Presentation of written exercise 3

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02433 Hidden Markov Models

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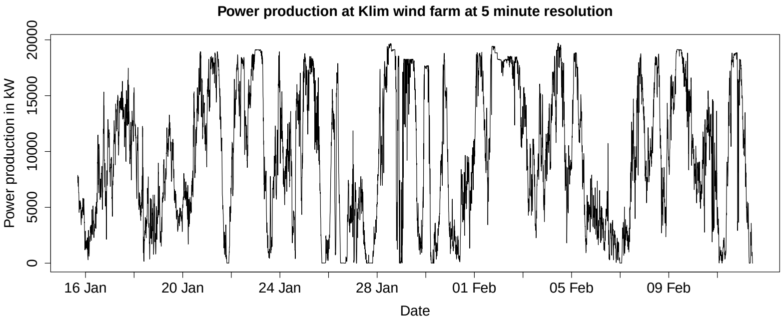
A few remarks about the report

- The notation for random variables is inconsistent. When discussing exponential smoothing and ARIMA y_t is used. When discussing Hidden Markov Models Y_t is used.
- In the tables with parameter estimates, the indices of the transition probabilities are interchanged. So γ_{ij} is really γ_{ji} for all i, j.

The data



- Measurements of power production at Klim wind farm
- Data is bounded on the interval $[0, 21000]$
- First 8000 measurements used as training data. 2000 used for performance evaluation.

The task

The problem description was open ended. In short form the task was to analyse the data, using various methods for time series analysis.

Exponential smoothing

Using the framework presented in (Hyndman08[1]), a simple exponential smoothing was found to best forecast the data. The smoothing constant was found as $\alpha = 0.9999 \approx 1$, which gives the model

$$Y_t = Y_{t-1} + e_t$$

which is just a random walk. The one-step prediction is $\widehat{Y}_{t|t-1} = Y_{t-1}$ and based on that, the RMSE of the forecasts on the testset was

$$R_{\text{RandomWalk}} = 921.33$$

[1] *Forecasting with Exponential Smoothing - The State Space Approach*, Rob Hyndman et. al.

ARIMA

Using the ACF and PACF estimates the best ARIMA model was found to be an ARIMA(1,1,2) model. Estimating the parameters gave the model

$$\nabla Y_t = 0.80 \nabla Y_{t-1} - 0.61 e_{t-1} - 0.21 e_{t-2} + e_t$$

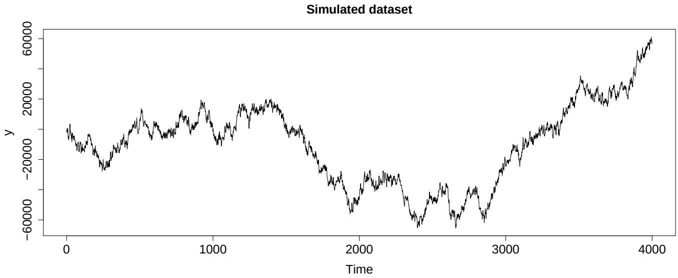
One step predictions were calculated and the RMSE was found to be

$$R_{\text{ARIMA}(1,1,2)} = 874.73$$

A large improvement compared with the exponential smoothing forecasts.

ARIMA model check

To make an emperical check of the found ARIMA model, a few datasets were simulated from the found model.



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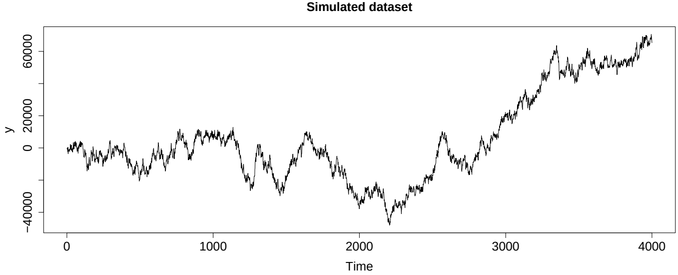

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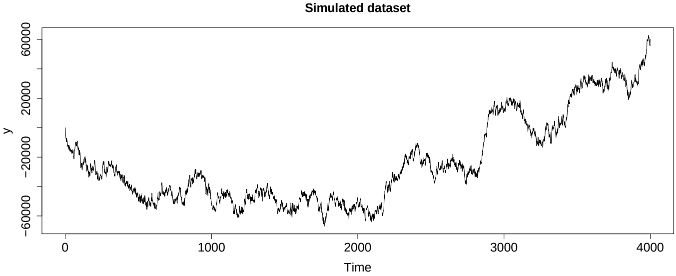

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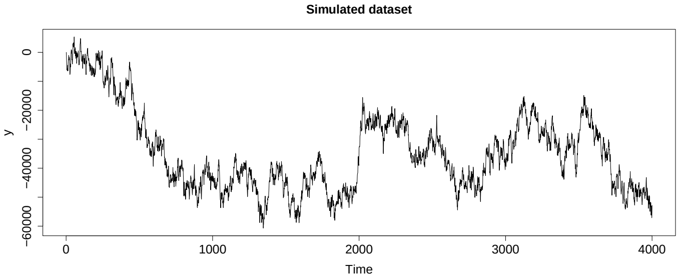

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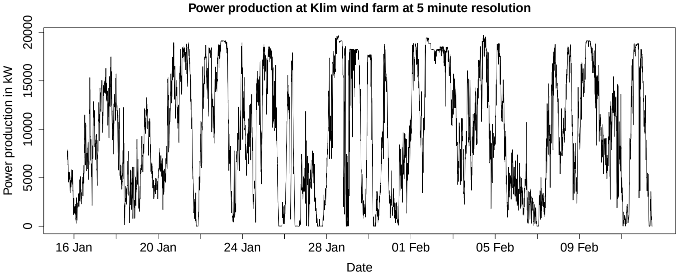
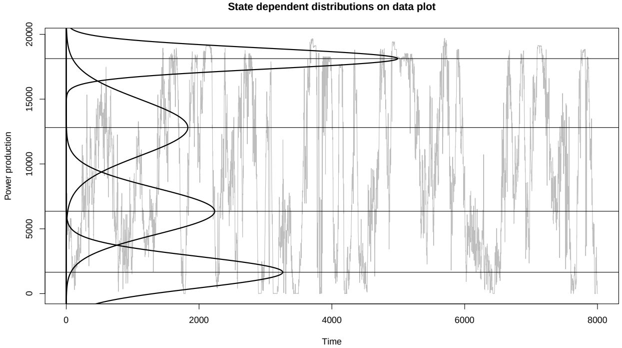


Hidden Markov Models

2-, 3-, and 4-state HMM were fitted to the data. The state dependent distributions as chosen as

$$Y_t \mid C_t = i \;\sim\; N(\mu_i, \sigma_i^2)$$

The best found model was the 4-state model with the state dependent distributions



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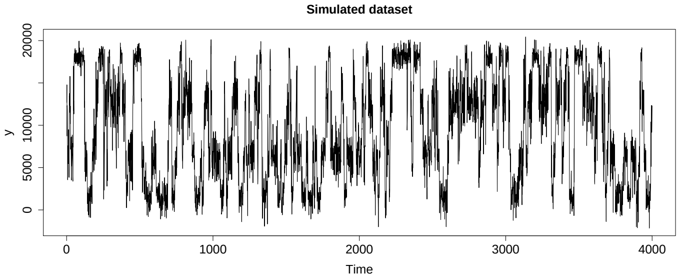

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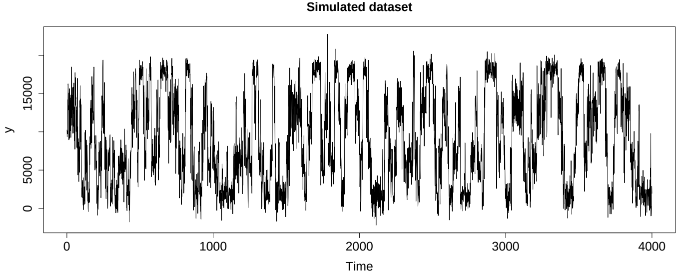

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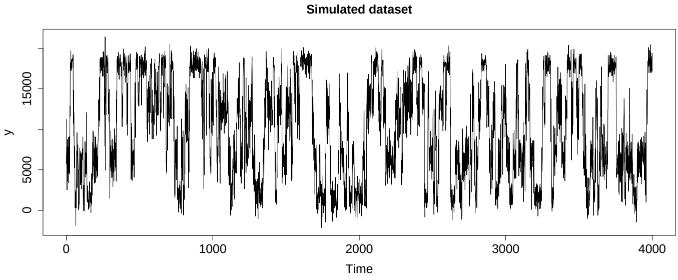

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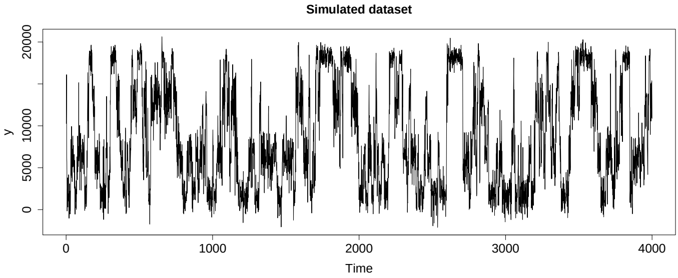

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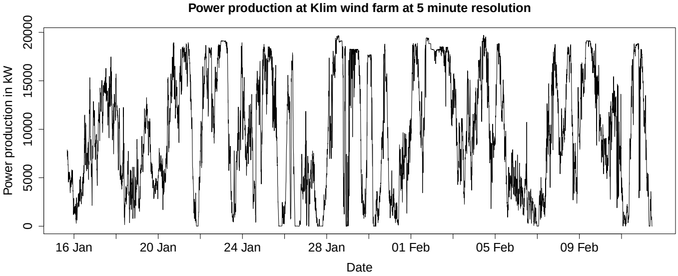


One-step predictions

Had troubles with all one-step prediction distributions being almost identical. Maybe it can be explained by the fact that

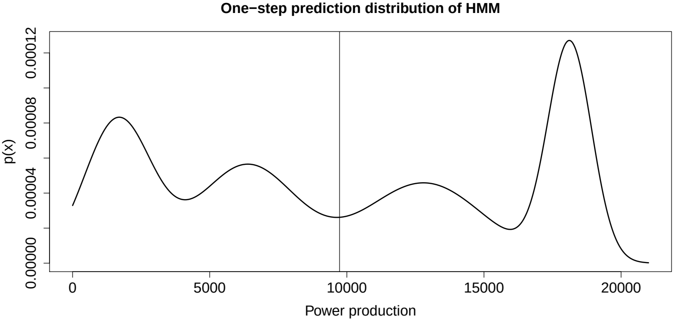
$$\Pr(Y_{t+1} = y \mid \boldsymbol{Y}^{(t)} = \boldsymbol{y}^{(t)}) = \boldsymbol{\phi}_t \boldsymbol{\Gamma} \boldsymbol{P}(y) \mathbf{1}'$$

where

$$\boldsymbol{\phi}_t = \frac{\boldsymbol{\alpha}_t}{\boldsymbol{\alpha}_t \mathbf{1}'}$$

If eg. the absolute value of the elements in $\boldsymbol{\alpha}_t$ are of the order 70000, then differences between elements in $\boldsymbol{\alpha}_t$ and $\boldsymbol{\alpha}_{t+1}$ of the order 100 will not change $\boldsymbol{\phi}_t$ much.

One-step predictions continued



Also $\phi_t \approx \boldsymbol{\delta}$, so the prediction distributions was almost the marginal distribution of Y_t.

Questions

Time for some questions...