Presentation of written exercise 2

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02433 Hidden Markov Models

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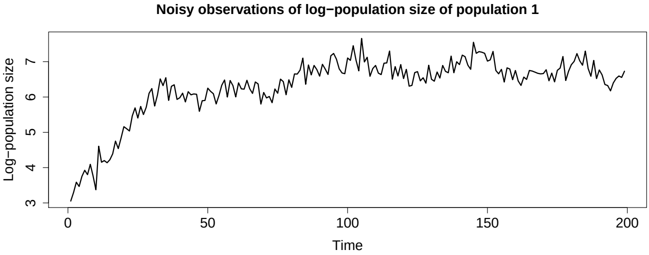
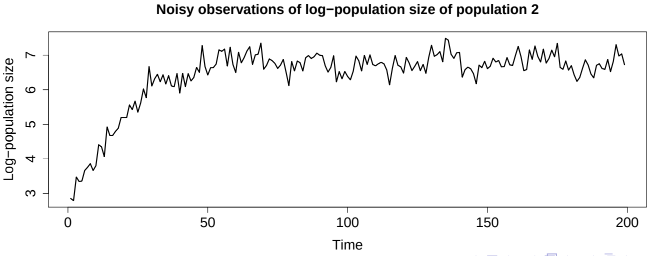
The data



Noisy observations of log−population size of population 1
Log−population size
3
4
5
6
7
0
50
100
150
200
Time



Noisy observations of log−population size of population 2
Log−population size
3
4
5
6
7
0
50
100
150
200
Time

The Model

Non-linear state space model.

$$P_t = P_{t-1} + r_0 \left(1 - \left[\frac{\exp\left(P_{t-1}\right)}{K} \right]^{\theta} \right) + e_t$$
$$X_t = P_t + u_t$$

- P_t is log-population size. $e_t \sim N(0, Q)$ and $u_t \sim N(0, R)$ are iid.
- The parameter vector is $\boldsymbol{\lambda} = (\theta, r_0, K, Q, R)$ and all parameters are assumed positive.

Discretization of model

- Assume the state P_t is bounded on an interval $[a_0, a_1]$
- Partition $[a, b]$ into m intervals $\Omega_i = (b_{i-1}, b_i)$
- The width of each interval is then $w = (a_1 - a_0)/m$
- And the boundaries are $b_i = a_0 + wi$
- Introduce discrete, integer valued state-space variables C_t.
- If $P_t \in \Omega_i$ then $C_t = i$.
- To link P_t and C_t the discrete state i is represented by the midpoint $p_i = a_0 + w(i - 0.5)$

Discretization of model continued

- The state dependent distribution is then given by

$$X_t \,|\, C_t = i \;\sim\; N(p_i, R)$$

- And the transition probabilities as

$$\begin{aligned} P(C_t = j \,|\, C_{t-1} = i) &= \int_{\Omega_j} n(p_t, \mu_i, Q)\, dp_t \\ &\approx \frac{w}{2}\left(n(b_{j-1}, \mu_i, Q) + n(b_j, \mu_i, Q)\right) \end{aligned}$$

where $\mu_i = p_i + r_0 \left(1 - \left[\frac{\exp(p_i)}{K}\right]^{\theta}\right)$

Computing the likelihood

The markov chain is not assumed stationary so instead

$$\phi_1 = \frac{\mathbf{1}\boldsymbol{P}(x_1)}{\mathbf{1}\boldsymbol{P}(x_1)\mathbf{1}'}$$

is used as the initial distribution.
Since all parameters $\boldsymbol{\lambda} = (\theta, r_0, K, Q, R)$ are assumed positive they can easily be transformed to unconstrained working parameters by

$$\boldsymbol{\tau} = \log \boldsymbol{\lambda} \quad \text{and back} \quad \boldsymbol{\lambda} = \exp \boldsymbol{\tau}$$

Parameter estimates

Parameter estimates for dataset 1

	Estimate	95% Conf.Int		Std.Dev
θ	0.4615	-0.3769	1.3000	0.4278
r_0	0.1423	-0.0315	0.3160	0.0886
K	822.9574	631.7430	1014.1718	97.5584
Q	0.0090	0.0037	0.0144	0.0027
R	0.0407	0.0304	0.0510	0.0053

Is it sensible to assume that θ and/or r_0 is 0?

Is it sensible?

The model is then

$$P_t = P_{t-1} + e_t$$
$$X_t = P_t + u_t$$

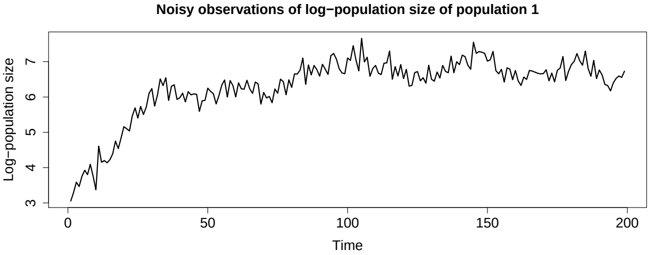
which gives

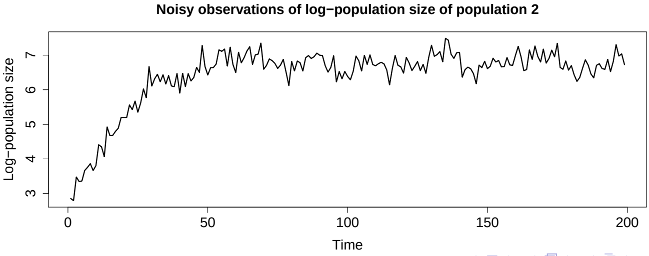
$$\begin{aligned} X_t &= X_{t-1} + P_t - P_{t-1} + u_t - u_{t-1} \\ &= X_{t-1} + e_t + u_t - u_{t-1} \end{aligned}$$

and since $\epsilon_t = e_t + u_t - u_{t-1} \sim N(0, 2R + Q)$ is just white noise, we get the random walk process

$$X_t = X_{t-1} + \epsilon_t$$

Is it sensible?





Correlation of estimates

The model

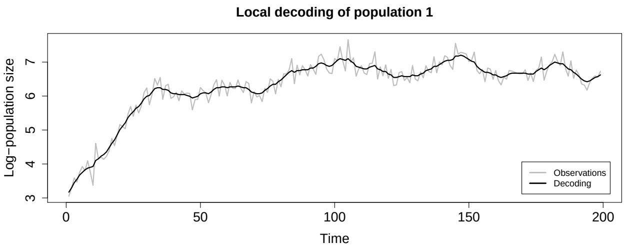
$$P_t = P_{t-1} + r_0 \left(1 - \left[\frac{\exp\left(P_{t-1}\right)}{K} \right]^{\theta} \right) + e_t$$

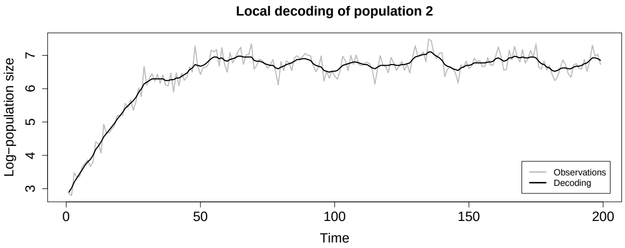
$$X_t = P_t + u_t$$

Correlation of parameter estimates

$$\mathrm{Corr}[\widehat{\theta}, \widehat{r_0}] = -0.954,\ \mathrm{Corr}[\widehat{Q}, \widehat{R}] = -0.315,\ \mathrm{Corr}[\widehat{\theta}, \widehat{Q}] = 0.247$$

Local decoding





Questions

Time for some questions...