Markov and the Mean Reversion Framework

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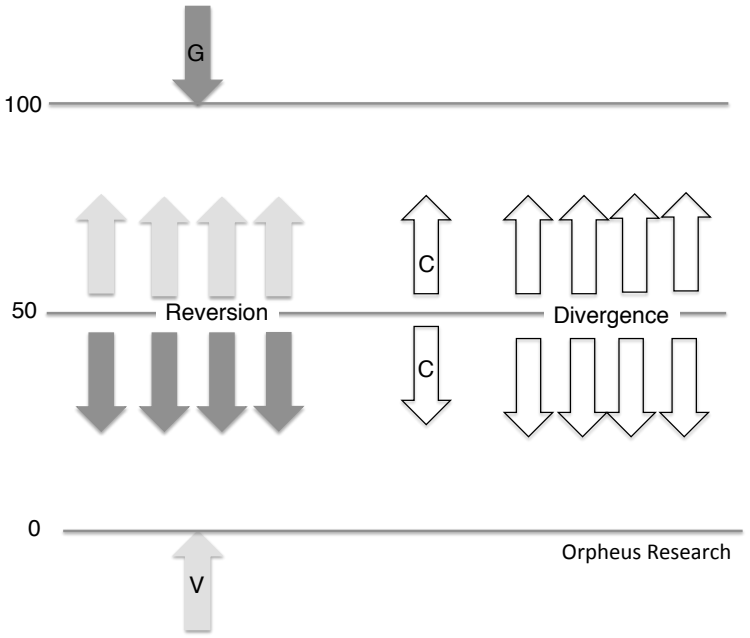
Abstract

Natural systems witness reversion and divergence simultaneously across different periods of time. This paper tests the performance proxy as mentioned in a previous paper on the 'Mean Reversion Framework' for Markov's transition probabilities. The framework exhibits a stable pattern when tested for STOXX 50, S&P 100, Nikkei 225 and FTSE 100 components across different periods of time from 20 days to 3750 days. The three bin classifications of the framework; value, growth and core exhibit a consistency in growth and decay pattern. Both value and growth exhibit persistence compared to the core bin and tends to decay slowly. While the core bins show a symmetric decay across other bins. Such a probabilistic behavior in group components leads the author to believe that the mean reversion framework is indeed converging and diverging leading to a robust expression of a stock market system. The framework could work across data sets from various domains, confirming the proposed universality of the mean reversion framework.

The Mean Reversion Framework and the Jiseki proxy

The framework as explained in 'Mean Reversion Framework', Pal, 2015 is the relative price performance proxy consisting of five bins. The bottom 0-20 percentile bin is considered value, 80-100 percentile is considered growth and the rest of the core (middle) bins are considered transition bins 20-40, 40-60 and 60-80.



The illustration explains how a mean reversion framework consists of both reversion and divergence. The V value bin (light grey) and G growth bin (dark grey) components have a tendency to move above the relative 50 percentile mean ranking and below 50 rankings respectively. An average of nearly 50% value and growth components move above and below the 50 mean. This was the Galtonian mean reversion. It's the section on the right hand side marked 'Divergence' that completes the framework. The number of components in the 40-60 Core ranking move toward 0-20 Value bin at the bottom extreme and 80-100 Growth at the top extreme. This case of Divergence illustrates that just like Reversion, there are components in the Jiseki proxy that illustrate divergence from the mean. The summary illustration suggests that Jiseki proxy experiences both reversion and divergence, a classic expression in case of natural systems.

Markov Chain

Since the proxy is a sequence of steps, it can thus be used for describing systems that follow a chain of linked events, where what happens next depends only on the current state of the system. A Markov chain named after Andrey Markov, is a random process that undergoes transitions from one state to another on a state space. It must possess a property that is usually characterized as "memoryless": the probability distribution of the next state depends only on the current state and not on the sequence of events that preceded it. This specific kind of "memorylessness" is called the Markov property. Markov chains have many applications as statistical models of real-world processes.

Natural systems as probabilistic systems

According to the mean reversion framework, reversion and divergence processes can not only be reconciled in natural systems but also a proxy can be applied understand a natural system. It's the interaction of reversion and divergence processes that create randomness, which is seen across natural systems across times. Assuming the proxy to be a robust framework to explain randomness and order we tested it for Markov Transition probabilities. We assumed market to be in a state of five bins at point zero and we studied how the bin components behave over different set of time durations. We referred to the bottom 0-20 bin as Value, 20-80 as three core (transition) bins and 80-100 as top growth bin.

Period and data tested

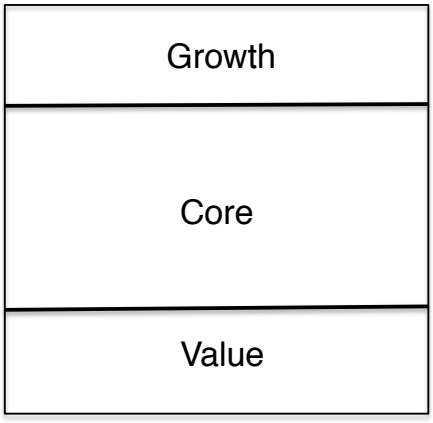
The FTSE UK (F) 100, Nikkei 225 Japan (N), STOXX 50 (S), STOXX Bank Sector (SB), S&P 100 (U) components were tested for 15 years of data for 20,40, 60, 125, 250, 500, 750, 1000, 1250, 1500, 2500, 3750 days. The stock prices were ranked on a quarterly performance and ranked on a scale of 0 to 100 and then tested for percentage changes in components in the respective bins over the various time durations.

Interpreting probability

The numbers were expressed in percentage terms. So a move of 5% from 0-20 Value bin to 20-40 Core bin was considered as a transition probability of the value bin component to move to 20-40 core bin.

Universal patterns

A similar pattern was seen across various stock market groups over different durations starting 20 days to 3750 days. The decay or transformation process for the Growth and Value bins were consistent, decaying non-normally (exponentially) in time. While the Core bins were symmetrically decaying across the proxy. Similarity of the pattern suggests universality. Extending the Markov transition probability test to other non-capital market data sets will strengthen the case of the Jiseki



Proxy (Mean Reversion Framework) as universality

Some questions one may ask regarding the universality; Why are value and growth extremes similar in their transformation process? Why does divergence from core decay in a symmetric form? Why is there persistence in behavior when it comes to core, value and growth? Why is value and growth expression different from that of core? Does diffusion of the pattern suggest disintegration in the integrity of the natural system cluster? Does the diffusion suggest that over time the predictability of the cluster is weak?

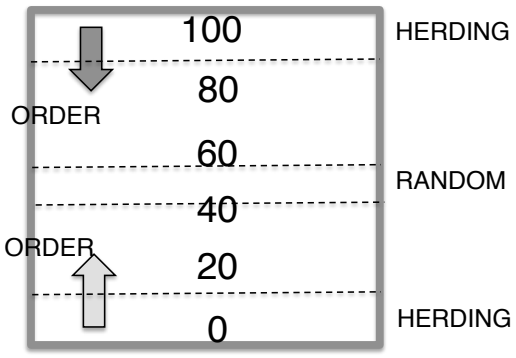
Pareto and Galton

Power Law (Paretoian expression) and Normality (Galtonian expression) are the two forces of reversion and divergence coexisting in all natural systems. They are prone to transform into each other leading to the dynamism of natural system. The consistency of the probability patterns as seen across our database suggests that the proxy not only maps the behavior of components of the natural system but also the interaction between the three characteristic bins.

Stock Market Cases

Below we have compared the transition proxy for STOXX 50, S&P 100, FTSE and Nikkei 225 components. These are different markets with different components. But despite the difference the Jiseki proxy allows for standardization and comprehension of component specific behavior as it assumes the characteristic of the value-core-or growth bin. One can see a clear persistence in value and growth bins in both cases (number in green boxes). The 0-20 and 80-100 continue to retain a higher value (darker green) initially, as the transition core bins transform and give away components faster, out of the bin. This clearly illustrates that Value and Growth bin behave differently compared the middle core bins from 20 to 80 for the initial period, showing more stickiness and then slowly the persistence starts to give away.

The transition analysis illustrates how the decay and growth pattern between bins is similar to both the groups under study. The Fig 1 shows a similar pattern for both the groups. Both of them show a consistent initial persistence and then exponential decay after a certain duration. Even bin specific; growth, value, 20-40 Core, 40-60 Core, 60-80 Core show similar expression. This confirms the proxy as a driving influence on group components. If something is in value and growth, it's going to behave differently than if that component is in core. The value and growth component will show initial persistence, the reason why growth will continue to deliver for a period of 9 to 12 months. And then after that period, Value will start pushing out of the bin as both growth and value decay. The value decay being favorable for the portfolio's long value components.



Core divergence

The symmetry of core decay confirms the view that divergence is happening more in the core. Divergence in terms of a two way decay, both from 40-60 towards the 80-100 growth extreme and from 40-60 Core to the 0-20 value extreme. The very fact that this is a two way process makes it hards to understand component trends, whether some component is headed higher or lower. Over weight and equal weight core selection hence is a harmful exercise for portfolios that don't differentiate between the characteristics of value-core and growth classifications.

Longer durations

Another interesting trend one can observe here is that over longer durations, the difference in probabilities become marginal. This suggests that going beyond a certain duration without rebalancing the portfolio robs the portfolio of the opportunity of excess risk weighted alpha; the beneficial risk coming from the persistence of growth in the early transition and transformation of value in the later part of the transition. The longer durations also brings in more randomness and unpredictability to the portfolio process. The durations could change as the group size or composition changes, the patterns of which will become more clear as we study more on the probability maps related to the Jiseki proxy.

STOXX 50 Markov transition

	20 days					40 days					60 days					125 days				
	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100
0-20	89	11	0	0	0	81	16	2	0	0	76	19	3	1	0	64	23	8	4	2
20-40	10	75	14	0	0	15	61	20	3	1	18	53	22	6	1	23	38	23	11	4
40-60	0	14	70	15	0	2	21	54	22	2	3	24	45	24	4	7	26	34	24	10
60-80	0	0	15	75	10	0	3	22	60	16	0	5	25	51	18	3	11	25	38	23
80-100	0	0	0	10	89	0	0	2	16	81	0	1	4	20	75	1	4	9	26	59
	250 days					500 days					750 days					1000 days				
0-20	47	21	16	8	7	25	16	17	15	26	19	19	17	18	26	21	20	15	19	22
20-40	22	27	21	19	11	15	20	22	22	21	16	25	22	19	18	20	20	20	23	18
40-60	14	25	25	20	16	17	23	23	21	16	19	22	20	21	18	16	21	21	22	20
60-80	9	19	23	26	24	20	19	21	23	16	22	17	20	22	20	21	19	21	21	19
80-100	4	10	16	28	41	19	23	16	21	20	19	19	21	22	19	17	22	23	19	19
	1250 days					1500 days					2500 days					3750 days				
0-20	20	17	15	23	23	18	20	18	21	22	21	22	17	19	21	17	20	14	25	25
20-40	17	19	22	22	21	19	21	20	20	20	22	22	17	19	19	16	20	14	26	24
40-60	16	20	23	21	19	15	20	22	22	20	20	22	20	20	19	22	16	19	18	24
60-80	21	20	21	19	18	20	20	21	21	18	15	21	23	22	18	20	20	26	17	16
80-100	21	25	19	17	18	24	19	19	18	20	12	16	23	23	25	21	28	28	12	10

US 100 Markov transition

	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100
0-20	95	5	0	0	0	90	9	1	0	0	86	12	2	0	0	79	16	4	1	0
20-40	4	92	5	0	0	7	85	8	1	0	9	80	9	1	0	13	71	14	2	0
40-60	0	5	90	5	0	1	8	82	9	0	1	10	76	12	1	2	14	66	17	1
60-80	0	0	4	92	3	0	0	8	85	6	0	0	11	80	8	0	2	16	69	12
80-100	0	0	0	3	96	0	0	0	6	93	0	0	0	8	91	0	0	1	12	86
	250 days					500 days					750 days					1000 days				
0-20	71	19	8	2	1	58	23	12	6	1	50	25	15	7	2	47	24	16	11	3
20-40	18	61	18	3	1	24	48	19	7	1	27	40	21	9	3	29	34	22	10	5
40-60	4	16	55	23	2	6	19	44	26	5	7	20	38	26	9	9	20	34	26	11
60-80	1	4	20	58	18	2	7	24	45	23	3	10	25	39	23	4	14	24	33	24
80-100	0	0	3	17	78	1	1	6	21	69	2	3	8	23	63	3	3	10	25	56
	1250 days					1500 days					2500 days					3750 days				
0-20	45	22	16	13	4	44	19	17	13	7	24	14	17	20	24	15	11	13	31	29
20-40	29	33	21	11	7	25	35	21	11	9	34	25	18	15	8	28	23	15	19	15
40-60	9	21	32	26	13	11	19	31	25	14	11	28	23	23	15	18	24	17	19	22
60-80	6	16	24	30	23	6	18	24	29	22	10	17	25	22	25	16	21	27	21	14
80-100	4	4	13	26	52	5	6	13	26	48	7	13	20	25	31	11	15	29	18	23

FTSE 100 Markov transition

	20 days					40 days					60 days					125 days				
	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100
0-20	97	3	0	0	0	94	6	0	0	0	92	7	0	0	0	89	10	1	0	0
20-40	3	93	4	0	0	5	87	8	0	0	6	83	10	1	0	10	75	13	2	0
40-60	0	4	91	4	0	0	8	84	8	0	1	10	80	10	0	1	13	72	14	1
60-80	0	0	4	93	3	0	0	8	87	5	0	1	10	82	7	0	1	14	75	10
80-100	0	0	0	3	97	0	0	0	5	94	0	0	0	7	92	0	1	1	9	88
	250 days					500 days					750 days					1000 days				
0-20	85	12	2	1	0	80	16	2	1	1	75	18	4	2	1	71	20	5	2	2
20-40	13	67	17	3	0	17	57	19	5	1	20	50	22	5	2	23	46	23	6	3
40-60	2	17	61	19	1	2	21	53	20	4	3	24	47	20	6	4	27	42	20	8
60-80	1	3	18	65	13	1	5	22	56	16	2	7	22	50	20	3	8	22	47	20
80-100	0	1	1	9	88	0	1	2	18	76	0	1	3	24	68	0	1	6	26	64
	1250 days					1500 days					2500 days					3750 days				
0-20	69	19	6	3	3	68	18	7	3	4	55	19	8	7	8	38	14	6	5	5
20-40	24	45	19	8	5	23	43	18	10	5	25	31	21	14	8	18	22	15	10	5
40-60	4	25	42	20	9	5	27	38	20	9	9	31	28	18	14	6	22	19	13	10
60-80	3	10	22	43	22	3	9	26	40	21	6	17	28	27	23	4	12	19	19	16
80-100	0	2	9	26	59	1	2	11	27	57	4	2	15	34	44	3	2	10	24	31

Nikkei 225 Markov transition

	20 days					40 days					60 days					125 days				
	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100
0-20	96	4	0	0	0	92	8	0	0	0	89	10	0	0	0	83	15	1	0	0
20-40	4	90	6	0	0	8	82	10	0	0	10	75	13	1	0	16	63	18	3	0
40-60	0	6	88	6	0	0	11	77	12	0	0	14	69	16	1	1	20	55	22	2
60-80	0	0	7	89	4	0	0	12	80	7	0	1	16	74	9	0	3	22	62	13
80-100	0	0	0	4	96	0	0	0	7	93	0	0	1	9	90	0	0	3	12	85
	250 days					500 days					750 days					1000 days				
0-20	77	19	3	1	0	70	21	6	2	1	65	23	7	4	2	62	23	8	5	2
20-40	20	52	21	5	1	23	42	23	9	3	25	37	23	12	4	25	34	24	11	5
40-60	3	24	44	25	5	5	25	35	28	7	8	22	36	25	8	9	21	32	28	9
60-80	1	6	26	52	16	2	11	26	42	19	2	16	24	39	19	4	18	22	37	19
80-100	0	1	5	16	77	0	3	8	17	71	1	4	8	19	68	1	6	12	18	64
	1250 days					1500 days					2500 days					3750 days				
0-20	59	24	9	5	3	57	24	11	5	3	49	25	13	9	5	43	21	15	12	9
20-40	25	32	25	13	6	24	31	25	14	6	25	27	24	15	9	23	26	20	20	11
40-60	10	23	29	27	11	10	24	28	25	13	13	23	22	26	15	14	19	24	25	17
60-80	6	17	23	34	20	6	17	24	32	21	10	18	26	27	19	12	19	24	24	21
80-100	1	7	11	20	60	2	8	10	23	57	4	9	12	24	51	7	16	16	19	41

Average

	20 days					40 days					60 days					125 days				
	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100	0-20	20-40	40-60	60-80	80-100
0-20	94	6	0	0	0	89	10	1	0	0	86	12	1	0	0	79	16	3	1	1
20-40	5	88	7	0	0	9	79	11	1	0	11	73	14	2	0	15	62	17	4	1
40-60	0	7	85	8	0	1	12	74	13	1	1	14	68	15	1	3	18	57	19	4
60-80	0	0	8	87	5	0	1	13	78	9	0	2	16	72	11	1	4	19	61	14
80-100	0	0	0	5	95	0	0	1	9	90	0	0	1	11	87	0	1	3	15	79
	250 days					500 days					750 days					1000 days				
0-20	70	18	7	3	2	58	19	9	6	7	52	21	11	8	8	50	22	11	9	7
20-40	18	52	19	7	3	20	42	21	11	7	22	38	22	11	7	24	33	22	13	8
40-60	5	21	46	22	6	8	22	39	23	8	9	22	35	23	10	9	22	32	24	12
60-80	3	8	22	50	18	6	11	23	41	19	7	12	23	37	20	8	15	22	34	21
80-100	1	3	6	18	71	5	7	8	19	59	5	7	10	22	54	5	8	13	22	51
	1250 days					1500 days					2500 days					3750 days				
0-20	48	21	12	11	8	47	20	12	11	9	37	20	14	14	14	28	16	12	18	17
20-40	24	32	22	13	9	22	32	21	14	10	26	26	20	16	11	21	23	16	19	14
40-60	9	22	32	23	13	10	22	30	23	14	13	26	23	22	16	15	20	20	19	18
60-80	9	16	23	32	21	9	16	24	30	21	11	18	26	25	21	13	18	24	20	17
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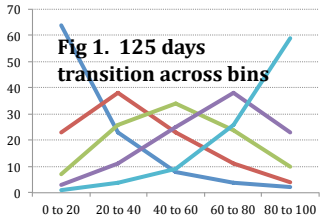
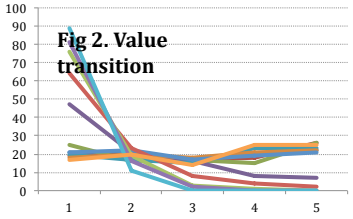
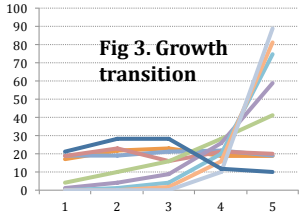
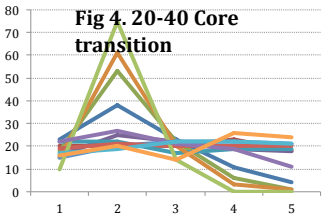
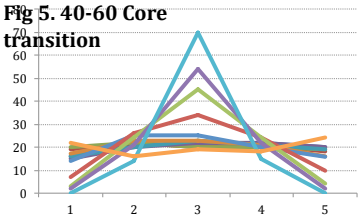
STOXX 50 transition analysis

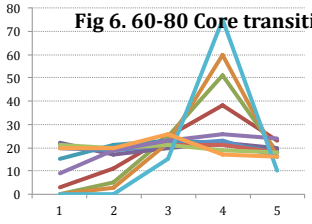


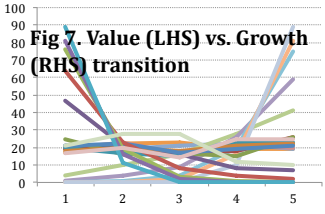


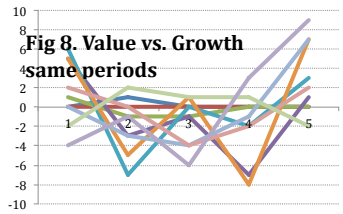


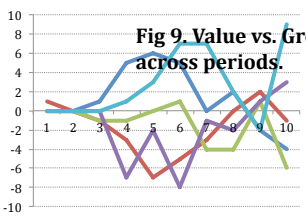




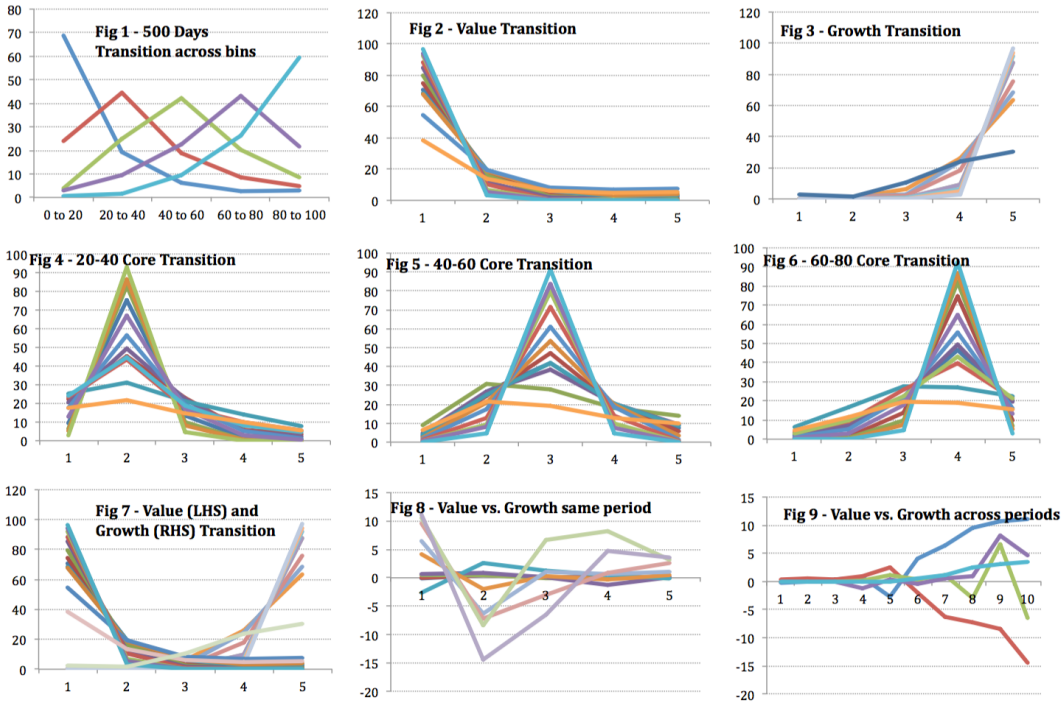




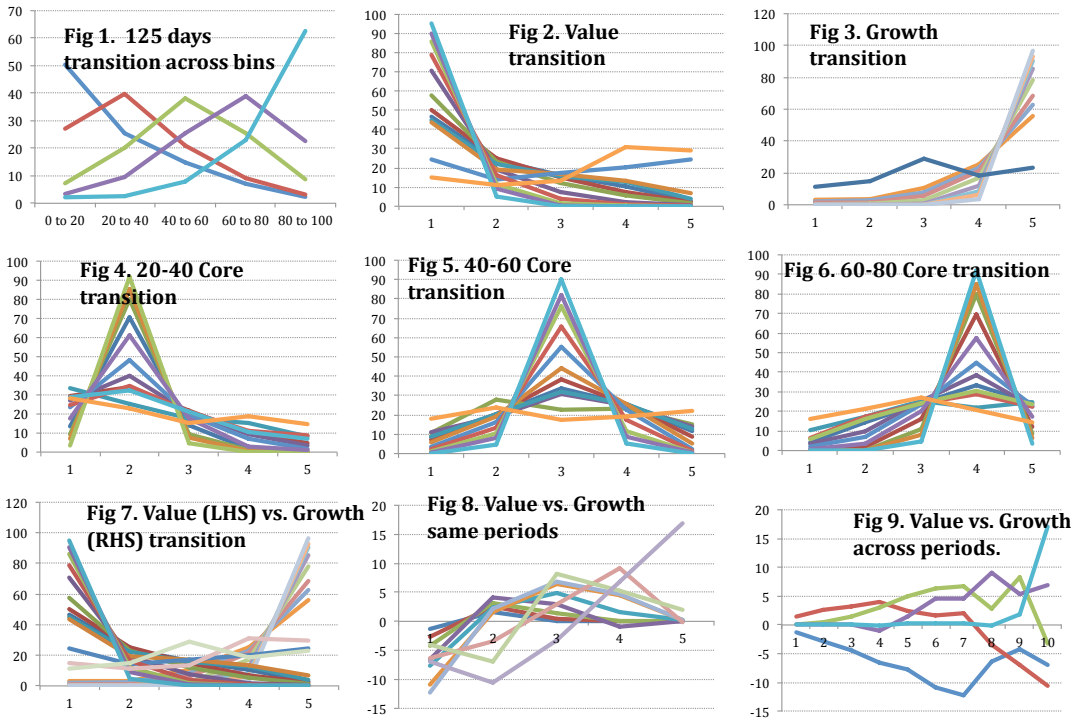




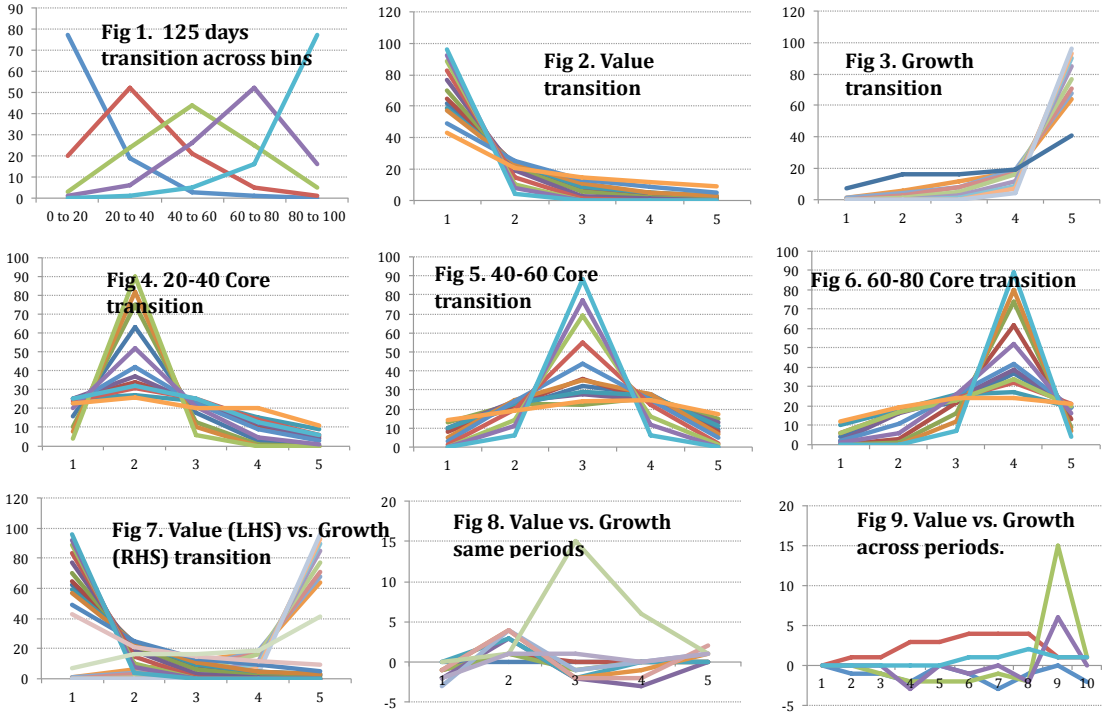
US 100 transition analysis



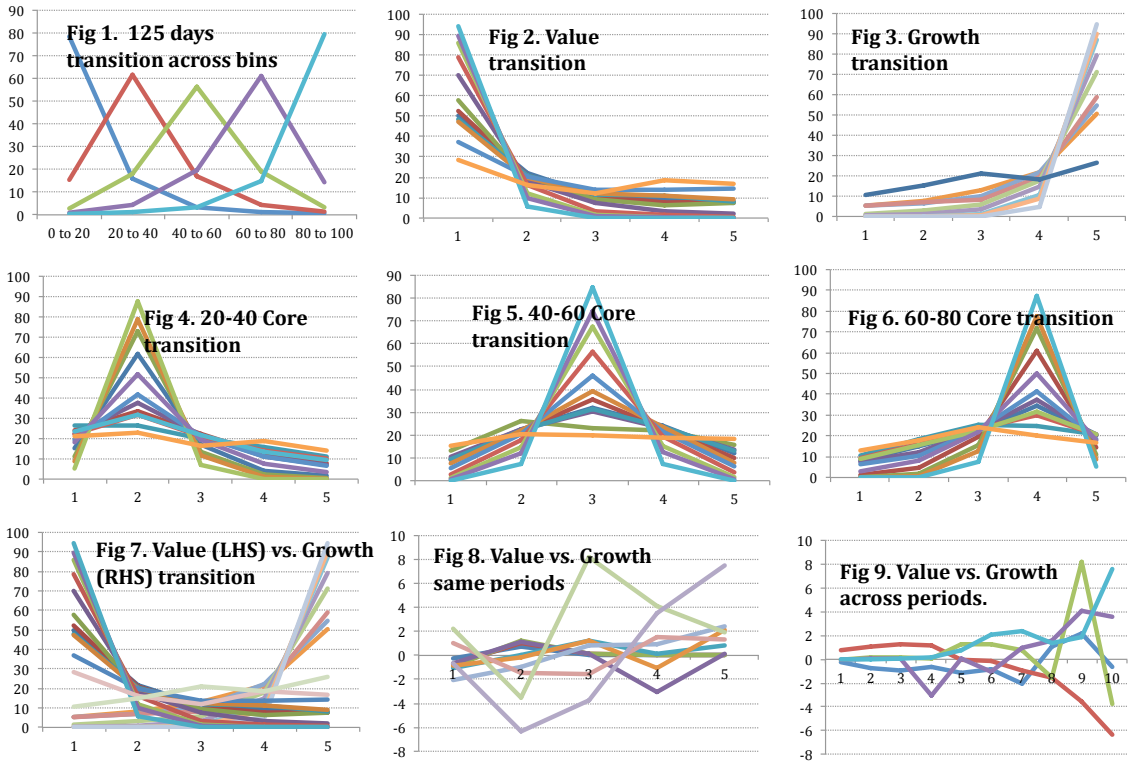
FTSE 100 transition analysis



Nikkei 225 transition analysis



Average



Data Universality and potential applications

Innovations built on the universal proxy should bring out an original way to look at markets and it's components, suggesting a data universality. Since the proxy is primarily a data innovation, the pattern should be domain agnostic. The transition matrix should show similar pattern across domains like sports, politics, finance, auto, health, and anything data. The proxy should be able to anticipate probabilistic growth and decay trends in groups components across different periods of time.

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