What is Value?

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Abstract

The work of Jules Regnault, Francis Galton, John Rae and Vilfredo Pareto covered Duration, Behavior, and Value. Regnault talked about stock market science, statistical nature of Value, duration importance and price behavior. Galton laid the foundation for the robust behavior of Reversion in natural phenomenon. Rae introduced the idea of intertemporal choices which showcased time inconsistency in human behavior and Pareto talked about another robust behavior now known to be ubiquitous in natural systems as a power law. Duration, Behavior, and Value are inseparable. Statistical behavior of natural systems (e.g. stock markets) expresses themselves durationally. And because stock market systems exhibit uncertainty and order, this creates inconsistencies (anomalies). Instead of acknowledging the statistical behavior of stock market systems, a few generations of researchers have focussed on explaining these inconsistencies through behavioral biases, leading to a polarized debate around efficiency and inefficiency of markets.

This debate has many casualties, one of the key being the global investor and how he(she) understands Value. If stock market systems function statistically, value creation and its transformation into growth are statistical phenomena rather than driven by fundamental, psychological or economic factors. Value is misunderstood by the global investor. Despite the fact that Value stocks move from an inexpensive state to expensive state while Growth underperforms and drags in performance over the longer durations moving from an expensive state to a less expensive state, Value and Growth are interpreted as disconnected ideas which are assumed to be only defined fundamentally. This narrow definition of Value has added to the academic confusion around inconsistencies and created an investing style bias.

Investing styles are at the heart of the investment business, which brings along with it new factors like ‘Size’. These various factors overlap with each other. On occasions, it has been even seen that the factors are a proxy for each other. This raises the question regarding the theoretical foundation driving the respective factors. If Value can be explained statistically, it will also explain factors like Growth, Size, Momentum and other factors and hence bring in a needed clarity of how markets function and whether there is a universal factor that drives stock market systems.

1697-1790 Dojma to American Stock Exchange
1834-1886 Value, Behavior, Duration is Statistical
1920-1934 Value is Fundamental and Cheap
1952 Linear Assumptions
1958 Is Capital Structure Relevant or Irrelevant?
1962 The Non-Linear Model
1964 Divergence of Risk from Return

1966 Outperformance
1977 - 2001 The Size Proxy
1988-2015 Is Reversion Statistical?
2000 - Power Law Criticism and Ising Model
2006-2014 The Duration Factor
2007 Divergence Cyclicality
2010, Mean Reversion Framework
2015 Reversion and Diversion Hypothesis
2015 Is Smart Beta Dumb?
2016 Conclusion

1697- 1790 Dojma to American Stock Exchange

The Dōjima Rice Exchange was established in 1697.The federal government issued $80 million in bonds to repay Revolutionary War debt, marking the birth of the U.S. investment markets. Two years later, 24 stockbrokers sign the “Buttonwood Agreement” and eventually move to the Tontine Coffee House to trade. It was in 1750’s the first Axe-Houghton Index was formed followed by the Dow Industrial and then the S&P500. Passive investing came much later after the society came to terms with depression and started understanding the benefits of diversification. The first Index fund can be traced back to 1970. As of 2014 Index funds are estimated to drive nearly 20% of the mutual fund assets in the US. This focus and trend towards ETF growth and Index based passive investments are driven by investment styles like Value, Growth, Size, Momentum, Dividend etc. Now with a move towards sentiment analytics-driven investment funds, the debate on factors is creating more confusion rather than clarity.

1834-1886 Duration, Behavior, and Value

It is essential to revisit the history of research before we answer the questions about Value. The Value was defined statistically 100 years before value investing got a fundamental label. Though on one side going back in time to identify the roots of Value can help appreciate the contribution of various thinkers in refining big picture thinking, while on the other side one can also see that a piecemeal thinking creates confusion and distortion of historical facts. One can assume this misinterpretation to be unintentional as deciphering historical research required a multidisciplinary knowledge and in some cases even knowledge of various languages.The aim of this paper is to dig deeper into historical research and relook at those seminal ideas which formed the basis of the modern finance. Jules Regnault (1863), Francis Galton (1886), Vilfredo Pareto (1896) and John Rae (1834) were the first to lay down the statistical foundations for duration, behavior and value.

Duration, John Rae, 1834

Intertemporal choice is the study of how people make choices about what and how much to do at various points in time when choices at one time influence the possibilities available at other points in time. These choices are influenced by the relative value people assign to two or more payoffs at different points in time.

Behavior, Galton, 1884

Galton was the first to describe and explain the common phenomenon of regression toward the

mean, which he first observed in his experiments on the size of the seeds of successive generations of sweet peas.

Jules Regnault, 1863

Jules Augustin Frédéric Regnault was a French stock broker's assistant who first suggested a modern theory of stock price changes in Calcul des Chances et Philosophie de la Bourse (1863) and used a random walk model. He is also one of the first authors who tried to create a "stock exchange science" based on a statistical and probabilistic analysis. His hypotheses were used by Louis Bachelier.

Vilfredo Pareto, 1886

The Pareto distribution, named after the Italian civil engineer, economist, and sociologist Vilfredo Pareto, is a power law probability distribution that is used in the description of social, scientific, geophysical, actuarial, and many other types of observable phenomena.

Galton and Regnault

Galton though a contemporary of Regnault was more focussed on the behavior towards mean (mediocrity) in various systems different than stock markets. Regnault's experience in markets assisted him in seeing stock market systems and the science in them. Both Regnault and Galton talked about mean and the variance around it. For Jules Value was connected to the idea of statistical reversion, duration relevance, and random walk. Markets according to Regnault were about reversion to true Value. Regnault's work laid the basis for the random walk for Louis Bachelier. Both the ideas involving random walk and reversion to mean are an integral part of modern finance.

Regnault, Bachelier and Einstein

Bell Curve history starts from, 'The doctrine of chances', Abraham de Moivre in 1738. This was followed by Carl Friedrich Gauss's, 'Theoria motus corporum coelestium in sectionibus conicis solem ambientium' in 1809. Although Gauss was the first to suggest the normal distribution law, Pierre - Simon Laplace made significant contributions in his 'Mémoire sur la probabilité des causes par les événements, in 1774. And it was only in 1860 that James Clerk Maxwell, in 'Illustrations of the dynamical theory of gases' illustrated that normal distribution is not just a convenient mathematical tool, but may also occur in natural phenomena. It was Francis Galton, who illustrated the idea in 'Regression towards mediocrity in hereditary stature' in 1886. Louis Jean-Baptiste Alphonse Bachelier, The Theory of Speculation, 1900 was the first to model the stochastic process now called the brownian motion, which is normally distributed.

Does God practice a random walk? The 'financial physics' of a nineteenth-century forerunner, Jules Regnault, Franck Jovanovic and Philippe Le Gall, 2001

"*Bachelier had anticipated many of the mathematical results developed in Albert Einstein's 1905 paper' (LeRoy 1989: 1587). Random processes happen to have been first considered in the context of economics, in 1900, by Louis Bachelier' (Mandelbrot 1966: 243)…Though his contributions were ignored for sixty years, the first statement and test of the random walk model was that of Bachelier' (Fama 1970: 389)."*

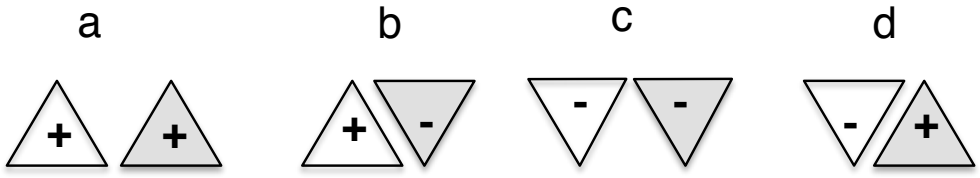
Calcul des Chances et Philosophie de la Bourse (1863)

"…we suspect that such an identification of the origin of the random walk hypothesis in finance, and consequently of the modern approaches to capital markets, is historically imperfect and mistaken. In this paper, we indeed claim that in his book, Jules Regnault, laid the basis of modern stochastic models of price behavior."

It is not the first time that acknowledgments have alluded many historical thinkers. There was 60 years delayed acknowledgment of Bachelier's work, which was based on Regnault's earlier thoughts on Random Walk. It was Jovanovic and Gall who analyzed the work of Jules Regnault in 2001.

Why is Regnault's work important to understand 'Value'? Regnault's contribution along with Rae, Pareto and Galton form the foundation of understanding randomness, behavior, and their statistical relationship. Though Mandelbrot was the one who acknowledged Bachelier for his contributions on Random Walk, he challenged the Bell Curve as an expression in stock markets, "Bell Curve is Nonsense" in The (Mis)Behaviour of Markets: A Fractal View of Risk, Ruin, and Reward. Mandelbrot was a proponent of Pareto distributions. The exaggeration of trends and failure of trends to revert have come to be seen as the predominance of Pareto in stock markets. Because reversion phenomenon was so widely accepted, the appearance of exaggerated trends came to be seen as a failure of mean reversion as a robust law. This fueled the argument for Pareto believers who came to see the statistical distribution as a challenger to Galtonian reversion, even coming to suggest it as irrelevant for natural systems.

In, Is smart beta dumb?, Pal, 2015, the author explained how Pareto and Galton together defined the framework for natural systems. Vilfredo Pareto, father of microeconomics created the Pareto curve explaining wealth distribution in Italy. The popular rebranded 80-20 law suggests that "80% of effects come from 20% of the causes". In stock market terms it means, "Winners and losers persist", "Momentum is a natural continuum". Pareto is also referred to as the Power law, the behavior observed in stock markets and also used to make a case against the normally distributed bell curve. Though the bell curve has been challenged as the natural expression for stock markets and Pareto has been stated to be the real expression, markets express both Pareto and Galton. As markets express the four states (a,b,c,d); Past winners tend to lose, past winners tend to win, past losers tend to lose, past losers tend to win. Momentum persists and fails just like reversion. Both these diametrically opposite observances may seem incomplete, but they do occur often at the same time. This durational aspect links back to John Rae and his work on behavior and inter-temporal choices.



Behavioral Finance acknowledges Reversion in many popular research papers, "Does stock market overreact?" DeBondt and Thaler (1985)

Scientific and Deterministic

According to Regnault, the scientific and deterministic were synonymous and inseparably connected. The deviation, noise, the error was an intrinsic part of the scientific determination.

Le Jardin au Noyer. Pour un Nouveau Rationalisme. Paris: Le Seuil, Jovanovic, Franck (2000)

"..great part of Bachelier's reasoning came directly from Regnault most importantly the random walk hypothesis….Regnault undoubtedly followed Quételet's program of social physics, designed to form the foundation for an exact science of human societies, and approached 'scientifically' the issue of speculation. His work tried to show that short-term speculation based on the search for immediate gains leads to ruin but that, by contrast, another kind of speculation, based on the long-term gains, was socially useful…But his approach to economics was also certainly pioneering: as early as 1863, his work took the shape of theoretical and mathematical models that were however discussed in literary terms and, at a time when the application of statistics to economic affairs was controversial in France, some of these models were combined with statistical procedures…the construction of a 'scientific' approach to the stock market, Regnault followed a statistical and probabilistic path…His aim was the identification of 'new laws of the variations on the stock market'. It has to be noted that he saw his investigations as undoubtedly scientific his agenda was the construction of a 'Science of the stock market'."

"His aim was thus, in a scientific perspective, to separate two kinds of speculation: short-term speculation and long-term speculation…Regnault believed that the terms 'scientific' and 'deterministic' were synonymous and inseparably connected…good reasons to refer to Laplace in the early pages of his book. His fundamental postulate was that chance does not exist: 'in nature, nothing is arbitrary'. Nature was characterized by 'general and unchanging [immutables] laws', society was 'a huge machine made up of springs that are connected'."

Bounded Knowledge

Jovanovic and Gall, explain how Regnault was much ahead of his time. Here he talks about bounded knowledge, which is imperfect and approximated. Though Regnault talks about how neither the variations nor previous prices were useful to predict future variations, he also mentions how future inexorably originates in the past, just like the effect constantly originates in the cause. The knowledge about laws is an art of approximation and uncertainty arose because of errors and deviations from these laws. Putting it differently, Jules meant that laws are an expression of human approximation of nature and to take them on face value and not embrace them with their failures and weaknesses is bounded knowledge or ignorance. Suggesting that Galtonian Reversion is redundant and Pareto is the only law points to academic ignorance.

A conceptual history of the emergence of bounded rationality, Matthias Klaes, ESHET conference, Paris, 2003.

The idea of bounded rationality has roots in limited intelligence 1840, finite intelligence in 1880, and Herbert Simon 1957.

"…'nothing can happen without a previous cause' and that 'anytime, everywhere, the same causes produce the same effects'..second, the deterministic laws were only accessible to us in

a state of perfect knowledge: it would then be possible to discover what the future had in store for us. However, individuals have bounded knowledge and have to content themselves with an approximate knowledge of these laws. This imperfection of knowledge has two consequences. On the one hand, individuals can only try to tame the future through the elaboration of probabilities that are, of course, of a subjective nature: 'Chance does not exist, only our ignorance exists'. 'Our whole calculations are only based on our personal observation'. On the other hand, he emphasized that the underlying uncertainty favored errors and deviations: 'Ignorance, that . . . maintains our illusions and errors, is the first cause of our excesses, our passions, our misfortunes'."

Mean Value, Statistics, and Society

Here Jovanovic and Gall explain how Regnault explained 'Mean Value' as equilibrium in the social world, a sign of order, a deterministic law that governed human affairs. He connected societal behavior with a statistical mean.

"On the other hand, the mean value represented a harmonious equilibrium in the social world, the ideal in morals: it was the sign for an order that he supposed to prevail beneath…The mean was seen as the manifestation of the deterministic laws that rule natural and human affairs, and statistics was seen as a means to approximate determinism. Once these exact laws were discovered by scientists, the individuals could then reach 'a stable and quiet state'. Deviations should thus be reduced through a progress of civilization or of advancement in scientific knowledge."

Stock Markets Variations and Universal Laws

Here Jovanovic and Gall explain how Regnault expresses societal activities in the context of a multitude of unrelated events while still looking at the society as a part of a unified conception of nature, through time and space. For Regnault the moral world ruled by the same laws as the physical world, the reason he talked about stock market science explaining the fundamental wheels of nature.

"The variations of the Stock market are ruled by constant mathematical laws! Events that are generated by the caprices of men, the most unpredictable shocks of politics, the most cleverly studied financial combinations, the result of a multitude of events that are not related, all these effects are tied up in an admirable set, and chance is now a meaningless word.…A unified conception of the world: analogies and transfers at work another feature of Regnault's deterministic framework is to be found in the idea of reductionism: the various bodies that constitute the universe were seen as ruled by the same kinds of laws. A unified conception of nature means here that some laws were constantly at work, through time and through space, and at very different levels, and that various sciences and disciplines have to share principles, methods and also laws in common. It is well known that several social scientists of the nineteenth century and of the turn-of-the- century era developed similar ideas, and were consequently led to approach economics at the light of the natural sciences…that 'the moral world is ruled by the same laws as the physical world'. Regnault to ensure that his own work in the social field was connected to fundamental wheels of nature."

Short Term Speculation

Here the authors explain how Regnault considered speculation as short term, as moral with perfect equal chances to make profits. Speculators had no advantage as price behavior was a random walk. It was Regnault's work that formed the basis of a contemporary theory of informational efficiency of financial markets. Information as well as its interpretation were seen as two kinds of causes that generate the short-term variations of stock prices.

"Regnault aimed here at constructing a model of moral short-term speculation, i.e. at showing that speculation is moral because each agent has here perfectly equal chances to make profits 'at any moment, no advantage is existing for one possibility or for the other'. In such a case, the expected profit is zero for each operation and consequently he aimed at showing that the stock market should not be condemned per se. In a very pioneering way, price behavior took the shape of a random walk model although he never used the word….This was undoubtedly an important step in the construction of financial theory as well as in its theoretical basis. Bachelier will follow this path in his 1900 dissertation in which he first offered a formalization of the Brownian motion and thus initiated the mathematical theory of stochastic processes in continuous time (Mandelbrot 1966). Models of that kind will also be used in the construction and the test of the theory of informational efficiency that was synthesized by Fama (1970). Given this relation between the theory of efficient markets and the random walk model, it is not surprising to discover in Regnault's book the main lines of the contemporary theory of informational efficiency of financial markets: information as well as its interpretation were seen as two kinds of causes that generate the short-term variations of stock prices."

The author believes that it was Regnault's information hypothesis that forms the basis of Efficient Market Hypothesis. Regnault was more focussed on information irrelevance as for him information was more a factor of short-term speculation. It was Kenneth Boulding who laid a more comprehensive framework in 1966.

"Regnault's originality lies in this association of these causes with the new information that arrives on the market. In addition, he claimed that at every moment of time, the price contains the whole information:..he deduced that the probability for the price to increase is equal to the probability for the price to decrease, i.e. a half. If such was not the case, agents could arbitrage and choose systematically the strategy that has the greatest probability…In all the games of chance that contain two opposite chances, relative equality precisely results from the possibility for the player to choose one chance or the other: moreover, these two conditions cannot be separated, because if a possibility would generate a greater advantage than the other one, it would be constantly chosen..like in a game of heads or tails, the movements of stock prices are independent: neither the variations, nor previous prices were useful to predict future variations… analysis of the price behavior in terms of a random walk. However, another particular feature of the short- term behavior is the subjective nature of the probabilities…The movement of prices and the subjective evaluation of new information. The subjective nature of the probabilities affects the evaluation of securities as well as their price, but Regnault suggested that the evaluations made by individuals were following a normal law….evaluation of the height of a building by a group of individuals, and was led to think that these evaluations were ruled by a law close to the Gaussian one…"

Value, Duration, Reversion

Regnault was the one to not only talk about Random walk model but also connect it to the law of variations through time. Though the challenge to the Random walk emerged as prices were shown to be non-normally distributed the approximation that price variance is a factor of square root of time holds well. This was the first ever proof of the co-existence of non-normal distribution in prices and normal distribution in variance; both Pareto and Galton coexisting together. The security, through its variations, was in search for true value and this it did by reverting around the mean variation.

$$\sqrt{t\sigma^2} = \sqrt{t}\sqrt{\sigma^2} = \sigma\sqrt{t}$$

"These assumptions thus enabled him to construct a symmetrical random walk model. We must now show the way he obtained from this model a law of variations of the price through time… The law of deviations thus showed that there was an equal probability of a half for the price to increase or decrease at each moment of time. He then investigated the way the prices were varying through time, and he discovered a relation between the mean deviation of prices and time…In the latter case, a mean deviation for a given period can be calculated, he looked for a relation between time and that deviation. He remarked that the mean deviations for a given period of time were approximately equal and that the shorter the period of time considered, the smaller the deviations…if the period of time is only half as much, the deviation is divided by a number inferior to two. …the deviation of the prices increases with the square root of time… Otherwise stated, it is not possible for an agent to anticipate in an exact way the future price of a security, but he can anticipate its mean deviation for a given period of time."

"The security, in its variations, is constantly in search for its true price, or an absolute price, that we can represent as the center of a circle, the radius of which would represent the deviation that can equally appear in the one or the other direction, and on each point of the surface, in a period of time consequently equal to the surface, and the whole points of its circumference would represent the limits of the deviations. In its variations, the security is only moving away or getting closer to the center, and from the basic notions of geometry, we know that the radius or the deviations are proportional to the square roots of the area or of time."

Law of Ruin

Jovanovic and Gall explain how Regnault believed that it was the frequency of trades that caused the ruin for the speculator. He referred to it as the law of ruin. This resonates with the current debate regarding passive investing versus active money management and a case for tactical passive investment management.

"However, Regnault constructed a second model of short-term speculation that incorporated the transaction costs and that enabled him to demonstrate the inexorable ruin of the short-term speculation….Other things the same, the unfavorable possibilities are largely reduced on a large market, where the business is very important, and they proportionally increase on a small and especially a sensitive market….Regnault thus had in his hands the elements that could pave the way for a 'law of ruin'…The chances of loss are increasing the power of the inverse ratio of time…The frequency of operations is an abuse; and since the unique motivation of each

exchange is and has to be usefulness, each time this usefulness disappears, there is an error or a bad use; from this, we can thus clearly separate the game from the speculation…the transaction costs represent for him twenty times more, that is 2.5%. As a consequence, since the gain and the loss equilibrates, no more that forty operations will lead to the loss of his capital."

Non Random Long Duration

Regnault was also the one suggesting persistent trends, which he referred to as deterministic laws that rule the stock market, existed in the longer term. As the duration became larger, there was anticipation, predictability and social usefulness of statistical approach for the social world. Stock prices had a longer term expression, which differentiated itself from the short term noise game. Simply putting, Regnault talked about Random walk in the shorter term and statistical reversion in value over the longer term, a non-random behavior.

"…In the long term, deterministic laws rule the stock market. We shall first examine the way long-term speculation was defined, the way it was seen as representing a social usefulness, and explain its relations with constant causes. Then, it will be possible to shed some light on Regnault's methodological choices: he thought that the accidental causes compensate on the long term and consequently that the underlying order that rules society could be discovered. In Quételet's style, his approach was closely associated with the determination of mean values and this was an opportunity to claim the usefulness of a statistical approach to the social world…Some speculation about long-term causal laws…The construction of this second model originates in the belief that true deterministic laws were permanently acting although masked by the accidental causes…The knowledge of such a constant cause was indeed delineating the outlines of a moral behaviour that leads to a social usefulness. At that stage, Regnault clearly opposed the short-term game to long-term speculation."

Reversion is statistical, Diversion is a vice

Statistics was a way for Regnault to approximate deterministic laws. The moral expression and its vices which cause deviations are brought in balance with statistical longer term regularity. Galton continued (1886), similar work, regarding variation away and back to mean generationally keeping natural systems in a balance. Regnault rather accepted divergences expressed as social vices as a necessary evil for the statistical longer term laws of reversion to work. Galton ignored the role of deviations and divergence as being essential component to the mean reversion process (Pal, 2015).

"He extensively discussed its applications to numerous issues and focused on the resulting possibility of forecasting. More precisely, statistics was for him a way to discover and to approximate deterministic laws. Statistical laws are not only the concern of material things. . . . Most importantly, they are also and as rigorously the concern of the moral facts, those which are precisely the less likely to belong to a stable or normal state. Births, marriages, diseases, suicides, crimes, etc. can fluctuate from year to year under the influence of accidental causes, but on a rather long period, they will appear as regular. . . .Most surprisingly, our mistakes, our distractions, our biases, and even our caprices are ruled by the law of probabilities. The human mind can appear as indiscernible. . . . Yet, the phenomena that produce it . . . appear as more regular than the physical phenomena when men are free, that is when they are not disturbed by private causes of personal interest….Like Quételet, the statistical approach Regnault had in

mind was closely associated with the calculus of mean values, seen as signs of stability and order every kind of variation from an average was considered as an error or a vice."

In the long duration, Past is connected to future

This role of divergence, deviation and variance was why there was a dependence between past and future. The future originated from the past, as while short-term deviations cancel each other in the longer term behavior was admitted regular around the use of averages. A history of researchers has cited the longer term cycles including Shiller's CAPE (1988)

"The future inexorably originates in the past, just like the effect constantly originates in the cause….He thus showed that all data was including a short-term or accidental component and a long-term or constant component…The short-term components were seen as the product of causes that 'inexorably cancel each other'. By contrast, the long-term components are 'admirably regular', and 'the demonstration of this fact is given by the use of averages'…In accordance with his deterministic views, he thus believed that some averaging procedures, associated with the law of large numbers, should be useful to reveal the long-term tendencies. Although he did not construct a formal decomposition of time-series such as those devised during the early twentieth century, Regnault deserves a place in the history of time-series analysis"

Reversion stronger than Diversion

Regnault was also the first thinker to talk about the symmetry of reversion and diversion and how there was a less frequent, but more marked and stronger tendency to revert towards to mean i.e. statistical Value compared to the more frequent tendency to diverge and increase away from the mean.

"He then statistically examined the robustness of this hypothesis. Of course, he had no testing procedures available; however, he proceeded just like turn-of-the-century pioneers of econometrics did and used a graphical approach…Regnault then investigated the way these probable errors were distributed around the mean value and an analysis of the observed deviations led him to think that no symmetry was prevailing…The fluctuations in the prices of public bonds are variable, and their extraordinary deviations reveal a more marked tendency to be inferior to the mean value than to be superior to it. We can thus consider that the increasing state, more than the decreasing state, is a normal one; the decreasing state is more intense, but less durable. . . . In other words, the causes that generate the decline are less frequent than the causes that generate the increase, but their strength is superior."

Deterministic Law

For Regnault, unlike the deviations, there was a statistical attraction mechanism that constantly moves the prices toward the mean. The attraction centers that brought balance were driving the universal law and this is why it was necessary to wait. In summary, Regnault talked about divergence, its connection with duration and the determinism brought by the universality of the longer term back to the mean.

"The final step: the law of attraction The final step was that of the deduction of a deterministic law that rules long-term speculation: a kind of attraction mechanism that constantly moves the price toward the mean…In the first part of his book, Regnault presented a law of deviations, according to which 'The deviation of the prices increases with the square root of time' and is also independent of the price itself. This law could not, of course, contribute to explain a possible relation between the observed price and a mean price. However, on the basis of the preceding steps, he put forward the existence of a second law that stipulates a relation between these deviations and the mean price. The normal distributions around the 'attraction centers' led him to think that a kind of 'force' was acting. This second law at work states that the price, in all its deviations, is permanently attracted to its mean price, and this attraction is increasing with the square of its distance…Apart from this example, Regnault himself dealt cautiously with this law and confessed that he ignored 'the measure of this force, and the very moment when it will be at work' (1863: 188). However, he had good reasons to suppose the existence of this law. The 'attraction law' was indeed seen as a 'universal law' (1863: 168)…This 'law of attraction' was undoubtedly the law Regnault was looking for. It crystallized the various constant causes at work and, consequently, aimed at proving that the short-term speculation was a mere illusion: in spite of its apparent randomness, the price of the French 3 per cent bond was dependent on a deterministic law. The short-term speculator was thus considered as 'blind', whereas the 'true speculator' behaves 'in relation with the interests; he does not focus on the current circumstances, but on the future'. The end of the story was thus a moral one: the short-term speculator was favoring and took advantage of – the existing (moral) deviations of society, but 'the extreme things always lead individuals and people to their loss'. The deterministic laws Regnault put forward had thus to induce a (moral) change in the behaviour of the economic agents: they have to learn the crucial importance of time – 'it is necessary to WAIT', since 'Time will always lead the price to their true value and will correct the deviations of speculation'. The knowledge of these laws could then lead to a kind of convergence towards a state of certainty – 'the constant feature of events invariably leads to a convergence in a more or less immediate future'.

1920-1934, Value is Fundamental and Cheap

Investing styles nomenclature has a history going back to 1920s. Though Graham and Dodd never used the phrase, "value investing", they are credited for the same. The term was coined later to help describe their ideas. The Graham approach was to recommend low-risk entry inexpensive (cheap) stocks. They advocated a systematic approach to selection (bargains). They sought to buy businesses trading at a discount to net current asset values, what has been subsequently referred to as 'net-nets'. intrinsic value rather than price momentum.

Though inexpensive (cheap) selection can be looked at as value, it can not be the definition of Value. Value is a broader concept than inexpensive equity selection. Moreover, an identification is connected to availability. Markets always create opportunities, but it's primarily the exaggeration of irrationality (crisis) that creates value opportunities, low-risk entry. Graham and Dodd value may concern value in equities, but Value as an opportunity is multi-asset (multi-domain) in nature. The assumption that Value is primarily equity is incorrect and because of the history of published literature connected with stock picking. It's also because there is not enough research done to explain what Value means for commodities, bonds, currencies etc. The idea of inexpensiveness Value, suggests a low-risk entry, which should be asset-agnostic. Value investors and researchers in equity can remain oblivious to this or accept it. If they do accept it we are talking of a redefinition of what constitutes Value. This not only makes Value

broader in it's appeal but also global in it's coverage. With access to information global, the portfolio definition has also moved beyond equity and more multi - asset and encompassing various risk preferences.

1952, Linear Assumptions

Portfolio Selection, Markowitz 1952

"Consider expected return a desirable thing and variance of returns an undesirable things.""The law of large numbers will insure that the actual yield of the portfolio will be almost the same as the expected yield." "This rule is a special case of the expected returns - Variance of returns rule." "Diversification cannot eliminate all variance. there is a rate at which investor can gain expected return by taking on variance, or reduce variance by giving up on expected return." "We assume static probability beliefs, probability distributions are a function of time." "The rules serves better, investment as distinguished from speculative behavior." "If the two original portfolios have equal variance then typically the variance of the resulting portfolio will be less than the variance of the either portfolio." "It is necessary to avoid investing in securities with high covariances among themselves." "We should diversify across industries because firms in different industries, especially industries with different economic characteristics, have lower covariances than firms within an industry." "Not only the E-V hypothesis implies diversification, it implies the "right kind" of diversification for the "right reason"

Simplification was a necessity and hence the model assumptions. However, many of these assumptions stand challenged. According to Markowitz, the expected yield converges, owing to LLN (Law of Large Numbers). LLN follows CLT (Central Limit Theorem), which suggests normality, a regression to mean. Markowitz does not mention his view on the stationary or non-stationary behavior of variance, whether variance just like returns converged or diverged in time.

Many of the challenges to MPT are built around the dynamic nature of markets and how markets are not static but witnesses convergence and divergence in time. Challenges to MPT assumptions; Mismatch of exponential utility assumption with normally distributed returns. The constancy assumption of correlations between assets. The assumption of static probability in a context of probability distributions that are a function of time only suggest the difficulties in modeling dynamic systems, which converge and diverge in time. In the Single-Period Mean-Variance analysis in a changing world, Markowitz, Dijk (2003) argue that calculating the optimum strategy is well beyond foreseeable computing capabilities.

One of the assumptions in their 2003 paper is that "good months would tend to be followed by good months, bad months by bad months." The assumption of a continuity of trend is linear thinking. Markets also have good months followed by bad months and vice versa. This stifled approach in not efficient investing and the reason for the challenges faced by the respective model.

Markets are dynamic systems, where momentum is transformed and followed by reversion. Something that his high risk could continue to offer higher returns or chose to disappoint. Any selection model that chooses to ignore time varying volatility will remain ineffective. The dynamic nature of stock market behavior over time translates into different variances. When components are interacting at multi-durational level identifying behavior based on historical averages, is like shooting in the dark.

This means that though Markowitz is correct in identifying a stage of efficiency in portfolio construction, the solution fails to live up to the dynamic test of real markets. If markets are dynamic systems, the point we identify an efficient portfolio is the moment we should be getting out if it. Static systems though perfect in their portrayal are inefficient in their functioning. The simplification hence leads to the limitations of MPT.

1958, Capital Structure Relevant or Irrelevant?

Time Dependence of the Optimal Capital Structure, Pal, Mitroi 2016

The capital structure was considered relevant in (Gordon, 1959) and irrelevant in (M&M, 1961). The relevance and irrelevance of Capital Structure are not competing thoughts but in line with the reconciliation of market efficiency (Fama, 1965) and inefficiency debate (Ball and Brown, 1968). The relevance-irrelevance of capital structure is based on the relevance-irrelevance of information (Reversion Diversion Hypothesis, Pal 2015). The relevance and irrelevance of the capital structure would bring forward the idea of markets indeed being natural systems (Mean Reversion Framework, Pal 2015), and capital structure irrelevance is an incomplete idea.

Dividend Relevance

Dividends, Earnings and Stock Prices, MJ Gordon, 1959
Dividend Policy: Its Influence on the Value of the Enterprise, James E. Walter, 1963
Optimal Investment and Financial Policy, Journal of Finance, MJ Gordon, May 1963, 264-272

Myron talks about perception

"*Dividend paying companies are viewed positively hence the positive influence on the share price. Relevance is connected to the certainty of the cash flow. While Walter talks about dividend reinvestment and resulting higher returns. He set up a linear equation comparing the opportunity cost with the cost of capital. If the cost of capital was higher than opportunity cost, the company should pay out 100% as dividend and vice versa. Gordon suggests that corporation's share price (or its cost of capital) is not independent of the dividend rate."*

An empirical evaluation of accounting income numbers, Ball and Brown (1968)
Post Earnings Announcement Drift, Bernard & Thomas, (1989)

Though simplistic in there approach (Walter, Gordon, Myron) the relevance idea here is similar to the one mentioned by Ball and brown (1968). Ball and Brown considered the content in accounting information, the flow of information, the relevance of information, its predictive powers and its continuity and time dependence. The observed reversion was used as a validation of predictive content in earnings. This relevance was later shown to cause a drift, an anomaly, un unexplainable behavior of a market. Bernard and Thomas (1989) showcased a seasonal positive autocorrelation in partial periods connected to the news followed by a seasonal negative autocorrelation.

Dividend Irrelevance

Merton H. Miller and Franco Modigliani, Dividend Policy, Growth and the Valuation of Shares, Journal of Business, October 1961, 411-433

"*Given a firm's investment policy, its dividend policy was irrelevant to its current market valuation."*

Dividend, Earnings, Leverage and Stock Prices, and the supply of capital to Corporations, Review of Economics and Statistics, John Lintner, August 1962, 243-269

"We have found that in general theoretical models, non-linearities, complex interactions, and inequalities leading to market preferences, abound…Models which substitute linear for non-linear functions and straight-jacket variable and interacting parts into constant sums can encompass practical reality to an acceptable approximation."

The Cost of Capital, Modigliani-Miller 1958

Skimming the butter may seem to have little common with systems which express reversion and divergence (deviations), but a closer look explains how dividend irrelevance has a lot to do with a dynamic system with extremes and a natural central tendency.

Modigliani and Miller (1959) argue that capital structure irrelevance proposition describes the central tendency of the real world capital market given that mechanisms exist to promote equilibrium. But this does not exclude temporary deviations from equilibrium.

"Much like their work on the capital-structure irrelevance proposition, Modigliani and Miller also theorized that, with no taxes or bankruptcy costs, dividend policy is also irrelevant. This is known as the "dividend-irrelevance theory", indicating that there is no effect of dividends on a company's capital structure or stock price. MM's dividend-irrelevance theory says that investors can affect their return on a stock regardless of the stock's dividend. For example, suppose, from an investor's perspective, that a company's dividend is too big. That investor could then buy more stock with the dividend that is over the investor's expectations. Likewise, if, from an investor's perspective, a company's dividend is too small, an investor could sell some of the company's stock to replicate the cash flow he or she expected. As such, the dividend is irrelevant to investors, meaning investors care little about a company's dividend policy since they can simulate their own."

Dividend irrelevance can only be driven by an intrinsic efficiency of the environment in which the company or set of companies operates. The environment which delivers an order despite it's uncertainty.

"Target Reversion" or "Mean reversion" is the continuous process of adjusting capital structure toward the target ratio (Shyam-Sunder and Myers 1999; Frank and Goyal 2003). Chang and Dasgupta show that even with Random financing and with no apparent target, leverage may appear to be mean reverting. Target Debt Level; Debt ratios; Leverage confirm mean reversion. In a regression of change in book leverage vs. target leverage, the evidence usually confirms mean reversion (Fama and French 2002; Kayan and Titman 2007)."

"*No sufficient explanation size as to what determines the size of the risk discount and how it varies in response to the changes in other variables. Under uncertainty, there corresponds to each decision of the firm not a unique profit outcome, but a plurality of mutually exclusive outcomes which can at best be described by a subjective probability distribution. The profit outcome, in short, has been a random variable and as such it's maximization no longer has an operational meaning. For decisions which affect the expected value will also tend to affect the*

dispersion and other characteristics of the distribution of the outcomes. Accordingly, the extrapolation of the profit maximization criteria of the certainty model has tended to evolve into utility maximization, sometimes explicitly, more frequently in a qualitative and heuristic form….permits us to develop a theory of investment of the firm under conditions of uncertainty."

Moving beyond subjectivity towards a system that embraces conditions of uncertainty

"Notice also that the uncertainty attaches to the mean value over time of the stream of profits and should not be confused with variability over time of the successive elements of the stream. That variability and uncertainty are two different concepts should be clear from the fact that the elements of a stream can be variable even though known with certainty. …Various shares within the same class differ, at most, by a 'scale factor'. Accordingly if we adjust for the difference in scale, by taking the ratio of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class. It follows that all relevant properties of a share are uniquely characterized by specifying 1) the class to which it belongs. 2) It's expected returns. The significance of this assumption is that it permits us to classify firms into groups within which the shares of different firms are "homogeneous", that is perfect substitutes for one another…In any given class the price of every share must be proportional to it's expected return….all bonds are intact substitutes on a scale factor…."

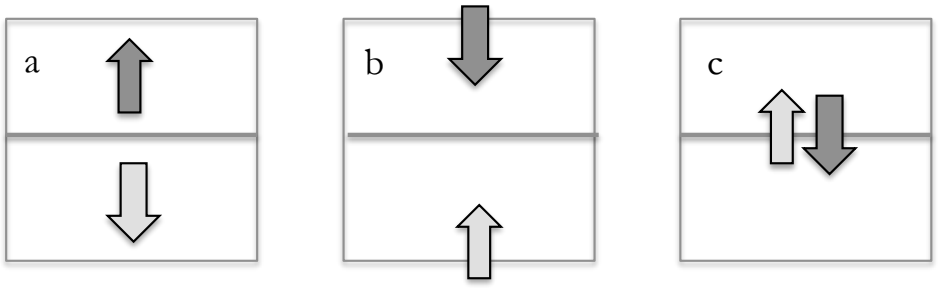
Here the authors differentiate mean returns from variability, certainty in an uncertain environment. The similarity in properties and commonality owing to the scale factor.

"Our propositions as noted earlier, do not depend for their validity on any assumption about individual risk preferences. Nor do they involve any assertion as to what is an adequate compensation to investors for assuming a given degree of risk. They rely merely on the fact that a given commodity can not consistently sell at more than one price in the market; or more precisely that the price of the commodity representing a "bundle" of two other commodities cannot be consistently different from the weighted average of the price of the two components (the weights being equal to the proportion of the two commodities in the bundle). This assertion is equal to the proposition that, under perfect markets, a dairy farmer cannot, in general, earn more for the milk he produces by skimming some of the butter fat and selling it separately, even though butter fat per unit of weight, sells for more than whole milk. The advantage of skimming the milk rather than selling the whole milk would be purely illusory; for what would be gained from selling the high-priced butter fat, would be lost in selling the low-priced residue of thinned milk."

"Our propositions describe the central tendency around which the observations will scatter- but also there are large and systematic imperfections in the market which permanently bias the outcome. In any event, whether such prolonged, systematic departures from equilibrium really exist or whether our propositions are better descriptions of the long-run market behavior can be settled only by empirical research."

M&M's theory independent from any assumption of individual risk preferences illustrates the disconnect between risk and return, but still emphasizing behavior. The behavior which suggests that a certain portfolio price can diverge in time from the weighted average portfolio price (mean), but over the longer period that divergence can't sustain away from the natural central tendency towards the mean. The no-arbitrage argument is period dependent and if in the long run the process of divergence and reversion can cause the irrelevance of the dividend

structure, the understanding of the dynamic process becomes more important to understand natural behavior which drives behavior in an uncertain environment.



1962, The Non-Linear Model

Random vs. Systematic Changes, Paul H Cootner, 1962

The elements of the non-linear model were cited as early as 1962 by Cootner. Though he pinpointed the functioning of a non - linear model of markets very well, he could not reconcile the idea of randomness with non - random behavior. Cootner's work is significant for the dynamic (non - linear) school because dynamism is about looking at random and non-random behavior together.

"Their profits will come from observing the random walk of the stock market prices produced by the non-professionals until the price wanders sufficiently far from the expected price that they can expect the future prices to force prices towards their mean more often than not."

Cootner was very close to understanding the role extremities (extremes) played in a dynamic system. The farther the extreme went the stronger the recoil. This is what Galton (1886) talked in his seminal paper on "Regression towards mediocrity in hereditary stature".

"If prices fell to the lower limit, however, the rate at which prices moves back to the expected price is governed by the random process which operates within the barriers, so even if their expectations are correct, there profit rate is still a stochastic variable."

Cootner acknowledged that though there was a recoil (reversion), it was governed by the random process. This again illustrates that he could not explain the co-existence of random with non-random behavior.

"There is much random behavior in such a series but it is substantially different from a random walk. One implication of this, of course, is that it would be difficult to detect the significance of any weekly price change if the series were truly a random walk. But the high variance also means that any professional who did feel knowledgeable about the mean price would still want to set his buying price considerably away from the mean to protect against risks. If the lower edge of the barrier were to average several weekly standard deviations away from the mean, by far the greater portion of the successive weekly price changes will be totally uncorrelated with each other. On the other hand, when prices neared the barrier there would be a tendency for some negative autocorrelation since movements to the barrier would be more likely to be followed by the movements in the opposite direction. The net effect would be a moderately

negative correlation near the boundary which would be heavily diluted by the number of cases when the prices were near the mean which should show no so much negative autocorrelation."

Cootner saw that things were random but substantially different, a behavior with a noise, which created opportunities and possibilities for arbitrage. He identified negative autocorrelation at extremes which diluted near the mean. There was a clear difference between the center and the extremes. Cootner's explanation is Galton's organic system.

" In addition to this negative correlation, the effect of the barriers would be to produce more small price changes than would be expected from a normal distribution of price changes. When unencumbered, by the barrier, the central limit theorem would tend to ensure that the total weekly effect of a large number of individual transaction price changes would be approximately normal. The existence of the barrier, however, would cut short some of the price movements towards the barriers without restricting as much the very large price movements which could still occur in the direction of away from the barriers. We would expect the distribution of price changes over short periods of time to be more leptokurtic under such conditions than the normal distribution."

Cootner also explained two set of probability distributions, one near the mean expressing normal distribution, while near the extreme more leptokurtic. This again expresses the complexity of a natural system behaving both normal and leptokurtic at the same time, depending on which classification of the group was studied, near the mean, or near the extreme. A framework could classify the extreme different from the mean and how the system oscillates from one end to the other.

"I have spoken of stock price series as composed of several trends of different slopes. As we lengthen the period over which we take differences the mean becomes more important relative to the standard deviation. Furthermore, the mean of each of the component trend becomes more distinguishable from the group mean.This will result in an increasing element of positive autocorrelation as long as the time interval of the differencing is less than the length of the trends. That is the successive changes in an uptrend will all tend to be higher than the overall mean and the successive changes in a downtrend will all tend to be lower. As the differencing interval exceeds the length of some of the trends, this positive autocorrelation will begin to disappear. The positive autocorrelation is also present the in changes over one week periods, but over such short periods the absolute magnitude of the difference in means is so small relative to the standard deviation that the effect is negligible. It simple becomes increasingly prominent particularly when measured against the negative serial correlation induced by the barriers as the time interval increases."

Above Cootner explains the idea of the coexistence of several trends, with different holdings. And how mean, a function of duration was more prominent than deviation. He also explained how the dynamic system, trends, durations, and boundaries caused continuity and reversal of trends.

"All the tests of the autocorrelation of weekly stock price changes have consistently shown deviations from random behavior."

Deviations from random behavior…

The shift from an excessive tendency for reversals to an excessive tendency for trends takes place relatively uniformly as the interval increases.

Duration increase and extreme(s) reversion…

Cootner just like Galton were so close in identifying the workings of a natural system, but just like Galton who identified reversion in the first place but failed to detail the role of divergence with reversion, Cootner failed to reconcile the random with the non - random.

1964, Divergence of Risk from Return

Capital Asset prices, William F. Sharpe, 1964

"At present, there is no theory describing the manner in which the price of risk results from the basic influences of investor preferences, the physical attributes of capital assets, etc. Moreover, lacking such a theory, it is difficult to give any real meaning to the relationship between the price of a single asset and its risk. However, since the proper test of a theory is not the realism of its assumptions but the acceptability of its implications,and since these assumptions imply equilibrium conditions which form a major part of classical financial doctrine, it is far from clear that this formulation should be rejected-especially in view of the dearth of alternative models leading to similar results."

Capital Asset Price Model says that expected return on any portfolio (or stock) should earn a premium above the risk-free rate. In simpler words it said, low risk meant lower return and vice versa. The CAPM was introduced by Jack Treynor, William Sharpe, John Lintner and Jan Mossin, building on the work of Harry Markowitz. Sharpe, Markowitz, and Merton Miller got the 1990 Nobel Memorial Prize in Economics. Just like Markowitz model, CAPM's linearity assumption is the primary reason it has been challenged.

"*Much of the early work in financial economics dealt with markets in which the interaction of a large number of individuals, each equally informed, determined prices. In this sense, the work followed the tradition of competitive equilibrium theory in economics. More recently, attention has focused on markets in which there are few participants and/or in which different individuals have different sets of information. The CAPM is, of course, a theory in the earlier tradition of the field. It is a positive theory, incorporates assumptions about investors' utility functions, and assumes a market with a large number of participants, each of whom has access to the same set of information. Each December from 1923 to 2003 they estimate a beta for every stock on the NYSE, AMEX and NASDAQ, using two to five years of prior monthly returns. Ten portfolios are then formed based on beta and the returns and tracked over the next 12 months. The figure plots the average return for each decile against its average beta. The straight line shows the predictions from the CAPM. The model predictions are clearly violated. CAPM woefully under predicts the returns in the low beta stocks and massively overestimates the returns in high-beta stocks. This might suggest that investors might be well advised to consider a strategic tilt towards low-beta and against high-beta, a strategy first suggested by Fishcher Black in 1993. Suggesting simply that low-risk could deliver a higher return and vice versa."*

James Montier (1990) rechristened CAPM as CRAP (Completely Redundant Asset Pricing) in a research paper. No doubt behavioral experts had insights into the market behavior, but somewhere there is an academic bias that creeps in, making academicians more positively

biased about their body of work. History is full of literature where new theorists have not been objective about the previous body of work. Montier strengthened his case against CAPM assumptions by illustrating the low beta and high beta portfolio behavior. He illustrates Fama and French's 2004 review of CAPM.

"Each December from 1923 to 2003 they estimate a beta for every stock on the NYSE, AMEX and NASDAQ using 2-5 years of prior monthly returns. Ten portfolios are then formed based on Beta and the returns and tracked over the next 12 months. The figure plots the average return for each decile against its average Beta. The straight line shows the predictions from the CAPM. The model predictions are clearly violated. CAPM woefully under predicts the returns in the low beta stocks and massively overestimates the returns in high beta stocks."

This might suggest that investors might be well advised to consider a strategic tilt towards low beta and against high beta – a strategy first suggested by Fishcher Black in 1993. Suggesting simply that low-risk could deliver higher – return and vice versa.

What is wrong with the CAPM assumptions? According to Montier, investors prefer to benchmark to market returns rather than to risk-free rate. This is why the investing community is more focused on tracking error than on risk-free interest rate. Which is correct, but having the risk free rate as a benchmark may not make it realistic, but it does not make it redundant. Fama and French improved the model by adding Value, Size (capitalization) variables to the CAPM variables. Though testing suggested that the new variables enhanced the understanding of the market behavior, widening the definition of Beta but it still failed to explain how low beta gave high returns and vice versa. Even newer models with Momentum as a factor failed to establish rules and address the CAPM failures. The paradox was that CAPM worked in Japan but failed in a majority of the markets, a poor formulation but not redundant.

So if CAPM was still valid and far from completely redundant, why was it referred to as CRAP? Did we need to look at CAPM beyond the academic bias? A lot of data interpretation focuses on causally explaining mean reversion failures, or simply putting divergence from idealized cases. This is why a divergence from a proposed behavior, made CAPM a poor idealization. We continue to seek better-idealized scenarios, but somewhere we forget that markets are not made of one idealization, but a set of idealizations. In this case, both CAPM and Fama and French being two sets of idealizations.

Data Universality Proxy explanation of Risk and Return Divergence

If it's about mean reversion failure, it's all about models failing to explain divergence. Data Universality (Pal, 2010) explains the power of proxy and how data behavior is universal irrespective of the variables be it financial or non-financial. A simple performance ranking can be a good proxy to explain Value, Growth, Momentum, Reversion, Beta, Volatility etc. in a certain universe of assets prices or simply any natural data set. The test included a proxy percentile performance ranking of worst (bottom fifth) and best in a group of assets (top fifth) for the S&P 100 components for periods ranging from 20 days to 1200 days. Even after 1200 days of holding nearly 20% of the worst losers and best winners continued to remain worst and best respectively. This proved that though there was a tendency for the worst to outperform and for the best to underperform, this was not a rule. This could be extended to the idea of low beta stocks could continue to stagnate, which in other words meant that low-risk could continue to deliver low-return or simply suggesting that CAPM was not crap, but a relevant case of market

behavior. The results also suggested that predictability of reversion or its failure is never at a component level but primarily at a portfolio level.

1966, Outperformance

Can Mutual Funds Outguess the Market?, Treynor, Mazuy, 1966

These are the 1966 summary points of Treynor and Mazuy's paper. The investing process is dependent on market fluctuations. This suggests the affect of market dynamics on the investing process. The dynamics mean the random and non-random characteristics of the market. Hence, the manager's skill is connected to the ability to identify underpriced industries and companies. Or in other words, the extreme opportunities come from the diverged asset prices from the relative mean. Hence outguessing was a fruitless activity. No one could do it. In conclusion, the authors suggested not to hold fund managers responsible for failing to foresee. This also implicitly indicated the need for systems.

"Our results suggest that the investor in Mutual funds is completely dependent on fluctuations in the general market. This is not to say that a skillful fund management cannot provide the investor with a rate of return that is higher in both bad times and good that the return provided by the market averages, but it does suggest the improvement in rate of return will be due to the fund manager's ability to identify underpriced industries and companies, rather than to any ability to outguess turns in the market as a whole. The fact that only one of the 57 mutual funds in our sample has a characteristic line suggesting curvature indicates perhaps that no investor - professional or amateur - can outguess the market. This finding has clear significance for the man in the street managing his own portfolio, for the man with the fiduciary responsibility for a private estate, for the president of a manufacturing company responsible for it's pension fund, and for a college treasurer managing an endowment. It means that probably the best assumption they can make is that the investment managers have no ability to outguess the market and should not try to. It also means that they should not hold fund managers responsible for failing to foresee changes in the market climate."

Beating the market impossibility argument made 50 years back still fails to get a serious acceptance. Even with a dismal record of hedge fund performance, the active money is still at it. We can assume there are arbitrage and speculative opportunities, which keep driving the active money against the argument that beating the market is an impossibility.

This is an idea of extreme importance for a society that not only gives undue weight to financial risk, does not understand risk, but also relies on the return and growth that accompanies calculated risk-taking.

Though financial models have the limited history, the risk has traditionally been under-judged and might never be completely understood. We can't pinpoint the source of the problem because markets evolve and what seemed risky yesterday is not that relevant today. Risk, like many other social parameters, is a moving target. Many risk parameters have moved from reverence to irreverence, as they failed to pass the test of time.

Behavioural finance was the first to challenge the status quo and break illusions built around beta and benchmarks. Framing errors were showcased among fund managers comparing their

portfolio with benchmarks that showed enhanced performance. Then, of course, we had research suggesting the beta (relative performance) was dead.

1977 - 2001; The Size Proxy

The Size Proxy, Pal 2015

Though 'Size' is the most important factor explaining stock market returns the possibility of Size being a proxy was first mentioned in Banz (1978). Even after forty years of factor investing the industry is still looking for answers. This paper chronologically lists the research on 'Size' and why the question regarding 'The Size Proxy' has never been so relevant.

History of Size

After Ball and Brown (1968) work on information content in earnings, researchers looked for further confirmation in financial data relevance. Basu (1977) suggested that Price Earnings had information content. Although it was not clear, Size seemed to subsume other variables. Barr and Marathe (1976) observed that macroeconomic events on individual securities could be captured through microeconomic characteristics essentially common factors. While confirmation regarding information relevance continued, it was Bawa and Klien (1977) who suggested that P/E could also fail as an indicator i.e. information content could fail or be insufficient. This was followed by Banz (1978) who discussed fundamental value as a predictive indicator and questioned whether Size is just a proxy. He also highlighted the lack of theoretical foundation regarding Size. Statmann (1980) observed that Book Value was a key factor while Kiem (1980) suggested that Book Value was a proxy for Size. Arbel and Strebel (1983) suggested that small Size could be a special case of neglect effect. Since Size and book value were still debated, Fama's argument regarding the ineffectiveness of Beta was challenged by Lakshinok (1993).

Year	Researcher	Idea
1966	Boulding	Information
1976	Rosenberg and Marathe	Microeconomic determinants for macroeconomic events
1977	Basu	P/E has information content
1977	Klien	Failure of P/E
1978	Banz	Relationship between return and market value of common stocks
1980	Banz	Relationship between return and market value of common stocks
1980	Stattman	Book values and stock returns
1980	Kiem	Size related anomalies
1983	Arbel and Strebel	Neglect effect
1993	Lakshinok	Beta dead or alive
1996	Fama	Multifactor explanations
2000	Fama	Dividends and Size

Beta was not redundant. In 1996, Fama and French accepted that they have stumbled on the factors and do not know why the factors work but continued to highlight the importance of Size. Dividend policy irrelevant but driven by Size, Fama, and French (2000).

Investment performance of common stocks in relation to their P/E ratios, Basu, 1977

"Contrary to the growing belief that publicly available information is instantaneously impounded

in security prices, there seems to be lags and frictions in the adjustment process. As a result publicly available P/E ratios seem to possess information content and may warrant and investor's attention at the time of portfolio formation or revision…Although it is not clear that the anomalous returns derive explicitly from the failure of the CAPM to account for firm Size, several studies have shown that anomalous return behavior associated with firm-specific variables is largely subsumed under the 'Size effect'."

The research focussed on outperforming low P/E strategy vs. high P/E strategy. P/E was considered to have information content. This seemed in line with the Ball and Brown (1968) work on information content in earnings. For example, Reinganum (1981) finds that the relation between abnormal returns and P/E ratios reported by Basu (1977) appears to vanish after controlling for Size.

Common Factors in Security Returns: Microeconomic Determinants and Macroeconomic Correlates, Barr Rosenberg and Vinay Marathe 1976

The research developed the theory that the effects of macroeconomic events on individual securities could be captured through microeconomic characteristics essentially common factors, such as industry membership, financial structure, or growth orientation.

The Effect of Limited Information and Estimation Risk on Optimal Portfolio Diversification, Bawa and Klein 1977

"If insufficient information is available about a subset of securities, investors will not hold these securities because of estimation risk, i.e. uncertainty about the true parameters of the return distribution. If the amount of information is generated by small firms is smaller, many investors will choose not to hold such firms. There is inconsistency in the functioning of P/E. Over time measures are known to lose their predictive strength. There is enough literature on the failure of P/E as a predictive tool and as a factor"

The relationship between return and market value of common stocks, Banz, 1978

"The securities held by only a subset of the investors have higher risk-adjusted returns than those considered by all investors. Why are small firms less risky than the large firms in the UK, while the opposite is true in the US. The size effect has been in existence for at least forty years and is evidence that the capital asset pricing model is misspecified. The size effect is not linear in market value; the main effect occurs for very small firms while there is little difference in return between average sized and large firms. It is not known whether size per se is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size.There is no theoretical foundation for such an effect…offer some conjectures and even discuss some factors for which size is suspected to proxy. P/E effect a proxy for size. the P/E-ratio effect is a proxy for the size effect and not vice versa."

Book values and stock returns, Stattman D., 1980

The research finds a positive relationship between average return and BTM for U.S. stocks, as do Rosenberg, Reid, and Lanstein (1985). Chan, Hamao, and Lakonishok (1992) find that BTM is useful in explaining Japanese stock returns.

Size Related Anomalies and Stock Return Seasonality, Kiem, 1980

The research found a significant negative relation between abnormal returns and the degree to which market value of equity exceeds the book value of equity, and also interpret this relation as a proxy for the Size effect.

The neglected and small firm effects, Arbel and Strebel, 1983

The research paper argued that the small size effect is just a special case of "neglect effect"; firms that are neglected by security analysts exhibit superior market performance because intensive analyst coverage raises stock prices and hence lower returns.

Is beta dead or alive?, Lakshinok, 1993

"The stock market is so noisy that, with the existing data, one cannot generally draw clear-cut conclusions as Fama and French do that Beta is dead. Unless the difference in returns is large and the period is long, no conclusive statements can be made. Noisy stock returns are a problem not only when we try to figure out if Beta works. Consider a star money manager who beats the market by 200 basis point a year with a tracking error of 5 percent a year. It would take 25 years to determine whether this manager is smart or lucky. Needless to say, plan sponsors are not so patient. We examined whether the very noisy and constantly changing environment generating stock returns permits strong statements about the importance of Beta… The noisy, dynamic environment generating stock returns clouds our ability to reach firm conclusions with respect to the compensation for beta risk….downside risk is the major concern…not found a single money manager who seems to be concerned about the various multidimensional risk measures consistent with the arbitrage pricing models. If downside is a major concern of investors, beta is a good measure of risk…The discussion thus far indicates that burying beta might be premature. An alternative explanation is behavioral and institutional factors that are unrelated to risk, play a major role in generating stock returns, thereby confounding the relationship between risk and return."

Market noise and dynamic environment, limited testing time of 20 years, makes it hard to measure the effectiveness of Beta. This does not make it redundant. Is smart beta dumb?, Pal 2015 explained how Beta was relevant and how it is the smart beta thinking that is inaccurate and needs to evolve.

Multifactor explanations of Asset Pricing Anomalies, Fama, 1996

"In other words, without knowing why, we have stumbled on explanatory portfolios that are close to three factor MMV…we don't know without knowing a) are there new common factors b) how correlated are the new common factors."

Disappearing Dividends: Changing Firm Characteristics or Lower Propensity to Pay?, Fama and French, 2000

"Dividend policy may be irrelevant, but it's driven by Size. The evidence suggests that three fundamentals – profitability, investment opportunities, and size – are factors in the decision to pay dividends. Dividend payers tend to be large profitable firms with earnings on the order of investment outlays. Firms that have never paid are smaller and they seem to be less profitable

than dividend payers, but they have more investment opportunities…and their investment outlays are much larger than their earnings. The salient characteristics of former dividend payers are low earnings and few investments."

Size Inconsistency

It is sometime explained as neglect effect owing to information unavailability. The factor might be important, but it's not universal, and the possibility that size is still a proxy, leaves the door open for future research. Any factor that challenges size effect consistently would resolve this debate of what is the proxy that drives size and what is more important than size. This factor would also leave the information embedded in other subfactors as unimportant. Since returns tend to run in cycles, there have been long periods when large-cap stocks have outperformed smaller stocks. Small-cap stocks also have higher price volatility, which translates into higher risk. Some investors choose the middle ground and invest in mid-cap stocks seeking a tradeoff between volatility and return.

Size just a proxy was mentioned by Banz (1978), still the industry has not moved beyond size effect. The effect exists in a certain set of market situations and vanishes in a different set of situations. The premium may be visible, but it has failed to express itself as a universal factor, the reason we still see more factors emerging. The other question is if Beta is really driven by a sequence of factors, which one is more important that the other.

Though there was a clear need to adopt non-linear thinking, a generation of researchers focussed on understanding the linear sensitivity of asset returns to various factors. This was the reason for overlapping research regarding the workability and non-workability of static models to explain the dynamic, noisy, multi-durational nature of the market. The confusion between relevance and irrelevance of factors continued to dominate the research from Ball and Brown (1968). The factor school remains fragmented with more than a few reasons to believe that there is a factor beyond the Size factor and despite industry clustering behind Size, the question asked by Banz (1978) has the potential to change everything we have assumed about the Size factor.

1988-2015; Is Reversion Statistical?

There is no disagreement regarding the statistics of mean reversion. What goes up comes down and vice versa. Campbell and Shiller (1988) said that the simple theory of mean reversion was basically right. Fama and French (1989) also suggest that valuation ratios forecast five-year returns with quantifiable accuracy. It is the failure of reversion (divergence) that has not been reconciled with the expression of reversion. Why is the society so keen to accept reversion as statistical, but its failure as behavioral? John Bogle's (2001), Star, Comets and the Sun analogy to the idea of reversion around investing styles reinforces the idea, that all failure of reversion is an error while reversion is a statistical reality. The reason Bogle suggests not to focus on investing styles, as markets are anyway going to recoil back to the statistical mean. Thaler (1999), goes a step further explaining reversion failure as driven by behavioral errors. How strong is the behavioral case? Should reversion and diversion both be statistical?

Reality of Reversion

Stock Price, Earnings and Expected Dividends, John Y. Campbell, Robert Shiller, 1988

“The research suggested that a long moving average of real earnings helps to forecast future real dividends, which in turn are correlated with returns on stocks. The idea is to take a long-term average of earnings (typically 5 or 10 years) and adjust for inflation to forecast future returns. The long-term average smooths out the short-term volatility of earnings and medium-term business cycles in the general economy and they thought it was a better reflection of a firm's long-term earning power…despite all the evidence that stock returns are hard to forecast in the short run, this simple theory of mean reversion is basically right.…Thus, it seems natural to give at least some weight to the simple mean-reversion theory that when stock prices are very high relative to these indicators, as they have been recently, then the prices will eventually fall in the future to bring the ratios back to more normal historical levels. Both Siegel and Glassman– Hassett emphasize that stock returns have historically had a lower risk at long horizons than at short horizons. This is a manifestation of the same mean-reversion…Graham and Dodd recommended an approach that…shifts the original point of departure or basis of computation, from the current earnings to the average earnings, which should cover a period of not less than five years, and preferably seven to ten years.”

“Index dividend yields and cyclically adjusted P/E ratios (CAPE's), among the other aggregate variables can predict future equity returns. High dividend yields and low CAPEs tend to predict above-average future returns. Conversely, low yields and high CAPE's signal below average returns at some point in time. In Campbell and Shiller, 1987b that both real dividend growth and the log-dividend price ratio follow the stationary stochastic process so that they have fixed means.”

Shiller CAPE spectacularly forecasted the carnage of the 2000 tech because it looked at the historical CAPE average and expected reversion. The idea of expensive or inexpensiveness being tied to the extremity of a ratio and its reversion to the mean. The science of predicting trends and allocating investments based on extremities, which in turn are based on limited social or market history seems naive.

Moreover, there are assumptions in CAPE and in its interpretation. First; CAPE as a measure is good and will predict; Second; Extremes in CAPE are bound to reverse; Third; Extremes though subjective should work objectively, and work as a timing indicator annually. Are the fundamentalists and behavioral experts behaving like longer trend anticipating cyclists? The indicator has reached an extreme, and it should turn now, bringing in underperformance.

Using P/E as a way to value the stock market is of limited value if research has showcased P/E to be a proxy for size. And size could itself could be a proxy as explained in the Size Proxy (Pal, 2015).

Eventuality of Reversion

The Stock Market UniverseStars, Comets, and the Sun, John Bogle, 2001

“*Will the valuation ratios revert to their historical means? One should think of the P/E ratio as a stochastic process that will continue to cycle, but within a higher range. Thus, substantial fluctuations in valuation ratios - albeit around a higher mean. In each decade, the top-quartile funds tumbled sharply - and rather consistently - in terms of their excess returns over the S&P 500 Index. One reason the top quartile funds fall back to only slightly behind the market while*

the rise of the bottom quartile funds usually fails even to return them to the market's return is that top-quartile funds have below-average operating expenses and bottom-quartile funds have above-average operating expenses. While gross performance reverts to the mean, fund expense ratios do not. So funds with lower expenses garner the advantage in net performance. Clearly, reversion to mean (RTM) rules the mutual fund seas."

Despite this powerful, self-evident pattern of mean reversion, the mutual fund industry both revels in it and panders to it. Why? Because past performance attracts investor attention and investor assets. The industry aggressively promotes past fund returns—but only when they have been extraordinary. The net result is that money pours into a high-performing (i.e. hot) fund only after the performance is achieved. To make matters worse, when the seemingly inevitable reversion to the mean (and usually well below it) takes place, investors' illusions are shattered, and the money flow first dries up and then turns negative, as investors depart the sinking ships.

"*Money cannot flow into or out of technology stocks, for each purchase of a technology share by one investor must represent a sale by another. But in the mutual fund industry, not only can cash flows exist from one style to another, they can be accurately measured. So it is easy to observe money pouring into growth funds and out of value funds, or vice versa. And pour in and out it does!…Speaking for myself, I have the ability to forecast neither how much of this recent reversion to the mean in favor of Value remains, nor when it will end. If you are smart enough to know, please be my guest and act accordingly. Good luck!"*

Bogle claims to lack the ability to forecast. All his argument is around reversion to mean. Don't bother about Value or Growth, be with the low-cost market… take beta and be happy..no point of heading into new research…timing Value or growth is impossible…smart beta is an illusion. Do we need to reconcile smart beta with the beta? This is in line with the argument that Reversion to mean does not exist alone without Reversion failure (Divergence).

Divergence

'Arbitraging the Anomalies' (Pal, 2015)', Relying on behavioral explanations to explain diversion (anomalies, failure of reversion) did not further the cause of behavioral investing. The case for what causes inefficiency, what explains it has been simplified by the behavioral economists, using behavioral reasons for anomalies (cases of inefficiency). Unfortunately, this has lead to the subject becoming more of a subjective discourse rather than something objective. Ideas like anomalies are here to stay, markets can't be predicted, but some forms of extreme anomalies can be profited from lead to the idea of behavioral funds. Adjusted for risk, behavioral funds were tantamount to value investing. Behavioral finance fund performance proved that anomalies can't be identified and exploited on a persistent basis. The behavioral model also accepts it's temporal limitations.

End of behavioral Finance, Thaler, 1999

The five aspects Thaler points out in his paper 'End of behavioral finance' (a term he confidently used to suggest that behavioral finance will be the only form of finance left) are 1) The equity premium puzzle, 2) Predictability, 3) Dividends, 4) Volatility and 5) Volume myth. All of these five aspects can be explained as mean reversion failures.

First; the equity premium puzzle is that the undue premium equities get over treasuries are

more than justified by the inherent risk in equities. So, the question behavioral finance is asking here is why equity premium (above the risk premium) does not revert to the mean (vanish), or why don't equities erase the respective premium vs. treasuries over a certain period. Second; behavioral finance suggests that predictability in markets is a factor of mispricing. When value gets mispriced versus glamor, it invariably corrects and delivers abnormal returns. Here behavioral finance suggests that because a mispriced asset reverts to mean it delivers returns. This again is a case of a mean reversion failure followed by a regular mean reversion.

Third; dividends, i.e. why do most large companies pay cash dividends? And why do stock prices rise when dividends are initiated or increased when companies can make their taxpaying shareholders better off by repurchasing shares rather than paying dividends? Here behavioral finance seems to be questioning why dividend stocks earn a premium when they shouldn't. Or, in other words, why dividend premium should not revert to a mean value (vanish)? Fourth and fifth; volatility and volume are other cases of mean reversion failure. Both volatility and volume are unexplained, exhibit extreme behavior and don't adhere to any standard models.

Are behavioral finance equity funds a superior investment?, Goodfellow et al. 2013

"*U.S. behavioral funds outperformed during bull markets but underperformed in bear markets. The funds don't outperform passive benchmarks; they do outperform active funds in general. Either stock markets are more efficient, or fund management is worse than behavioral funds advertise. Adjusted for risk, behavioral funds were tantamount to value investing. There was no clear evidence of outperformance on a risk-adjusted basis. Behavioral finance fund performance proved that anomalies can't be identified and exploited on a persistent basis and costs could be a partial contributor."*

If behavioral funds are like fundamental funds where does this lead us? This tells us that anomalies that behavioral finance is trying to exploit are like deep Value. And for fundamentalists, this means that Value has something to do with behavioral biases. The idea of reversion in Value also becomes a commonality for both fundamentalists and behavioral finance. Both fundamentalists and behavioral finance underplay the idea of reversion. They have chosen to accept reversion and ignore divergence as a behavioral error or simply an error.

If the behavioral model was correct and it had moved beyond temporal limitations or shown excess returns above benchmark there would have been no debate regarding divergence (reversion) being driven by behavioral errors, but behavioral finance has failed to challenge the market efficiency theories. Shiller's argument only strengthens the case of reversion more than the one for CAPE. Bogle's relying on reversion is a weak case against factor investing. Bogle only brings back home the argument that researchers can not choose to explain reversion as a natural phenomenon and divergence as something based on behavioral errors or irrational exuberance. If reversion and divergence are natural phenomenon that can be explained statistically then we would could look beyond behavioral errors to systems where failure of reversion and reversion are both statistical in nature.

2000 - Ising, Non-Normality and Power Law Criticisms

Despite its popularity, the power law has not been without its failures and has rather come under criticism. In the paper 'Scale-dependent price fluctuations for the Indian stock market', Matia K, Pal M, Salunkay H, Stanley HE (2004), the authors explained how Indian stock market may

belong to a universality of class different than that observed in developed markets.

In her book, Complexity: A Guided Tour, Melanie Mitchell, 2009 mentions how too many phenomenon are being described as power law or scale free. Data used by Laszlo Barbasi and colleagues for analysing metabolic networks came from a web based database to which biologists from all over the world contributed information. Such biological databases, while invaluable to research are invariably incomplete and error ridden. A number of networks previously identified to be “scale free” using curve fitting techniques have later been shown to in fact to have non-scale free distributions. Considerable controversies over which real world networks are scale free.

Evelyn Fox Keller mentioned that the current assessments of the commonality of power laws are probably overestimates. According to Cosma Shalizi,“Our tendency to hallucinate power laws is a disgrace…Preferential attachment is not necessarily the one that naturally occurs in nature. There turn out to be nine and sixty ways of constructing power laws, and every single of them is right. It’s not obvious how to decide which ones are the mechanisms that are actually causing the power law mechanism in real world.” Even for networks that are actually scale free, there are many possible causes for power law degree distributions in networks.

Normality can not be junked

Non-Normality facts and fallacies, Esch, JOIM, 2010

David N Esch in the Journal of Investment Management addresses the non-normality facts and fallacies. The author reinitiates the century-old debate by suggesting that normal efficient models can’t be simply rejected i.e. market rationality can’t be just junked.

Power-law distributions are not alone

Power law is, as we have seen, impressively ubiquitous, but they are not the only form of broad distribution. Gaussian distributions tend to prevail when events are completely independent of each other. As soon as you introduce the assumption of interdependence across events, Paretian distributions tend to surface because positive feedback loops tend to amplify small initial events. For example, the fact that a website has a lot of links increases the likelihood that others will also link to this website.

Beyond Gaussian averages: Redirecting organization science toward extreme events and power laws, Andriani, McKelvey 2007

“Gaussian distributions can morph into Paretian distributions under two conditions – when tension increases and when the cost of connections decreases. In our globalizing economy, tension rises as competitive intensity increases and as business landscapes evolve faster than the capacity of most organizations to adapt. At the same time, costs of connections are rapidly decreasing as public policy shifts towards the freer movement of goods, money and ideas and rapid improvements in the price-performance of IT infrastructures dramatically reduce the cost of information transmission. Bottom line: Paretian distributions become even more prevalent.”

In the papers, ‘Power laws, Pareto distributions and Zipf's law, M. E. J. Newman (2006) and N. Jan, L. Moseley, T. Ray, and D. Stauffer, Is the fossil record indicative of a critical system? Adv.

Complex Syst. 2, 137–141 (1999), the authors explain how Pareto and Galton could be reconciled.

The two distributions can be reconciled. For example if we consider one of the most famous systems in theoretical physics, the Ising model of a magnet. In its paramagnetic phase, the Ising model has a magnetization that fluctuates around zero. Suppose we measure the magnetization 'm' at uniform intervals and calculate the fractional change 'δ = (Δm)/m' between each successive pair of measurements. The change 'Δm' is roughly normally distributed and has a typical size set by the width of that normal distribution. The 1/m on the other hand produces a power-law tail when small values of m coincide with large values of Δm.. Natural systems are replete with phase changes like Ising model. And since aspects of the same natural systems can exhibit the two respective distributions, this suggests that both distributions could not only co-exist in natural systems but also could be linked with its dynamic nature.

2006-2014, Duration Factor

Time Varying Returns

Though there is no guarantee of the persistence of premium, the behavioral school continues to look for reasons for size premium and believes that premiums could be based on the human irrationality. This idea fails to stand on its own as both irrationality and rationality itself are time varying. There is a lot of literature written about time varying volatility and behavioral finance also accepts its temporal limitation. This pushes us back into the acceptance of diversion (noise, error, fluctuations, divergence) as driven by the time-varying volatility.

Causal Explanations

The Relationship between the Value Effect and Industry Affiliation, Banko, Conover (2006)
Size Related Anomalies and Stock Return Seasonality, Kiem, 1980

"*Recent research in finance has revealed stock price behavior that is inconsistent with the predictions of familiar models. The research on time series predictability, as a whole, is convincing evidence that expected returns are not constant through time. There are reasonable business conditions stories that can account for time variation in expected returns. However, some of the temporal patterns in returns in particular those relating to calendar turning points are troubling as they defy economic interpretations."*

"*The evidence on cross-sectional anomalies poses a significant challenge to well-established asset pricing paradigms. Yet despite mounting evidence, there is little consensus on alternative theoretical models. As such, the focus of future research should be on the development of such models. Indeed, one of the most significant contributions of this strand of research has been the recognition of potential alternative sources of risk (e.g., risk related to financial distress) and of the potential importance of behavioral models. Importantly, researchers must recognize that the existence of this anomalous evidence does not constitute proof that existing paradigms are 'wrong.' There is the issue of data snooping much of the empirical research on financial market anomalies is predicated on previous research that documented similar findings with the same data. And although many of these effects have persisted for nearly 100 years, this in no way guarantees their persistence in the future. More research is necessary to resolve these issues."*

If stock price behavior that is inconsistent with the predictions of familiar models. Time series predictability research, evidence that expected returns are not constant through time. Some temporal patterns in returns defy economic interpretations. Cross-sectional anomalies poses a significant challenge. Little consensus on alternative theoretical models. Anomalous evidence does not constitute proof that existing paradigms are wrong. Data snooping can create illusions. Many effects have persisted for nearly 100 years, no way guarantees their persistence in the future.

Effect of Liquidity on Size Premium, Frank Torchio and Sunita Surana, 2014

"*The size premium compensates investors for the systematic risk of holding small capitalization companies. Researchers have recognized that the measurement of size premiums can also include the effects of the lack of liquidity that disproportionately affects stocks of smaller sized companies. The lack of liquidity for small-sized firms causes transactions costs of trading a share of stock to be greater, which in turn results in a premium to properly compensate investors for holding these stocks relative to more liquid stocks."*

Manager performance is also prone to reversion

The Selection and Termination of Investment Management Firms by Plan Sponsors, Goyal et al. (2008)

"*We find that plan sponsors hire investment managers after the superior performance but on average, post-hiring excess returns are zero. Plan sponsors fire investment managers for many reasons, including but not exclusively for underperformance. But, post-firing excess returns are frequently positive and sometimes statistically significant. Our sample of round-trips shows that if plan sponsors had stayed with fired investment managers, their excess. Subtracting a constant from the mean return obviously does not change the standard errors and will "make" the excess returns statistically significant. returns would be no different from those actually delivered by newly hired managers."*

Temporal Changes in Shiller's Exuberance

Robert Shiller's Paper on 'The Volatility of Stock markets Prices' published in 1987 uses dividend data and real interest rates to seek evidence that true investment value changes through time sufficiently to justify the price changes. His paper concluded that most of the volatility of the stock market prices appear unexplained. Shiller volatility or fluctuations prove that behavior of markets is not normal. Non-normal distribution series is a widely followed proof of inefficiency in prices.

In the paper, Pal, Mitroi, Shah (2011) the author's analyzed Shiller's data not for the change but for the rate of change. The rate of change in dividend values, interest rates, and market price is used to isolate temporal changes (time durations) defined in days. Though on one side the time duration data illustrate a non-normal distribution and confirms Shiller's non-normalcy finding within the value (fundamental data) and market data, it opens a larger debate suggesting temporal changes to be the reason for market volatility and inefficiency.

The temporal fluctuation

Fluctuations are present all over the place, be it the universe, stock markets or Google search data. A performance divergence plot from Shiller's exuberance can be juxtaposed with a divergence in scientific or social data. It was not just stock market data that was fluctuating; every other organic data set was fluctuating, frequently. A host of data sets tested for stationarity (a statistical test for mean reversion) and found confirming results (Fortune Index, Pal, 2013). The fluctuating outliers were reverting temporally, suggesting Shiller's hypothesis of fluctuation owing to social mood could be incorrect. Inefficiency could be linked with time rather than chaos or psychology.

2015 Divergence Cyclicality

Now that we understand diversion (divergence) as an intrinsic aspect of the natural system which can we explained as instances when mean reversion fails to happen or the instances when prices deviate away from the intrinsic mean. Divergence which may also be referred to as dispersion, diversity, fluctuation etc. is a law of nature and just like reversion is witnessed in all natural data sets. This is how nature and society works.

The whole idea of change is built around the dynamic movement of divergence and reversion. The need for balance comes from the constant push towards imbalance. Which suggests that reversion itself can not be assumed to be reaching in a state of constancy. The natural system is in a constant move between reversion and diversion. A constancy, as a state of utopia, is a static perspective of an underlying ever-changing dynamic system.

Divergence is a part of all natural systems. It's both inter and intra-domain, process that drives natural variables from growth to value or vice versa, a process that powers momentum and reversion in natural variables, be it price, sentiment etc.. A universal law working across different time durations. The reason why natural systems witness outliers, extremities, and the reason why the same natural variables are brought back in balance by reversion. In some natural contexts, we call this balancing events as an economic or social crisis.

Divergence Fallacy

There is nothing wrong with considering divergence as occurring randomly, but its very nature is to create noise that creates deviation, fluctuations, extremities and outliers, which in turn gives the natural system an opportunity to express dynamism, non-linearity, uncertainty, and unpredictability. This is followed by reversion which brings in order, predictability, structure etc.. Till we look at deviations and reversions as a part of the same system we will be looking at the incomplete picture. Both divergence and reversion are universal. Stock Market science is the quest to determine a framework that can explain the co-existence of diversion and reversion together. Analysts, economists, technicians have traditionally been studying and looking at price spreads and divergence for a long time. The spread between economic data, or between traditionally correlated assets. Over a period of time, high correlations between assets prices have given way to poor correlation.

Divergence Cyclicality

In another paper, ‘The Divergence Cyclicality’ Pal, Nistor (2010), the authors explained how grouping assets and ranking them could understand performance divergence. Divergence cyclicality could not only harness reversion and divergence but also extend the case for quantifying relative pair performance.

The paper discussed long-short cases between best and worst ranked assets and creates a divergence cycle indicator to pin point performers and underperformers in a case of global assets. The paper empirically showcased how divergence cyclicality could be used to understand performance divergence in markets. And since divergence is ubiquitous, the divergence cyclicality could be a good proxy for the natural growth and decay process across natural systems, witnessed through the respective data sets. Divergence cyclicality conceptually extended the idea of natural systems that not only expressed reversion, but extreme(s) reverting back to a relative mean.

2010, Mean Reversion Framework

Mean Reversion Indexing, Pal 2012

In their 1985 paper ‘Does the stock market overreact?', DeBondt and Thaler explained the idea of mean reversion and how it leads to the Loser’s portfolio of 3 years outperforming the Winner’s portfolio of the same time. Based on mean reversion, this paper illustrates a new stock selection and trend determining approach. The paper uses an innovative approach to convert price performance data into non-price ranking data, which is positively tested for mean reversion and stationarity.

Mean Reversion Framework, Pal 2015

The original work by Galton on mean reversion in 1886 emphasized relative before absolute, talked about the relation of the variable with the sample average, pointed out the balance between convergence and divergence and showcased cross-domain expression of mean reversion. Though mean reversion as an idea has been in the open domain for 130 years, there has been no attempt to extend the Galtonian definition of natural systems into a framework that could allow for better understanding and functioning of natural systems and also explain the failures of reversion. Any proxy that expresses Galtonian reversion should be simple, relative and universal. This paper takes a stock market case and defines a framework that builds on the Galtonian explanation of a natural system and incorporates the idea of relative ranking, relative average, balancing forces of convergence and divergence, and the universal workability of the framework across domains.

Pal 2015, Momentum and Reversion

Momentum and Reversion have always been seen as independent of each other and never as a composite. This study explains how the two behaviors are not only connected but also get transformed into each other. This dynamic drives not only stock market systems but all natural systems. One reason researchers did not see this composite behavior is because of the focus on independent components (asset prices) rather than a group of components (a collection of stock prices) and because of a lack of an adequate framework to illustrate the two key

behaviors together. Based on the author's previous work on the 'Mean Reversion Framework', which explains how natural systems witness reversion and divergence simultaneously across different periods of time, this paper re-examines the framework for absolute trends. The trends are defined as the percentage number of components that transform themselves out of there original framework classification as Value, Core or Growth. All the three bins exhibit a consistency in transformation over time. More than 70% of all the components in the three bins transform out of their original position to other states. The absolute trends confirm that the Framework is not only a good proxy for understanding group behavior, but is also the reason why momentum gets transformed into reversion and vice versa. This opens up a new approach to constructing investment portfolios, as a combination of momentum and reversion extends to a combination of a trend - counter trend, fundamental value - growth, low-high beta, small-big size, low-high book value and various other styles.

Stock Market Stationarity, Pipas, Pal 2015

Stationarity tests are used to detect mean reversion in a certain dataset. Mean Reversion processes suggest a non-random behavior in a time series (Lo and MacKinley, 1988). Previous research has focused on studying mean reversion at stock price level (Debondt and Thaler, 1985; Lindemann et al., 2004) and considers stationary assumptions to be restrictive for a financial time series (Lo and MacKinley, 1988). The authors look at the concept of stationarity at group specific level as previously defined by the 'Mean Reversion Framework'. The group approach allows for a different interpretation of stationarity, as it overcomes limitations of stationary tests on time series and the problems regarding trend and difference stationarity when it comes to finite data (Cochrane, 1987). The groups approach to look at stationarity also offers an easy way to prove the co-existence of non-random and random behavior in a group (stock market). The stationarity trends are defined as the percentage number of components that exhibit stationarity at the Value, Core or Growth bin levels in the 'Framework'. The trends observed were consistent and showcase a duration dependency. More than 50% of all the components in the three bins exhibit stationarity, suggesting that the 'Framework' is a good proxy for complex and natural systems, which express both random and non-random behavior. The authors combine the absolute trends of bin transformation with the stationarity trends to draw parallels to understand how reversion and divergence co-exist in the 'Framework'. Stationarity at a group level strengthens the case of markets as a complex system and 'Framework' as a good proxy to understand the behavior of such systems.

Markov and the Mean Reversion Framework, Pal 2015

Natural systems witness reversion and divergence simultaneously across different periods of time. This paper tests the performance proxy as mentioned in a previous paper on the 'Mean Reversion Framework' for Markov's transition probabilities. The framework exhibits a stable pattern when tested for STOXX 50, S&P 100, Nikkei 225 and FTSE 100 components across different periods of time from 20 days to 3750 days. The three bin classifications of the framework; value, growth and core exhibit a consistency in growth and decay pattern. Both value and growth exhibit persistence compared to the core bin and tends to decay slowly. While the core bins show a symmetric decay across other bins. Such a probabilistic behavior in group components leads the author to believe that the mean reversion framework is indeed converging and diverging leading to a robust expression of a stock market system. The framework could work across data sets from various domains, confirming the proposed universality of the mean reversion framework.

2015, Reversion Diversion Hypothesis

Reversion Diversion Hypothesis, Pal, 2015

Information is an assumption for modern finance. The Efficient Market Hypothesis uses information to back its case of efficiency. The EMH case is weak, but as Martin Swell (2011) explains that until a flawed hypothesis is replaced by better hypothesis, criticism is of limited value. This paper challenges the information assumption in EMH based on the idea laid out first by Kenneth E. Boulding (1966), highlights the body of work discussing information relevance, information irrelevance, information content since Ball and Brown (1968) and illustrates how 'Mean Reversion Framework' (2015) can be used to re-explain the transformation of information from relevance to irrelevance, also referred to as the 'Reversion Diversion Hypothesis'.

Value and Information

We can not understand 'Value' without understanding information. Information or market efficiency on one side suggests that there is no advantage that can be drawn from information to the extent of beating the market as markets are random and markets reflect all available information. While on the other information or market inefficiency is the contrary view where markets do not reflect available information and there continues to be relevance in information. The relevance or irrelevance of information is not a debate since the conflict between market efficiency and inefficiency is extensively observed and cited and the asset pricing model has faced challenges regarding its validity.

There could be two ways to build a new hypothesis and challenge the EMH. First at the product level and second by dealing with information at the conceptual level.

Granger (1992) mentioned, *"To build a method that consistently produces positive profits after allowing for risk correction and transaction costs and if this method has been publicly announced for some time, then this would possibly be evidence against EMH…Only if a profitable rule is found to be widely known and remains profitable for an extended period can the efficient market hypothesis be rejected…Benefits can arise from taking a longer horizon, from using disaggregated data, from carefully removing outliers or exceptional events, and especially from considering non-linear models.".*

An investment hypothesis in Pal (2015), 'Is smart beta dumb?' showcased how a new indexing methodology can be built by combining value and growth and dual rebalancing. This paper develops the second approach by reexamining information as a building block underlying EMH to illustrate how EMH's weakness as a hypothesis could be owing to its assumptions regarding information.

Information Relevance

It was in 1968 that Ball and Brown considered the content in accounting information, the flow of information, the relevance of information, its predictive powers and its continuity and time dependence. The observed reversion was used as a validation of predictive content in earnings. This relevance was later shown to cause a drift, an anomaly, un unexplainable behavior of a market. Bernard and Thomas (1989) showcased a seasonal positive autocorrelation in partial periods connected to the news followed by a seasonal negative autocorrelation.

An empirical evaluation of accounting income numbers, Ball and Brown (1968)

"Of all the information about an individual firm which becomes available during a year, one-half or more is captured in that year's income number. Its content is, therefore, considerable. If the income forecast error is negative, we define it as bad news and predict that if there is some association between accounting income numbers and stock prices, then the release of the income number would result in the return of that firm's securities being less than would have been expected. Such a result would be evidenced by negative behavior in the stock return residuals around the annual report announcement date. The converse should hold for a positive forecast error."

From establishing content in information to defining bad and good news to connecting it to negative and positive stock behavior (prediction), the authors not only showcased relevance of information but also connected it with predictive behavior.

"If the information is useful in forming capital asset prices, then the market will adjust asset prices to that information quickly and without leaving any opportunity for further abnormal gain. If as the evidence indicates, security prices do in fact adjust rapidly to new information as it becomes available, then changes in security prices will reflect the flow of information to the market. An observed reversion of stock prices associated with the release of the income report would thus provide evidence that the information reflected in the income numbers is useful. Our method of relating accounting income to stock market prices builds on this theory and evidence by focussing on the information, which is unique to a particular firm. Especially we construct two alternative models of what the market expects income to be and then investigate the markets reaction when its expectations prove false...Historically the income of the firms has tended to move together. One study found that half of the variability in the level of an average firm's earnings per share (EPS) could be associated with economy-wide effects. In light of this evidence, at least, part of the change in a firm's income from one year to the next is to be expected. If, in prior years, the income of a firm has been related to the income of the other firms in a particular way, then knowledge of that past relation, together with a knowledge of the incomes of those other firms for the present year, yields a conditional expectation for the present income of the firm. Thus apart from confirmation effects, the amount of new information conveyed by the present income number can be approximated by the difference between the actual change in income and its conditional expectation."

The 1968 paper showcased the utility of information, information flow, and how market's reaction to information. However, the usefulness linked with behavior predictability was validated with observed reversion. If the expected earnings were higher than actual, the news was bad and hence was preceded by a reversion in future stock market returns and vice versa. The idea of reversion linked to the forecasting error showcased dependency of past with future, and how information had influence.

"But not all this difference is necessarily new information. Some changes in income result from financing and other policy decisions made by the firm. We assume that, to a first approximation, such changes are reflected in the average change income through time. King (1966) it was estimated that about 30-40 percent of the variability in a stock's monthly rate of return over the period March 1944 - Dec 1960 could be associated with market-wide effects. Market-wide variations in stock returns are triggered by the release of information which concerns all

firms.Thus, since the market has been found to adjust quickly and efficiently to new information, the residual must represent the impact of new information…persistence in the drifts beyond the announcement month. Since the efficiency of the capital market is largely determined by the adequacy of its data sources, we do not find it disconcerting that the market has turned to other sources which can be acted upon more promptly than annual net income."

There was subjectivity linked with information. There was a news which was market wide and hence adjusted quickly, and there was new information, which was believed to be causing drifts. The idea of new information was open ended and the focus was shifted to quality of data sources. This could have started the information industry wave and its perceived connection with stock market behavior.

Information Drift

In financial economics and accounting research, post–earnings-announcement drift, or PEAD (also named the SUE effect) is the tendency for a stock's cumulative abnormal returns to drift in the direction of an earnings surprise for several weeks (even several months) following an earnings announcement.

"The nagging general question is what kind of equilibrium would support market prices that only partially reflect information as widely disseminated as and free available as earnings. Why the market would appear to react with a surprise to earnings information that is predictable, based on earnings for the prior quarter… Once a firm's current earnings become known, the information content should be quickly digested by investors and incorporated into the efficient market price. However, it has long been known that this is not exactly what happens. For firms that report good news in quarterly earnings, their abnormal security returns tend to drift upwards for at least 60 days following their earnings announcement. Similarly, firms that report bad news in earnings tend to have their abnormal security returns drift downwards for a similar period. This phenomenon is called post-announcement drift."

The drift was an observed anomaly. What kind of market structure caused such an anomaly? What was the reason for a 60-day drift? Why the positive and negative drift? Why was it symmetrical? Why public information was not fully reflected instantly? What was the reason for this partial organic assimilation of news? Why do we call it a surprise? Why the anomaly? Even after nearly 50 years, we have limited clarity on the above questions. The question of the interaction of information with market systems could possibly throw some light.

"The counterargument to market efficiency theory, PEAD is considered a robust finding and one of the most studied topics in financial market literature. The most widely accepted explanation for the effect is investor under-reaction to earnings announcements. According to Bernard & Thomas (1990), PEAD patterns can be viewed as including two components. The first component is a positive autocorrelation between seasonal difference (i.e., seasonal random walk forecast errors – the difference between the actual returns and forecasted returns) that is strongest for adjacent quarters, being positive over the first three lag quarters. Second, there is a negative auto correlation between seasonal differences that are four quarters apart."

Anomalies pose a potential risk if they remain unexplainable for a generation. The interesting part is regarding the symmetry of the drift and the tendency for reversion. The fact that a positive trend of three-quarters changes to a negative tendency is not surprising. In our focus on

the influence (non) of information and the predictability (non) of errors, we could have ignored the conceptual structure of the market leading to its behavior.

Information Efficiency

Random Walks in Stock - Market Prices, Eugene F. Fama, 1965

"The assumption of the fundamental analysis approach is that at any point in time an individual security has an intrinsic value (or, in the terms of the economist, an equilibrium price) which depends on the earning potential of the security. If actual prices tend to move toward intrinsic values, then attempting to determine the intrinsic value of a security is equivalent to making a prediction of its future price; and this is the essence of the predictive procedure implicit in fundamental analysis. An "efficient" market is defined as a market where there are large numbers of rational profit-maximizers actively competing, with each trying to predict future market values of individual securities, and where important current information is almost freely available to all participants. In other words, in an efficient market at any point in time, the actual price of a security will be a good estimate of its intrinsic value. Now in an uncertain world, the intrinsic value of a security can never be determined exactly. Thus, there is always room for disagreement among market participants concerning just what the intrinsic value of an individual security is, and such disagreement will give rise to discrepancies between actual prices and intrinsic values. In an efficient market, however, the actions of the many competing participants should cause the actual price of a security to wander randomly about its intrinsic value. The discrepancies between actual prices and intrinsic values are systematic rather than random in nature, then knowledge of this should help intelligent market participants to better predict the path by which actual prices will move toward intrinsic values."

Ball, Brown and Fama were contemporaries. Ball and Brown's challenge to the efficient view linked with information were based on content, relevance, and prediction. But Fama's argument regarding information was based on unpredictability, leading to information irrelevance at a performance (beating the market) level. According to Fama information's predictive content was an illusion because intrinsic value itself was a moving target. Information and market participants consistently pushed the equilibrium value, so much so that Fama believed that the current security price was the equilibrium price. The level of disagreement between the players and the discrepancies made prediction a failed effort. Moreover, there was a neutralizing of the effort leading to prices wandering around the intrinsic value. The past information had no meaning, hence, independence was to be expected. Though Ball and Brown's arguments were substantiated and perfectly contrasting with a robust anomaly backing it, Fama's EMH continued to stand firm for a few decades. The reason could be because Fama's Hypothesis was based on a framework which explained how market participants and information interacted with the market. Now that Ball and Brown have more support and the 'Inefficient' school's strong research backed argument has further weakened the EMH hypothesis, a new framework explaining information and market could lead to a new hypothesis.

"When the many intelligent traders attempt to take advantage of this knowledge, however, they will tend to neutralize such systematic behavior in price series. Although uncertainty concerning intrinsic values will remain, actual prices of securities will wander randomly about their intrinsic values. Of course, intrinsic values can themselves change across time as a result of new information. This says that the "instantaneous adjustment" property of an efficient market implies that successive price changes in individual securities will be independent. A market

where successive price changes in individual securities are independent is, by definition, a random-walk market. Most simply the theory of random walks implies that a series of stock price changes has no memory - the past history of the series cannot be used to predict the future in any meaningful way. The future path of the price level of a security is no more predictable than the path of a series of cumulated random numbers. It is unlikely that the random-walk hypothesis provides an exact description of the behavior of stock-market prices. For practical purposes, however, the model may be acceptable even though it does not fit the facts exactly. Thus, although successive price changes may not be strictly independent, the actual amount of dependence may be so small as to be unimportant."

The mention of intrinsic value and equilibrium points to Fama's acceptance that market does need a dynamic mean to oscillate too. As efficiency means a move towards value, which is intrinsic to the system under study. Fama also talks about disagreements between market participants leading to discrepancies, noise, or divergence from intrinsic value, the very reason market needs intrinsic value for equilibrium. The move towards equilibrium is key for Fama's hypothesis.

His argument revolves around the divergence from and reversion to intrinsic value, which is not something static, but something constantly changing in time. This was a similar thought when Galton laid out in his 1884 paper, 'Regression towards mediocrity in hereditary stature'. The 'Mean Reversion Framework' (2015), explained how Galton laid down a framework for functioning of natural systems with a balance of reversion and diversion, he primarily focussed on illustrating reversion and did not build a case on divergence. Fama just like Galton acknowledges diversion (discrepancy) but does not focus on it. The consideration of discrepancy as noise is the reason, EMH afford's to accept anomalies and ignore inefficiency, relevance and dependence choosing to stick to independence, irrelevance and efficiency of the system to revert over the composite of reversion and divergence, a broader framework more close to actual market functioning.

History of Debate

Even with the Joint hypothesis problems, attacks on randomness, research which highlighted the benefits of collection information outweighed the costs, proof that there was a section of the market which traded anything but information, market's innate ability to generate information, markets reaction to new information, existence of predictability with randomness the EMH remains the weak but not redundant hypothesis. This is unlike the CAPM, which has a stronger case of redundancy against it. The table below illustrates the history of the debate in the context of the information. A part of the chronology has been updated from Swell (2011), 'History of the Efficient Market Hypothesis'.

Information Framework

Information drives the knowledge process. Even if a part of it keeps getting reflected by the market, the information generation is a continuous process, which makes information an entity which can never reach an ideal state of being fully reflected in the market. The 'fully reflected" state can not be considered an ideal state, it is one of the many states which the interaction of information and market brings. Our understanding of information should improve in time, but it is important to consider alternative ways to look at information and its interaction with the market entity. Kenneth Boulding not so coincidentally was also a contemporary of Ball, Brown and

Fama. While Ball and Brown were looking at information relevance, Fama was looking at the process of information becoming irrelevant, Kenneth laid down the guidelines that could assist in structuring an information framework.

Year	Researcher	Idea	Efficient	Inefficient	Both
1965	Sameulson and Fama	Properly antcipated prices fluctuate randomly	*		
1966	Boulding	Information			*
1968	Ball and Brown	Information content		*	
1970	Fama	"fully reflects"	*		
1971	Kemp and Reid	Share price movement non-random		*	
1971	Bagheot and Hirshleifer	Strong hypothesis true	*		
1971	Niederhoffer	Impactless news headlines	*		
1976	Grossman	Efficiency eliminated private incentive to collect information	*		
1978	Ball	Excess Returns after announcements		*	
1979	Shiller	Voltatility greater than predicted		*	
1978	Jensen	EMH is valid	*		
1979	Radner	EMH linked to finite information	*		
1980	Grossman and Stiglitz	Impossible for market to be perfectly informationally efficient		*	
1981	Shiller	Stock market fluctuations		*	
1982	Milgrom and Stokey	Private information cannot create incentives to trade	*		
1986	Black	Noise traders trade on anything other than information		*	
1986	French and Roll	Market generates its own news		*	
1989	Cultler	News does not explain market movement	*		
1989	Eu and Shin	Interdependence exists among markets		*	
1991	Jackson	Equilbrium possible with fully revealing prices and costly information	*		
1996	Chan	Market responds gradually to new information		*	
1999	Bernstein	Information's marginal benefits increase its marginal costs		*	
2005	Malkiel	Managers do not outperform	*		
2015	Pal	Mean Reversion Framework			*
2015	Pal	Momentum and Reversion co-exist			*
2015	Pal and Ferent	Stock markets are natural dynamic systems			*

Boulding talked about this transformation of information into knowledge and how the assumption of knowledge being costless was incorrect. He explained how the income of arbitrageurs might be regarded as the cost of acquiring knowledge, which is necessary to operate the market and the other people are willing to pay this rather than become arbitrageurs themselves. What is an anomaly today does not mean it will stay an anomaly in future. An anomaly proves that our understanding of how market interacts with information are limited and our current systems are limited in their potential to address these gaps.

John C. Bogle has argued that no value premium exists, claiming that Fama and French's research is period dependent. *"While gross performance reverts to the mean self-evident pattern of mean reversion. Yet as we observe these extended cycles of mean reversion, it must occur to you that investors ought to be able to capitalize on them, riding one horse until it tires, then leaping to the other."*

Boulding talked about how static systems were weak in capturing the dynamics of systems. There was a clear dependence of past information with the future. Decision-making theory which could not consider how inputs from the past could determine the future was "pretty empty". Knowledge for Boulding was integral to the dynamic system, one could not separate the dynamic system from the information it faced and the information it generated. The system thinking was more important than the content of the information. The system was above the information and its content. This was a conceptual thinking ahead of its time. While randomness was getting rediscovered, dependence and independence of subsequent price changes were debated, EMH took center stage, Boulding's work on information remained relevant.

The Economics of Knowledge and the Knowledge of Economics, Kenneth E Boulding, 1966

"The absence of any unit of knowledge itself, however, and perhaps the intrinsic heterogeneity of its substance makes it very difficult to think of the price of knowledge as such, and indeed has probably contributed to a certain resistance which we feel to thinking of knowledge as a commodity…The theory of economic development is part of the general problem of evolutionary change, and its poor condition reflects the general poverty of theory of dynamic systems. Throughout the sciences, physical, biological, and social, we are still really more at home with equilibrium systems than we are with dynamic systems…The recognition that development is essentially a knowledge process has been slowly penetrating the minds of the economists, but we are still too much obsessed by mechanical models, to the neglect of the study of the learning process which is the really key to development…The decision is always the choice among alternatively perceived images of the future. The study of the decision, therefore, must concentrate on how these images of the future are derived from the information inputs from the past. The epistemological theory of decision making is, of course, pretty empty unless we can specify ways in which the inputs of the past determine the present images of the future."

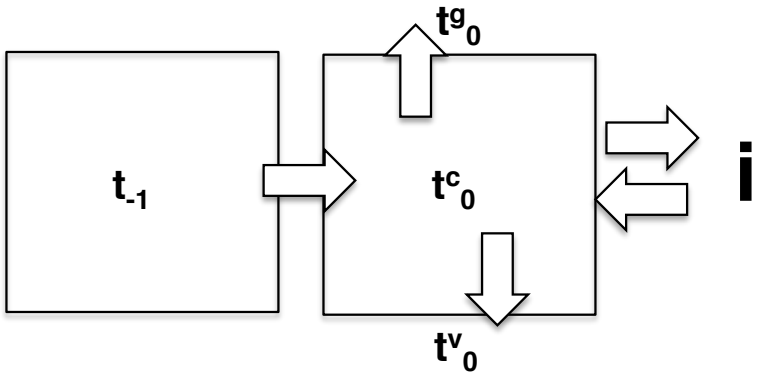
There are challenges to measuring information and related knowledge owing to its heterogeneity and society's limited understanding of dynamic systems. Researchers have higher comfort with equilibrium compared to something dynamic. Dynamic systems could have multiple states of equilibrium. The linkage of past with future is integral to decision making, so independence of subsequent changes is an empty argument.

"We have here a certain epistemological paradox, that where knowledge in an essential part of the system, knowledge about the system changes the system itself. This is a kind of generalized

Heisenberg Principle, which is particularly troublesome in the social sciences. What this means, of course, is not that the knowledge is unattainable, but that we must regard it as a part of a total dynamic system. That is to say, we are not simply acquiring knowledge about a static system which stays put, but acquiring knowledge about a static system which stays put, but acquiring knowledge about a whole dynamic process in which acquisition of the knowledge itself is a part of the process…Thus in the case of the operations of a market and the behavior involved in buying and selling, it is doubtful whether the knowledge of economics as such makes very little difference. I am inclined to attribute a good deal to good luck and noneconomic forces. An enormous intellectual task still awaits the economist. We are a very long way from writing finis to this chapter of the human enterprise. We still can not handle some of the most elementary problems regarding economic development, economic dynamics, the function of the price system."

Knowledge about the system changes the system, which means there are both internal and external influences. This indicates stages of dependence and independence between price movements. Equilibrium is essential for any system. Galton's 1884 work illustrating a natural system which moves towards equilibrium was an observation that laid the foundation for linear regression. Fama's work also acknowledges the dynamic nature of stock markets but assumes equilibrium to be an intrinsic value, summarizing all information and activity around it. Just like CAPM, though not redundant, EMH is one of the many idealized states, which stock market systems witness. Markets are dynamic systems with both internal and external informational influences. These influences drive the constant and consistent reversion and diversion process. It is in its many transient states that information goes through cycles of relevance and irrelevance, making markets efficient and inefficient. The assumption of independence between subsequent price movements is not incorrect but incomplete. It is time for markets to accept that random and non-random behavior coexist, the very reason for a new framework.

Building on what Boulding mentioned, information leading to knowledge is integral to the 'Framework'. It is hard to separate the system from the information (i) it interfaces and generates. The system is above the information and its content. The current state of the system is connected to its past state and the component or group specific information it generates is a function of the current state of the group and its components.



Reversion Diversion Hypothesis

The 'Reversion Diversion Hypothesis' is intrinsic to natural systems including stock market systems. The hypothesis is based on the 'Mean Reversion Framework', which explains how reversion and diversion can not be seen independently. It is the reversion and diversion process that drives assets into price momentum and into price reversion. The 'Framework' classifies price momentum into 'Value' and 'Growth' and illustrate how 'Value' and 'Growth' could be statistically driven. The Reversion Diversion Hypothesis addresses the failings of EMH in addressing discrepancies, by building a system approach to explaining how the idea of intrinsic value could be extended from the current price to something relative and dynamic. EMH accepts that the market needs a dynamic mean to oscillate to but does not develop a comprehensive framework to reconcile the arguments against market efficiency. EMH fails to address information in a comprehensive way and its insistence on market efficiency and independence of past from the future despite a large body of work on market inefficiency renders it incomplete. The 'Framework' was tested across time frames and across various groups for absolute and stationarity trends. Its transformation across states was orderly and significant. The Reversion Diversion Hypothesis proves that random and non-random systems, inefficient and efficient market systems, dependence and independence, relevance and irrelevance of information could co-exist.

Adaptive Market Hypothesis

Adaptive Market Hypothesis (AMH) embraces Efficient Market Hypothesis (EMH) as an idealization that is economically unrealizable, but which serves as a useful benchmark for measuring relative efficiency. AMH's adaptability to changing dynamics of the market suggests that investors are potentially capable of an optimal dynamic allocation. There is nothing wrong here in the direction pointed by Andrew Lo. However the assumption that human innovation driven adaptability, is the way ahead is an open-ended solution. This leaves little room for system thinking and overrules the possibility that natural systems could explain human behavior rather than vice versa. AMH just like EMH is based on a set of assumptions, which are good for illustrating market idealizations but lack in terms of addressing contradictions. This makes both AMH and EMH a system philosophy rather than a system framework. Reversion Diversion Hypothesis (RDH) (Pal, 2015) reconciles the contradictory assumptions into a statistical framework that addresses the limitations of EMH, AMH and extends the idea of a natural system functioning to markets.

Biological Paradox

Adopting a Darwinian approach in the time of complex networks may not necessarily be a step back, but focusing on human decision making as a driver for stock market anomalies, which are assumed to correct in time, is a very simplistic way to map dynamic systems. If we talk about biological and natural systems, natural selections, sociobiology, evolutionary dynamics then we should first define the working of a natural system outside the context of risk preference. Giving markets a context of risk preference is too narrow a definition of contextually-dependent dynamic systems. Before we talk about learning and adapting, we should understand the very nature of natural systems that witness reversion and diversion behavior irrespective of the fact whether it's price, height or emotion.

Unlike AMH, RDH explains human decision making as a part of the reversion - diversion

statistical framework. The human adaptiveness or innovation as Lo suggests is not capable of altering the reversion - diversion multi durational process as first mentioned by Schumpeter in Business Cycles. Human innovation and decision-making in the context of the reversion - diversion framework can allow for better understanding of group behavior and the better understanding of uncertainty that is a part of every natural system.

Why the bubble burst: US stock market performance since 1982, Evans (2003)

"LeRoy (1989) offers one of the most information and extensive treatments of the history of the theory of efficient capital markets and points out that random walk hypothesis was itself an insufficient theory of equity price determination. For one, Cowles (1933) had already produced work that suggested that stock prices exhibited serial correlation. This evidence appeared to be inconsistent with the random walk hypothesis and at odds with the theory of perfect competition as it applied to equity markets. However, the research of Working (1960) produced evidence that serial correlation was not necessarily incompatible with the random walk hypothesis, illustrating that if the data generated by a unit root process 'were averaged over time spurious correlation between successive changes would result (p. 1587). Thus, such correlation in stock price behavior was the predictable and uneventful consequence of its random behavior."

Conclusion

It all started in 1850's when Jules Regnault talked about stock market science and laid the foundations of Random Walk Hypothesis. His hypothesis was used by Louis Bachelier who had anticipated many of the mathematical results developed by Albert Einstein in1905. Regnault explained how short-term deviations were balanced by long-term reversion, social behavior was connected to the price mean, reversion was a stronger force than deviations, there was an essential order in stock market systems and though anticipating of future from past was hard, the approximation of laws was based on the understanding of the past. The idea of random walk emerged from the coexistence of deviation and reversion at a multi duration level.

Francis Galton (1884) and Vilfredo Pareto (1886) brought statistical behavior into the mainstream. The idea of normality and non-normality took mass fancy as a generation of researchers chose to be on either side of the debate. John Rae's (1834) work on durational inconsistency in human behavior saw the emergence of the behavioral explanations. Value, Behavior, and Duration are inseparable ideas but the narrow definition of Value as primarily fundamental has added to the academic confusion around inconsistencies and created an investing style bias.

Graham and Dodd's (1920-1934) approach was about inexpensive stock picking which was deviating away from Value. Their approach was not scientific but more about common sense. This definition of Value was not broad enough and needed redefinition.

MPT (1952) assumptions are challenged. There was a mismatch of exponential utility assumption with normally distributed returns. The constancy assumption of correlations between assets was for model simplification. The observation that good months would tend to be followed by good months, bad months by bad months illustrates naivety. Markets also have good months followed by bad months and vice versa. Research on capital structure debated on its relevance and irrelevance. Ball and Brown (1968) saw relevance in earnings while Bernard and Thomas (1989) saw information drift (anomaly). The popularity of dividend despite its

academic irrelevance is considered as a behavioral bias. A few even consider such linear modeling as an acceptable approximation (Lintner, 1962).

M&M's theory independent from any assumption of individual risk preferences illustrates the disconnect between risk and return, but still emphasizing behavior. The behavior which suggests that a certain portfolio price can diverge in time from the weighted average portfolio price (mean), but over the longer period that divergence can't sustain away from the natural central tendency towards the mean. This suggested that dividend irrelevance can only be driven by an intrinsic efficiency of the environment in which the company or set of companies operates. The environment which delivers an order despite it's uncertainty. Chang and Dasgupta (2009) showed that even with random financing and with no apparent target, leverage may appear to be mean reverting.

Cootner (1962) was very close to understanding the role extremities (extremes) played in a dynamic system. The farther the extreme went the stronger the recoil. This is what Galton (1886) talked in his seminal paper on "Regression towards mediocrity in hereditary stature". Though Cootner acknowledged that the recoil (reversion) was governed by the random process and there was co-existence of random with non-random behavior, he saw that things were random but substantially different, a behavior with a noise, which created opportunities and possibilities for arbitrage. He identified negative autocorrelation at extremes which diluted near the mean (recoil owing to reversion). For Cootner there was a clear difference between the center and the extremes. He explained two set of probability distributions, one near the mean expressing normal distribution, while near the extreme more leptokurtic. This again expresses the complexity of a natural system behaving both normal and leptokurtic at the same time, depending on which classification of the group was studied, near the mean, or near the extreme. Classifying the extreme and the mean and how the system oscillates from one end to the other, Cootner talked about the idea of the coexistence of several trends, with different holdings and how Mean, a function of duration was more prominent than deviation. He also explained how the dynamic system, trends, durations and boundaries caused continuity and reversal of trends. As the duration increased the extreme(s) reversion increased. Cootner just like Galton was so close in identifying the workings of a natural system, but just like Galton who identified reversion in the first place but failed to detail the role of divergence with reversion, Cootner failed to reconcile the random with the non - random as a framework.

CAPM (1964) was again under attack for the same reasons. The model explained how low risk delivered a low return and vice versa. But the reality of markets witnessed other cases where low risk delivered higher returns and vice versa. Markets were behaving as they were supposed to do, winners sometime win and sometime lose while losers sometime lose and sometime win, but CAPM failed to acknowledge the non-linearity of the markets, which encompassed both reversion and diversion (creating momentum). The paradox was that CAPM worked in Japan but failed in the majority of the markets, a poor formulation but not redundant.

Treynor and Mazuy's (1966) suggested how the investing process is dependent on market fluctuations and on market dynamics. Hence, the manager's skill is connected to the ability to identify underpriced industries and companies. Or in other words, the extreme opportunities come from the diverged asset prices from the relative mean. Hence outguessing was a fruitless activity. No one could do it. One could not hold fund managers responsible for failing to foresee.

This is when research reached factors. Though 'Size' is the most important factor explaining

stock market returns the possibility of Size being a proxy was first mentioned in Banz (1978). For forty years the factor investing industry has looked for answers. Size inconsistency is sometime explained as neglect effect owing to information unavailability. The factor might be important, but it's not universal, and the possibility that Size is still a proxy, left the door open for future research. The factor that challenges the Size effect consistently can resolve this debate of what is the proxy that drives Size and what is more important than Size. This new Size replacing factor would also leave the information embedded in other subfactors as unimportant. Since returns tend to run in cycles, there have been long periods when large-cap stocks have outperformed smaller stocks. Small-cap stocks also have higher price volatility, which translates into higher risk. Some investors choose the middle ground and invest in mid-cap stocks seeking a tradeoff between volatility and return. The Size effect exists in a certain set of market situations and vanishes in a different set of situations. The premium may be visible, but it has failed to express itself as a universal factor, the reason we still see newer factors emerging.

The other question is if Beta is really driven by a sequence of factors, which one is more important that the other. Though there was a clear need to adopt non-linear thinking, a generation of researchers focussed on understanding the linear sensitivity of asset returns to various factors. This was the reason for overlapping research regarding the workability and non-workability of static models to explain the dynamic, noisy, multi-durational nature of the market. The factor school remains fragmented with more than a few reasons to believe that there is a factor beyond the Size factor and despite industry clustering behind Size, the question asked by Banz (1978) has the potential to change everything we have assumed about the Size factor.

While all this confusion continued, there was no disagreement regarding the statistics of mean reversion. What goes up comes down and vice versa. Campbell and Shiller (1988) said that the simple theory of mean reversion was basically right. Fama and French (1989) also suggest that valuation ratios forecast five-year returns with quantifiable accuracy. Moreover, there were assumptions in CAPE and in its interpretation. First; CAPE was considered as a good measure that could predict; Second; Extremes in CAPE were expected to reverse; Third; Extremes though subjective were expected to work objectively, as a timing indicator annually. Despite such strong evidence of reversion and related extremities, Shiller and Campbell did not delve into the statistical nature of market systems.

John Bogle (2001) wonders why despite this powerful, self-evident pattern of mean reversion, the mutual fund industry both revels in it and panders to it. Bogle's relying on reversion is a weak case against factor investing because he commits the same mistake of focusing on one side of the framework, the reversion to the mean, ignoring the science that along with reversion comes diversion and extremities which indeed create opportunities. Bogle considers it vain to analyze extremities ignoring the historical work on deviations and extremities around mean and how a generation of researchers have shown persistent anomalies.

Relying on behavioral explanations to explain diversion (anomalies, failure of reversion) did not further the cause of behavioral investing. The case for what causes inefficiency, what explains it has been simplified by the behavioral economists, using behavioral reasons for anomalies (cases of inefficiency). Unfortunately, this has lead to the subject becoming more of a subjective discourse rather than something objective. Ideas like anomalies are here to stay, markets can't be predicted, but some forms of extreme anomalies can be profited from lead to the idea of behavioral funds. Adjusted for risk, behavioral funds were tantamount to value investing. Behavioral finance fund performance proved that anomalies can't be identified and exploited on

a persistent basis. The behavioral model also accepts it's temporal limitations. The five aspects Thaler points out in his paper 'End of behavioral finance' (a term he confidently used to suggest that behavioral finance will be the only form of finance left) are 1) The equity premium puzzle, 2) Predictability, 3) Dividends, 4) Volatility and 5) Volume myth. All of these five aspects can be explained as mean reversion failures.

Though there is no guarantee of the persistence of premium, the behavioral school continues to look for reasons for Size premium and believes that premiums could be based on the human irrationality. This idea fails to stand on its own as both irrationality and rationality itself are time varying. There is a lot of literature written about time varying volatility and behavioral finance also accepts its temporal limitation. This pushes us back into the acceptance of diversion (noise, error, fluctuations, divergence) as driven by time-varying volatility. Manager selection is also prone to reversion, Goyal et al. (2008). Robert Shiller's Paper on 'The Volatility of Stock markets Prices' published in 1987 uses dividend data and real interest rates to seek evidence that true investment value changes through time sufficiently to justify the price changes. His paper concluded that most of the volatility of the stock market prices appear unexplained. Shiller's volatility or fluctuations prove that behavior of markets is not normal. Non-normal distribution series is a widely followed proof of inefficiency in prices. In the paper, Pal, Mitroi, Shah (2011) the author's analyzed Shiller's data not for the change but for the rate of change. The rate of change in dividend values, interest rates, and market price is used to isolate temporal changes (time durations) defined in days. Though on one side the time duration data illustrate a non-normal distribution and confirms Shiller's non-normalcy finding within the value (fundamental data) and market data, it opens a larger debate suggesting temporal changes to be the reason for market volatility and inefficiency. This is not surprising considering the temporal limitation of behavioral reasoning.

This is not the first time that non-normality has required more than a behavioral explanation. Looking for non-normality and power law everywhere have recently come under criticism. Despite its popularity, the power law has not been without its failures. Evelyn Fox Keller mentioned that the current assessments of the commonality of power laws are probably overestimates. According to Cosma Shalizi, the preferential attachment is not necessarily the one that naturally occurs in nature. It's not obvious how to decide which ones are the mechanisms that are actually causing the power law mechanism in the real world. According to Hagel (2007), Gaussian distributions tend to prevail when events are completely independent of each other. As soon as you introduce the assumption of interdependence across events, Paretian distributions tend to surface because positive feedback loops tend to amplify small initial events. Beyond Gaussian averages: Redirecting organization science toward extreme events and power laws, Andriani, McKelvey 2007

Jan et al (1999) explained how the two distributions can be reconciled for example if we consider one of the most famous systems in theoretical physics, the Ising model of a magnet. In its paramagnetic phase, the Ising model has a magnetization that fluctuates around zero. Suppose we measure the magnetization 'm' at uniform intervals and calculate the fractional change '$\delta = (\Delta m)/m$' between each successive pair of measurements. The change 'Δm' is roughly normally distributed and has a typical size set by the width of that normal distribution. The 1/m on the other hand, produces a power-law tail when small values of m coincide with large values of Δm. Natural systems are replete with phase changes like Ising model. And since aspects of the same natural systems can exhibit the two respective distributions, this suggests that both distributions could not only co-exist in natural systems but also could be linked with its

dynamic nature.

In another paper, ‘The Divergence Cyclicality’ Pal, Nistor (2010), the authors explained how grouping assets and ranking them could understand performance divergence. Divergence cyclicality could not only harness reversion and divergence but also extend the case for quantifying relative pair performance.

The original work by Galton on mean reversion in 1886 emphasized relative before absolute, talked about the relation of the variable with the sample average, pointed out the balance between convergence and divergence and showcased cross-domain expression of mean reversion. Though mean reversion as an idea has been in the open domain for 130 years, there has been no attempt to extend the Galtonian definition of natural systems into a framework that could allow for better understanding and functioning of natural systems and also explain the failures of reversion. Any proxy that expresses Galtonian reversion should be simple, relative and universal. This paper takes a stock market case and defines a framework that builds on the Galtonian explanation of a natural system and incorporates the idea of relative ranking, relative average, balancing forces of convergence and divergence, and the universal workability of the framework across domains.

The framework (Pal, 2015) explains how stock market systems could work in a relative ranking behavior expressing ‘Herding’, ‘Randomness’ and ‘Order’ (Parunak et al., 2004). Granger (1992) mentioned, “To build a method that consistently produces positive profits after allowing for risk correction and transaction costs and if this method has been publicly announced for some time, then this would possibly be evidence against EMH…Only if a profitable rule is found to be widely known and remains profitable for an extended period can the efficient market hypothesis be rejected…Benefits can arise from taking a longer horizon, from using disaggregated data, from carefully removing outliers or exceptional events, and especially from considering non-linear models.”.

The ‘Reversion Diversion Hypothesis’ RDH (Pal, 2015) is intrinsic to natural systems including stock market systems. The hypothesis is based on the ‘Mean Reversion Framework’, which explains how reversion and diversion can not be seen independently. It is the reversion and diversion process that drives assets into price momentum and into price reversion. The ‘Framework’ classifies price momentum into ‘Value’ and ‘Growth’ and illustrate how ‘Value’ and ‘Growth’ could be statistically driven. The Reversion Diversion Hypothesis addresses the failings of EMH in addressing discrepancies, by building a system approach to explaining how the idea of intrinsic value could be extended from the current price to something relative and dynamic. EMH accepts that the market needs a dynamic mean to oscillate to but does not develop a comprehensive framework to reconcile the arguments against market efficiency. EMH fails to address information in a comprehensive way and its insistence on market efficiency and independence of past from the future despite a large body of work on market inefficiency renders it incomplete. The ‘Framework’ was tested across time frames and across various groups for absolute and stationarity trends. Its transformation across states was orderly and significant. The RDH proves that random and non-random systems, inefficient and efficient market systems, dependence and independence, relevance and irrelevance of information could co-exist.

Adaptive Market Hypothesis (AMH) embraces Efficient Market Hypothesis (EMH) as an idealization that is economically unrealizable, but which serves as a useful benchmark for

measuring relative efficiency. AMH's adaptability to changing dynamics of the market suggests that investors are potentially capable of an optimal dynamic allocation. There is nothing wrong here in the direction pointed by Andrew Lo. However the assumption that human innovation driven adaptability, is the way ahead is an open-ended solution. This leaves little room for system thinking and overrules the possibility that natural systems could explain human behavior rather than vice versa. AMH just like EMH is based on a set of assumptions, which are good for illustrating market idealizations but lack in terms of addressing contradictions. This makes both AMH and EMH a system philosophy rather than a system framework. RDH (Pal, 2015) reconciles the contradictory assumptions into a statistical framework that addresses the limitations of EMH, AMH and extends the idea of a natural system functioning to markets.

Unlike AMH, RDH explains human decision making as a part of the reversion - diversion statistical framework. The human adaptiveness or innovation as Lo suggests is not capable of altering the reversion - diversion multi durational process as first mentioned by Schumpeter in Business Cycles. Human innovation and decision-making in the context of the reversion - diversion framework can allow for better understanding of group behavior and the better understanding of uncertainty that is a part of every natural system.

In the book, Why the bubble burst: US stock market performance since 1982, Evans (2003), the author cites, "LeRoy (1989) offers one of the most informational and extensive treatments of the history of the theory of efficient capital markets and points out that random walk hypothesis was itself an insufficient theory of equity price determination. For one, Cowles (1933) had already produced work that suggested that stock prices exhibited serial correlation. This evidence appeared to be inconsistent with the random walk hypothesis and at odds with the theory of perfect competition as it applied to equity markets. However, the research of Working (1960) produced evidence that serial correlation was not necessarily incompatible with the random walk hypothesis, illustrating that if the data generated by a unit root process were averaged over time spurious correlation between successive changes would result (p. 1587). Thus, such correlation in stock price behavior was the predictable and uneventful consequence of its random behavior."

The question of beta vs. smart beta has evoked a lot of debate. Investment businesses have thrived on both sides of the question. The smart beta has carved a new business for itself calling the beta dumb, justifying the need for smart beta solutions and advocating a need for moving away from the popular benchmarks that are constructed on market capitalization methodology. The smart beta is built on the premise that popular cap-weighted benchmarks are wrong and inefficient. Even smart beta itself has come under attack. But there has been limited work explaining smart beta and beta together. The debate has added to the list of other debates like; efficient-inefficient markets, random-nonrandom behavior, bell curve or power law etc.. On one side researchers cite proofs that beta is an inefficient measure of risk and on the other side there are proofs that beta is not redundant. The framework explains the debate on both sides to explain why the beta is neither dead nor dumb and it's the smart beta thinking that is inaccurate and needs to evolve.

This study illustrates how history has been ignored to validate partial beliefs regarding the statistical nature of markets, which exhibit both reversion and diversion. For over 150 years, not only the idea of randomness (deviation) has been shown to exhibit reversion and normality, but both deviation and reversion have existed because natural systems (Groups) work between stages of imbalance and balance. Trend and no-trend are not incompatible. Random and non-

randomness work together. The aspects of negative and positive correlation in time is possible and observable. This is why Value is statistical in nature and fundamental content of Value is an insufficient and narrow definition. If Value is statistical, factors like Size are mere proxies. It's the statistical nature of Value that explains its transformation to Growth and its selection as a part of a portfolio (Group). We can choose to ignore what LeRoy (1989) mentioned about the compatibility of serial correlation with random walk hypothesis, but the evidence is clear that the theory of pure randomness is incomplete if we don't look at it from the reversion-diversion process.

Granger (1992) and Swell (2011) said that the only way to challenge the redundancy of current modern finance theory and create a new theory was to explain the conceptual framework, validate it statistically, and build an index that beats the market. The generation of researchers has neglected the historical thinking around the reversion-diversion reconciliation. The Mean Reversion Framework presents the concept and its statistical validation. The author has constructed the RMI™ Index using the framework and illustrated risk weighted excess returns vs. the benchmark. Academic thought now has an opportunity to clarify the misconceptions about Value and the functioning of markets.

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