decided on in some internal logic. All messages in InterDyne are passed through a harness; this does not alter the messages or delay them,<sup>3</sup> but does store the messages and their order which can be used in post analyse.

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<sup>3</sup>Unless instructed to, using a static delay.

Each of the agents within an InterDyne simulation can be completely unique and modelled to different levels of complexity, as is the case with most agent-based models. As a whole InterDyne simulations are deterministic, repeated experiments will return identical results. However non-determinism can be added via the agents, making some part of an agent stochastic, and will lead to repeated experiments on the whole returning different results. A pseudo-random element can also be added by instructing InterDyne to randomly sort the message order for any agent receiving multiple messages in one time step. This is only pseudo-random as, as long as the same seed is used each run of the simulation will order the rearranged messages in the same way.<sup>4</sup>

InterDyne is created to be particularly amenable to dynamic analyses of its simulations, this is achieved in part by all messages being sent via the harness allowing them to be recorded for post processing.

### **2.3.1 Applicability to Finance**

InterDyne has been designed with modelling flash crashes in the financial markets in mind and has a number of features that make it well suited to this purpose.

#### **2.3.1.1 Deterministic**

The deterministic nature of InterDyne allows for experiments to be run multiple times with the same result always returned, this allows for changes to the experiment setup to be investigated. For example changing the number of traders in the market and comparing this to a previous run allows for an investigation into how many traders are required for emergent behaviour to be observed.

This becomes particularly interesting when comparing the interactions between market makers to that of the n-body problem. Like with this problem one could expect emergent behaviour might occur to some extent in a large group of market makers, however the question of whether emergence persists in a comparable market to the three-body problem and how this compares to a larger market can be investigated.

#### **2.3.1.2 Message Delays**

Allowing for messages to be delayed is needed to facilitate hypotheses involving delays as a factor for emergent behaviour. Delays have been suggested to have caused “hot potato” trading during a flash crash, these delays can exist due to processing time and transmission time of messages [8].

InterDyne allows for both symmetric and asymmetric, delays. These delays can be static or dynamic, with dynamic delays requiring a special intermediary agent.

#### **2.3.1.3 Storing Messages**

InterDyne facilitates analyses of simulations by allowing for the messages between agents to be stored and viewed as a trace file. This can include all messages as well as timings, and messages can also be sent directly to the harness which will be added to the trace file.

#### **2.3.1.4 Message Ordering**

The order in which messages are processed can be very important. For example in an exchange, it can change whose limit order has priority at a given price and whose market order executes the lowest prices. Changing these factors can make or break feedback loops within the system, meaning if message ordering is not properly dealt with the correct emergent behaviour may not be observed. Hence InterDyne stores messages in the order they are received by an agent, taking into account delays to the messages. This however can not be done when multiple messages are

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<sup>4</sup>If an agent receives multiple messages at the same time step and the pseudo-random element is not being used, these messages will order based on the identifiers of their sender agents.



received at the same time step, due to the nature of discrete time there is no way for the agents to know which message arrived first. Therefore two options are presented by InterDyne; messages are ordered according to their agent identifier or messages are randomised and executed in the emerging order.

### 2.3.2 InterDyne Detailed Operation

InterDyne is written in the functional language Haskell. The structure of an InterDyne model is shown in Fig. 4, this structure contains a number of agents and a simulator harness. These agents send two types of messages, either one-to-one or one-to-many (broadcast) messages. Both these messages are sent to the harness, the harness then resends these messages to the appropriate agents, one-to-one messages are sent to their target agent and one-to-many messages are sent to any agent subscribed to the relevant broadcast channel. Messages can also be sent directly to the harness and not rerouted to another agent. At the end of the simulation the harness will produce a trace file containing information on all the messages sent for post-hoc analysis.

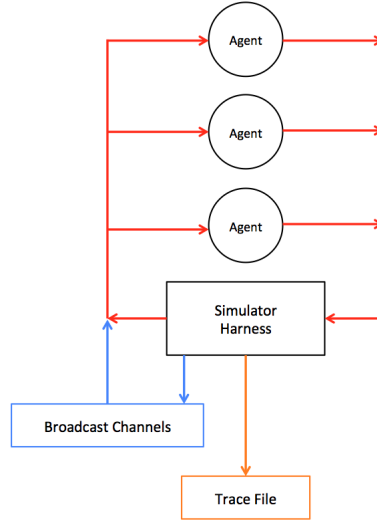


Figure 4: *Structure of an InterDyne simulation containing three agents. The messages sent by the agents to the harness and from the harness are coloured red. The trace file and messages sent to the trace file are coloured orange. The messages sent from the harness to the broadcast channels and then back to the agents are coloured blue [71].*

InterDyne has an intrinsic delay, which means any message sent will not be received until the next time step; message sent at  $t$  will be received the earliest at  $t + 1$ . This is in part due to the fact that the harness compiles all messages from a single time step before sending them to their targets and hence initiating the next time step. The harness can be seen as the driver in the simulation sending out the message and forcing the agents to send their next message.

Longer delays can also be added using InterDynes topology, this allows for any two agents to be selected (a sender and a receiver) and for a delay between them to be given. This topology takes the form of a directed graph, with the agents acting as the nodes and the interaction routes as the links, this allows for the delays to be asymmetrical. Using this, a delay for a path could also be set as an abort message, making a particular communication route unusable.

Adding delays is done through run time arguments, here two run time arguments must be added, one stating the maximum delay in the system and the other listing the delays. To show

the maximum delay the argument ( $Arg(Str\ "maxDelay", 10)$ ) is used, where 10 is the maximum delay. To add the list of delays the argument ( $DelayArg(Str\ "DelayArg", delay)$ ) is used, where  $delay$  is a function that takes two agents and returns the delay between the first and the second.

Messages sent between agents can in theory be as complex as needed, these messages do however have to comprise of the following components:

- A tag indicating the type of message being sent, e.g. a broadcast message.
- A tuple of two integers, with the first being the sending agents ID and the second being either the receiving agents ID or the receiving broadcast channels ID.
- The data that is being sent.

An example of this is a one-to-one message, for example  $Message\ (1, 2)\ data$ , where  $data$  is the information being sent from agent 1 to agent 2

A broadcast message, for example  $Broadcastmessage\ (3, 1)\ broadcastdata$ , can be received by agents subscribed to its channel, in this case 1. All agent subscriptions to broadcast channels have to be announced at the beginning of a simulation and can not be changed during it. This is done by adding the subscription channel to the list in a tuple passed as an argument, for example this could look like  $(agent1, [1])$ , for agent 1 subscribed to broadcast channel one.

Empty messages can also be sent, if the message is required to be known to be intentionally empty and not a mistake, the a message can be sent containing "Hiaton". This is done at the beginning of the run to allow for the harness to send a first message.

InterDyne is designed in such a way that agents both take in and produce a seemingly infinite list of messages. This is achieved through lazy evaluation in Haskell, which in short means that any element in the list will not be calculated until it is absolutely needed. This allows for an infinite list as long as no agent tries to read further into the list than what has already been calculated. In achieving this an agent must at each time step, read in a message (even if it is then not used) and produce a list of messages (even if this is an empty list). An agent will hence iterate over the list of incoming messages, at each time step adding a new output message to its list of output messages.

Agents in InterDyne are typically, though not required to be, written in two parts, a wrapper function and a logic function. The wrapper will manage the reading of inbound messages to the agent and generate its outbound messages, it will also update the local state of the agent. While a logic function is called by the wrapper to calculate the messages to be sent and their content. An agent written following this design could have a form similar to that of the agent show in Fig. 5. This agent's wrapper recursively calls itself on the inbound message list, consuming the head of the list<sup>5</sup> and producing a list of outbound messages using a logic function.

```

1 wrapper st args ((t, msgs, bcasts) : rest) myid = [m] : (wrapper st args rest myid)
2                                     where
3                                     m = logic (t, msgs, bcasts)
```

Figure 5: *Simple wrapper function, which creates a list of output messages by iterating through a list of inbound messages and calling the sub-function logic.*

The simulator harness is an intrinsic part of the simulator, it drives the simulation, handles message passing and produces the output trace file.

This has the type shown in Fig. 6, this type illustrates the functions input of a number of steps and runtime arguments (with agents and the broadcast channels they are subscribed to).

<sup>5</sup>This design dictates that the head of the list is the messages for the current time step.

The output from this function can be seen to be of type  $IO()$ , this is writing the trace information to file.

```
1 Int -> [Arg-t] -> [(Agent-t, [Int])] -> IO()
```

Figure 6: *Type definition of the function sim.*

The simulator harness is embodied in a function called `sim`, this function contains code associated with a number of administrative operations. For the purpose of this project a reduced specification of the function `sim` will be shown. This specification captures all of the core functionality for managing message passing for agents and driving evaluation of the simulation. This reduced InterDyne `sim` harness can be seen in Fig. 7, with its function type shown in Fig. 8. This reduced function is shown outputting a single integer value, that is the first message sent by the first agent at time step  $x$ . This is just for illustration purposes since the normal output of `sim` is a trace file and is not being shown here. In the reduced type definition it can be seen that the output is now an integer instead of an  $IO()$  type, the list passed to the `sim` that contains the subscription channels is now of unknown type as no channels are passed.

```
1 sim x runtime_args agentinfo = (getargval.hd.getmsgargs) (((snd (outputs!x))!0)!0)
2   where
3   outputs = (0,[]) : timed_transposed_msgs
4   timed_transposed_msgs = zip [1..] transposed_msgs
5   transposed_msgs = transpose allmessages
6   allmessages = map apply_to_args (zip [1..]
7     agentinfo)
8   where
9   apply_to_args (agentid, (
10    agentfn, broadcastsubscriptions)) = agentfn Emptyagentstate runtime_args (
11    redacted_msgs agentid outputs) agentid
12   redacted_msgs id outputs =
13   map (f id) outputs
14   f x (time, xs) = (
15   time, concat (map (filter (h x)) xs))
16   h x (Message (f,t) args) = (
17   t==x)
```

Figure 7: *Reduced InterDyne simulator. This sim function outputs, the value of the first messages from the first agent at the final time step.*

```
1 Int -> [Arg-t] -> [(Agent-t, [*])] -> Int
```

Figure 8: *Type definition of the reduced function sim.*

### 3 Description and Analysis of the Problem

Investigating emergent behaviour originating from interaction dynamics, within a complex system, relies on viewing the communication between different components of the system as messages being passed. These messages can then be analysed to assess the communication and its pattern that lead to the creation of the emergent behaviour; this approach is amenable to the discovery of phenomena, such as dynamic feedback loops. A model investigating emergent behaviour hence must be able to view communication as messages being passed between different components. For results from this model to be broadly accepted, in fields such as economics, the model as a whole must also be well defined [72].

Agent-based models naturally support a message passing view making them well suited to analysing and investigating emergent behaviour. However most agent-based models, though having each individual agent fully specified, do not have a well defined formalisation for the system as a whole. This makes it challenging to perform system wide analytics on most agent-based model. Recurrence relations naturally support a system wide analysis, due to the full definition of their models. However their formulation does not support a message passing view of communication between components, making it difficult to analyse emergent behaviour with them directly.

This creates a problem as neither model is perfect for the task at hand. A resolution to this problem suggested by [57], is to give a formal definition of the specification of an agent-based model as a set of recurrence relations.

This section discusses how relating these two models can be achieved, reviews literature on work done formalising agent-based models, and discusses challenges related to the method presented in this dissertation.

#### 3.1 Method for Achieving a Translation

To allow for recurrence relations to be used in formalising an agent-based model, the two paradigms must be connected. This connection must be able to demonstrate some sort of equivalence between these two models.

It has been shown informally by [17], that most agent-based models can be expressed as a set of partial recursive functions. Therefore it can be inferred that a connection between a recurrence relation model and an agent-based model can be created in the form of a translation, from one to the other.

A correctness-preserving translation between a recurrence-relation model of a system and an agent-based model of the same system provides a formalism for the agent-based model if it can be said that the recurrence-relation model defines the meaning of the agent-based model. Correctness preserving in this context is used to mean that both models representing an identical system and will produce identical results; so that both views of the system should be completely interchangeable.

An important consideration in creating a translation between the two methods is the direction in which this translation should be effected. Ideally, correctness-preserving translations would be defined in both directions and this would prove the semantic equivalence of the two models. However, defining two complete translations is beyond the scope of this project. Defining the translation from a recurrence-relation model to an equivalent agent-based model has been chosen, since it is believed that those who prefer recurrence-relation models would prefer to start with a recurrence-relation model; the agent-based model then becomes an equivalent view within which dynamic analysis of emergent behaviour can be undertaken.

The next consideration is how this translation will be achieved. Inspiration can be drawn from other works investigating the design of translations, such as [73], [74] and [75]. These three papers discuss translations between program representations in a way in which, the final representation will faithfully represent the initial. The approach taken by these papers is to

increment the translation in step-by-step manner, where each step slightly transforms the previous to be closer to the final form. The benefit of this method is that if each step on the translation can be proven to be correct then the whole translation can be seen as correct. This approach relies on the correctness of the steps being transitive; that a correctness preserving translation from state 1 to state 2, and then another from state 2 to state 3, will produce a correctness preserving translation from state 1 to state 3.<sup>6</sup>

A connection between these two models can hence be created by translating in a step-by-step fashion from a recurrence relation language to an agent-based model. The agent-based model used in this translation is InterDyne and the recurrence relation language used is discussed in Section 4. InterDyne was selected due to a number of factors, one being that from its conception it was designed with modelling and investigate emergent behaviour with the financial markets in mind (with research existing already using it to this end [9]), this software was also a convenient choice due to access to the source code and local expertise.

### 3.2 Review of Similar Work

The idea of using recurrence relations to give a formal definition to an agent-based model is not a novel one and falls under the general heading of using formal systems to prove properties of agent-based models. Work in this area has covered game theory, discrete systems, classifier agents and equation-based models [76]. The use of recurrence relations falls under the classification of an equation-based model, these have been used in a number of papers looking to give a formalisation to agent-based models. Here three papers [72], [57] and [69], looking at this area will be reviewed and discussed, along with DEVS formalism introduced by Zeigler in [77].

Before reviewing these papers, it is worth considering the more obvious question of does a formalism need to be given by another modelling technique to allow for the analyses of an agent-based model? North [76] present's a formalism for the analysis of agent-based models, looking in particular at performance. This paper looks to prove findings from agent-based models, in particular the focus is on existing agent-based software such as Swarm, Repast 3 and others. This paper notes that these pieces of software use a modular imperative architecture with factored agents, spaces, a scheduler, logs, and an interface. Hence this paper presents a theoretical formalism for analysing a modular imperative agent-based model.

This formalism is presented using an abstract machine specification of the modular imperative agent-based modelling architecture. A Random Access Stored-Program (RASP) architecture was chosen for the abstract machine, this architecture had been created in an earlier paper [78] and was chosen due to its similarities to modern computer systems. This is opposed to other options including Turing machines and  $\lambda$ -calculus, as they are less well matched with modern computer systems.

One of North's motivations for using this technique over an equation-based model formalisation, is that he claims that a formulation of the agent-based model in terms of an equation-based model would be unnatural and possibly unrecognisable to those accustomed to agent-based models. This consideration is not as important to the work presented in this dissertation, since here the aim is to familiarise those accustomed with equation-based models to agent-based models and not the reverse. The same argument applies to the use of RASP over alternatives such as  $\lambda$ -calculus, this dissertation is more interested in creating a formalism which is relatable to economists over computer scientists.

In [72] Costa et al consider the formalisation of agent-based models by creating their implementation guided by equation-based models. This paper looks at an example with interacting entities with given strategies, however these strategies could change upon interactions. In this example the equation-based model took a probabilist and macro-view approach where the agent-

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<sup>6</sup>A suggested method for further work in this area is given in Section 7.1.

based model, due to its nature, took a micro-view approach. The analyses of these two models showed that though the systems appeared to be the same the micro-view approach could create slightly different outcomes depending on the number of agents chosen. This difference comes from the fact that the individual agents in the agent-based model were created to match a macro-level description given by the equation-based model, however due to the autonomous nature of the agents only their general behaviour can match and in certain circumstances even this did not correctly produce the behaviour. This issue spawns from trying to recreate the behaviour of a probabilistic model using a deterministic model.

This paper concludes summarising the benefits of this approach in validating an agent-based model, here an agent-based model is known to be a faithful representation of the system as long as the equation-based model is a faithful representation. However the question of whether the agent-based model is correctly implemented still rests with that model.

The main difficulties faced by this paper appear to be the transformation from a macro-level view (equation-based model) to a micro-level (agent-based model) view, this is particularly due to the statistical nature of the equation-based model. Though they succeeded in creating a formalisation of their model the differences in results between runs still raises questions over the equality between the two view points. To avoid this issue, a more micro-level focussed equation-based model should be taken.

The next paper is [57], this paper is based on a similar principle to this project in that it seeks to bridge the conceptual gap around agent-based models in economics. It tackles this problem by introducing an equation-based formalism to describe both agent-based models and analytical models, though of interest here is their work on the former.

The agent-based model is formalised by specifying how each agent's state within the model changes between each time step. This is done by defining agent attributes using vectors of state variables and then applying time-indexed difference equations to specify each agents state change. This allows for expression of the agent-based model in terms of difference equations, which they then use to look at parameter estimation.

Their work shows the ability to provide estimations on agent-based models, counter to some criticism that has been levelled at agent-based models. Some difficulty in this approach can come from agent-based models with complicated algorithmic behaviour within the agents, specifying these models as difference equations can be a challenging task. The work in this dissertation does not directly suffer from this problem as all systems are first described as recurrence relations before being translated into agent-based models, meaning that the equation view will only be as complex as the original writer intends.

Macal [69] investigates relating Forrester's System Dynamics (SD) model to an agent-based model. System dynamics models consist of a set of difference equations, so this can be seen as a comparison between difference equations and agent-based models. This paper presents a formal specifications for both the Forrester's and agent-based models, these specifications are then compared to show their equivalence. Once equivalence has been proven the specification of a Forrester's model can be used to derive an agent-based model that is a correct representation of the same system.

This technique is applied to an SIR epidemic model, creating a Forrester's and agent-based model representation. These two descriptions of the system are then evaluated and have their results compared. The findings show that though both models show similar behaviour, there is not an exact match between them. This is due to the probabilistic nature of the system being modelled and the different approach that an agent-based model takes to these kind of systems.

This paper, though showing equivalence, does so in a more abstract manner than a direct translation between the two models. The difference in experimental results from the two models is down to the use of describing the system in a probabilist fashion, this can be avoided by describing a system in a more micro-level view.

A discussion relating to the formalism of agent-based models would be amiss without mentioning the Discrete Event System Specification (DEVS) [77]. DEVS is a theoretical formalism designed for analysing complicated simulations, which it does by describing simulations as hierarchical Moore machines, with additional timing information added to the state transitions. DEVS has a few variants including classic DEVS (this is the core abstraction), coupled DEVS (allows for hierarchical nesting of the Moore machines [79]) and Dynamic Structuring DEVS (attempts to allow for dynamic behaviour [80]).

DEVS is able to provide a theoretical foundation for analyses of a number of critical computational properties of agent-based models, including properties such as decidability. In theory DEVS has the ability to represent any computational model, including agent-based models. However though DEVS is capable of representing an agent-based model it is not necessarily the best approach. Despite the work done in [80], DEVS is still not an ideal match for models that have dynamic behaviour, such as agent-based models where agent behaviour can be dynamically generated. Classic DEVS imposes boundaries between states and behaviours making dynamic behaviour difficult to represent, where as Dynamic Structuring DEVS introduces new constraints that have a similar effect in limiting utility. These constraints and difficulties make it challenging to use DEVS as a formal basis for agent-based modelling [76].

### 3.3 Challenges with the Translation Approach

The approach taken in this project of creating a step-by-step translation between a recurrence relation description of a system to a agent-based model description on the same system has a number of challenges associated with it. In this section the difficulties with this project will be described.

As noted in Section 3.2, when looking at work done by other researchers in this area, a main challenge was how to model systems that have no counterpart, or are challenging to create a counterpart for, in an agent-based model. An example of this is modelling probabilist systems and macro-level systems, which are difficult to establish an equivalent agent-based model for. This project avoids these challenges by using micro-level recurrence relation models for the translation.

A problem overshadowing most aspects of this project is that the two paradigms being translated between are completely different and in some ways opposing. Recurrence relation models take an analytic (where the system evolution is defined statically.) approach where as agent-based models take a dynamic one (where the system evolution is given in simulation).

The formulation of how information is exchanged within these two models is also in contrast. Recurrence relations take a static view with information passed via function calls, opposed to the agent-based model approach of dynamically sending the information between agents using message passing. This represents a difference in the way information is seen in these models, in recurrence relations information is freely available and the equation wishing to use it simply has to make a function call. Where as in agent-based modelling information is given by its creator to only other agents selected by its creator, hence agents can not use information which has not been already sent to them.

The agent-based model selected for use, InterDyne, operates by making every agent take in an infinite list of their inputs and output an infinite list of their outputs. Creating this view of the system from a recurrence relation model, which is created as a snapshot of a current time step, is challenging.

In InterDyne both direct and broadcast messages exist, these two messaging types can be seen as the use of private and public data respectively. Recurrence relations do not have an inbuilt method of distinguishing between different types of data as all information is passed by direct function calls, reconciling these two view points creates another challenge.

In recurrence relations information from any previous time step can be easily accessed, whereas in InterDyne information is only available for the time step at which it was received. This

creates the idea of time-limited information with InterDyne that is not present within a recurrence relations model.

Agent-based models have name control, indicating the different agents and functions within them. However introducing name control to recurrence relations, that have no sense of agents, in a way in which it is tangible to the agent-based model is a challenge.

Recurrence relations access information directly as a value where needed; agent-based models however receive special item called a message that contains this value along with other information (e.g. the senders id). Translating between these two different ways of viewing values poses difficulties.

There has been criticism that agent-based models splinter a model, finding a way to split the recurrence relations into agents with out splintering the model creates another challenge.

The approach taken in this dissertation is to create a series of small steps between a recurrence relation model and an agent-based model, the smaller the steps are the easier it is for a reader to follow the process. However a challenge is finding how to create these small steps as it may often be easier to create large but less comprehensible steps.



## 4 Bespoke Recurrence Relation Language

This section introduces a bespoke recurrence relation model, created to be able to model complex systems with interacting elements. This recurrence relation model is used as the initial description of the system in the translation into an agent-based model and hence needs to be able to express the system being modelled both statically and dynamically.

A bespoke recurrence relation language is created for three reasons. Firstly, an automated translator needs to have a well-defined input that it can parse. Secondly, the functionality of the language needs to be restricted to keep it as simple as possible, to allow for the language to be readily understandable and easy to parse, only introducing complexity where necessary. Thirdly, the language still needs to maintain sufficient functionality to model complex systems. Creating a bespoke language offers a straightforward method satisfying these three criteria.

A set of recurrence relations can be used to statically model a interacting system, however this model does not give a time-dependent experiment required for dynamic modelling. To add time dynamics to this model the values of the recurrence relations need to be traced through time.

To allow for tracing, initial values must be provided (noting that different experiments might have different initial values), and a main expression must be defined to control demand for the evaluation of the relevant traces.

This section first gives the concrete syntax, then keywords and naming conventions, finally a well formed program in this language is described and an example is given.

### 4.1 Syntax

The formal concrete syntax for this bespoke recurrence relation language is defined in Backus-Naur form (BNF), shown in Fig. 9. The main aspects of this notation, in this context, are described as follows:

- Expressions of the design of language are represented inside  $\langle \rangle$ ; these are high level design aspects of the language (e.g. the simulation as a whole, or the main block defining the experiment).
- These high level expressions are given meaning by  $::=$ , the expression on the left hand side of this must take the form given on the right hand side (e.g. in  $\langle a \rangle ::= \langle b \rangle$ ,  $\langle a \rangle$  has to take the form given by  $\langle b \rangle$ ).
- The right hand side of  $::=$ , can have multiple options, these are split by  $|$ . This can be seen to representing an *or* operation, with the left hand side being able to be represented by any of the options given on the right hand side (e.g. in  $\langle a \rangle ::= \langle b \rangle | \langle c \rangle$ ,  $\langle a \rangle$  can be either  $\langle b \rangle$  or  $\langle c \rangle$ ).
- The speech marks “” are used to denote actual writing in the language, this allows the syntax to describe how the language can be written (e.g. connecting the expression  $\langle where \rangle$  to the word “where” in the language). In this notation “” are used for strings of characters, and ‘’ are used for single characters (e.g. “where” and ‘w’).
- Here .. is used to denote any item that would logically appear between the left hand side and right hand side of the .., these items are separated by an  $|$  (e.g. ‘1’..‘4’ gives ‘1’ | ‘2’ | ‘3’ | ‘4’).
- The character ‘\n’ is used to denote a new line in the language.
- Parameterised nonterminals are used to allow for recursive calls with in the syntax, these are noted by a second item with the  $\langle \rangle$  (e.g.  $\langle indent\ n \rangle ::= \langle space \rangle \langle indent\ n - 1 \rangle$ , for  $n = 3$  will produce  $\langle space \rangle \langle space \rangle \langle indent\ 1 \rangle$ ).

```

1 <simulation> ::= <mainblock> <newline> <initblock> <newline> <whereblock>
2 <mainblock> ::= <main> <newline> <expression>
3 <numbers> ::= <number> | <number> <space> <numbers>
4 <initblock> ::= <init> | <init> <newline> <valdefinitions>
5 <whereblock> ::= <where> | <where> <newline> <recreldefinitions>
6 <valdefinitions> ::= <valdef> <newline> | <valdef> <newline> <valdefinitions>
7 <valdef> ::= <valname> <space> <equals> <space> <expression>
8 <valname> ::= <letters>
9 <recreldefinitions> ::= <recreldef> <newline> | <recreldef> <newline> <
    recreldefinitions>
10 <recreldef> ::= <newline>
11 | <recrelname> <space> <t> <space> <equals> <space> <
    expression>
12 | <recrelname> <space> <t> <space> <args> <space> <
    equals> <space> <expression>
13 <recrelname> ::= <letters> <underscore> <letters>
14 <args> ::= <argname> | <argname> <space> <args>
15 <argname> ::= <letters> | <number> | <lparenthesis> <number> <
    rparenthesis>
16 <expression> ::= <simple_expression> | <simple_expression> <
    expr_whereblock>
17 <simple_expression> ::= <number>
18 | <valname>
19 | <argname>
20 | <t>
21 | <localvalname>
22 | <list>
23 | <lparenthesis> <simple_expression> <rparenthesis>
24 | <cond> <simple_expression> <simple_expression> <
    expression>
25 | <simple_expression> <operation> <simple_expression>
26 | <internop> <simple_expression>
27 | <recrelname> <simple_expressions>
28 | <localfuncname> <simple_expressions>
29 <expr_whereblock> ::= <newline> <indent n> <where> <newline> <localdefinitions
    n> <newline> <indent n> <endwhere> <newline>
30 <localdefinitions n> ::= <indent n> <localvaldef> <newline>
31 | <indent n> <localfuncdef> <newline>
32 | <indent n> <localvaldef> <newline> <localdefinitions n
    >
33 | <indent n> <localfuncdef> <newline> <localdefinitions
    n>
34 <localvaldef> ::= <localvalname> <space> <equals> <space> <expression>
35 <localfuncdef> ::= <localfuncname> <space> <args> <space> <equals> <space>
    <expression>
36 <simple_expressions> ::= <simple_expression> | <simple_expression> <space> <
    simple_expressions>
37 <localfuncname> ::= <underscore> <letters>
38 <localvalname> ::= <underscore> <letters>
39 <list> ::= <lbracket> <rbracket> | <lbracket> <listitems> <rbracket>
    >
40 <listitems> ::= <simple_expression> | <simple_expression> <comma> <space>
    > <listitems>
41 <internop> ::= <head> | <tail> | <not>
42 <number> ::= <zero> | <posnumber> | <minus> <posnumber>
43 <posnumber> ::= <firstdigit> <digits> | <firstdigit> <digits> <dot> <
    digits>

```

(a) First half of the concrete BNF syntax for the bespoke recurrence relations.

```

1 <operation> ::= <plus> | <minus> | <times> | <divide> | <lessthan> | <
  greaterthan> | <equals> | <notequals> | <lessthanorequalto> | <
  greaterthanorequalto> | <cons> | <index>
2 <indent 1> ::= <space>
3 <indent n> ::= <space> <indent n-1>
4 <letters> ::= <letter> | <letter> <letters>
5 <letter> ::= <lowercaseletter> | <uppercaseletter>
6 <digits> ::= <digit> | <digit> <digits>
7 <main> ::= "main"
8 <init> ::= "init"
9 <where> ::= "where"
10 <endwhere> ::= "endwhere"
11 <firstdigit> ::= '1' .. '9'
12 <digit> ::= '0' .. '9'
13 <lowercaseletter> ::= 'a' .. 'z'
14 <uppercaseletter> ::= 'A' .. 'Z'
15 <equals> ::= '='
16 <space> ::= ' '
17 <newline> ::= '\n'
18 <underscore> ::= '_'
19 <lparenthesis> ::= '('
20 <rparenthesis> ::= ')'
21 <lbracket> ::= '['
22 <rbracket> ::= ']'
23 <comma> ::= ','
24 <head> ::= "hd"
25 <tail> ::= "tl"
26 <not> ::= '~'
27 <dot> ::= '.'
28 <plus> ::= '+'
29 <minus> ::= '-'
30 <times> ::= '*'
31 <divide> ::= '/'
32 <lessthan> ::= '<'
33 <greaterthan> ::= '>'
34 <notequals> ::= "~="
35 <lessthanorequalto> ::= "<="
36 <greaterthanorequalto> ::= ">="
37 <cons> ::= ':'
38 <index> ::= '!'
39 <t> ::= 't'

```

(b) Second half of the concrete BNF syntax for the bespoke recurrence relations.

Figure 9: Concrete BNF syntax for the bespoke recurrence relation language, split between Fig. 9a and Fig. 9b.

A simulation written in this language takes the structure shown in Fig. 10, this structure is defined to have the same meaning as the Miranda expression shown in Fig. 11; where the BNF syntax for the items encapsulated by `<>` is shown in Fig. 12.

```

1 ' 'main' '
2 <exp>
3 ' 'init' '
4 <idefs>
5 ' 'where' '
6 <fdefs>

```

Figure 10: Layout of a simulation in bespoke recurrence relation language.

```

1 main = <exp>
2     where
3     <idefs>
4     <fdefs>

```

Figure 11: *Miranda definition of the layout shown in Fig.10.*

```

1 <exp>      ::= <expression>
2 <idefs>    ::= <valdefinitions>
3 <fdefs>    ::= <recreldefinitions>

```

Figure 12: *BNF syntax for items shown in Fig. 10, with further definitions defined in Fig. 9.*

The syntax given in this section dictates how a simulation written in this bespoke recurrence relation must be formatted and what items are allowed to be used within this language. The functionality of this language, given by the BNF, will be described more fully in the Section 4.2. Any program that does not fit the shape described by the concrete BNF can not be considered to be a model written in the language defined in this section.

## 4.2 Naming Conventions

This language has a strict naming convention, as well as some restricted names. Naming conventions are in place for recurrence relation definitions, where blocks, sub-functions, variables, initial conditions and if statements.

Restricted names are those that cannot be used other than for their designated purpose (given in Fig. 9), these are:

- The use of ‘\_’, unless it is being used for its designated purpose in recurrence relations and internal functions.
- The use of the name “*main*”, unless it is used to define the main block.
- The use of the name “*init*”, unless it is to define the init block.
- The use of the name “*where*”, unless used to make a where statement.
- The use of the name “*endwhere*”, unless used to end a where statement.
- The use of symbols, unless for their defined purpose.

A recurrence relation is defined in four parts, *name arguments = expression*. The name must be formatted in the manner *group.name*, where *group* is the name of the group the recurrence relation belongs to and *name* is the name of the particular recurrence relation. Multiple recurrence relations can belong to the same group, and hence have the same group name, however only one relation in each group can have a set name. This language requires that all recurrence relations take at least one argument, and that time *t* has to be the first argument. Time *t* does not need to be utilised within the relation but it must be in the definition.

A recurrence relation can have sub definitions relating to it, this is done through the use of a where block. A where block is written after the expression for a recurrence relation and starts with the keyword *where*. After this local definitions for internal functions and variables can be defined.

Internal functions must be formatted in the manner *\_name*, where *name* is the name for the function. These functions can only be accessed by the recurrence relation they are within. Local variables can be defined using the formate *name*, this name can be anything as long as it does not clash with restricted names or other variable names with in the same where block.

The driving expression is given in a defined section of the model, shown in Section 4.3, and is written as *group\_name arguments*. The *group\_name* must correspond to a recurrence relation listed in the model and the *arguments* must be given as values, variables that are listed as initial conditions, or variables listed in a local where block, allowing the recurrence relation to be evaluated.

Initial conditions are listed in a special location in the layout, which will be show in Section 4.3, and are defined in three parts *name = expression*. This name can be seen as a variable and can be anything not in the restricted name list or used by any other variable (even those within where blocks).

This language allows for both *arguments* and *expressions* to be written using: lists, numbers, variable names, recurrence relation names, internal function names, mathematical operators, brackets, some inbuilt functions, and conditional statements.

Lists in this language are contained within [], items within these lists are then separated using ,.

This language allows the use of both integers and decimal numbers, with . used in decimalisation.

Variables that have been defined and are in scope can be called by simply calling their name.

Recurrence relations that are in scope can be called by calling their name with a relevant set of arguments.

Internal functions that are in scope can be called by calling their name and a relevant set of arguments, if applicable.

This language allows for the mathematical operates shown in Fig. 9.

Expression can be incased in brackets by using ().

This language allows for the use of three inbuilt functions: *hd* (returns the head of a list), *tl* (returns the tail of the list), and *:* (adds an item as the new head of a list). The functions *hd* and *tl* are particularly special as unlike the other operators, they only take one value.

Conditional statements with in this language are written as *cond condition expression expression*. Where if the condition is true, then the first expression is run and if not the second expression is.

### 4.3 Well Formed Programs

This language has a strict style and layout, this covers a number of aspects: layout, experiment, initial conditions and recurrence relations.

An experiment is written as a function call, with a certain set of arguments. Figure 13 shows a experiment that will return the value of the recurrence relation *i\_f1* at time 3.

```
1 i_f1 3
```

Figure 13: *Function call for value at time 3 in recurrence relation 1\_f1.*

The initial conditions, which use will be shown later, are defined as shown in Fig. 14. There is no defined limit to the amount of initial conditions that can be defined.

```

1 q = 4
2 k = 16

```

Figure 14: *Definition of two initial conditions,  $q$  and  $k$ .*

A simple recurrence relation can be written as shown in Fig. 15, this recurrence relation belongs to group  $i$ , is named  $f1$  and takes one argument which is time  $t$ . The expression of this relation is to call itself at the previous time step and to add that to a call for another recurrence relation, from group  $j$ , at the previous time step.

```

1 i_f1 t = ( i_f1 (t-1) ) + ( j_f1 (t-1) )

```

Figure 15: *A simple recurrence relation, belonging to group  $i$  that adds its value at the previous time step to that of  $j\_f1$  at the previous time step.*

Initial conditions can be added to recurrence relations using pattern matching, as shown in Fig. 16. The recurrence relation is first written with the value for which the initial condition will be sent and this is sent to equal the initial condition, then the full recurrence relation is written.

```

1 i_f1 0 = q
2 i_f1 t = ( i_f1 (t-1) ) + ( j_f1 (t-1) )

```

Figure 16: *Recurrence relation containing pattern matching for initial conditions.*

An if statement, shown in Fig. 17, can contain very complex components it is hence good practice to encase each expression with in a set of brackets to maintain legibility and correct functionality.

```

1 j_f1 t = cond (t<0) (k) (25 + ((j_f1 (t-1)) * (i_f1 (t-1))) + (j_f2 (t-1)))

```

Figure 17: *Recurrence relation using an if statement.*

A where statement is used to provide additional functionality to a recurrence relation and can be used to make an expression more legible and help reduce human errors. Figure 18 shows the use of a where statement to add an internal variable and an internal function to a recurrence relation.

```

1 i_f1 t = a * (_sf1 t)
2   where
3     a      = 5
4     _sf1 x = x*2
5   endwhere

```

Figure 18: *A recurrence relation containing a where statement to add an internal function and an internal variable.*

This style and layout is brought together to produce a full simulation, as shown in Fig. 19.

```

1 main
2 i_f1 3
3
4 init
5 q = 4
6 k = 16
7
8 where
9 i_f1 0 = q
10 i_f1 t = (i_f1 (t-1) ) + (j_f1 (t-1))
11
12 j_f1 0 = k
13 j_f1 t = cond (t<0) (k) (25 + ((j_f1 (t-1)) * (i_f1 (t-1))) + (j_f2 (t-1)))
14 j_f2 0 = k+q
15 j_f2 t = (j_f1 (t-3)) + 27

```

Figure 19: *A simulation in the bespoke recurrence relation language.*

## 5 Design of a Translation from Recurrence Relations to InterDyne

This section details the design of a step-by-step translation between the previously described bespoke recurrence relation language and a reduced form of the InterDyne simulator. This is broken down into eight individual steps, with each of these steps altering the output of the previous step to resemble more closely an InterDyne simulation.

The eight steps required for this translation are to introduce:

1. Infinite list output. Solving the issue of InterDyne agents producing infinite lists of their outputs, compared with the recurrence relation model where outputs are calculated when given an input.
2. Grouping into agents. Solving the issue of InterDyne containing agents, where the recurrence relation model a list of recurrence relations.
3. A global output list. Begins to solve the issue of InterDyne containing a harness that mediates all the messages; by creating a function to mediate the messages.
4. Agent input lists. Begins to solve the issue of agents in InterDyne taking in messages, compared with the function calls in the recurrence relation model; by giving each group of recurrence relations an input to use.
5. Targeted information passing. Continues to solve the issue of message passing; by allowing information produced by a recurrence relation to have a defined group of recurrence relations as a target.
6. A simulator harness. Continues to solve the issue of InterDyne containing a harness; by grouping the functionality for producing the global output list, and the sorting of inputs for the recurrence relation groups, into a single function.
7. Runtime arguments. Increases the produced agent-based models similarity to InterDyne; by adding a number of arguments to be passed to the harness and between the harness and agents, and by changing the structure to be more comparable with an InterDyne simulation.
8. Message communication. Solves the issue of message passing in InterDyne; by adding messages that correspond to those found within InterDyne. Adding these messages also allows for the harness to be restructured to correspond to that of a reduced InterDyne simulation.

These steps are conducted in a way, so that each step builds upon the translation made by its predecessor; as such the order in which these steps are organised can not be rearranged. This order also allows for the bespoke recurrence relation language to evolve into an agent-based model in a comprehensible manner; with each steps translation building on the last and hence increasing the overall complexity in small increments.

These particular translation steps have been chosen for this translation following two requirements; each step should change the target state of the previous step (with the first step changing the initial bespoke recurrence relation language) in a small enough manner that it is easy to follow, and the target state produced by each step should be a functional model (a model that can be translated into Miranda code and executed to produce an output). Additionally each step was chosen to introduce either a single larger piece of functionality to the model or a number of smaller pieces.

Though in this design eight steps have been chosen, this is by no means the only formulation that can be taken. Some of the steps described below can be translated in more steps, and some of the steps can be combined to create a large jump in the translation. However the



steps chosen follow the requirements listed above and create a translation process that should be formulated in an digestible manner; increasing the number of steps would increase the length of the translation most likely without much improving its clearness, and decreasing the number of steps, though shortening the translation, would most likely make following it more challenging.

These eight steps translate the bespoke recurrence relation model into that of an InterDyne model, with some limited functionality. This translation also does not use the functionality of broadcast messages within InterDyne, all information passing within the recurrence relations is considered to be private and is hence sent as a one-to-one message. This translation does support expanded recurrence relation that take more then one argument, as well as recurrence relations that are first degree or higher.<sup>7</sup>

Another translation step could be added to translate the final target state into the Haskell syntax, matching that of InterDyne. However this step is not necessary in creating an agent-based model that can be seen to be equivalent to a reduced version of InterDyne.

These steps will be shown in this section by translating an example written in the bespoke recurrence relation language, shown in Fig. 20. This example consists of four recurrence relations (*i-f1*, *j-f2*, *j-f2* and *k-f1*) split into three groups (*i*, *j* and *k*). Each of these relations uses the allowed pattern matching to define their initial conditions in terms of *q* and *k*, which are defined in the section *init*. The experiment, defined under *main*, is to return the value of the relation *i-f1* at the time step 3.

```

1 main
2 i_f1 3
3
4 init
5 q = 4
6 k = 16
7
8 where
9 i_f1 0 = q
10 i_f1 t = (i_f1 (t-1)) + (j_f1 (t-2))
11 j_f1 (-1) = k
12 j_f1 0 = k
13 j_f1 t = cond (t<0) (k_f1 (t-1)) (25+((j_f1 (t-1))*(i_f1 (t-1))) + (j_f2 (t-1)))
14 j_f2 0 = k+q
15 j_f2 t = cond (t<3) 27 ((j_f1 (t-1)) + 27)
16 k_f1 0 = 0
17 k_f1 t = k + q + (j_f1 (t-1))

```

Figure 20: *Example of an experiment written in the bespoke recurrence relation language, using a set of four recurrence relations.*

## 5.1 Infinite List Output

In InterDyne each agent outputs an infinite list containing the outputs of that agent at each time step. Here each recurrence relation is changed to output its values as an infinite list in time.

For this to be achieved each recurrence relation is wrapped into a parent function, denoted by adding *\_list* to the end of the relations name. This parent function returns an infinite list of the outputs of its sub recurrence relation outputs, with each item in the list being the value for this relation at each time step. This is done using the sub function *\_createlist*, this function takes in a function and a time marker. These arguments are then used to add the value of the function

<sup>7</sup>A first-degree recurrence relation references the previous time step ( $t - 1$ ), a second-degree relation references the one prior to this ( $t - 2$ ) and so on.

at that time maker to the head of a least, where the next value is created by recursively calling `_createlist` but with the next time step, as shown in Fig. 21.

```
1 _createlist f t = (f t):(_createlist f (t+1))
```

Figure 21: `_createlist` function, used to create an infinite list in time of a recurrence relation.

Since the recurrence relations are now contained within parent functions (that do not take time as an input) that are outputting infinite lists, to obtain a value at a certain time step these list have to be indexed into. This indexing using time now replaces the previous method of calling a relation with time as an argument. Indexing is done with the bang operator (!), this applied to a list and followed by a value will return the numbered item in the list. When numbering the list, zero is considered to be the first item, so `[1,2,3]!1` would return a value of 2. Therefore all calls within other functions are changed to this new notation as well as the call in main for the experiment. Once these changes have been applied Fig. 20 will be translated into Fig. 22.

```
1 main
2 i_f1_list ! 3
3
4 init
5 q = 4
6 k = 16
7
8 where
9 i_f1_list = _createlist i_f1 0
10     where
11         _createlist f t = (f t):(_createlist f (t+1))
12         i_f1 0 = q
13         i_f1 1 = (i_f1_list ! 0) + k
14         i_f1 t = (i_f1_list ! (t-1)) + (j_f1_list ! (t-2))
15     endwhile
16 j_f1_list = _createlist j_f1 0
17     where
18         _createlist f t = (f t):(_createlist f (t+1))
19         j_f1 0 = k
20         j_f1 t = cond (t<0) (k_f1_list ! (t-1))
21                     (25 + ((j_f1_list ! (t-1)) * (i_f1_list ! (t-1))) + (
22             j_f2_list ! (t-1)))
23     endwhile
24 j_f2_list = _createlist j_f2 0
25     where
26         _createlist f t = (f t):(_createlist f (t+1))
27         j_f2 0 = k+q
28         j_f2 t = cond (t<3) 27 ((j_f1_list ! (t-1)) + 27)
29     endwhile
30 k_f1_list = _createlist k_f1 0
31     where
32         _createlist f t = (f t):(_createlist f (t+1))
33         k_f1 0 = 0
34         k_f1 t = k + q + (j_f1_list ! (t-1))
35     endwhile
```

Figure 22: Figure 20 translated to contain relations that produce infinite lists throughout time and functionality added for relations to index into these lists.

## 5.2 Grouping into Agents

InterDyne contains agents which are the entities that interact, here agents are created by grouping together recurrence relations in the same assigned group.

This is done by creating a wrapper function which contains the previously created *\_list* functions. Each wrapper, which is labeled as *group\_wrapper*, contains the *\_list* functions assigned to the same group. These wrappers output an infinite list, each item within this list is another list containing the outputs of the internal *\_list* functions at that time step. Therefore a wrapper containing two functions, *a\_list* and *b\_list*, would produce an infinite list of the form,  $[[a\_list!0, b\_list!0], [a\_list!1, b\_list!1], \dots, [a\_list!\infty, b\_list!\infty]]$ . This is done using the internal function *transpose*, this function takes a list of lists and returns a list containing lists, where the first list is a list of the heads of the initial lists and the next list is a list of the heads of the remainder of the initial list and so on. An example of the use of *transpose* is *transpose*  $[[1, 2, 3], [4, 5, 6]]$ , which returns  $[[1, 4], [2, 5], [3, 6]]$ .

Since the *\_list* functions are now contained within a wrapper function calls to these functions must now be calls to the wrapper function which they are part of. The exception to this rule is for functions that are part of the same wrapper, these can still make direct calls to *\_list* functions that share the same group. Since the wrapper outputs an infinite list in time, first this list must be indexed to the right time step and then this list must be indexed for the right internal function. For example, to return the value of the first recurrence relation in a group called *g* at time *t*, the call  $((g\_wrapper!t)!0)$  would be used. This will change the notation of all relation calls by other recurrence relations and the recurrence relation call listed in *main*.

These changes will translate the example in Fig. 22 to that shown in Fig. 23.

```

1 main
2 (i-wrapper ! 3) ! 0
3
4 init
5 q = 4
6 k = 16
7
8 where
9 i-wrapper = (transpose [i-f1-list])
10   where
11     i-f1-list = _createlists i-f1 0
12     where
13       _createlists f t = (f t):(_createlists f (t+1))
14       i-f1 0 = q
15       i-f1 1 = (i-f1-list ! 0) + k
16       i-f1 t = (i-f1-list ! (t-1)) + (((j-wrapper ! (t-2)) ! 0)
17     endwhile
18   endwhile
19 j-wrapper = (transpose [j-f1-list, j-f2-list])
20   where
21     j-f1-list = _createlists j-f1 0
22     where
23       _createlists f t = (f t):(_createlists f (t+1))
24       j-f1 0 = k
25       j-f1 t = cond (t < 0) ((k-wrapper ! (t-1)) ! 0)
26       ! (t-1)) ! 0) + (j-f2-list ! (t-1))
27     endwhile
28     j-f2-list = _createlists j-f2 0
29     where
30       _createlists f t = (f t):(_createlists f (t+1))
31       j-f2 0 = k+q
32       j-f2 t = cond (t < 3) 27 ((j-f1-list ! (t-1)) + 27)
33     endwhile
34   endwhile
35 k-wrapper = (transpose [k-f1-list])
36   where
37     k-f1-list = _createlists k-f1 0
38     where
39       _createlists f t = (f t):(_createlists f (t+1))
40       k-f1 0 = 0
41       k-f1 t = k + q + ((j-wrapper ! (t-1)) ! 0)
42     endwhile
43   endwhile

```

Figure 23: Translation of Fig. 22 to allow for grouping of recurrence relations into agents.

### 5.3 A Global Output List

In InterDyne a simulator harness is used to mediate the messages sent between different agents, here the first step for the creation of this harness will be added in the form of a global output list.

This is done by creating a list of the simulation outputs, *s\_outputs*, that collects the outputs from all the agents and returns an infinite list containing these outputs, the structure of this function is shown in Fig.24. Because InterDyne has a circular structure for its message passing through time, requiring the agents to take a message at the beginning of every time step, a initial set of messages is added to the front of this list. These initial values are set as zero to represent a null value, but to still give an agent a message to consume on the first time step. The simulation outputs list, with the addition of null values is shown in Fig. 25.

```
1 s_outputs = (transpose [i-wrapper , j-wrapper , k-wrapper])
```

Figure 24: *An infinite list in time containing the values of every recurrence relation at each time step.*

```
1 s_outputs = niloutputs : (transpose [ i wrapper , j wrapper , k wrapper ])
2       where
3       niloutputs = [[ nullvalue ] , [ nullvalue , nullvalue ] , [ nullvalue
4       ]]
5       nullvalue = 0
6       endwhere
```

Figure 25: *An infinite list in time containing the values of every recurrence relation at each time step, with null values at the start of the list.*

This new list of all agents outputs can now be used to refer to values produced by other agents. The list produced by *s\_outputs* is an infinite list in time. Containing lists for each time step, which contain a list for each agent, containing all the values for that time step produced by that agent. Since this list contains null values at the head, to get a value for a time step of  $t - 1$ , the list needs to be indexed by  $t$ .

Translating Fig. 23 to contain this global output list and to index into it for retrieving values from other agents produces Fig. 26.

```

1 main
2 ((s-outputs!(3+1))!0)!0
3
4 init
5 q = 4
6 k = 16
7
8 where
9 s_outputs = niloutputs : (transpose [i-wrapper, j-wrapper, k-wrapper])
10     where
11         niloutputs = [[nullvalue], [nullvalue, nullvalue], [nullvalue]]
12         nullvalue = 0
13     endwhile
14 i-wrapper = (transpose [i_f1_list])
15     where
16         i_f1_list = _createlists i_f1 0
17         where
18             _createlists f t = (f t):(_createlists f (t+1))
19         i_f1 0 = q
20         i_f1 1 = (i_f1_list ! 0) + k
21         i_f1 t = (i_f1_list ! (t-1)) + j_f1_2last_timestep
22         where
23             j_f1_2last_timestep = (((outputs!(t-1))!1)!0)
24         endwhile
25     endwhile
26 endwhile
27 j-wrapper = (transpose [j_f1_list, j_f2_list])
28     where
29         j_f1_list = _createlists j_f1 0
30         where
31             _createlists f t = (f t):(_createlists f (t+1))
32         j_f1 0 = k
33         j_f1 t = cond (t<0) k_f1_last_timestep
34             (25 + ((j_f1_list ! (t-1)) *
35                 i_f1_last_timestep) + (j_f2_list ! (t-1)))
36         where
37             i_f1_last_timestep = (((outputs!t)!0)!0)
38             k_f1_last_timestep = (((outputs!t)!2)!0)
39         endwhile
40     endwhile
41 j_f2_list = _createlists j_f2 0
42     where
43         _createlists f t = (f t):(_createlists f (t+1))
44         j_f2 0 = k+q
45         j_f2 t = cond (t<3) 27 ((j_f1_list ! (t-1)) + 27)
46     endwhile
47 k-wrapper = (transpose [k_f1_list])
48     where
49         k_f1_list = _createlists k_f1 0
50         where
51             _createlists f t = (f t):(_createlists f (t+1))
52         k_f1 0 = 0
53         k_f1 t = k + q + j_f1_last_timestep
54         where
55             j_f1_last_timestep = (((outputs!t)!1)!0)
56         endwhile
57     endwhile
58 endwhile

```

Figure 26: Figure 23 translated to contain the global output list, *s\_outputs*, and to use this list to retrieve values from other agents.

## 5.4 Agent Input Lists

In InterDyne an agent is only able to access information sent to it in the form of messages. These messages take one time step to arrive at the agent and hence have the effect of only allowing the agent to use information sent to it one time step ago (if a message is sent at  $t - 1$  the agent will receive it for use at  $t$ ). Here the beginnings of message passing will be added, by allowing each wrapper to take inputs that contain the outputs list and then internally forcing the wrappers to only use the information in this list from the previous time step.

The output list, *s\_outputs*, will still produce an infinite list in time containing the outputs of all the wrappers, however to do this it now has to give inputs to the wrapper functions. Each wrapper function will receive two inputs, the outputs list and an initial time value. The initial time value is zero, this initialises recursion within the wrappers that will be discussed later. This will lead to *s\_outputs* similar to that shown in Fig. 27.

```
1 s_outputs = niloutputs : (transpose [i-wrapper s_outputs 0, j-wrapper s_outputs 0,
    k-wrapper s_outputs 0])
```

Figure 27: *Example of part of the s\_outputs form after addition of inputs for the wrapper functions.*

The wrapper functions now takes two inputs, a list of all the outputs from all the wrappers (*inputs*) and a time stamp ( $t$ ). When first called by *s\_outputs* this list is complete and the time stamp is equal to zero. The head of the *inputs* list contains just the information produced during the last time step, in this case this is the *niloutputs*, therefore to restrict the wrapper to just use the information from the last time step only the head can be used. However as the time steps increase this will no longer be true, so to create the output list required from the wrapper, the wrapper recursively calls its self using the tail of the inputs and increasing the time stamp by one. This means on the next call the head of the list will represent the allowed information for that time step, as this will be the head of the tail of the list, and so on as the time increases. The output list created by the agent, now is achieved by producing a list containing the values of the logic for that time step and adding this to the front of the recursive list. A wrapper written in this way would have a similar form to that shown in Fig. 28

```
1 i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper future_messages (
    t+1))
2
3     where
4     inbound_messages_this_timestep = hd inputs
5     future_messages                = tl inputs
6     outbound_messages_this_timestep = [(logic1 t), (logic2 t)]
7     logic1 t = function (inbound_messages_this_timestep)
8     logic2 t = function (inbound_messages_this_timestep)
    endwhere
```

Figure 28: *Example of part of a wrapper function form after addition of inputs for the wrapper functions.*

As mentioned a wrapper now only has access to information sent at the previous time step,  $t - 1$ , however some logic requires the use of information from time steps prior to this, such as  $t - 2$ . Allowing a wrapper to access this information can be done by delaying its arrival at the wrapper. This can be done by sending a message through a delay agent, this agent will then delay the value until the time it needs to be accessed by the wrapper requiring it.

A delay agent is a new wrapper named in the format *functionname<sub>delay</sub>*, where *functionname* is the name for the recurrence relation value that it is delaying, such *jf1*. Use of this delay agent

requires reverse engineering of the model, the agent needs to know what value is being delayed and for how long, and the wrapper requiring the delay now needs to use the value returned by the delay agent, at the current time step, instead of the wrapper that produces the initial value.

The delay agent needs to have access to the initial values of the logic it is delaying. The wrapper then takes in the value that it is delaying and then outputs it. This process delays a value to  $t - 2$ , as any value the delay agent reads in is already from  $t - 1$  and when another wrapper reads the value from the delay agent this will be delayed another time step to  $t - 2$ . For delays greater than  $t - 2$  the delay agent will output two or more values and read in two or more values, with the output list being values delay in the following manner,  $[.., t - 4, t - 3, t - 2]$ . The head of this list is read by the wrapper requiring the delayed value, all other items in this list are read by the delay agent, which moves each item up the list and adds the current value of the value being delayed to the end of the list.

A delay agent, for delaying a value to  $t - 2$ , will have a form similar to that shown in Fig. 29. Since this agent is treated the same as any other wrapper, it will also be added to the outputs lists and be part of the inputs.

```

1 jf1 delay inputs t = outbound messages this timestep : (jf1 delay future messages (
   t+1))
2
3       where
4       inbound messages this timestep = hd inputs
   future messages = tl inputs outbound messages this timestep =
   [( jf1 delayed t) ]
5       jf1 delayed 0 = k
6       jf1 delayed t = (( inbound messages this timestep !1) !0)
7       endwhere

```

Figure 29: *Example of a delay wrapper that delays the first value from the second wrapper, so that a value from  $t - 2$  is received at  $t$ .*

In this translation all information is passed as an input to the wrapper so even internal functions are referred to through the input list. The list head is taken at each time step and contains a list where each item is the output by each wrapper defined in the experiment, with delay wrappers added to the end of this list.<sup>8</sup> Each output by a wrappers is a list of the values produced by that wrapper at the previous time step. To access a required value this input list must be indexed twice, first return the values from the correct wrapper and then to return the correct value outputted by that wrapper. Translating Fig. 26 in this way will produce Fig. 30.

<sup>8</sup>If there are multiple delay wrappers, reverse engineering is used to make sure that the wrappers are referencing the correct delay agents.



```

1 main
2 ((s-outputs!(3+1))!0)!0
3
4 init
5 q = 4
6 k = 16
7
8 where
9 s_outputs = niloutputs : (transpose [i-wrapper s-outputs 0, j-wrapper s-outputs 0,
   k-wrapper s-outputs 0, jfl_delay s-outputs 0 ])
10     where
11         niloutputs = [[nullvalue], [nullvalue, nullvalue], [nullvalue], [
   nullvalue]]
12         nullvalue = 0
13         endwhere
14 i-wrapper inputs t = outbound_messages_this_timestep : (i-wrapper future_messages (
   t+1))
15     where
16         inbound_messages_this_timestep = hd inputs
17         future_messages = tl inputs
18         outbound_messages_this_timestep = [(i_fl t)]
19         i_fl 0 = q
20         i_fl t = cond (t<0) 0
21         ( ((inbound_messages_this_timestep!0)!0) +((
   inbound_messages_this_timestep!3)!0))
22     endwhere

```

(a) From Fig. 30, main, init, the outputs list, and *i* wrapper.

```

1 j-wrapper inputs t = outbound_messages_this_timestep : (j-wrapper future_messages (
    t+1))
2         where
3         inbound_messages_this_timestep = hd inputs
4         future_messages                 = tl inputs
5         outbound_messages_this_timestep = [(j_f1 t), (j_f2 t)]
6         j_f1 0 = k
7         j_f1 t = cond (t<0) k_f1_last_timestep
8                 (25 + (j_f1_last_timestep * i_f1_last_timestep)+
9         j_f2_last_timestep)
10        where
11        i_f1_last_timestep = ((inbound_messages_this_timestep
12        !0)!0)
13        j_f1_last_timestep = ((inbound_messages_this_timestep
14        !1)!0)
15        j_f2_last_timestep = ((inbound_messages_this_timestep
16        !1)!1)
17        k_f1_last_timestep = ((inbound_messages_this_timestep
18        !2)!0)
19        endwhere
20        j_f2 0 = k+q
21        j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
22        where
23        j_f1_last_timestep = ((inbound_messages_this_timestep
24        !1)!0)
25        endwhere
26        k-wrapper inputs t = outbound_messages_this_timestep : (k-wrapper future_messages (
27        t+1))
28        where
29        inbound_messages_this_timestep = hd inputs
30        future_messages                 = tl inputs
31        outbound_messages_this_timestep = [k_f1 t]
32        k_f1 0 = 0
33        k_f1 t = k + q + j_f1_last_timestep
34        where
35        j_f1_last_timestep = ((inbound_messages_this_timestep
36        !1)!0)
37        endwhere
38        jf1_delay inputs t = outbound_messages_this_timestep : (jf1_delay future_messages (
39        t+1))
40        where
41        inbound_messages_this_timestep = hd inputs
42        future_messages                 = tl inputs
43        outbound_messages_this_timestep = [(jf1_delayed t)]
44        jf1_delayed 0 = k
45        jf1_delayed t = ((inbound_messages_this_timestep!1)!0)
46        endwhere

```

(b) From Fig. 30,  $j$  and  $k$  wrapper functions.

Figure 30: Translation of Fig. 26 to allow wrappers to access information through input values, and to limit the information given to only that from the previous time step. Figure 30a shows the experiment, the outputs list and the  $i$  wrapper, and Fig. 30b shows the  $j$  and  $k$  wrappers.

## 5.5 Targeted Information Passing

The next step in creating message passing is to have the wrappers send messages addressed to each other instead of just producing a blanket output for any other wrapper. In InterDyne a message

has three main components, these are a sender, a receiver and a value. In this setup a wrapper can only access a value if it is sent directly to it, therefore if more then one wrapper uses a value then that many messages need to be sent. Here these three item messages will be added along with the functionality for a wrapper to distinguish which messages are meant for it.

The changes required to the output list function (*s\_outputs*) is purely related to the *nullvalue* aspect of it. The *nullvalue*, which were previously zero now have to be a list of three value messages. These three items will be stored in a tuple, in the format (*from*, *to*, *value*), for the *nullvalue* this the message list will be a single message containing zeros, [(0, 0, 0)].

The wrapper functions now output messages in the same form, so a message from the first wrapper function will contain its id, which is one, the targeted wrapper id and the value it wishes to send. The wrappers id's can be found by looking at where that wrapper appears in the order of wrappers, with the first wrapper having id one and so on. The the id of the receiving wrapper is found by analysing the call functions made by each wrapper to discern, which wrappers they are drawing information from. Once this is found messages can be created in these wrappers to send the information to the wrappers that require it. This reverse engineering as it were, can be used to deduce what message need to be sent to what wrappers. An example of the form for sending a messages from a wrapper can be seen in Fig. 31.

```

1 i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper future_messages (
    t+1))
2
3           where
4           inbound_messages_this_timestep = hd inputs
5           outbound_messages_this_timestep = [ [ (1,1,logic1),(1,2,logic1
6           )]]
7
8           logic1 t = function (inbound_messages_this_timestep)
9           endwhere

```

Figure 31: *Example of part of a wrapper that is sending two messages containing the same value. These messages come from this wrapper, which is the first wrapper (1), and are sent: to itself (1), and to the second wrapper (2). Here logic1 is just used to show where the logic would be and that this produces the output values, as well as what inputs this takes in.*

Though a wrapper receives all messages sent it should only use those that are sent directly to it. To do this the wrapper sorts the inbound messages to return only those with its id as the target. The sorting function can be seen in Fig. 32, this takes the head of the list, so the current time step and then uses the function *map* with the input *f* on it. This function applies the the function *f* to each item in the list it is passed and then returns the new list, for example *map hd* [[1,2],[3,4]] = [1,3]. The function *f* in this case then maps the function *g* across all the items in the lists that it is mapped across. The function *g* then uses a function called *filter* takes a condition and a list and returns a list of all elements that matched the condition. In this case the condition is that the message tuple is sent this this wrapper (*id* = 1) or () that it is a null message from the *s\_outputs* function.

```

1 i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper future_messages (
    t+1))
2
3     where
4     inbound_messages_this_timestep = map f (hd inputs)
5                                     where
6                                     f xs = map g xs
7                                     g xs = filter h xs
8                                     h (a,b,c) = ((b=1) \/\ (a,b,c
9
10                                     ) = (0,0,0))
11                                     endwhile
12     future_messages = tl inputs
    outbound_messages_this_timestep = [ [messages]]
    logic t = function (inbound_messages_this_timestep)
    endwhile

```

Figure 32: *Example of part of a wrapper that is used to sort the input off all messages and only return the ones directed at this wrapper.*

Since the format that the information is passed in, has changed the experiment function in main must change, as well as any calls to *inbound\_messages\_this\_timestep* within wrappers. These calls now must first index to the correct message (this is again found through reverse engineering), then take the head of this list (as it will be a list just containing one message) and then take the third item from the tuple (which is the value). To take the head and the third item the command *thd3.hd* is used, this is equivalent to *thd3(hd input)*, where the command *thd3* returns the third item in a three item tuple. Translating Fig. 30 with these steps will lead to Fig. 33.

```

1 main
2 (thd3.hd) (((s_outputs!(3+1))!0)!0)
3
4 init
5 q = 4
6 k = 16
7
8 where
9 s_outputs = niloutputs : transpose [ i_wrapper s_outputs 0 ,j_wrapper s_outputs 0
    ,k_wrapper s_outputs 0, jf1_delay s_outputs 0 ]
10     where
11         niloutputs = [[newnullvalue], [newnullvalue, newnullvalue], [
newnullvalue], [newnullvalue]]
12         nullvalue = [(0,0,0)]
13     endwhile
14 i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper future_messages (
t+1))
15     where
16         inbound_messages_this_timestep = map f (hd inputs)
17         where
18             f xs = map g xs
19             g xs = filter h xs
20             h (a,b,c) = ((b=1) \ / (a,b,c)
) = (0,0,0))
21         endwhile
22         future_messages = tl inputs
23         outbound_messages_this_timestep = [ [ (1,1,res) ,(1,2,res) ] ]
24         where
25             res = i_f1 t
26         endwhile
27         i_f1 0 = q
28         i_f1 t = cond (t<0) 0 (i_f1_last_timestep +
j_f1_2last_timestep)
29         where
30             i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
31             j_f1_2last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!3)!0)
32             endwhile
33         endwhile

```

(a) From Fig. 33, main, init, the outputs list and *i\_wrapper*.

```

1 j-wrapper inputs t = outbound_messages_this_timestep : (j-wrapper future_messages (
  t+1))
2
3       where
4       inbound_messages_this_timestep = map f (hd inputs)
5                                     where
6                                     f xs = map g xs
7                                     g xs = filter h xs
8                                     h (a,b,c) = ((b=2) \ / (a,b,c)
9
10      ) = (0,0,0))
11
12      future_messages = tl inputs
13      outbound_messages_this_timestep = [ [ (2,4,res1) ,(2,2,res1)
14      ,(2,3,res1) ], [(2,2,res2) ] ]
15
16      where
17      res1 = j_f1 t
18      res2 = j_f2 t
19      endwhile
20
21      j_f1 0 = k
22      j_f1 t = cond (t<0) k_f1_last_timestep
23              (25 + (j_f1_last_timestep * i_f1_last_timestep)+
24      j_f2_last_timestep)
25
26      where
27      i_f1_last_timestep = (thd3.hd) ((
28      inbound_messages_this_timestep!0)!0)
29      j_f1_last_timestep = (thd3.hd) ((
30      inbound_messages_this_timestep!1)!0)
31      j_f2_last_timestep = (thd3.hd) ((
32      inbound_messages_this_timestep!1)!1)
33      k_f1_last_timestep = (thd3.hd) ((
34      inbound_messages_this_timestep!2)!0)
35      endwhile
36
37      j_f2 0 = k+q
38      j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
39
40      where
41      j_f1_last_timestep = (thd3.hd) ((
42      inbound_messages_this_timestep!1)!0)
43      endwhile
44
45      endwhile

```

(b) From Fig. 33, *j\_wrapper*.

```

1 k_wrapper inputs t = outbound_messages_this_timestep : (k_wrapper future_messages (
    t+1))
2
3     where
4     inbound_messages_this_timestep = map f (hd inputs)
5                                     where
6                                     f xs = map g xs
7                                     g xs = filter h xs
8                                     h (a,b,c) = ((b=3) \ / (a,b,c)
9
10                                     ) = (0,0,0))
11
12     future_messages = tl inputs
13     outbound_messages_this_timestep = [[(3,2,k_f1 t)]]
14     k_f1 0 = 0
15     k_f1 t = k + q + j_f1_last_timestep
16     where
17     j_f1_last_timestep = (thd3.hd) ((
18     inbound_messages_this_timestep!1)!0)
19     endwhile
20
21     endwhile
22 jf1_delay inputs t = outbound_messages_this_timestep : (jf1_delay future_messages (
23     t+1))
24
25     where
26     inbound_messages_this_timestep = map f (hd inputs)
27                                     where
28                                     f xs = map g xs
29                                     g xs = filter h xs
30                                     h (a,b,c) = ((b=4) \ / (a,b,c)
31
32                                     ) = (0,0,0))
33
34     future_messages = tl inputs
35     outbound_messages_this_timestep = [[(4,1,jf1_delayed t)]]
36     jf1_delayed 0 = k
37     jf1_delayed t = (thd3.hd) ((inbound_messages_this_timestep!1)
38
39     !0)
40
41     endwhile

```

(c) From Fig. 33, *k\_wrapper* and *jf1\_delay*.

Figure 33: Translation of Fig. 30 to allow messages to be sent as a tuple in the form (from, to, value) and for wrappers to sort these messages as to only used ones addressed to them. Figure 33a shows the experiment, the outputs list and the *i* wrapper, Fig. 33b shows the wrapper function for *j*, and Fig. 33c shows the wrapper function for *k* and the wrapper function for the delay agent *jf1\_delay*.

## 5.6 A Simulator Harness

As mentioned earlier InterDyne has a harness that mediates the message passing between the agents, this harness covers aspects such as driving the simulation by requiring an output from the agents, receiving and redistributing the messages and redacting the inputs to each agent so that the only receive the messages sent to them. Here a harness is added that takes control of this functionality.

The harness function, called *h\_sim*, takes one input, which is time. The return of this function is the value of *i\_f1* at the time step given to the harness function, this value is found by indexing into the output list. The output list itself is now created within the harness function, along with its first set of messages *niloutputs*. The rest of the list is now *redactedoutputs*, this contains the same values as the prior rest of the list, however in this formulation each wrapper is only passed the messages directed to it. This is because the harness has taken over the role of sorting the messages, from the wrapper functions. Hence the input values labeled as *group\_inputs*

contain just the messages sent to the specific wrapper. An example of how a harness is formatted is shown in Fig. 34.

```

1 h_sim x = outputfunction (x)
2   where
3     newnullvalue = [(0,0,0)]
4     niloutputs   = listfuction (newnullvalue)
5     outputs      = niloutputs : redactedoutputs
6     redactedoutputs = transpose [ i_wrapper i_inputs 0 ,j_wrapper j_inputs 0]
7   where
8     i_inputs  = map (map (map (filter (h 1)))) outputs
9     j_inputs  = map (map (map (filter (h 2)))) outputs
10    h x (a,b,c) = ((b=x) \ / (a,b,c) = (0,0,0))
11    endwhile
12  endwhile

```

Figure 34: *Example of how the harness function  $h\_sim$  is formatted for two wrappers. This function produces the output for the system and in the process mediates the passing of messages between wrappers, only allowing a wrapper to receive messages sent to it. The function  $listfuction$  represents the null value list, which uses the value  $newnullvalue$ .*

Now that the  $h\_sim$  can return the experiment value  $main$  can be changed to call this function at the correct time step, since the output list starts with a null value the time step required is  $3 + 1$ .

Since the  $h\_sim$  sorts the inputs to the wrapper functions so that they are only receiving messages sent to them, the wrappers no longer need to deal with this internally. The *inbound\_messages\_this\_timestep* in the wrapper now simply takes the head of the *inputs* list to access the messages at the current time step.

Translating Fig. 33 to contain a harness function with this functionality will result in the experiment shown in Fig. 35.



```

1 main
2 h.sim (3+1)
3
4 init
5 q = 4
6 k = 16
7
8 where
9 h.sim x = (thd3.hd) (((outputs!x)!0)!0)
10     where
11         newnullvalue = [(0,0,0)]
12         niloutputs   = [[newnullvalue], [newnullvalue, newnullvalue], [
13         newnullvalue], [newnullvalue]]
14         outputs      = niloutputs : redactedoutputs
15         redactedoutputs = transpose [ i-wrapper i_inputs 0 ,j-wrapper j_inputs 0
16         ,k-wrapper k_inputs 0, jfl_delay jfl_inputs 0 ]
17     where
18         i_inputs  = map (map (map (filter (h 1)))) outputs
19         j_inputs  = map (map (map (filter (h 2)))) outputs
20         k_inputs  = map (map (map (filter (h 3)))) outputs
21         jfl_inputs = map (map (map (filter (h 4)))) outputs
22         h x (a,b,c) = ((b=x) \ / (a,b,c) = (0,0,0))
23     endwhile
24 endwhile

```

(a) From Fig. 35, main, init, and the sim wrapper.

```

1 i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper future_messages (
  t+1))
2
3   where
4   inbound_messages_this_timestep = hd inputs
5   future_messages                = tl inputs
6   outbound_messages_this_timestep = [ [ (1,1,res) ],(1,2,res) ] ]
7                                   where
8                                   res = i_f1 t
9                                   endwhile
10
11   i_f1 0 = q
12   i_f1 t = cond (t<0) 0 (i_f1_last_timestep +
j_f1_2last_timestep)
13
14   where
15   i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
16   j_f1_2last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!3)!0)
17   endwhile
18   endwhile
19
20 j_wrapper inputs t = outbound_messages_this_timestep : (j_wrapper future_messages (
  t+1))
21
22   where
23   inbound_messages_this_timestep = hd inputs
24   future_messages                = tl inputs
25   outbound_messages_this_timestep = [ [ (2,4,res1) ],(2,2,res1)
,(2,3,res1) ],[(2,2,res2) ] ]
26
27   where
28   res1 = j_f1 t
29   res2 = j_f2 t
30   endwhile
31
32   j_f1 0 = k
33   j_f1 t = cond (t<0) k_f1_last_timestep
j_f2_last_timestep)
34
35   where
36   i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
37   j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
38   j_f2_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!1)
39   k_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!2)!0)
40   endwhile
41   j_f2 0 = k+q
42   j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
43   where
44   j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
45   endwhile
46   endwhile

```

(b) From Fig. 35, *i\_wrapper* and *j\_wrapper*.

```

1 k_wrapper inputs t = outbound_messages_this_timestep : (k_wrapper future_messages (
    t+1))
2         where
3         inbound_messages_this_timestep = hd inputs
4         future_messages                 = tl inputs
5         outbound_messages_this_timestep = [[(3,2,k_fl t)]]
6         k_fl 0 = 0
7         k_fl t = k + q + j_fl_last_timestep
8         where
9         j_fl_last_timestep = (thd3.hd) ((
    inbound_messages_this_timestep!1)!0)
10        endwhile
11        endwhile
12 jfl_delay inputs t = outbound_messages_this_timestep : (jfl_delay future_messages (
    t+1))
13         where
14         inbound_messages_this_timestep = hd inputs
15         future_messages                 = tl inputs
16         outbound_messages_this_timestep = [[(4,1,jfl_delayed t)]]
17         jfl_delayed 0 = k
18         jfl_delayed t = (thd3.hd) ((inbound_messages_this_timestep!1)
    !0)
19        endwhile

```

(c) From Fig. 35, *k\_wrapper* and *jfl\_delay*.

Figure 35: Translation of Fig. 33 to contain a harness function that deals with message sorting and passing, as well as outputting the experiment value. Figure 35a shows the experiment and sim wrapper, Fig. 35b shows the wrapper functions for groups *i* and *j*, and Fig. 35c shows the wrapper function for *k* and the wrapper function for the delay agent *jfl\_delay*.

## 5.7 Runtime Arguments

At the current stage of this translation a form of agent-based model has been created, this agent-based model however is still missing a number of features that InterDyne has. These features include: passing the agent id to each wrapper, having input messages and time as a tuple, incrementing time within the harness, passing local state augments to the wrappers, passing run time arguments to the harness, and passing agent information to the harness. Here these features are added to the translation.

The harness, *h\_sim*, now takes three arguments, the first is still time followed by the run time arguments and then the agent information. The run time arguments, labeled as *runtime\_args*, now take the place of the initial conditions defined under *init*, the list of runtime arguments define both *q* and *k* with their respective arguments. To signify an argument a data type is used, signified with the constructor *Arg*, the initial condition *q* would hence be given as *Arg* (“*q*”, 4). The agent information argument, labeled as *agentinfo*, is a list containing tuples, there is one tuple for each wrapper and the tuple contains the wrappers name and an empty list, e.g. (*i\_wrapper*, []). The empty list in this tuple is for broadcast channels and would be used to list the channels that this agent is subscribed to, but since no broadcast messages are being implemented this list is here to make this translation closer to a InterDyne formulation.

The harness now also bunches messages sent to each agent with a time stamp, this is done using the *zip2* function. This function takes to list add combine each consecutive items in the list into tuples, for example *zip2* [1, 2, 3] [4, 5, 6] = [(1, 4), (2, 5), (3, 6)]. This function is also used to pass the agent id to each agent, with the first agent being id equal one. The harness also passes the agents the arguments and a local state (which is unused), as well as adding the functionality for broadcast subscriptions, which are also unused. These changes create a *h\_sim* function that is

very similar to the simulator harness within InterDyne, a harness of this form is shown in Fig. 36.

```

1 h_sim x args agents = (thd3.hd) (((snd (outputs!x))!0)!0)
2   where
3     newnullvalue      = [(0,0,0)]
4     niloutputs        = listfunction (newnullvalue)
5     outputs           = zip2 [0..] (niloutputs : (redactedoutputs))
6     redactedoutputs   = transpose (map apply_to_args (zip2 [1..]
7       agents))
8   where
9     apply_to_args (agentid, (f,
10      broadcastsubscriptions)) = f Emptyagentstate args (myoutputs agentid) agentid
11     myoutputs id = map (f id) outputs
12     f x (t, xs) = (t, map (map (filter (h x))) xs)
13     h x (a,b,c) = ((b=x) \ / (a,b,c) = (0,0,0))
14   endwhere
15 endwhere

```

Figure 36: *Example of how the harness function `h_sim` is formatted when using runtime arguments. The function `listfunction` represents the null value list, which uses the value `newnullvalue`.*

The wrapper functions four inputs: the local state (*localstate*), the arguments (*args*), the input messages and time step (*inputs*), and the agent id (*id*). The arguments are then read in the wrapper using *getargstr* which returns the argument name when applied and *getargval* which returns the argument value when applied. The input list is unpacked using *fst* and *snd*, these return the first (time) and second (messages) items in a tuple respectively. The id passed to the wrapper is now used to mark the sender in outbound messages. A wrapper function with these changes takes the form shown in Fig. 37.

```

1 i_wrapper localstate args inputs id = outbound_messages_this_timestep : (i_wrapper
2   localstate args future_messages id)
3   where
4     q = (getargval.hd) (filter ((="q").getargstr)
5       args)
6     k = (getargval.hd) (filter ((="k").getargstr)
7       args)
8     inbound_messages_this_timestep = snd (hd
9       inputs)
10    t = fst (hd
11      inputs)
12    future_messages = tl inputs
13    outbound_messages_this_timestep = [ [ (id,1,
14      logic), (id,2,logic) ] ]
15    logic t = function (inbound messages this
16      timestep)
17    endwhere

```

Figure 37: *Example of how a wrapper function is formatted when passing the arguments: `localstate`, `args`, `inputs` and `id`. Here `logic1` is just used to show where the logic would be and that this produces the output values, as well as what inputs this takes in.*

Since the inputs to the harness function have changed the call in *main* must also change to include these arguments, the addition of runtime arguments has also replaced the *init* section of the experiment. Translating Fig. 35 using these steps results in the experiment shown in Fig. 38.

```

1 main
2 h.sim (3+1) runtime_args agentinfo
3
4 where
5 runtime_args = [(Arg ("q",4)), (Arg ("k", 16))]
6 agentinfo = [(i-wrapper, []), (j-wrapper, []), (k-wrapper, []), (jfl-delay, [])]
7 h.sim x args agents = (thd3.hd) (((snd (outputs!x))!0)!0)
8   where
9     newnullvalue = [(0,0,0)]
10    niloutputs = [[newnullvalue], [newnullvalue, newnullvalue], [
11      newnullvalue], [newnullvalue]]
12    outputs = zip2 [0..] (niloutputs : (redactedoutputs))
13    redactedoutputs = transpose (map apply_to_args (zip2 [1..]
14      agents))
15    where
16      apply_to_args (agentid, (f,
17        broadcastsubscriptions)) = f Emptyagentstate args (myoutputs agentid agentid
18          myoutputs id = map (f id) outputs
19          f x (t, xs) = (t, map (map (filter (h x))) xs)
20          h x (a,b,c) = ((b=x) \ / (a,b,c) = (0,0,0))
21          endwhere
22    endwhere
23  i-wrapper localstate args inputs id = outbound-messages_this_timestep : (i-wrapper
24    localstate args future-messages id)
25    where
26      q = (getargval.hd) (filter ((="q").getargstr)
27        args)
28      k = (getargval.hd) (filter ((="k").getargstr)
29        args)
30      inbound-messages_this_timestep = snd (hd
31        inputs)
32      t = fst (hd
33        inputs)
34      future-messages = tl inputs
35      outbound-messages_this_timestep = [ [ (id,1,
36        res) ,(id,2,res) ]]
37      where
38        res = i_f1
39      t
40      endwhere
41      i_f1 0 = q
42      i_f1 t = cond (t<0) 0 (i_f1_last_timestep +
43        j_f1_2last_timestep)
44      where
45        i_f1_last_timestep = (thd3.hd) ((
46          inbound-messages_this_timestep!0)!0)
47        j_f1_2last_timestep = (thd3.hd) ((
48          inbound-messages_this_timestep!3)!0)
49      endwhere
50    endwhere

```

(a) From Fig. 38, main, init, the sim wrapper, and the *i* wrapper.

```

1 j-wrapper localstate args inputs id = outbound_messages_this_timestep : (j-wrapper
  localstate args future_messages id)
2       where
3       q = (getargval.hd) (filter ((="q").getargstr)
4       args)
5       k = (getargval.hd) (filter ((="k").getargstr)
6       args)
7       inbound_messages_this_timestep = snd (hd
8       inputs)
9       t = fst (hd
10      inputs)
11      future_messages = tl inputs
12      outbound_messages_this_timestep = [ [ (id,4,
13      res1),(id,2,res1),(id,3,res1)],[ (id,2,res2)]]
14      where
15      res1 = j_f1
16      res2 = j_f2
17      endwhere
18      j_f1 0 = k
19      j_f1 t = cond (t<0) k_f1_last_timestep
20              (25 + (j_f1_last_timestep *
21      i_f1_last_timestep)+j_f2_last_timestep)
22      where
23      i_f1_last_timestep = (thd3.hd) ((
24      inbound_messages_this_timestep!0)!0)
25      j_f1_last_timestep = (thd3.hd) ((
26      inbound_messages_this_timestep!1)!0)
27      j_f2_last_timestep = (thd3.hd) ((
28      inbound_messages_this_timestep!1)!1)
29      k_f1_last_timestep = (thd3.hd) ((
30      inbound_messages_this_timestep!2)!0)
31      endwhere
32      j_f2 0 = k+q
33      j_f2 t = cond (t<3) 27 (j_f1_last_timestep +
34      27)
35      where
36      j_f1_last_timestep = (thd3.hd) ((
37      inbound_messages_this_timestep!1)!0)
38      endwhere
39      endwhere

```

(b) From Fig. 38, *j\_wrapper*.

```

1 k_wrapper localstate args inputs id = outbound_messages_this_timestep : (k_wrapper
  localstate args future_messages id)
2   where
3     q = (getargval.hd) (filter ((="q").getargstr)
4       args)
5     k = (getargval.hd) (filter ((="k").getargstr)
6       args)
7     inbound_messages_this_timestep = snd (hd
8       inputs)
9     t = fst (hd
10      inputs)
11     future_messages = tl inputs
12     outbound_messages_this_timestep = [(id,2,
13      k_fl t)]
14     k_fl 0 = 0
15     k_fl t = k + q + j_fl_last_timestep
16     where
17       j_fl_last_timestep = (thd3.hd) ((
18         inbound_messages_this_timestep!1)!0)
19     endwhile
20   endwhile
21 jfl_delay localstate args inputs id = outbound_messages_this_timestep : (jfl_delay
  localstate args future_messages id)
22   where
23     inbound_messages_this_timestep = snd (hd
24       inputs)
25     future_messages = tl inputs
26     t = fst (hd
27       inputs)
28     outbound_messages_this_timestep = [(id,1,
29      jfl_delayed t)]
30     k = (getargval.hd) (filter ((="k").getargstr)
31       args)
32     jfl_delayed 0 = k
33     jfl_delayed t = (thd3.hd) ((
34       inbound_messages_this_timestep!1)!0)
35     endwhile

```

(c) From Fig. 38, *k\_wrapper* and *jfl\_delay*.

Figure 38: Translation of Fig. 35 to contain *args* and *agents* as an input to the harness, and to contain *localstate*, *args*, *inputs* and *id* as inputs for the wrapper functions. Figure 38a shows the *experiment*, *sim wrapper* and *i wrapper*, Fig. 38b shows the wrapper functions for group *j*, and Fig. 38c shows the wrapper function for *k* and the wrapper function for the delay agent *jfl\_delay*.

## 5.8 Message Communication

The last part of the translation that is needed to make this experiment comparable with InterDyne is the addition of the message type. This type is used to signal that a set of data is a message and to encompass the information of the sender, receiver and data being sent. Here the use of this message type is added creating a experiment equivalent to a reduced version of InterDyne.

The main aspect of the wrappers transformation is the addition of this type into their output messages, instead of sending a message of the form (*from*, *to*, *value*) they will now send messages of the form *Message* (*from*, *to*) [(*Arg*(“*fun\_name*”, *value*))]. The key word *Message* is the type constructor and takes two values, the first is a tuple containing the from and to information and the second is a list of argument values. Arguments are signified with the constructor *Arg*, which takes a tuple with the arguments name (in this case the arguments are the internal functions so name for example could be *i\_fl*) and the value associated with that argument (so the result

of *i\_f1* at that time step). A wrappers output at each time step will now be a list of messages written in this form.

With this new message form, the assigning of message values to variables within the wrapper can be more generalised. Previously this was hard coded using reverse engineering to know where in the list the message would appear, since there were no identifiers in the messages to deduce what data it held. Now that the argument name is passed with the data this can be used to assign the data without needing to know where in the list of messages it appears.

This is done by first filtering the messages to return only those from the sender of interest, the arguments from these messages are then turned into a list, which is in turn filtered to return the value in the argument associated with the function name of interest. The functions *getmsgfrom* and *getmsgargs* are used, these return, the from ids and the arguments, respectively. The function *concat* is also used, this turns a list of lists into a list. An example of this process is shown in Fig. 39.

```

1 msgs_from_i      = filter ((=1).getmsgfrom) inbound_messages_this_timestep
2 args_from_i      = concat (map getmsgargs msgs_from_i)
3 i_f1_last_timestep = (getargval.hd) (filter ((=" i_f1 ").getargstr) args_from_i)

```

Figure 39: *Example of the filtering process for returning message values, here the value from function i\_f1 sent by wrapper one is being searched for, when it is found it will be saved to the variable i\_f1\_last\_timestep.*

The harness has also been altered to make it more general, allow for this new message type and make it more similar to that of the harness in InterDyne, this is shown in Fig. 40. This harness creates an output list (*outputs*) where each item is a tuple of a time stamp and all the messages sent at that time. This list is created by adding an tuple for time zero with no messages to the head of the *timed\_transposed\_msgs* list, this list is a list of tuples with a time stamp and all the messages at that time, after time zero (for example this list will look like [(1, *t1\_outputs*), (2, *t2\_outputs*), ...]). This is created by using the *zip2* function to add a time stamp to the list containing lists of messages at each time (*transposed\_msgs*). This list is created from the *allmessages* list, this is a finite list containing a tuple of an agent id and an infinite list all the messages sent, for each agent, this is created from the *agentinfo* passed to the harness. The *apply\_to\_args* function is used to turn the input information of the different agents into this list of outputs.



```

1 h_sim x runtime_args agentinfo = (getargval.hd.getmsgargs) (((snd (outputs!x))!0)
2                               !0)
3                               where
4                               outputs = (0,[]):timed_transposed_msgs
5                               timed_transposed_msgs = zip2 [1..] transposed_msgs
6                               transposed_msgs = transpose allmessages
7                               allmessages = map apply_to_args (zip2 [1..]
8                               agentinfo)
9                               where
10                              apply_to_args (agentid, (agentfn,
11 broadcastsubscriptions)) = agentfn Emptyagentstate runtime_args (redacted_msgs
12 agentid outputs) agentid
13                              redacted_msgs id outputs = map (f id
14                              ) outputs
15                              f x (time, xs) = (time
16                              , concat (map (filter (h x)) xs))
17                              h x (Message (f,t) args) = (t=x)
18                              endwhere
19                              endwhere

```

Figure 40: *Simulator harness for translation comparable to InterDynes simulator harness.*

After these translations are applied to the experiment in Fig. 41 they create the experiment shown in Fig. 41.

```

1 main
2 h_sim (3+1) runtime_args agentinfo
3
4 where
5 runtime_args = [(Arg ("q",4)), (Arg ("k", 16))]
6 agentinfo = [(i_wrapper,[]), (j_wrapper, []), (k_wrapper, []), (jfl_delay, [])]
7 h_sim x runtime_args agentinfo = (getargval.hd.getmsgargs) (((snd (outputs!x))!0)
8                               !0)
9                               where
10                              outputs = (0,[]):timed_transposed_msgs
11                              timed_transposed_msgs = zip2 [1..] transposed_msgs
12                              transposed_msgs = transpose allmessages
13                              allmessages = map apply_to_args (zip2 [1..]
14                              agentinfo)
15                              where
16                              apply_to_args (agentid, (agentfn,
17 broadcastsubscriptions)) = agentfn Emptyagentstate runtime_args (redacted_msgs
18 agentid outputs) agentid
19                              redacted_msgs id outputs = map (f id
20                              ) outputs
21                              f x (time, xs) = (time,
22                              concat (map (filter (h x)) xs))
23                              h x (Message (f,t) args) = (t=x)
24                              endwhere
25                              endwhere

```

(a) From Fig. 41, main, init, and the sim wrapper.

```

1 i_wrapper localstate args inputs id = outbound_messages_this_timestep : (i_wrapper
  localstate args future_messages id)
2       where
3       q = (getargval.hd) (filter ((="q").getargstr)
  args)
4       k = (getargval.hd) (filter ((="k").getargstr)
  args)
5       inbound_messages_this_timestep = snd (hd
  inputs)
6       t = fst (hd
  inputs)
7       future_messages = tl inputs
8       outbound_messages_this_timestep = [ Message (
  id,1) [(Arg ("i_f1",res))], Message (id,2) [(Arg ("i_f1",res))]]
9       where
10      res = i_f1
11      t
12      i_f1 0 = q
13      i_f1 t = cond (t<0) 0 (i_f1_last_timestep +
  j_f1_last_timestep)
14      where
15      msgs_from_i = filter ((=1).
  getmsgfrom) inbound_messages_this_timestep
16      args_from_i = concat (map
  getmsgargs msgs_from_i)
17      i_f1_last_timestep = (getargval.hd)
  (filter ((="i_f1").getargstr) args_from_i)
18      msgs_from_jd = filter ((=4).
  getmsgfrom) inbound_messages_this_timestep
19      args_from_jd = concat (map
  getmsgargs msgs_from_j)
20      j_f1_2last_timestep = (getargval.hd)
  (filter ((="j_f1d").getargstr) args_from_jd)
21      endwhile
22      endwhile

```

(b) From Fig. 41, *i\_wrapper*.

```

1 j-wrapper localstate args inputs id = outbound_messages_this_timestep : (j-wrapper
  localstate args future_messages id)
2                                     where
3                                     q = (getargval.hd) (filter ((="q").getargstr)
  args)
4                                     k = (getargval.hd) (filter ((="k").getargstr)
  args)
5                                     inbound_messages_this_timestep = snd (hd
  inputs)
6                                     t = fst (hd
  inputs)
7                                     future_messages = tl inputs
8                                     outbound_messages_this_timestep = [ Message (
  id,4) [(Arg ("j_f1",res1))], Message (id,2) [(Arg ("j_f1",res1))], Message (id,3)
  [(Arg ("j_f1",res1))], Message (id,2) [(Arg ("j_f2",res2))]]
9                                     where
10                                    res1 = j_f1
11                                    res2 = j_f2
12                                    t
13                                    t
14                                    t
15                                    t
16                                    t
17                                    t
18                                    t
19                                    t
20                                    t
21                                    t
22                                    t
23                                    t
24                                    t
25                                    t
26                                    t
27                                    t
28                                    t
29                                    t
30                                    t
31                                    t
32                                    t
33                                    t
34                                    t
35                                    t

```

(c) From Fig. 41, *j\_wrapper*.

```

1 k_wrapper localstate args inputs id = outbound_messages_this_timestep : (k_wrapper
  localstate args future_messages id)
2   where
3     q = (getargval.hd) (filter ((="q").getargstr)
4     args)
5     k = (getargval.hd) (filter ((="k").getargstr)
6     args)
7     inbound_messages_this_timestep = snd (hd
8     inputs)
9     t = fst (hd
10    inputs)
11    future_messages = tl inputs
12    outbound_messages_this_timestep = [Message (
13    id,2) [(Arg ("k-fl",k-fl t))]]
14    k-fl 0 = 0
15    k-fl t = k + q + j-fl-last_timestep
16    where
17    msgs_from_j = filter ((=2).
18    getmsgfrom) inbound_messages_this_timestep
19    args_from_j = concat (map
20    getmsgargs msgs_from_j)
21    j-fl-last_timestep = (getargval.hd)
22    (filter ((="j-fl").getargstr) args_from_j)
23    endwhere
24  endwhere
25 jfl_delay localstate args inputs id = outbound_messages_this_timestep : (jfl_delay
  localstate args future_messages id)
26   where
27   k = (getargval.hd) (filter ((="k").getargstr)
28   args)
29   inbound_messages_this_timestep = snd (hd
30   inputs)
31   t = fst (hd
32   inputs)
33   future_messages = tl inputs
34   outbound_messages_this_timestep = [Message (
35   id, 1) [(Arg ("j-fl-d", jfl_delayed t))]]
36   msgs_from_j = filter ((=2).getmsgfrom)
37   inbound_messages_this_timestep
38   args_from_j = concat (map getmsgargs
39   msgs_from_j)
40   jfl_delayed 0 = k
41   jfl_delayed t = (getargval.hd) (filter
42   ((="j-fl").getargstr) args_from_j)
43   endwhere

```

(d) From Fig. 41, *k\_wrapper* and *jfl\_delay*.

Figure 41: Translation of Fig. 38 to contain the Message constructor and have a harness function comparable to that of InterDynes. Figure 41a shows the experiment and sim wrapper, Fig. 41b shows the wrapper function for groups *i*, Fig. 41c shows the wrapper function for *j*, and Fig. 41d shows the wrapper function for *k* and the wrapper function for the delay agent *jfl\_delay*.

## 6 Validation and Testing

Now that the design of the translation process has been described there are a few aspects relating to it that should be considered. These considerations can be formed as three questions: Is the final target state of the translation a good representation of a reduced-functionality version of InterDyne? Does this translation design produce an agent-based model that is a correct representation of the initial recurrence relation model? Can this translation process be automated?

This section covers these three questions, first by comparing the final target state of the translation to a reduced-functionality version of InterDyne, then by testing the numerical correctness of each translational step, and finally by showing the first steps in an automated version of this translation.

### 6.1 Translation Validation

This section attempts to validate whether the final output of the translation is a valid InterDyne simulation. It does this by:

1. A visual inspection of the output syntax and comparison with InterDyne syntax
2. Comparison of the type of the output's harness function with the type of InterDyne's harness function
3. Comparison of the type of each output agent function with the type of an InterDyne agent function

The reduced-functionality version of InterDyne being compared to has a harness of the form shown in Fig 42, a harness type shown in Fig. 43, and a agent type shown in Fig. 44.

```

1 sim x runtime_args agentinfo = (getargval.hd.getmsgargs) (((snd (outputs!x))!0)!0)
2                               where
3                               outputs          = (0,[ ]):timed_transposed_msgs
4                               timed_transposed_msgs = zip [1..] transposed_msgs
5                               transposed_msgs     = transpose allmessages
6                               allmessages         = map apply_to_args (zip [1..]
7                               agentinfo)
8                               where
9                               agentfn,broadcastsubscriptions)) = agentfn Emptyagentstate runtime_args (
10                               redacted_msgs agentid outputs) agentid
11                               redacted_msgs id outputs =
12                               map (f id) outputs
13                               time, concat (map (filter (h x)) xs)
14                               f x (time, xs) = (
15                               t==x)
16                               h x (Message (f,t) args) = (

```

Figure 42: *Harness for a reduced-functionality version of InterDyne.*

```

1 Int -> [Arg_t] -> [(Agent_t, [*])] -> Int

```

Figure 43: *Harness type for a reduced-functionality version of InterDyne.*

```
1 Agentstate_t -> [Arg_t] -> [(Int , [Msg_t])] -> Int -> [[Msg_t]]
```

Figure 44: *Agent type for a reduced-functionality version of InterDyne.*

Looking at the simulator harness for InterDyne in Fig. 42 and comparing this to the simulator harness in the final target stat in Fig. 40, one can see that these two harness are identical in form and very similar in syntax. The harness produced in the final target state of the translation has not yet been converted into Haskell (the language in which InterDyne is written), this explains the difference in syntax between the two harnesses.

The type of the harness produced by the translation process is show in Fig. 45, as can be seen this type adheres exactly to the type of InterDyne harness shown in Fig. 43.

```
1 Int -> [Arg_t] -> [(Agent_t , [*])] -> Int
```

Figure 45: *Type of the harness in the final target state of the translation.*

Since an agent can be formulated in almost any manner within InterDyne, it is only worth while to compare the types of the agents between InterDyne and the translation, and not the form. The type of an agent created in this translation is shown in Fig. 46, comparing this to the type of an InterDyne agent, shown in Fig. 44, it can be seen that these types exactly match.

```
1 Agentstate_t -> [Arg_t] -> [(Int , [Msg_t])] -> Int -> [[Msg_t]]
```

Figure 46: *Type of the agents in the final target state of the translation.*

The translation has explicitly implemented the message type used in InterDyne by creating messages using the constructor *Message*. This constructor is used for simple one-to-one messages and in InterDyne there are a large range of messages for different data. The translation has only implements this single type, again show casing the reduced functionality compared to InterDyne.

This assessment of the resemblance between the final product of this translation and an InterDyne simulation, show that this process does indeed create a InterDyne simulation, which is of reduced functionality. This reduced form of InterDyne is required to allow a matching to the initial recurrence relations and to limit excess complications during the translation.

## 6.2 Translation Semantic Considerations

Though the translation can be seen to produce an InterDyne simulation, the question still remains about the correctness of the steps in this process. Here the correctness in these steps is investigated from a extensionality view point; this view states that two objects are equal if they have the same external behaviour [81].

For this translation the output produced by running the target state of each step is compared. Following the logic of extensionality, if these outputs are equal then so to are the target states that produced them and by association the translation.

This numerical equivalence is assessed by translating each step into its Miranda equivalent and then executing the produced program. This conversion was done using a translation program (shown in Appendix A), the output of this program were then edited to match the Miranda syntax and to allow for an input value of *t* to replace the static value of 3 used in the

translation. These programs were then compiled into one file (shown in Appendix A) and executed for the time steps (0, 1, 2, 3 and 4).

Table 1 shows the results from these runs, as can be seen all steps produced the same output. This shows, at least for the values tested, there exists a numerical equivalence between each target state of the steps of the translation and hence between the initial recurrence relation model and the final agent-based model.

Table 1: *The output value of the initial recurrence relation model and each translation step at the five different time steps (0, 1, 2, 3, 4, 5).*

Input	Initial	Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7	Step 8
0	4	4	4	4	4	4	4	4	4
1	20	20	20	20	20	20	20	20	20
2	36	36	36	36	36	36	36	36	36
3	145	145	145	145	145	145	145	145	145
4	2377	2377	2377	2377	2377	2377	2377	2377	2377

This equivalence suggests that each step is correct and that the entire translation produces an agent-based model that numerically represents the initial recurrence relation model.

### 6.3 Automated Translation Implementation

Though the design of the translation has been shown, with an example being translated by hand, this is a long process to achieve for multiple experiments. The question hence arises, can this process be automated? Implementing this automation is outside the scope of this project, however the first three steps have been achieved and are discussed below (this work is shown in Appendix B).

The three aspects of implementation discussed are, the lexer, the parser, and the first translation step. These steps are written in the Miranda functional language and allows for a text file contain an experiment written in the bespoke language to be translated into the first translation step.

The selected text file is read in by the lexer, this tokenises the list of characters using the lexemes show Fig.47. These lexemes represent each item that is allowed as part of the bespoke recurrence relation language, these can be keywords or symbols. Tokenising these items adds meaning to them, for example a list of characters could be identified as a variable or as a recurrence relation, the lexemes hold this information.

```

1 lexeme ::= Init | Var [char] | Recrel [char] [char] | Cond | Main | Fwhere | Optl |
   Ophd | Intfunc [char] | Num num | Opcon | Opbng | Opdiv | Optim | Opmin |
   Opplus | Rbckt | Lbckt | Rlbkt | Libkt | Opequ | Opne | Oplt | Opgt | Oplet |
   Opget | Concomma | Newl | Endwhere

```

Figure 47: *Definition of allowed lexemes for tokenising a list of characters.*

When passing characters to the lexer it is assumed that they are in keeping with the rules of the bespoke recurrence relation language. The lexer has no knowledge of this language, passed its keywords and symbols, and will tokenise everything according to this. If a character or string of characters is passed that does not match any of the known keywords or symbols, it will be interpreted as a variable name and saved as such. This lexer does not use spaces so all tokenising

will be done strictly on the characters order and not their spacing, this also allows a experiment to be passed as a single continuous line if one wished.

The next step after tokenising the input is to add a structure to this list of tokens, this is done using a parser. The parser reads in the list of tokens and knows the BNF and structure a recurrence relation experiment should take. The parser uses this knowledge of the layout and the meaning of the tokens to translate the token list into a parse tree created to represents this experiment, shown in Fig. 48.

```

1 simulation      ::= Simulation mainblock initblock whereblock
2 mainblock      ::= Main expression
3 recrelname     ::= Recrelname [char]
4 initblock      ::= Init [valdef]
5 whereblock     ::= Where [recreldef]
6 valdef         ::= Valdef valname expression
7 valname        ::= Valnamedef [char]
8 recreldef      ::= Recreldef agentname funcname char [argname] expression
9 agentname      ::= Agentnamedef [char]
10 funcname       ::= Funcnamedef [char]
11 argname        ::= Argnamedef [char] | Argnumdef num
12 expression    ::= SimpleExpression simple-expression | WhereExpression
                  simple_expression expr_whereblock
13 simple-expression ::= Number num | Valname [char] | Argname [char] | Time char |
                  Localvalname [char]
14                  | List [listitem] | Bracketed simple-expression
15                  | Conditional simple-expression simple-expression
16                  simple_expression
17                  | Binaryop simple-expression operation simple-expression
18                  | Unaryop internop simple-expression
19                  | Application recrelname [simple-expression]
20                  | Localapplication localfuncname [simple-expression]
21 expr_whereblock ::= Whereblock [localdefinition]
22 localdefinition ::= Localvaldef localvalname expression | Localfuncdef
                  localfuncname [argname] expression
23 localfuncname  ::= Localfuncnamedef [char]
24 localvalname   ::= Localvalnamedef [char]
25 listitem       ::= Listitem simple-expression
26 internop       ::= Head | Tail | Not
27 operation      ::= Plus | Minus | Times | Divide | Lessthan | Greaterthan |
                  Equals | Notequals | Lessthanorequalto
                  Greaterthanorequalto | Cons | Index

```

Figure 48: *Parse tree, written in Miranda, to store the structure and data of the passed experiment.*

The parser expects for the tokens given to it to conform to the rules and layout of the bespoke recurrence relation language, this includes containing a *main* with an experiment, a *init* though this can be empty, and a *where* with a list of recurrence relations (this can be a single recurrence relation). Any formatting that does not match the formatting rules of this language will cause the parser to crash. However the actual function of the recurrence relations can be incorrect, causing black holes or other issues, but will still be parsed correctly.

Once in this structure a translation function can be used to translate the recurrence relation experiment into the first step in the translation (to allow for infinite list outputs). This is done by reading in this numeric type and then creating a new version of this numeric type that is translated identically apart from the aspects that need to be changed to allow for the translation step.

This translation step should never crash as the format passed to it should always be correct, with an incorrect format causing the parser to crash. This translation step also only



focusses on the format of the experiment and any actual functionality could be incorrect but will still be correctly translated.

This implementation can be seen to be syntactically complete as the parse tree shown above matches exactly with the concrete syntax given in Fig. 9. This means that there is no case in the concrete syntax which can not exist in the parse tree definition. Semantic completeness of this implementation is beyond the scope of this project, but would define what is not allowed because it does not make sense in the language despite being syntactically correct, such as a recurrence relation name that has no definition (but is correctly formed).

## 7 Summary and Conclusion

This project has looked at the use of recurrence relation models and agent-based models within economics. Both these models are widely used to model complex systems in many fields, including for investigating emergent behaviour within these systems. Agent-based models in particular offer an attractive method for analysing emergent behaviour within interacting systems, due to their use of message passing. This provides the ability to investigate message passing dynamics, which is something difficult to achieve with recurrence relation models. However in economics agent-based, unlike recurrence relation models, very rarely appear in top journals and in general are not widely accepted in this field.

This dissertation presents a method designed to increase the general acceptance of agent-based models within economics. Increasing the traction of this method within economics will benefit the investigation of numerous complex systems within this field, as well as helping work already done using this technique to be better received.

The work presented in this dissertation combats two main criticisms of agent-based models: their lack of a system-wide formalism and their lack of familiarity to those not accustomed to this method of modelling. This is done by adding a system-wide formalism to an agent-based model as well creating a process to allow for agent-based models to become more transparent to those unfamiliar with them.

A formalism is specified for the agent-based model in the form of an equivalent recurrence relation model, with the transparency given by a step-by-step (with each step being equivalent) translation from this recurrence relation model to the agent-based model.

To achieve this, this project has created a bespoke recurrence relation language and the design of a step-by-step translation between this language and an agent-based model.

The created bespoke recurrence relation language allows for experiments involving interacting recurrence relations to be fully specified. The language its self is defined with a concrete BNF syntax, details on creating well formed programs with it are also given.

A design for an eight step translation process was created, to translate experiments defined by the bespoke recurrence relations into an agent-based model that takes a reduced form of an InterDyne simulation. This design changes a few small aspects of the model at each step, gradually reshaping the recurrence relation model into an agent-based model, the eighth target state hence representing an agent-based model. This process was demonstrated by hand using an example experiment written in the bespoke recurrence relation and translating this into an agent-based model, showing its structure at each step.

The models presented at each of these steps during the translation, were rewritten as Miranda programs that could be executed for different time steps. These Miranda versions of the steps target states were then run for a selection of time steps to establish their numerical equivalency; with each step being shown to return the same output for a given time input. The initial recurrence relation model was also rewritten as a Miranda program and executed for the time time inputs, this program also returned the same values as each of the steps target states.

The project this dissertation describes, scope was to create both a bespoke recurrence relation language and the design for a step-by-step translation from this language to an agent-based model. Both these aims were achieved and additionally the first three steps of implementing this translation as an automated program have been accomplished. These steps were the creation, in the Miranda language, of a lexer, parser and the first translation step.

This implementation of the first steps, shows the method and potential for implementing the entire translational process. Though this implementation does not improve upon the concept of this project, it does allow for the translation processes to be achieved fair quicker.

This translation process allows for recurrence relations of first order and higher, with delay agents created to allow for the passing of the required delayed information. This translation

also allows for recurrence relations to have more inputs than just time, with these inputs being passed to the final agents as arguments.

The numerical equivalence between each step and between the initial recurrence relation model and final agent-based model, implies the correctness of this translation process and the equivalence of the two models.

The step-by-step nature of the translation allows for it to be smoothly followed and hence be more comprehensible to a reader. This allows those with knowledge of recurrence relation models to gain a deeper understanding of the formation and structure of agent-based models.

This translation also gives an numerically equivalent agent-based model for a given recurrence relation model, allowing this recurrence relation model to act as the system wide formalism for the produced agent-based model.

The achievements of this project are hence:

- The creation bespoke recurrence relation language and formal definition of its syntax.
- The design of a step-by-step translation process, turning a model written in the bespoke recurrence relation language into an agent-based model.
- The validation and testing of this step-by-step translation.
- The creation of the lexer, parser, and first translational step, in a program designed to automate the translation process.

The work presented in this dissertation gives a method that can be used to give a system-wide formalisation to an agent-based model as well as giving a method that allows for one familiar with recurrence relation models to increase their understanding of this technique. This allows for an increased understanding of agent-based models in economics and gives potential of an increase in the use of these models and acceptance of work based on them.

## 7.1 Discussion and Further work

This project builds the foundations for a number of additional extensions to be done. Four particular extensions to note are: translating delays to be internal, formally proving the correctness of the translation, implementing a the full translation, and creating a reverse translation and implementing it.

Though the implementation of delays given in this dissertation, is correct in preserving the meaning of the original model, for large simulations it could result in the creation of many more agents. This might produce a high over head for a simulation, due to the number of agents and messages, making it difficult to run the final agent-based model. A final step would be to convert from this agent delay method into another one allowed by InterDyne, where the delay agents are incorporated into the simulator harness.

The the correctness of each step, and of the two models, has been argued informally and numerically in this dissertation, however this has not been formally proved. A formal proof of the correctness of each step and hence the two models, is mostly readily achieved by translating each step into lambda calculus and then proving the correctness in this form.

The first three steps of implementation have been achieved in this project, however there are a remaining eight steps to be created. Seven of these steps are the remaining translational steps, for the implementation of these the parse tree storing the model will have to be altered as the translation progresses to allow for new items, such as tuples and types. The final step is to create a function that translates from the parse tree into a text file that lays out the models in a way that is accommodating to a user, this is will produce the layouts of the form shown during the translation in this dissertation.

Though the translating from a recurrence relation model to an agent-based model fits the aims of this dissertation, further work could be to create a reverse translation and implementing it. This translation would allow a given agent-based model to be translated into a recurrence relation model.

Implementing and combining both of these translation would give a method for readily viewing a system as either a recurrence relation model or an agent-based model, as is required. This method would be of particular use when wanting to preform both static and dynamic analysis on a system, as the most amenable view can be chosen and used.

# Appendix A: Program for Numerical Testing

## User Manual

Here the code that translates from the target states in the design steps shown in this dissertation is given, along with the code used to numerically test the target states.

This code is written in Miranda, therefore to run it Miranda must be installed; installation instructions and downloads for Miranda can be found through the following link: <http://miranda.org.uk/>.

The first code shown is for translating the target states in the translation steps into their Miranda form. This can be run after loading the file (using *mira file.m*) by running the command *tomira "import.m" "output.m"*, where the file names given are examples.

Though this code correctly translates the target states into their Miranda form, there are still a number of issues stop this form from then being run. The target states use some syntax that is different in Miranda, as well as some internal functions and types that need to be defined in Miranda.

To run numerical testing the outputs from this code were compiled into one program and edited to run in Miranda, definitions were also given to internal functions and types requiring them. Each step was given the name *main* with the step number appended to the end, for example the result from the first translation step is called *main1*. To run the numerical test the command *testvalue* is used and to test the numerical equivalence of these tests the command *testequiv* is used.

## Translation from Design to Miranda

```
1 tomira infile outfile = [Tofile outfile result]
2   where
3   input = read infile
4
5   result = "main = "++blk1++"\n"++"where\n"++blk2++"\n"++blk3
6   rest0   = losemain input
7   (blk1, rest1) = findblk1 rest0 []
8   (blk2, blk3)  = findblk2 rest1 []
9
10  losemain ('m': 'a': 'i': 'n': xs) = xs
11
12  findblk1 ('i': 'n': 'i': 't': xs) blk1 = (blk1, xs)
13  findblk1 (x:xs) blk1 = findblk1 xs (blk1++[x
14  ])
15
16  findblk2 ('w': 'h': 'e': 'r': 'e': xs) blk2 = (blk2, xs)
17  findblk2 (x:xs) blk2 = findblk2 xs (blk2
```

## Program for Numerical Testing

```
1
2
3
4
5
6 equivtest x = (main0 x = main1 x = main2 x = main3 x = main4 x = main5 x = main6 x =
7   main7 x = main8 x)
8 valuelist x = [main0 x, main1 x, main2 x, main3 x, main4 x, main5 x, main6 x, main7
9   x, main8 x]
10 testvalue = [(valuelist 0), (valuelist 1), (valuelist 2), (valuelist 3), (valuelist
11   4)]
```

```

10  testequiv = [(equivtest 0), (equivtest 1), (equivtest 2), (equivtest 3), (equivtest 4)]
11
12  || This Miranda script tests various stages of the translation from
13  || recurrence relations to an agent-based formulation
14  ||
15  || _____
16  ||
17
18  str == [char]
19  double == num
20  int == num
21  lob == [char]    || this is a placeholder – it is not used yet, and should be more
                    || structured
22
23  || Later (in main6 onwards) we will use arg_t, agentstate_t, msg_t etc. as follows
24  || :
25  arg_t ::= EmptyArg | Arg (str, double)
26
27  getargstr EmptyArg = error "Can't get argstring from empty arg"
28  getargstr (Arg (s,n)) = s
29
30  getargval EmptyArg = error "Can't get argval from empty arg"
31  getargval (Arg (s,n)) = n
32
33
34  agentstate_t ::= Emptyagentstate | Agentstate (double, double, int, lob)
35
36  msg_t ::= Hiaton | Message (int,int) [arg_t] | Datamessage (int,int) [char]
37  getmsgfrom Hiaton = 0 || Essentially: the sim harness is
                    || the from/to of all Hiatons
38  getmsgfrom (Message (f,t) args) = f
39  getmsgfrom (Datamessage (f,t) text) = f
40
41  getmsgto Hiaton = 0 || Essentially: the sim harness is
                    || the from/to of all Hiatons
42  getmsgto (Message (f,t) args) = t
43  getmsgto (Datamessage (f,t) text) = t
44
45  getmsgargs Hiaton = []
46  getmsgargs (Message (f,t) args) = args
47  getmsgargs (Datamessage (f,t) text) = error "Datamessage does not have data of type
                    || arg_t"
48
49  || agent_t == agentstate_t -> [arg_t] -> [(num, [msg_t])] -> num -> [[msg_t]]
50
51
52
53  || We need functions to get the components of a 3-tuple:
54
55  fst3 (a,b,c) = a
56  snd3 (a,b,c) = b
57  thd3 (a,b,c) = c
58
59
60  || Miranda doesn't have a distfix conditional "if ... then ... else ..."
61  || so we'll create something similar – a ternary conditional called "cond"
62
63  cond x y z = y, if x
64             = z, otherwise
65
66  || We also use the following function:
67
68  s_createlist f t = (f t):(s_createlist f (t+1))

```

```

69
70 || -----
71
72
73
74 || =====
75 || Original
76
77
78 main0 x
79   = i_f1 x
80   where
81     q = 4
82     k = 16
83     i_f1 0 = q
84     i_f1 t = (i_f1 (t-1)) + (j_f1 (t-2))
85     j_f1 (-1) = k
86     j_f1 0 = k
87     j_f1 t = cond (t<0) (k_f1 (t-1))
88              (25+((j_f1 (t-1))*(i_f1 (t-1))) + (j_f2 (t-1)))
89     j_f2 0 = k+q
90     j_f2 t = cond (t<3) 27 ((j_f1 (t-1)) + 27)
91     k_f1 0 = 0
92     k_f1 t = k + q + (j_f1 (t-1))
93
94
95
96
97 || =====
98 || Step 1
99
100 main1 x
101   = i_f1_list ! x
102   where
103     q = 4
104     k = 16
105     i_f1_list = s_createlist i_f1 0
106                 where
107                   i_f1 0 = q
108                   i_f1 1 = (i_f1_list ! 0) + k
109                   i_f1 t = (i_f1_list ! (t-1)) + (j_f1_list ! (t-2))
110     j_f1_list = s_createlist j_f1 0
111                 where
112                   j_f1 0 = k
113                   j_f1 t = cond (t<0) (k_f1_list ! (t-1))
114                          (25 + ((j_f1_list ! (t-1)) * (i_f1_list ! (t-1))) + (
115     j_f2_list ! (t-1)))
116     j_f2_list = s_createlist j_f2 0
117                 where
118                   j_f2 0 = k+q
119                   j_f2 t = cond (t<3) 27 ((j_f1_list ! (t-1)) + 27)
120     k_f1_list = s_createlist k_f1 0
121                 where
122                   k_f1 0 = 0
123                   k_f1 t = k + q + (j_f1_list ! (t-1))
124
125
126 || =====
127 || Step 2
128
129 main2 x
130   = (i_wrapper ! x) ! 0
131   where

```

```

132     q = 4
133     k = 16
134     i_wrapper = (transpose [i_f1_list])
135     where
136         i_f1_list = s_createlist i_f1 0
137         where
138             i_f1 0 = q
139             i_f1 1 = (i_f1_list ! 0) + k
140             i_f1 t = (i_f1_list!(t-1)) + ((j_wrapper!(t-2))!0)
141     j_wrapper = (transpose [j_f1_list, j_f2_list])
142     where
143         j_f1_list = s_createlist j_f1 0
144         where
145             j_f1 0 = k
146             j_f1 t = cond (t<0) ((k_wrapper!(t-1))!0)
147                     (25 + ((j_f1_list!(t-1)) * ((
148
149                         + (j_f2_list!(t-1)))
150                     j_f2_list = s_createlist j_f2 0
151                     where
152                         j_f2 0 = k+q
153                         j_f2 t = cond (t<3) 27 ((j_f1_list ! (t-1)) + 27)
154     k_wrapper = (transpose [k_f1_list])
155     where
156         k_f1_list = s_createlist k_f1 0
157         where
158             k_f1 0 = 0
159             k_f1 t = k + q + ((j_wrapper!(t-1)) ! 0)
160
161
162 || =====
163 || Step 3
164
165 main3 x
166     = ((outputs!(x+1))!0)!0
167     where
168         q = 4
169         k = 16
170         nullvalue = 0
171         niloutputs = [[nullvalue], [nullvalue, nullvalue], [nullvalue]]
172         outputs = niloutputs : (transpose [i_wrapper, j_wrapper, k_wrapper])
173         i_wrapper = transpose [i_f1_list]
174         where
175             i_f1_list = s_createlist i_f1 0
176             i_f1 0 = q
177             i_f1 1 = (i_f1_list ! 0) + k
178             i_f1 t = cond (t<0) 0 ((i_f1_list!(t-1)) + j_f1_2last_timestep
179
180             where
181                 j_f1_2last_timestep = (((outputs!(t-1))!1)!0)
182         j_wrapper = transpose [j_f1_list, j_f2_list]
183         where
184             j_f1_list = s_createlist j_f1 0
185             where
186                 j_f1 0 = k
187                 j_f1 t = cond (t<0) k_f1_last_timestep
188                     (25 + ((j_f1_list ! (t-1)) *
189
190                         + (j_f2_list ! (t-1)))
191                     where
192                         i_f1_last_timestep = (((outputs!t)!0)!0)
193                         k_f1_last_timestep = (((outputs!t)!2)!0)
194                     j_f2_list = s_createlist j_f2 0

```



```

193         where
194         j_f2 0 = k+q
195         j_f2 t = cond (t<3) 27 ((j_f1_list ! (t-1)) + 27)
196     k_wrapper = transpose [k_f1_list]
197         where
198         k_f1_list = s_createlist k_f1 0
199         where
200         k_f1 0 = 0
201         k_f1 t = k + q + j_f1_last_timestep
202         where
203         j_f1_last_timestep = (((outputs!t)!1)!0)
204
205
206 || =====
207 || Step 4
208
209 main4 x
210     = ((outputs!(x+1))!0)!0
211     where
212     q = 4
213     k = 16
214     nullvalue = 0
215     niloutputs = [[nullvalue], [nullvalue, nullvalue], [nullvalue], [nullvalue
216 ]]
217     outputs = niloutputs : transpose [ i_wrapper outputs 0 ,j_wrapper outputs 0
218     ,k_wrapper outputs 0,jf1_delay outputs 0]
219     i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper
220     future_messages (t+1))
221     where
222     inbound_messages_this_timestep = hd inputs
223     future_messages = tl inputs
224     outbound_messages_this_timestep = [(i_f1 t)]
225     i_f1 0 = q
226     i_f1 t = cond (t<0) 0
227             ( ((inbound_messages_this_timestep!0)!0)
228             +((inbound_messages_this_timestep!3)!0)
229
230 )
231     j_wrapper inputs t = outbound_messages_this_timestep : (j_wrapper
232     future_messages (t+1))
233     where
234     inbound_messages_this_timestep = hd inputs
235     future_messages = tl inputs
236     outbound_messages_this_timestep = [(j_f1 t), (j_f2 t)]
237     j_f1 0 = k
238     j_f1 t = cond (t<0) k_f1_last_timestep
239             (25 + (j_f1_last_timestep *
240     i_f1_last_timestep)+j_f2_last_timestep)
241     where
242     i_f1_last_timestep = ((
243     inbound_messages_this_timestep!0)!0)
244     j_f1_last_timestep = ((
245     inbound_messages_this_timestep!1)!0)
246     j_f2_last_timestep = ((
247     inbound_messages_this_timestep!1)!1)
248     k_f1_last_timestep = ((
249     inbound_messages_this_timestep!2)!0)
250     j_f2 0 = k+q
251     j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
252     where
253     j_f1_last_timestep = ((
254     inbound_messages_this_timestep!1)!0)
255     k_wrapper inputs t = outbound_messages_this_timestep : (k_wrapper
256     future_messages (t+1))
257     where

```

```

245         inbound_messages_this_timestep = hd inputs
246         future_messages                 = tl inputs
247         outbound_messages_this_timestep = [k_f1 t]
248         k_f1 0 = 0
249         k_f1 t = k + q + j_f1_last_timestep
250         where
251             j_f1_last_timestep = ((
inbound_messages_this_timestep!1)!0)
252         jf1_delay inputs t = outbound_messages_this_timestep : (jf1_delay
future_messages (t+1))
253         where
254             inbound_messages_this_timestep = hd inputs
255             future_messages                 = tl inputs
256             outbound_messages_this_timestep = [(jf1_delayed t)]
257             jf1_delayed 0 = k
258             jf1_delayed t = ((inbound_messages_this_timestep!1)!0)
259
260
261
262
263 || =====
264 || Step 5
265
266
267 main5 x
268     = (thd3.hd) (((outputs!(x+1))!0)!0)
269     where
270         q = 4
271         k = 16
272         newnullvalue = [(0,0,0)]
273         niloutputs = [[newnullvalue], [newnullvalue, newnullvalue], [newnullvalue],
[newnullvalue]]
274         outputs = niloutputs : transpose [ i_wrapper outputs 0 ,j_wrapper outputs 0
,k_wrapper outputs 0, jf1_delay outputs 0]
275         i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper
future_messages (t+1))
276         where
277             inbound_messages_this_timestep = map f (hd inputs)
278             where
279                 f xs = map g xs
280                 g xs = filter h xs
281                 h (a,b,c) = ((b=1)
\ / (a,b,c) = (0,0,0))
282             future_messages                 = tl inputs
283             outbound_messages_this_timestep = [ [ (1,1,res) ||
284
285                 ,(1,2,res)] ||
286
287                 ]
288             where
289                 res = i_f1 t
290
291             i_f1 0 = q
292             i_f1 t = cond (t<0) 0 (i_f1_last_timestep +
j_f1_2last_timestep)
293         where
294             i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
295             j_f1_2last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!3)!0)
296             j_wrapper inputs t = outbound_messages_this_timestep : (j_wrapper
future_messages (t+1))
297         where
298             inbound_messages_this_timestep = map f (hd inputs)
299             where

```

```

297                                     f xs = map g xs
298                                     g xs = filter h xs
299                                     h (a,b,c) = ((b=2)

\ / (a,b,c) = (0,0,0))

300                                     future_messages = tl inputs
301                                     outbound_messages_this_timestep = [ [ (2,4,res1) ||
302                                     from j to i                                     ,(2,2,res1) ||
303                                     from j to j                                     ,(2,3,res1)] ||
304                                     from j to k                                     ,[(2,2,res2)] ||
305                                     from j to j                                     ]
306                                     where
307                                     res1 = j_f1 t
308                                     res2 = j_f2 t
309                                     j_f1 0 = k
310                                     j_f1 t = cond (t<0) k_f1_last_timestep
311                                     (25 + (j_f1_last_timestep *
i_f1_last_timestep)+j_f2_last_timestep)
312                                     where
313                                     i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
314                                     j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
315                                     j_f2_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!1)
316                                     k_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!2)!0)
317                                     j_f2 0 = k+q
318                                     j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
319                                     where
320                                     j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
321                                     k_wrapper inputs t = outbound_messages_this_timestep : (k_wrapper
future_messages (t+1))
322                                     where
323                                     inbound_messages_this_timestep = map f (hd inputs)
324                                     where
325                                     f xs = map g xs
326                                     g xs = filter h xs
327                                     h (a,b,c) = ((b=3)

\ / (a,b,c) = (0,0,0))

328                                     future_messages = tl inputs
329                                     outbound_messages_this_timestep = [[(3,2,k_f1 t)]] ||

From k to j only

330                                     k_f1 0 = 0
331                                     k_f1 t = k + q + j_f1_last_timestep
332                                     where
333                                     j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
334                                     jf1_delay inputs t = outbound_messages_this_timestep : (jf1_delay
future_messages (t+1))
335                                     where
336                                     inbound_messages_this_timestep = map f (hd inputs)
337                                     where
338                                     f xs = map g xs
339                                     g xs = filter h xs
340                                     h (a,b,c) = ((b=4)

\ / (a,b,c) = (0,0,0))

341                                     future_messages = tl inputs
342                                     outbound_messages_this_timestep = [[(4,1,jf1_delayed t
)]]

```

```

343             jfl_delayed 0 = k
344             jfl_delayed t = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
345
346
347
348
349
350
351 || =====
352 || Step 6
353
354 main6 x
355     = sim (x+1)
356     where
357     q = 4
358     k = 16
359     sim x = (thd3.hd) (((outputs!x)!0)!0)
360     where
361     newnullvalue = [(0,0,0)]
362     niloutputs = [[newnullvalue], [newnullvalue, newnullvalue], [
newnullvalue], [newnullvalue]]
363     outputs = niloutputs : redactedoutputs
364     redactedoutputs
365     = transpose [ i_wrapper i_outputs 0 ,j_wrapper j_outputs 0 ,
k_wrapper k_outputs 0, jfl_delay jfl_inputs 0 ]
366     where
367     i_outputs  = map (map (map (filter (h 1)))) outputs
368     j_outputs  = map (map (map (filter (h 2)))) outputs
369     k_outputs  = map (map (map (filter (h 3)))) outputs
370     jfl_inputs = map (map (map (filter (h 4)))) outputs
371     h x (a,b,c) = ((b=x) \/\ (a,b,c) = (0,0,0))
372     i_wrapper inputs t = outbound_messages_this_timestep : (i_wrapper
future_messages (t+1))
373     where
374     inbound_messages_this_timestep = hd inputs
375     future_messages                = tl inputs
376     outbound_messages_this_timestep = [ [ (1,1,res)  ||
from i to i
377                                     ,(1,2,res)] ||
from i to j
378                                     ]
379     where
380     res = i_fl t
381     i_fl 0 = q
382     i_fl t = cond (t<0) 0 (i_fl_last_timestep +
j_fl_2last_timestep)
383     where
384     i_fl_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
385     j_fl_2last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!3)!0)
386     j_wrapper inputs t = outbound_messages_this_timestep : (j_wrapper
future_messages (t+1))
387     where
388     inbound_messages_this_timestep = hd inputs
389     future_messages                = tl inputs
390     outbound_messages_this_timestep = [ [ (2,4,res1)  ||
from j to i
391                                     ,(2,2,res1)  ||
from j to j
392                                     ,(2,3,res1)] ||
from j to k
393                                     ,[(2,2,res2)] ||

```

```

394 from j to j
395
396
397
398
399
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401
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440
441
442

```

```

]
where
res1 = j_f1 t
res2 = j_f2 t

j_f1 0 = k
j_f1 t = cond (t<0) k_f1_last_timestep
(25 + (j_f1_last_timestep *
i_f1_last_timestep)+j_f2_last_timestep)
where
i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
j_f2_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!1)
k_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!2)!0)
j_f2 0 = k+q
j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
where
j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
k_wrapper inputs t = outbound_messages_this_timestep : (k_wrapper
future_messages (t+1))
where
inbound_messages_this_timestep = hd inputs
future_messages = tl inputs
outbound_messages_this_timestep = [[(3,2,k_f1 t)]] ||

From k to j only
k_f1 0 = 0
k_f1 t = k + q + j_f1_last_timestep
where
j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
jfl_delay inputs t = outbound_messages_this_timestep : (jfl_delay
future_messages (t+1))
where
inbound_messages_this_timestep = hd inputs
future_messages = tl inputs
outbound_messages_this_timestep = [[(4,1,jfl_delayed t
)]]
jfl_delayed 0 = k
jfl_delayed t = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)

|| =====
|| Step 7

main7 x
= sim (x+1) runtime_args agentinfo
where
runtime_args = [(Arg ("q",4)), (Arg ("k", 16))]
agentinfo = [(i_wrapper,[]), (j_wrapper, []), (k_wrapper, []), (jfl_delay,
[])]
sim x args agents
= (thd3.hd) (((snd (outputs!x))!0)!0)
where
newnullvalue = [(0,0,0)]
niloutputs = [[newnullvalue], [newnullvalue, newnullvalue], [
newnullvalue], [newnullvalue]]
outputs = zip2 [0..] (niloutputs : (redactedoutputs))

```

```

443         redactedoutputs
444         = transpose (map apply_to_args (zip2 [1..] agents))
445         where
446         apply_to_args (agentid, (f,broadcastsubscriptions))
447             = f Emptyagentstate args (myoutputs agentid)
agentid
448         myoutputs id = map (f id) outputs
449         f x (t, xs) = (t, map (map (filter (h x))) xs)
450         h x (a,b,c) = ((b=x) \ / (a,b,c) = (0,0,0))
451         i-wrapper localstate args inputs id
452             = outbound_messages_this_timestep : (i-wrapper localstate args
future_messages id)
453         where
454         q = (getargval.hd) (filter ((="q").getargstr) args)
455         k = (getargval.hd) (filter ((="k").getargstr) args)
456         inbound_messages_this_timestep = snd (hd inputs)
457         t = fst (hd inputs)
458         future_messages = tl inputs
459         outbound_messages_this_timestep = [ [ (id,1,res) || from i to i
460                                             ,(id,2,res)] || from i to j
461                                             ]
462                                             where
463                                             res = i_f1 t
464         i_f1 0 = q
465         i_f1 t = cond (t<0) 0 (i_f1_last_timestep + j_f1_2last_timestep)
466         where
467         i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
468         j_f1_2last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!3)!0)
469         j-wrapper localstate args inputs id
470             = outbound_messages_this_timestep : (j-wrapper localstate args
future_messages id)
471         where
472         q = (getargval.hd) (filter ((="q").getargstr) args)
473         k = (getargval.hd) (filter ((="k").getargstr) args)
474         inbound_messages_this_timestep = snd (hd inputs)
475         t = fst (hd inputs)
476         future_messages = tl inputs
477         outbound_messages_this_timestep = [ [ (id,4,res1) || from j to i
478                                             ,(id,2,res1) || from j to j
479                                             ,(id,3,res1)] || from j to k
480                                             ,[ (id,2,res2)] || from j to j
481                                             ]
482                                             where
483                                             res1 = j_f1 t
484                                             res2 = j_f2 t
485         j_f1 0 = k
486         j_f1 t = cond (t<0) k_f1_last_timestep
487                 (25 + (j_f1_last_timestep * i_f1_last_timestep)+
j_f2_last_timestep)
488         where
489         i_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!0)!0)
490         j_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
491         j_f2_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!1)
492         k_f1_last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!2)!0)
493         j_f2 0 = k+q
494         j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
495         where
496         j_f1_last_timestep = (thd3.hd) ((

```

```

inbound_messages_this_timestep!1)!0)
497     k_wrapper localstate args inputs id
498     = outbound_messages_this_timestep : (k_wrapper localstate args
future_messages id)
499     where
500     q = (getargval.hd) (filter ((="q").getargstr) args)
501     k = (getargval.hd) (filter ((="k").getargstr) args)
502     inbound_messages_this_timestep = snd (hd inputs)
503     t = fst (hd inputs)
504     future_messages = tl inputs
505     outbound_messages_this_timestep = [[(id,2,k.f1 t)]] || From k to j
only
506     k.f1 0 = 0
507     k.f1 t = k + q + j.f1.last_timestep
508     where
509     j.f1.last_timestep = (thd3.hd) ((
inbound_messages_this_timestep!1)!0)
510     jf1_delay localstate args inputs id
511     = outbound_messages_this_timestep : (jf1_delay localstate args
future_messages id)
512     where
513     inbound_messages_this_timestep = snd (hd inputs)
514     future_messages = tl inputs
515     t = fst (hd inputs)
516     outbound_messages_this_timestep = [[(id,1,jf1_delayed t)]]
517     k = (getargval.hd) (filter ((="k").getargstr) args)
518     jf1_delayed 0 = k
519     jf1_delayed t = (thd3.hd) ((inbound_messages_this_timestep!1)!0)
520
521
522
523
524
525 || =====
526 || Step 8
527
528 main8 x
529     = sim (x+1) runtime_args agentinfo
530     where
531     runtime_args = [(Arg ("q",4)), (Arg ("k", 16))]
532     agentinfo = [(i_wrapper,[]), (j_wrapper, []), (k_wrapper, []), (jf1_delay,
[])]
533     sim x runtime_args agentinfo
534     = (getargval.hd.getmsgargs) (((snd (outputs!x))!0)!0)
535     where
536     outputs = (0,[]):timed_transposed_msgs
537
538     || timed_transposed_msgs is a list of outputs at each timestep,
539     || each being a tuple of a time and a list of outputs from all
agents
540     || [(1,t1-outputs), (2,t2-outputs),...]
541
542     timed_transposed_msgs = zip2 [1..] transposed_msgs
543
544     || transposed_msgs is a list of outputs at each timestep
545     || [t1-outputs, t2, outputs, ...]
546     || each being a list of outputs from all agents
547     || [t1-outputs_agent1, t1-outputs_agent2, ...]
548     || each item of which is a list of msg_t
549
550     transposed_msgs = transpose allmessages
551
552     || allmessages is a list of outputs from each agent
553     || [agent1-output, agent2-output, ...]

```

```

554         || each of which is a list of outputs
555         || per timestep, each being a list of msg-t
556
557     allmessages
558     = map apply_to_args (zip2 [1..] agentinfo)
559     where
560         apply_to_args (agentid, (agentfn, broadcastsubscriptions))
561             = agentfn Emptyagentstate runtime_args (
redacted_msgs agentid outputs) agentid
562
563         || redacted_msgs :: num -> [(num, [[msg-t]])] -> [(num, [msg-t])]
564         redacted_msgs id outputs = map (f id) outputs
565         f x (time, xs) = (time, concat (map (filter (h x)) xs)
)
566         h x Hiaton = False
567         h x (Message (f,t) args) = (t=x)
568         h x (Datamessage (f,t) text) = error "Datamessage not yet
implemented"
569         || i-wrapper :: agent-t
570         i-wrapper localstate args inputs id
571             = outbound_messages_this_timestep : (i-wrapper localstate args
future_messages id)
572         where
573             q = (getargval.hd) (filter ((="q").getargstr) args)
574             k = (getargval.hd) (filter ((="k").getargstr) args)
575             inbound_messages_this_timestep = snd (hd inputs)
576             t = fst (hd inputs)
577             future_messages = tl inputs
578             outbound_messages_this_timestep = [ Message (id,1) [(Arg ("i_f1",
res))] || from i to i
579                                     , Message (id,2) [(Arg ("i_f1",
res))] || from i to j
580                                     ]
581             where
582                 res = i_f1 t
583             i_f1 0 = q
584             i_f1 t = cond (t<0) 0 (i_f1.last_timestep + j_f1.last_timestep)
585             where
586                 msgs_from_i = filter ((=1).getmsgfrom)
inbound_messages_this_timestep
587                 args_from_i = concat (map getmsgargs msgs_from_i)
588                 i_f1.last_timestep = (getargval.hd) (filter ((="i_f1").
getargstr) args_from_i)
589                 msgs_from_j = filter ((=4).getmsgfrom)
inbound_messages_this_timestep
590                 args_from_j = concat (map getmsgargs msgs_from_j)
591                 j_f1.last_timestep = (getargval.hd) (filter ((="j_f1d").
getargstr) args_from_j)
592         || j-wrapper :: agent-t
593         j-wrapper localstate args inputs id
594             = outbound_messages_this_timestep : (j-wrapper localstate args
future_messages id)
595         where
596             q = (getargval.hd) (filter ((="q").getargstr) args)
597             k = (getargval.hd) (filter ((="k").getargstr) args)
598             inbound_messages_this_timestep = snd (hd inputs)
599             t = fst (hd inputs)
600             future_messages = tl inputs
601             outbound_messages_this_timestep = [ Message (id,4) [(Arg ("j_f1",
res1))] || from j to i
602                                     , Message (id,2) [(Arg ("j_f1",
res1))] || from j to j
603                                     , Message (id,3) [(Arg ("j_f1",
res1))] || from j to k

```



```

604                                     ,Message (id,2) [(Arg ("j-f2",
res2)))] || from j to j
605                                     ]
606                                     where
607                                     res1 = j_f1 t
608                                     res2 = j_f2 t
609                                     j_f1 0 = k
610                                     j_f1 t = cond (t<0) k_f1_last_timestep
611                                     (25 + (j_f1_last_timestep * i_f1_last_timestep)+
j_f2_last_timestep)
612                                     where
613                                     msgs_from_i      = filter ((=1).getmsgfrom)
inbound_messages_this_timestep
614                                     args_from_i      = concat (map getmsgargs msgs_from_i)
615                                     i_f1_last_timestep = (getargval.hd) (filter ((="i-f1").
getargstr) args_from_i)
616                                     msgs_from_j      = filter ((=2).getmsgfrom)
inbound_messages_this_timestep
617                                     args_from_j      = concat (map getmsgargs msgs_from_j)
618                                     j_f1_last_timestep = (getargval.hd) (filter ((="j-f1").
getargstr) args_from_j)
619                                     j_f2_last_timestep = (getargval.hd) (filter ((="j-f2").
getargstr) args_from_j)
620                                     msgs_from_k      = filter ((=3).getmsgfrom)
inbound_messages_this_timestep
621                                     args_from_k      = concat (map getmsgargs msgs_from_k)
622                                     k_f1_last_timestep = (getargval.hd) (filter ((="k-f1").
getargstr) args_from_j)
623                                     j_f2 0 = k+q
624                                     j_f2 t = cond (t<3) 27 (j_f1_last_timestep + 27)
625                                     where
626                                     msgs_from_j      = filter ((=2).getmsgfrom)
inbound_messages_this_timestep
627                                     args_from_j      = concat (map getmsgargs msgs_from_j)
628                                     j_f1_last_timestep = (getargval.hd) (filter ((="j-f1").
getargstr) args_from_j)
629                                     || k_wrapper :: agent_t
630                                     k_wrapper localstate args inputs id
631                                     = outbound.messages_this_timestep : (k_wrapper localstate args
future_messages id)
632                                     where
633                                     q = (getargval.hd) (filter ((="q").getargstr) args)
634                                     k = (getargval.hd) (filter ((="k").getargstr) args)
635                                     inbound_messages_this_timestep = snd (hd inputs)
636                                     t = fst (hd inputs)
637                                     future_messages = tl inputs
638                                     outbound.messages_this_timestep = [Message (id,2) [(Arg ("k-f1",
k_f1 t))]] || From k to j only
639                                     k_f1 0 = 0
640                                     k_f1 t = k + q + j_f1_last_timestep
641                                     where
642                                     msgs_from_j      = filter ((=2).getmsgfrom)
inbound_messages_this_timestep
643                                     args_from_j      = concat (map getmsgargs msgs_from_j)
644                                     j_f1_last_timestep = (getargval.hd) (filter ((="j-f1").
getargstr) args_from_j)
645                                     jf1_delay localstate args inputs id
646                                     = outbound.messages_this_timestep : (jf1_delay localstate args
future_messages id)
647                                     where
648                                     k = (getargval.hd) (filter ((="k").getargstr) args)
649                                     inbound_messages_this_timestep = snd (hd inputs)
650                                     t = fst (hd inputs)
651                                     future_messages = tl inputs

```

```

652         outbound_messages_this_timestep = [Message (id, 1) [(Arg ("j-f1d",
        jf1-delayed t))]]
653         msgs_from_j          = filter ((=2).getmsgfrom)
inbound_messages_this_timestep
654         args_from_j          = concat (map getmsgargs msgs_from_j)
655         jf1-delayed 0        = k
656         jf1-delayed t        = (getargval.hd) (filter (="j-f1").getargstr)
args_from_j)

```

## Appendix B: Implementation of Translation

### User Manual

Here the code for the implementation of the translation described in this dissertation is given; this implementation only covers the first three aspects of the automated translation. These three steps are the lexer, parser and first translation step, shown here in one file.

This code is written in Miranda, therefore to run it Miranda must be installed; installation instructions and downloads for Miranda can be found through the following link: <http://miranda.org.uk/>.

Once Miranda is installed, simply run the the code as a file using *mira file.m*. Once this is done the first translation step can be achieved by using the command *trans\_step1 (parser (lex input))*, where *input* is the character list defining the experiment to be translated (this can be created by reading in a text file).

### Lexer, Parser and First Translation Step Code

```
1 || Lexer
2 || Reads in the file and converts it to tokens
3
4 lexeme ::= Initl | Varl [char] | Recrell [char] [char] | Opcond | Mainl | Where1 |
      Optl | Ophd | Intfunc [char] | Numl num | Opcon | Opbng | Opdiv | Optim | Opmin
      | Opplus | Rbckt | Lbckt | Rlbkt | Llbkt | Opequ | Opne | Oplt | Opgt | Oplet
      | Opget | Concomma | Newl | Endwhere1
5
6
7 lex [] = []
8 lex ('>':xs) = Opget:(lex xs)
9 lex ('<':xs) = Oplet:(lex xs)
10 lex ('>':xs) = Opgt:(lex xs)
11 lex ('<':xs) = Oplt:(lex xs)
12 lex ('~':xs) = Opne:(lex xs)
13 lex ('=':xs) = Opequ:(lex xs)
14 lex ('[':xs) = Llbkt:(lex xs)
15 lex (']':xs) = Rlbkt:(lex xs)
16 lex ('(':xs) = Lbckt:(lex xs)
17 lex (')':xs) = Rbckt:(lex xs)
18 lex ('+':xs) = Opplus:(lex xs)
19 lex ('-':xs) = Opmin:(lex xs)
20 lex ('*':xs) = Optim:(lex xs)
21 lex ('/':xs) = Opdiv:(lex xs)
22 lex ('!':xs) = Opbng:(lex xs)
23 lex (' ':xs) = lex xs
24 lex ('\n':xs) = Newl:lex xs
25 lex (',':xs) = Concomma:(lex xs)
26 lex (':':xs) = Opcon:(lex xs)
27 lex (x:xs) = (Numl (numval a)): (lex b), if (isnumber a)
28             = (Intfunc (tl a)): (lex b), if (hd a) = '_'
29             = Ophd:(lex b), if a=['h','d']
30             = Optl:(lex b), if a=['t','l']
31             = Initl:(lex b), if a=['i','n','i','t']
32             = Endwhere1:(lex b), if a=['e','n','d','w','h','e','r','e']
33             = Where1:(lex b), if a=['w','h','e','r','e']
34             = Mainl:(lex b), if a=['m','a','i','n']
35             = Opcond:(lex b), if a=['c','o','n','d']
36             = (returnfunc a []):(lex b), if (isfunc a)
37             = (Varl a):(lex b), otherwise
38             where
39             (a,b)=f (x:xs) []
40             f [] a = (a,[])
41             f (' ':xs) a = (a,xs)
```

```

42         f (')':xs) a = (a, (')':xs))
43         f (',':xs) a = (a, (',':xs))
44         f ('+':xs) a = (a, ('+':xs))
45         f ('-':xs) a = (a, ('-':xs))
46         f ('*':xs) a = (a, ('*':xs))
47         f ('/':xs) a = (a, ('/':xs))
48         f ('<':xs) a = (a, ('<':xs))
49         f ('>':xs) a = (a, ('>':xs))
50         f ('=':xs) a = (a, ('=':xs))
51         f (']':xs) a = (a, (']':xs))
52         f (':':xs) a = (a, (':':xs))
53         f ('{':xs) a = (a, ('{':xs))
54         f ('}':xs) a = (a, ('}':xs))
55         f ('\n':xs) a = (a, xs)
56         f (x:xs) a = f xs (a++[x])
57
58 isnumber x = (removeall "0123456789." (mkset x)) = []
59
60 removeall xs [] = []
61 removeall xs (y:ys) = removeall xs ys, if member xs y
62 = y:(removeall xs ys), otherwise
63
64 isfunc [] = False
65 isfunc ('_':xs) = True
66 isfunc (x:xs) = isfunc xs
67
68 beforecore [] a = a
69 beforecore ('_':xs) a = a
70 beforecore (x:xs) a = beforecore xs (a++[x])
71
72 returnfunc [] a = Recrell a []
73 returnfunc ('_':xs) a = Recrell a xs
74 returnfunc (x:xs) a = returnfunc xs (a++[x])
75
76
77
78
79
80
81
82
83
84 || Parse tree
85 || Stores the structure of the inputted file
86
87
88 simulation      ::= Simulation mainblock initblock whereblock
89 mainblock       ::= Main expression ||recrelname [argname]
90 recrelname      ::= Recrelname [char]
91 initblock       ::= Init [valdef]
92 whereblock      ::= Where [recreldef]
93 valdef          ::= Valdef valname expression
94 valname         ::= Valnamedef [char]
95 recreldef       ::= Recreldf agentname funcname [argname] expression
96 agentname       ::= Agentnamedef [char]
97 funcname        ::= Funcnamedef [char]
98 argname         ::= Argnamedef [char] | Argnumdef num
99 expression      ::= SimpleExpression simple-expression | WhereExpression
100                simple-expression expr_whereblock
101 simple-expression ::= Number num | Valname [char] | Argname [char] | Time char |
102                Localvalname [char]
103                | List [listitem] | Bracketed simple-expression
104                | Conditional simple-expression simple-expression
105                simple-expression

```

```

103 | Binaryop simple-expression operation simple-expression
104 | Unaryop internop simple-expression
105 | Application recrelname [simple-expression]
106 | Localapplication localfuncname [simple-expression]
107 expr-whereblock ::= Whereblock [localdefinition]
108 localdefinition ::= Localvaldef localvalname expression | Localfuncdef
109 | localfuncname [argname] expression
109 localfuncname ::= Localfuncnamedef [char]
110 localvalname ::= Localvalnamedef [char]
111 listitem ::= Listitem simple-expression
112 internop ::= Head | Tail | Not
113 operation ::= Plus | Minus | Times | Divide | Lessthan | Greaterthan |
114 | Equals | Notequals | Lessthanorequalto | Greaterthanorequalto | Cons | Index
114
115
116
117
118
119
120
121
122
123
124 || Parser
125 || This turns the tokens into the parse tree
126
127 parser x = Simulation (get_main a) (get_init b) (get_lwhere c)
128 | where
129 | a = find_drive x []
130 | b = find_inits x
131 | c = find_rr x
132
133
134 find_drive (Initl:xs) dlist = dlist
135 find_drive (Mainl:xs) dlist = find_drive xs dlist
136 find_drive (x:xs) dlist = find_drive xs (dlist++[x])
137
138 find_inits (Initl:xs) = find_initssub xs []
139 find_inits (x:xs) = find_inits xs
140
141 find_initssub (Wherel:xs) ilist = ilist
142 find_initssub (x:xs) ilist = find_initssub xs (ilist++[x])
143
144
145 find_rr (Wherel:xs) = xs
146 find_rr (x:xs) = find_rr xs
147
148
149
150 get_main x = Main exp
151 | where
152 | (exp, none) = get_expr x
153
154 find_rrname ((Recrell a b):xs) = Recrelname (a++"_"++b)
155 find_rrname (x:xs) = find_rrname xs
156
157 find_rrargs ((Recrell a b):xs) = find_rrnamesub xs []
158 find_rrargs (x:xs) = find_rrargs xs
159
160 find_rrnamesub [] arglist = arglist
161 find_rrnamesub ((Num1 a):xs) arglist = find_rrnamesub xs (arglist++[(Argnumdef a)])
162 find_rrnamesub ((Var1 a):xs) arglist = find_rrnamesub xs (arglist++[(Argnamedef a)
163 | ])

```

```

164
165
166 get_init x = Init (find_indefs x [])
167
168 find_indefs [] dlist = dlist
169 find_indefs ((Varl a):(Opequ):xs) dlist = find_indefs nxs ndlist
170                                     where
171                                     ndlist = dlist ++ [ndef]
172                                     ndef = Valdef (Valnamedef a) (expr)
173                                     (expr, nxs) = get_expr xs
174
175
176 get_lwhere x = Where [Recreldef (Agentnamedef "a") (Funcnamedef "b") [] (
177     SimpleExpression (Number 1))] (find_rrlist x [])
178
179 find_rrlist [] rrlist = rrlist
180 find_rrlist ((Recrell a b):xs) rrlist = find_rrlist nxs nrrlist
181                                     where
182                                     nrrlist = rrlist ++ [nrr]
183                                     nrr = Recreldef (Agentnamedef a) (
184     Funcnamedef b) args expr
185                                     args = get_rrargs xs []
186                                     (expr, nxs) = get_expr (get_aftequ xs)
187
188 get_aftequ (Opequ:xs) = xs
189 get_aftequ (x:xs) = get_aftequ xs
190
191 get_rrargs (Opequ:xs) arglist = arglist
192 get_rrargs ((Numl a):xs) arglist = get_rrargs xs (arglist++[(Argnumdef a)])
193 get_rrargs ((Varl a):xs) arglist = get_rrargs xs (arglist++[(Argnamedef a)])
194
195
196 get_expr x = ((WhereExpression outw inw), rest), if is_where x
197 = ((SimpleExpression exp), rest), otherwise
198 where
199 outw = Number 1
200 inw = Whereblock []
201 (exp, rest) = get_expr x (Number 0)
202
203 is_where [] = False
204 is_where (Where1:xs) = True
205 is_where (x:xs) = is_where xs
206
207
208
209
210
211
212 get_exprision ((Numl a):xs) lst = get_exprision xs (Number a)
213 get_exprision ((Varl a):xs) lst = get_exprision xs (Valname a)
214 get_exprision ((Llbkt):xs) lst = get_exprision al (List il)
215 where
216 (il, al) = (find_list xs)
217
218 get_exprision ((Lbkt):xs) lst = get_exprision ab epr
219 where
220 epr = Bracketed eib
221 (eib, n) = get_exprision ib (Number 0)
222 (ib, ab) = (find_inbr xs [])
223
224 get_exprision ((Opcond):xs) lst = get_exprision r3 exp
225 where
226 exp = (Conditional fst snd trd)
227 (fst, r1) = get_exprision xs (Number 0)

```

```

226         (snd, r2) = get_exprision r1 (Number 0)
227         (trd, r3) = get_exprision r2 (Number 0)
228 get_exprision ((Ophd):xs)      lst = get_exprision rest nlst
229                                where
230                                nlst = Unaryop Head ex
231                                (ex, rest) = get_exprision xs (Number 0)
232 get_exprision ((Optl):xs)      lst = get_exprision rest nlst
233                                where
234                                nlst = Unaryop Tail ex
235                                (ex, rest) = get_exprision xs (Number 0)
236 get_exprision ((Recrell a b):xs) lst = get_exprision rest nlst
237                                where
238                                nlst = Application (Recrelname (a++"_"+b))
239
240         args
241 get_exprision ((Intfunc a):xs)  lst = get_exprision r1 nlst
242                                where
243                                nlst = Localapplication (Localfuncnamedef a)
244                                args
245 get_exprision (x:xs)           lst = (args, r1) = (get_eargs xs [])
246                                = (oper, rest), if is_op (x:xs)
247                                = (lst, (x:xs)), otherwise
248                                where
249                                oper = Binaryop lst (find_op x) nex
250                                (nex, rest) = get_exprision xs (Number 0)
251
252 find_list x = (list, rest)
253             where
254             list = get_li inl [] []
255             (inl, rest) = list_splitter x []
256
257 list_splitter (Rlbkt:xs) inl = (inl, xs)
258 list_splitter (x:xs)       inl = list_splitter xs (inl++[x])
259
260 get_li []                ce list = list
261 get_li (Concomma:xs) ce list = get_li xs [] nlist
262                                where
263                                nlist = list++[nnitem]
264                                nnitem = Listitem nitem
265                                (nitem, none) = get_exprision ce (Number 0)
266 get_li (x:xs)           ce list = get_li xs (ce++[x]) list
267
268
269 find_inbr (Rbckt:xs) inbrk = (inbrk, xs)
270 find_inbr (x:xs)       inbrk = find_inbr xs (inbrk++[x])
271
272
273 get_eargs (Rbckt:xs) args = (args, xs)
274 get_eargs x               args = get_eargs rest nargs
275                                where
276                                nargs = args ++[exp]
277                                (exp, rest) = get_exprision x (Number 0)
278
279 is_op [] = False
280 is_op (Opplus:xs) = True
281 is_op (Opmin:xs) = True
282 is_op (Optim:xs) = True
283 is_op (Opdiv:xs) = True
284 is_op (Oplt:xs) = True
285 is_op (Opgt:xs) = True
286 is_op (Opequ:xs) = True
287 is_op (Opne:xs) = True

```

```

288 is_op (Oplet:xs) = True
289 is_op (Opget:xs) = True
290 is_op (Opcon:xs) = True
291 is_op (Opbng:xs) = True
292 is_op x          = False
293
294
295
296 find_op Opplus = Plus
297 find_op Opmin  = Minus
298 find_op Optim  = Times
299 find_op Opdiv  = Divide
300 find_op Oplt   = Lessthan
301 find_op Opgt   = Greaterthan
302 find_op Opequ  = Equals
303 find_op Opne   = Notequals
304 find_op Oplet  = Lessthanorequalto
305 find_op Opget  = Greaterthanorequalto
306 find_op Opcon  = Cons
307 find_op Opbng  = Index
308
309
310
311 || first step translation
312
313
314 trans_step1 (Simulation mblock iblock wblock) = Simulation nmblok iblock nwblk
315                                           where
316                                           nwblk = add_wrapper cwblk
317                                           cwblk = add_callsw wblock
318                                           nmblok = add_callsm mblock
319
320 add_callsm (Main a) = Main na
321                   where
322                   na = add_expcall na
323
324 add_expcall (SimpleExpression a) = SimpleExpression na
325                               where
326                               na = addcalls_exprions a
327 add_expcall (WhereExpression a b) = WhereExpression na nb
328                               where
329                               na = addcalls_exprions a
330                               nb = addcalls_wl b
331
332 addcalls_wl (Whereblock defs) = Whereblock (adcalssdfs defs [])
333
334
335
336 adcalssdfs [] ndefs = ndefs
337 adcalssdfs ((Localvaldef vn exp):xs) ndefs = adcalssdfs xs nndfs
338                                           where
339                                           nndfs = ndefs++[nitem]
340                                           nitem = Localvaldef vn (add_expcall exp
341                                           )
342 adcalssdfs ((Localfuncdef vn arg exp):xs) ndefs = adcalssdfs xs nndfs
343                                           where
344                                           nndfs = ndefs++[nitem]
345                                           nitem = Localfuncdef vn arg (
346                                           add_expcall exp)
347
348
349 addcalls_exprions (Conditional a b c) = Conditional new_a new_b new_c

```



```

350         where
351         new_a = addcalls_exprions a
352         new_b = addcalls_exprions b
353         new_c = addcalls_exprions c
354 addcalls_exprions (Bracketed a) = Bracketed new_a
355         where
356         new_a = addcalls_exprions a
357 addcalls_exprions (List a) = List na
358         where
359         na = addcallslist a []
360 addcalls_exprions (Binaryop a b c) = Binaryop new_a b new_c
361         where
362         new_a = addcalls_exprions a
363         new_c = addcalls_exprions c
364 addcalls_exprions (Localapplication a b) = Localapplication a b
365 addcalls_exprions (Application a c) = Binaryop wrap Index fstarg
366         where
367         wrap = Application newb newc
368         newb = changename a
369         newc = tl (c)
370         fstarg = hd (c)
371 addcalls_exprions (Valname a) = Valname a
372 addcalls_exprions (Unaryop a b) = Unaryop a new_b
373         where
374         new_b = addcalls_exprions b
375 addcalls_exprions (Number a) = Number a
376
377
378
379
380 changename (Recrelname a) = Recrelname (a++"_list")
381
382 addcallslist [] list = list
383 addcallslist ((Listitem exp):xs) list = addcallslist xs (list++[(Listitem (
384     addcalls_exprions exp))])
385
386
387
388 add_callsw (Where x) = Where nx
389         where
390         nx = wlac x []
391
392 wlac [] list = list
393 wlac ((Recreldef a b c d):xs) list = wlac xs nlist
394         where
395         nlist = list++[(Recreldef a b c nd)]
396         nd = add_expcall d
397
398 add_wrapper (Where x) = Where nx
399         where
400         nx = adfunwraps x []
401
402 adfunwraps [] list = list
403 adfunwraps (x:xs) list = adfunwraps xs (list++[(addwrap x)])
404
405 addwrap (Recreldef a b c d) = Recreldef a nb [] ne
406         where
407         nb = chngnme b
408         ne = makewrapexp a b c d
409
410 chngnme (Funcnamedef a) = Funcnamedef (a++"_list")
411
412 makewrapexp a b c d = WhereExpression exp defs

```

```

413             where
414             exp = Localapplication (Localfuncnamedef "_createlist") [
Application nname [(Number 0)]
415             nname = createname a b
416             defs = Whereblock (mdeflist nname c d)
417 createname (Agentnamedef a) (Funcnamedef b) = Recrelname (a++b)
419
420
421
422 mdeflist a    b c = [item1, item2]
423             where
424             item1 = Localfuncdef (Localfuncnamedef "_createlist") [(
Argnamedef "f"), (Argnamedef "t")] exp1
425             exp1 = SimpleExpression lv1
426             lv1  = (Binaryop lv2 Cons lv3)
427             lv2 = (Localapplication (Localfuncnamedef "f") ([Valname "t"]))
428             lv3 = (Localapplication (Localfuncnamedef "_createlist") [
Localapplication (Localfuncnamedef "f") [Binaryop (Valname "t") Plus (Number 1)
429             ]])
430             item2 = (Localfuncdef (Localfuncnamedef (unpack a)) b c)
431
432 unpack (Recrelname a) = a

```

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