The Beta Maths

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Abstract

The two Nobel Prizes awarded in Economics in 1990 [1] and 2013 [2] define the boundaries of Modern Portfolio Theory (MPT). Size is the pillar for both the models. The 1990 winners assumed market to be driven by Market Capitalization (MCAP) [3] size, while the 2013 winner explained that factors like 'Small Size' [4] can explain portfolio performance better than 'Big Size' [5]. This conflict between the two ideas has bifurcated the industry into benchmark investing (MCAP) [6] and everything else not MCAP (Smart Beta) [7]. The fact that benchmark investing and smart beta is expected to be 50% of the USD 100 trillion investment management industry in 2020 [8] makes it imperative to seek a coherent argument and a conflict resolution.

This paper explains how a relative ranking on size [9] could reframe the argument between big and small size as a debate around why the RGR (Rich Get Richer) [10] and the PGR (Poor Get Richer) [11], which are expressions of the statistical law of 'Mean Reversion' [12] and its failure. The paper explains how benchmark investing and smart beta are not conflicting ideas and how a set of portfolio weights which allocate relatively shorter time (t) to big size companies and relatively longer time (T) to small size companies could have higher probability of offering more optimal set of portfolios among all the possible combinations involving different time periods and different sizes. The architecture [13] could bring Finance closer to Physics [14] and illustrate how assumptions of MPT [15] may be irrelevant today.

The Beta Maths

Size (S) is the top variable driving benchmark investing. Size is the other name for Market Capitalization. Market Capitalization is number of shares multiplied by price of the stock.

S (i)

P (Sb) = 1 (ii)

Benchmarking investing pioneered by Vanguard [16] assumes that popular MCAP weighted benchmarks efficiently allocate capital. In other words they assume that what is big in size will get bigger, or if Apple is overweighted in the S&P 500, it is not a problem, the stock will continue to become bigger in size. This claim is incorrect not only from an intuitive point of view, but is also based on a poor historical assumption which started in 1880's [17] with the creation of the S&P 500. The assumption suggests that benchmark investing is a certainty (ii), which is not true (iii) as the Rich do not always Get Richer (RGR), big companies not always get bigger, sometimes Cisco happens.

P (RGR) $\neq$ 1 (iii)

An exercise of relatively ranking yearly change of size [19] for the S&P500 stocks and monitoring the dynamics over successive periods can prove that the probability for change in size as a variable is indeed less than perfect (iv). The change of size will indicate that the big size company is not always growing faster than the small size company and vice versa. A study of relative price change is a good proxy to understand the dynamism and transformation of components from one extreme to another [18].

$P(\Delta Sb) < 1$ (iv)

Eugene Fama [20] explained that small size (Ss) companies had a tendency to do better than big size (Sb) companies. Fama referred to this as the 'Size Factor' [21]. The existence of the respective size inefficiency suggests that Fama believed that there is a high certainty for the 'Size Factor' to deliver, which means again a probability near 1 (v). This is again contestable if we look at the change in size as a variable and the dynamism of the variable as a reality, which can not be 100% sure that the bottom ranked size companies will always grow faster than their big sized peers.

$P(Ss) \sim 1$ (v)

This translates into an argument that probability of small size companies (Ss) continuing to drive portfolio returns better than big Size (Sb) is not perfect or near perfect. The respective probability is not equal to 1. Fama uses the world "unexplained" [22] more than a few times when discussing size factor and also highlights cases when the factor fails [23].

$P(PGR) \neq 1$ (vi)

This indicates that Fama's argument (Nobel Prize, 2013) which was built on the inefficiency of the CAPM model (Nobel Prize, 1990) i.e. small size (Ss) factor being better than the big size (Sb) factor (MCAP) is weak. The conflict between benchmark investing and factor investing, CAPM (Capital Asset Pricing Model) and Fama French Five Factor Model [24] hence becomes a contest around 'Size'. Are we certain that the small size (Ss) factor is better than the big size (Sb)?

The very fact that probability of small size (Ss) delivering is not equal to 1 and there are times that small size (Ss) may fail to deliver combined with the fact that big size (S)b also can deliver sometimes is the conflict between the two award winning ideas. The failure of size as an unequivocal factor is the reason why benchmark investing is thriving as an industry. If size factor was a certainty, Vanguard has no economic reason to exist and S&P 500 would be an irrelevant index. This is a confusion that I talked about in the, "How Physics Solved Your Wealth Problem!" (Pal, 2016) [25]. The equation vii summarizes this debate. Is the probability of small size as a variable driving portfolio returns higher than the probability of big size driving portfolio returns?

$P(Sb) > P(Ss); P(Sb) < P(Ss)$ (vii)

The equation vii translates into the RGR and PGR problem. If the relative ranking of yearly changes in size is a mechanism to explain size as a factor then the size conflict can be seen as a problem Physics has been struggling for nearly 120 years [26]. Is the probability of RGR (big size becoming bigger) higher than PGR (small size becoming bigger) or vice versa?

$P(RGR) > P(PGR); P(RGR) < P(PGR)$ (viii)

The equation viii translates into a statistical problem, which sees the RGR as a failure of mean reversion (FMR) and PGR as the phenomena of mean reversion (MR). It's not just Fama but even Markowitz, Kahneman, Thaler and a multitude of thinkers who never reframed the problem of a variable prediction and its failure in terms of mean reversion or the failure of it. This is why when Fama experiences failure of the 'Size Factor', he should have asked himself whether the failure of the small size anomaly was connected to the big size anomaly or the break down of the risk-return linear relationship. Failure of size as a variable like many other causal variables is a problem of the RGR and the failure of the phenomenon. In its true form the problem of RGR and its failure is the problem of mean reversion and the failure of mean reversion.

$P(FMR) > P(MR); P(FMR) < P(MR)$ (ix)

'Reversion' and 'Diversion' happen simultaneously [27]. There is no reversion without diversion and understanding of the phenomena is architecture dependent [28]. We sometimes need more than linear thinking to observe natural phenomena [29]. Diversion is the persistence of trends, which is the law of Pareto, the law of outliers, the law Rich Get Richer [30]. Only a relative ranking architecture can explain how RGR (Rich Get Richer), RGP (Rich Get Poorer), PGR (Poor Get Richer), PGP (Poor Get Poorer) work together.

The relative ranking architecture can also explain that time inconsistency is the missing link between the confusion between RGR and PGR. The only way the both the phenomena can co-exist is with inconsistent time [31]. When RGR takes a relative short time (t) compared to PGR which needs longer time (T), the systems allows inconsistent ideas to work seamlessly together. Mean Reversion needs longer time T while the Failure of Mean Reversion (FMR) needs relatively shorter time (t).

$P(FMR) = \delta(t)$; $P(MR) = \delta(T)$ (x)

Small size functioning along with big size can be understood relatively. When we rank yearly change of size, we can see that relative change in size is sometimes positive (RGR) and sometimes negative (PGR). Sometimes small size delivers and sometimes it's the big size which delivers. The only way for small size to work together with big size is to allow them to share a different inconsistency in time. It's relatively easier for a small size company to become bigger in time than for a big company to become bigger. The big size companies being on the top of the heap have an architectural disadvantage working against them. The idea that it's easy to top the charts but harder to stay at the top simplifies the explanation.

$P(FMR)t = P(RGR)t = P(Sb)t$ (xi)

$P(MR)T = P(PGR)T = P(Ss)T$ (xii)

The above equations suggest that a relative ranking architecture on yearly size along with shorter time (t) and longer time (T) can explain the conflict between big size (Sb) and small size (Ss). The debate about size is the debate on RGR, PGR, which is driven by the debate on why mean reversion sometimes works and sometimes fails. Benchmark investing is not in conflict with smart beta. Thinking 'Size' is a causality as just like 'Size' the architecture can explain 'Value' and other variables. Every natural systems works with RGR, PGR architecture and the architecture is stable because it allows for inconsistent time. The set of portfolio weights which allocate relatively shorter time to big size companies and relatively longer time to small size companies should have higher probability of offering more optimal set of portfolios compared to the probability of the universal set of portfolio constructions involving different time periods and different sizes.

$S[P(Sbt, SsT)] \geq U[P(Sbi, Ssj)]$ (xiii)

Since Size (S) is a function more of the changing price than the number of shares, the whole industry is looking at a proxy, while it's the relative price change that creates the dynamism between big size (Sb) and small size (Ss). This concludes the idea that 'Size' is an extrapolation of the RGR, PGR phenomena which is driven by mean reversion (MR) and its failure (FMR).

$S\ [P\ \{(RGR)t, (PGR)T\}] \geq U\ [\ P\ \{(RGR)ij, (PGR)ij)\}]$ (xiv)

In conclusion, the 'Beta Maths' Nobel Prize conflict is a non conflict if seen from a statistical architecture perspective. The architecture which explains mean reversion (MR) and it's failure (FMR) in the same framework. The set of portfolio weights which allocate relatively shorter time to factors that are failing to mean revert (FMR) and relatively longer time to factors that are mean reverting (MR) should have higher probability of offering more optimal set of portfolios compared to the probability of the universal set of portfolio constructions involving the combination of any factor (F) for any holding period (j). This argument offers a new way to not only look at MPT, but also finance, economics and every natural system.

$S\ [P\ \{(FMR)t, (MR)T\}] \geq U\ [\ P\ (F)ij\ ]$ (xv)

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