Presentation of written exercise 1

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02433 Hidden Markov Models

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Quick remarks

- I forgot to put a hat on the parameter estimates. The estimate of $\boldsymbol{\Gamma}$ should have been denoted $\widehat{\boldsymbol{\Gamma}}$ etc.

Dataset for the exercise

Count data so Poisson seems natural. But $\bar{x} = 5.44$ and $s^2 = 15.40$, so the data is overdispersed. Also the data is serially correlated so a simple Poisson mixture will not work. HMM to the rescue.

Poisson HMM

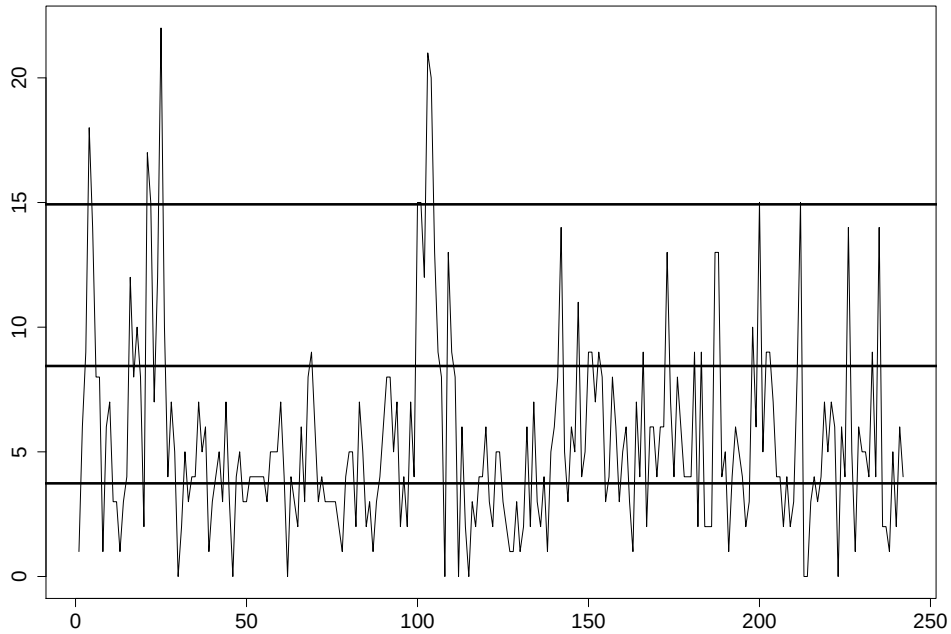
2-, 3- and 4-state Poisson HMM was fitted to the data.

	μ	σ^2	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ρ_7
Sample	5.442	15.401	0.392	0.250	0.178	0.136	0.038	0.044	0.052
2-states	5.429	13.784	0.329	0.178	0.097	0.052	0.028	0.015	0.008
3-states	5.421	14.721	0.407	0.268	0.178	0.120	0.081	0.055	0.037
4-states	5.418	14.779	0.380	0.241	0.157	0.103	0.067	0.044	0.029

	AIC	BIC
2-states	1245.337	1259.292
3-states	1239.043	1270.444
4-states	1240.528	1296.351

The 3-state model has the best AIC and the 2-state model has the best BIC. Better correspondance between the sample correlations and the correlations for the 3-state model makes the 3-state model the winner.

The 3-state model

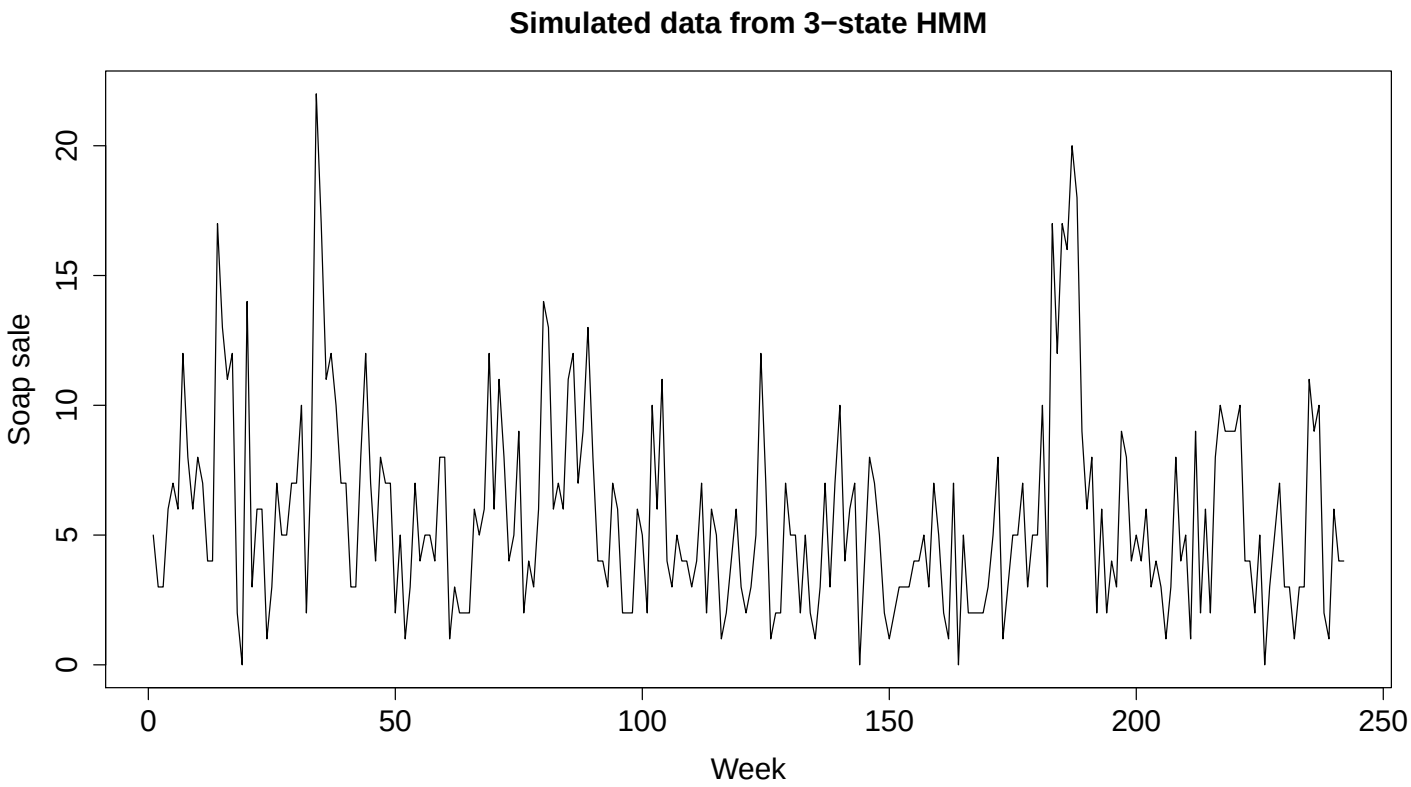


$$\boldsymbol{\Gamma} = \begin{pmatrix} 0.864 & 0.117 & 0.019 \\ 0.445 & 0.538 & 0.017 \\ 0.000 & 0.298 & 0.702 \end{pmatrix} \qquad \boldsymbol{\lambda} = \begin{pmatrix} 3.736 & 8.443 & 14.927 \end{pmatrix}$$

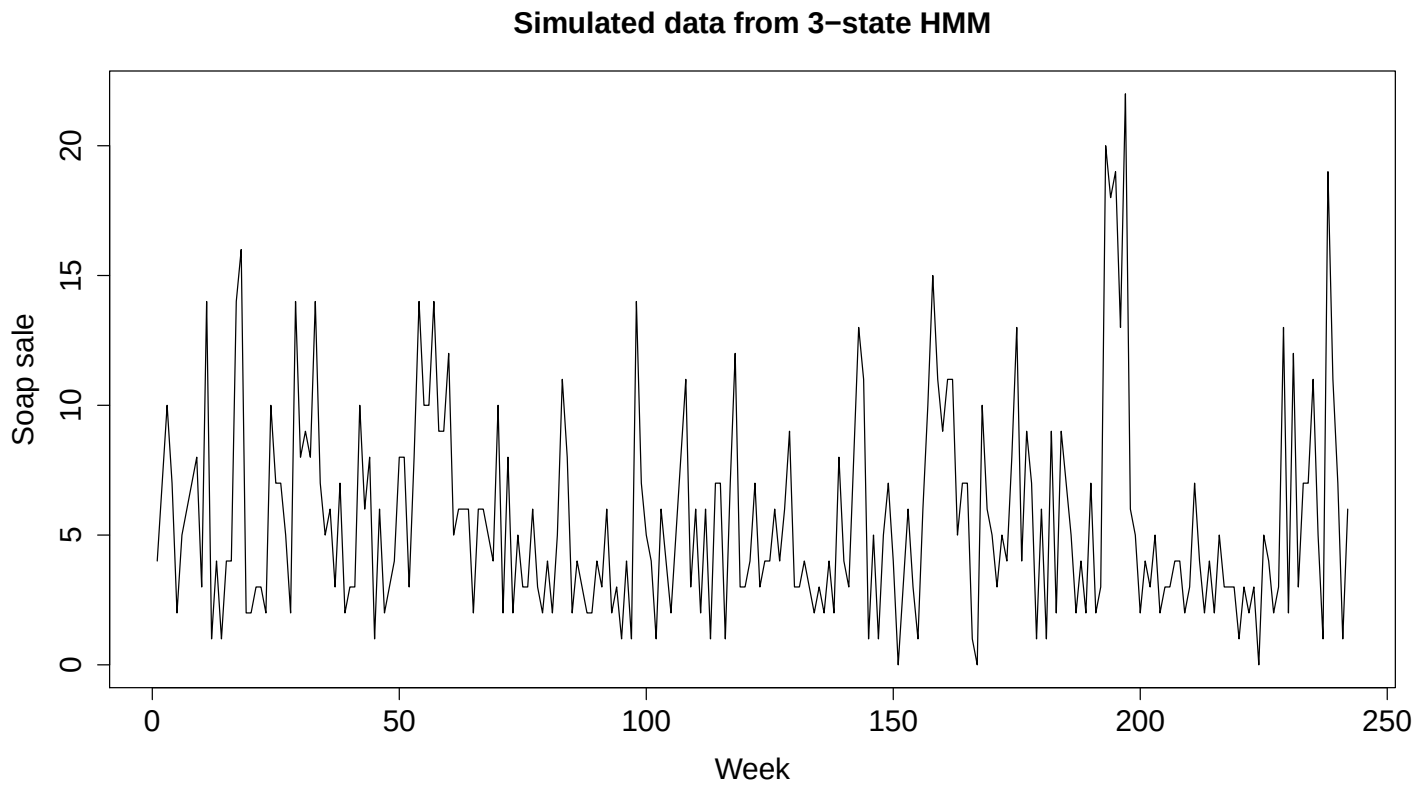
$$\boldsymbol{\delta} = \begin{pmatrix} 0.722 & 0.220 & 0.058 \end{pmatrix}$$

and still no hats on the parameter estimates

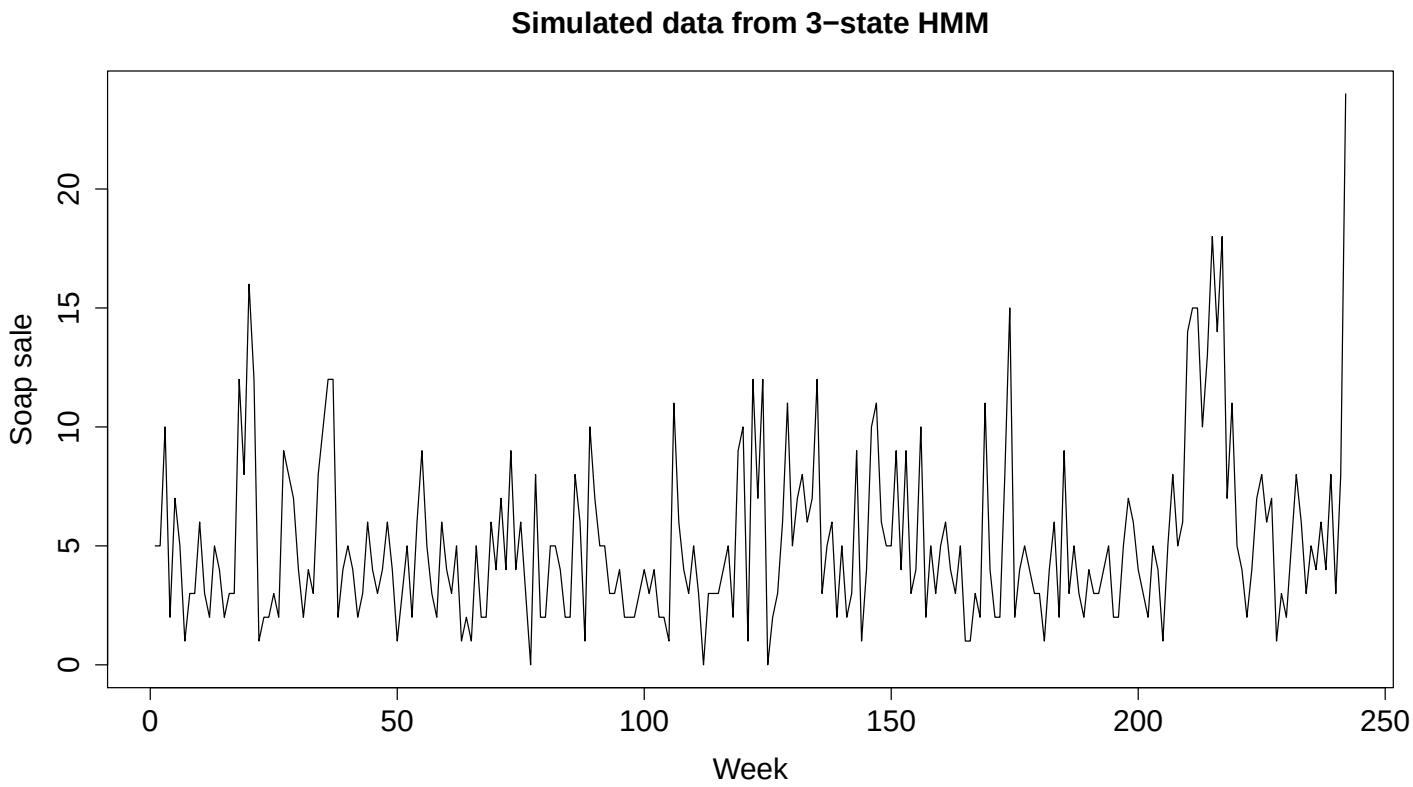
Model check


Simulated data from 3−state HMM
Soap sale
0
5
10
15
20
0
50
100
150
200
250
Week

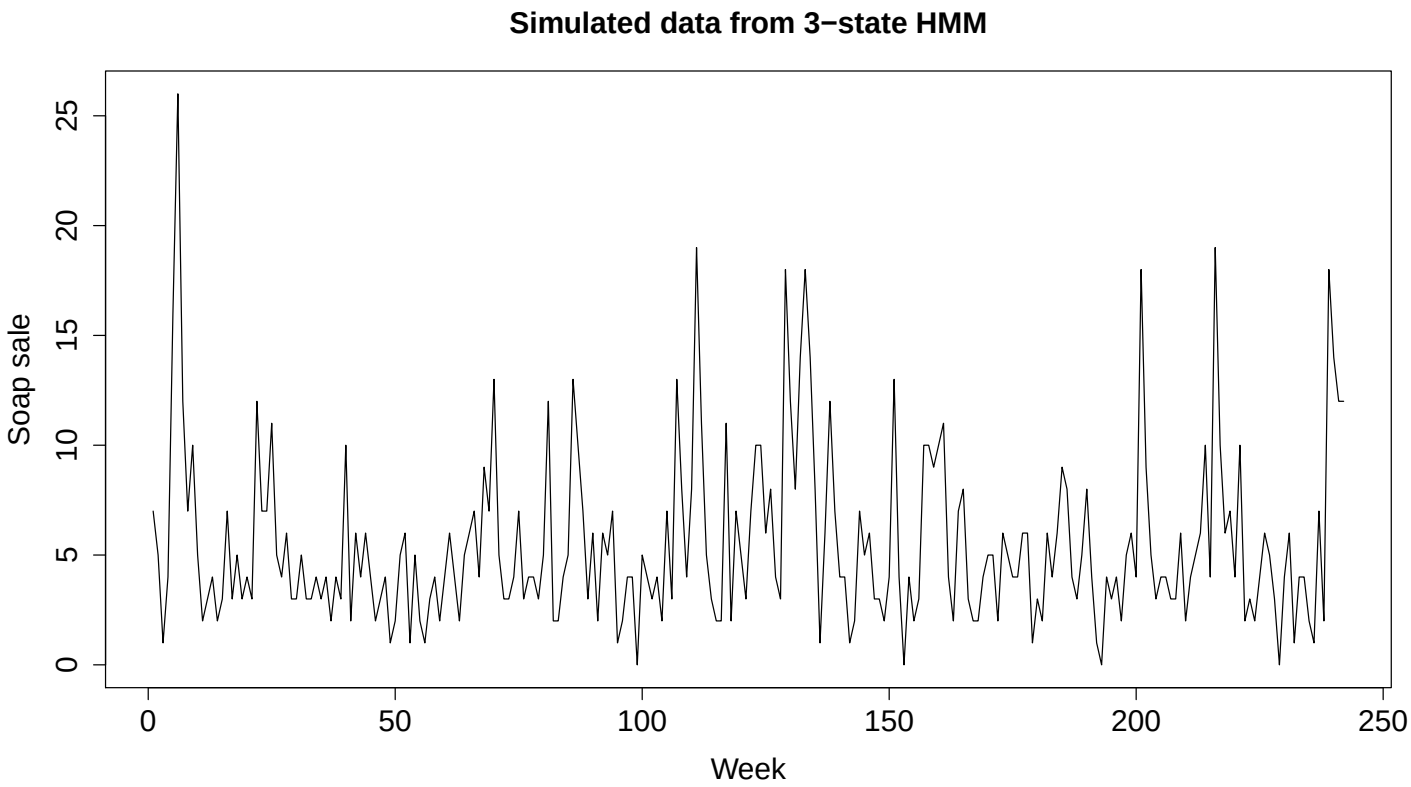
Model check


Simulated data from 3−state HMM
Soap sale
0
5
10
15
20
0
50
100
150
200
250
Week

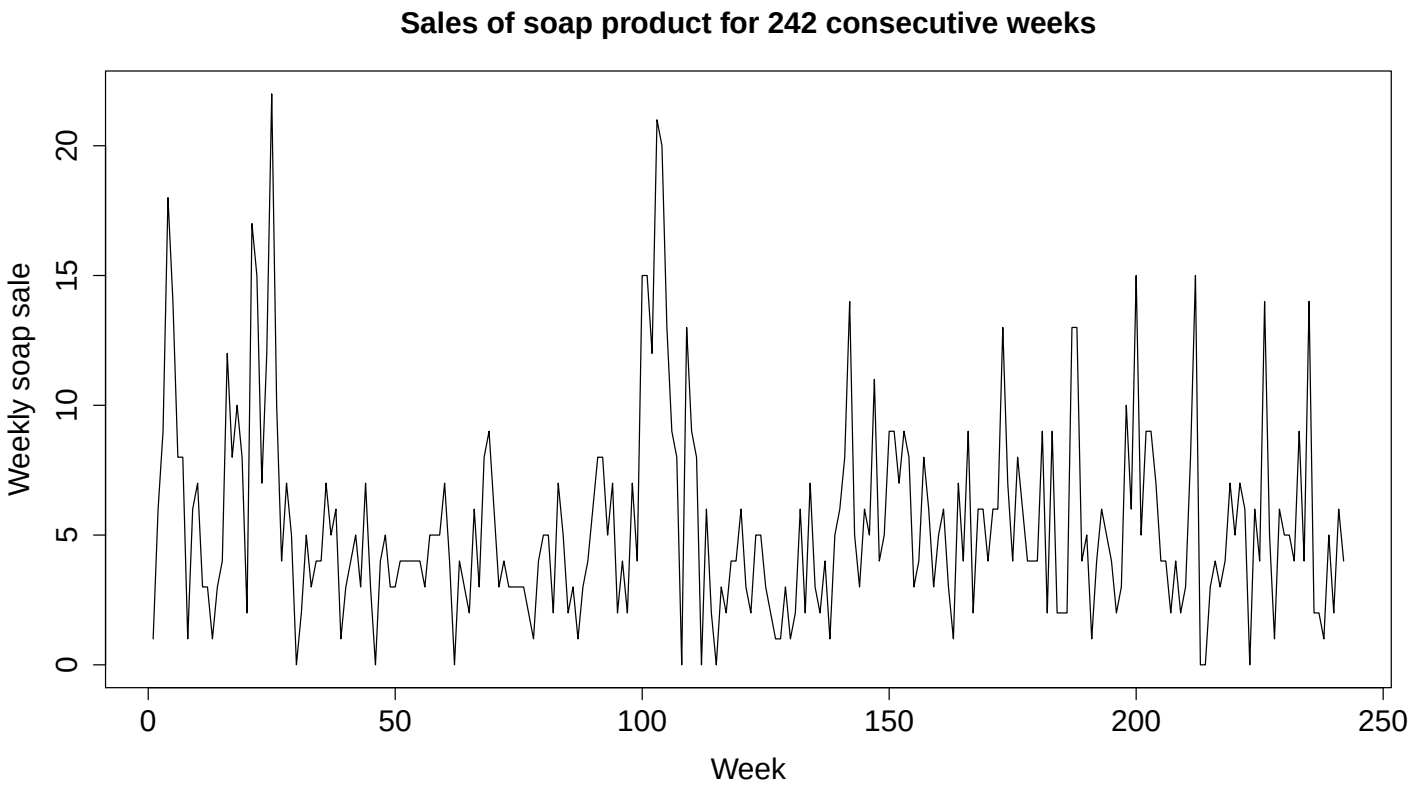
Model check


Simulated data from 3−state HMM
Soap sale
0
5
10
15
20
0
50
100
150
200
250
Week

Model check


Simulated data from 3−state HMM
Soap sale
0
5
10
15
20
25
0
50
100
150
200
250
Week

Model check


Sales of soap product for 242 consecutive weeks
Weekly soap sale
0
5
10
15
20
0
50
100
150
200
250
Week

Direct maximmization vs EM

Comparing direct maximization (DM) vs EM was the main focus of the exercise

- Similar parameter estimates was obtained
- The likelihood for the EM algorithm was a bit higher as expected, since stationarity was assumed for DM but not for EM
- In principle EM solves a harder problem, but $\boldsymbol{\delta}_0$ can be shown to be a unit vector, and m different simpler maximizations can be performed.
- For some initial parameter values the `nlm` function gave `NA` values when using the DM method.

Direct maximmization vs EM performance

- In report it is concluded that no significant difference in performance is noticed. This was a qualitative remark.
- In section 4.4 in the course text book the DM method is mentioned to converge faster than EM.
- Let's try to time the performance of the two methods using the `system.time` function in R.

Comparing performance of Direct Maximization vs EM

Runtime for the two algorithms. Each run is done with 9 different initial values.

Direct maximization

```
   user  system elapsed
  3.732   0.000   3.740
```

```
   user  system elapsed
 28.602   0.136  28.763
```

```
   user  system elapsed
 60.595   0.092  60.747
```

EM algorithm

```
   user  system elapsed
  3.784   0.008   3.795
```

```
   user  system elapsed
 20.326   0.020  20.363
```

```
   user  system elapsed
 26.138   0.076  26.234
```

Conclusion?

- Large difference in performance for 3-state and especially 4-state models.
- EM is seen to perform much better than DM for 4-states.
- Opposite of the remarks in section 4.4 in Zucchini 09

Questions

Time for some questions...