Variational Stereo Matching

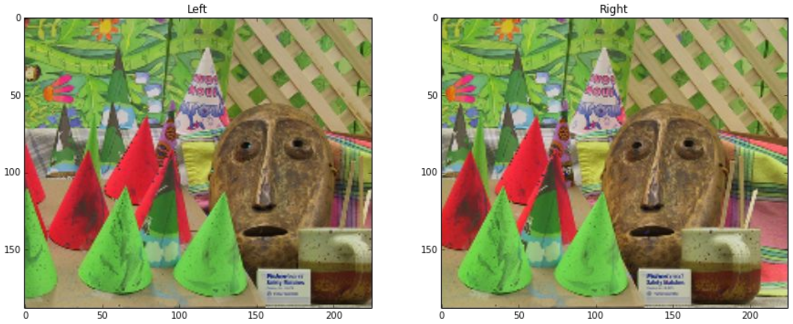
Gabriel Huang

MVA Compressed Sensing Final Project

What is Stereo Matching?

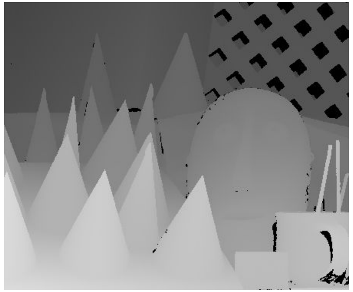
Input: Pair of Images

$$I_L(x), I_R(x) : \Omega \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$$



Perform Stereo Matching

IL(x) ~ IR(x+u(x))



Output: Disparity Map

$$u(x) : \Omega \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$$

Why Stereo Vision?

- Disparity is inversely proportional to depth
- A Passive system to measure distance
- Applications to 3D Reconstruction, autonomous car driving, optical flow estimation, improved vision algorithms

Variational Stereo Matching 1

$u(x) : \Omega \subset \mathbb{R}^2 \longrightarrow \mathbb{R}$

Idea: IL(x) ~ IR(x+u(x))

is the function that minimizes the functional

$$\min_u \quad \alpha \int_\Omega |\nabla u| dx + \int_\Omega |\rho(x, u)| dx$$

Regularity Term: Total Variation
Ensures, resulting image is smooth
(piecewise constant)

Data Term
Ensures warped right image IR(x+u(x))
matches left image

$$\rho(x, u) = |I_L(x) - I_R(x + u(x))|$$

Variational Stereo Matching 2

Idea: IL(x) ~ IR(x+u(x))

u is the function that minimizes the functional

$$\min_u \quad \alpha \int_\Omega |\nabla u| dx + \int_\Omega |\rho(x,u)| dx$$

Regularity Term: Total Variation
Ensures, resulting image is smooth
(piecewise constant)

Problem:
Data Term Non-Convex in u

Data Term
Ensures warped right image IR(x+u(x))
matches left image

$$\rho(x,u) = |I_L(x) - I_R(x + u(x))|$$

Characteristic Functions 1

Solution:

Enter the Characteristic Function

$$\phi(x,\gamma) : \Sigma = \Omega \times \Gamma \longrightarrow \{0,1\}$$

$$\phi(x,\gamma) = 1_{u(x)>\gamma}$$

Lives in set

$$D' = \{\phi(x,\gamma) : \Sigma \longrightarrow 0,1 \mid \phi(x,\gamma_{min}) = 1, \phi(x,\gamma_{max}) = 0\}$$

Is equivalent to **u**

layer cake formula

$$u(x) = \gamma_{min} + \int_\Gamma \phi(x,\gamma)d\gamma$$

New Problem

$$\min_{\phi \in D'} \int_\Sigma \alpha|\nabla_x \phi| + |\rho(x,\gamma)\nabla_\gamma \phi(x,\gamma)|d\gamma dx$$

Characteristic Functions 2

Problem:
Set D' is not Convex

$$D' = \{\phi(x,\gamma) : \Sigma \longrightarrow 0,1 \mid \phi(x,\gamma_{min}) = 1, \phi(x,\gamma_{max}) = 0\}$$

Characteristic Functions 3

Problem:
Set D' is not Convex

$$D' = \{\phi(x,\gamma) : \Sigma \longrightarrow 0,1 \mid \phi(x,\gamma_{min}) = 1, \phi(x,\gamma_{max}) = 0\}$$

Solution:
Relax **Phi** to be in **D** = ConvexHull(**D'**)

$$\boxed{\phi \in D = \{\phi(x,\gamma) : \Sigma \longrightarrow [0,1] \mid \phi(x,\gamma_{min}) = 1, \phi(x,\gamma_{max}) = 0\} \supset D'}$$

$$\min_{\phi \in D} \int_{\Sigma} \alpha |\nabla_x \phi| + |\rho(x,\gamma) \nabla_\gamma \phi(x,\gamma)| d\gamma dx$$

Characteristic Functions 4

THEOREM

whenever $\phi \in D$ minimizes

$$\min_{\phi \in D} \int_{\Sigma} \alpha |\nabla_x \phi| + |\rho(x,\gamma) \nabla_\gamma \phi(x,\gamma)| d\gamma dx$$

function $1_{\phi \geq \mu} \in D'$ minimizes

$$\min_{\phi \in D'} \int_{\Sigma} \alpha |\nabla_x \phi| + |\rho(x,\gamma) \nabla_\gamma \phi(x,\gamma)| d\gamma dx$$

Variational Problem is not differentiable 1.

Now...

$$\min_{\phi \in D} \int_{\Sigma} \alpha |\nabla_x \phi| + |\rho(x,\gamma) \nabla_\gamma \phi(x,\gamma)| d\gamma dx$$

Variational Problem is not differentiable 2

Now...

Problem:
Functional is not differentiable wrt **Phi**

$$\min_{\phi \in D} \int_{\Sigma} \alpha |\nabla_x \phi| + |\rho(x,\gamma)\nabla_\gamma \phi(x,\gamma)| d\gamma dx$$

Variational Problem is not differentiable 3

Problem:
Functional is not differentiable wrt **Phi**

$$\min_{\phi \in D} \int_{\Sigma} \alpha |\nabla_x \phi| + |\rho(x,\gamma) \nabla_\gamma \phi(x,\gamma)| d\gamma dx$$

Equivalent

Solution:
Introduce the dual variable **p** and use the dual norm

$$\min_{\phi \in D} \max_{p \in C} \int_{\Sigma} p^T \nabla_3 \phi \; d\gamma dx$$

$$C = \left\{ p(x,\gamma) : \Sigma \longrightarrow \mathbb{R}^3 \mid \sqrt{p_1^2 + p_2^2} \leq \alpha, |p_3| \leq \rho(x,\gamma), \quad \forall (x,\gamma) \in \Sigma \right\}$$

How to solve? 1

$$\min_{\phi \in D} \max_{p \in C} \int_{\Sigma} p^T \nabla_3 \phi \; d\gamma dx$$

Primal-Dual Proximal Point Algorithm

$$\phi^{t+1} \longleftarrow \min_{\phi \in D} \int_{\Sigma} (p^t)^T \nabla_3 \phi + \frac{1}{2\tau_\phi} \int_{\Sigma} (\phi - \phi^t)^2 \; d\gamma dx$$

$$p^{t+1} \longleftarrow \max_{p \in C} \int_{\Sigma} p^T \nabla_3 \phi^t - \frac{1}{2\tau_p} \int_{\Sigma} (p - p^t)^2 \; d\gamma dx$$

How to solve? 2

$$\min_{\phi \in D} \max_{p \in C} \int_{\Sigma} p^T \nabla_3 \phi \; d\gamma dx$$

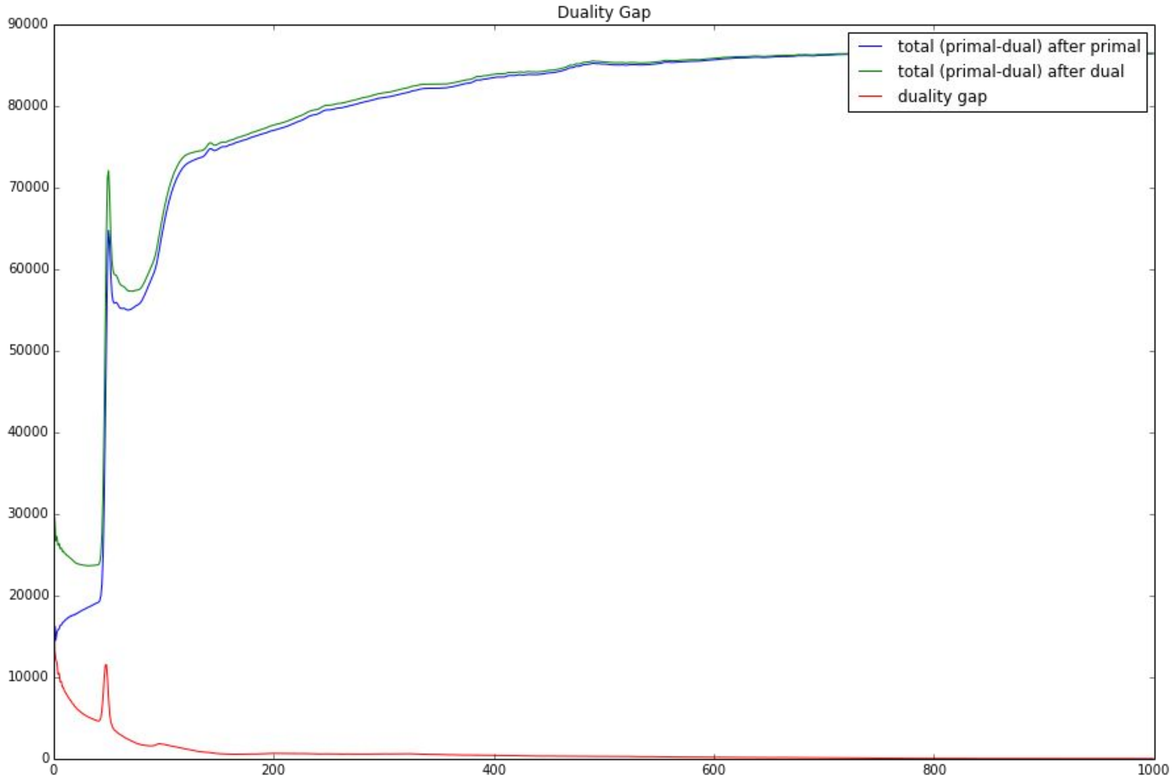
Primal-Dual Proximal Point Algorithm

$$\phi^{t+1} \longleftarrow \mathbf{proj}_D(\phi^t + \tau_\phi \mathbf{div} p)$$

$$p^{t+1} \longleftarrow \mathbf{proj}_C(p^t + \tau_p \nabla_3 \phi)$$

Results

Partial Primal-Dual Gap



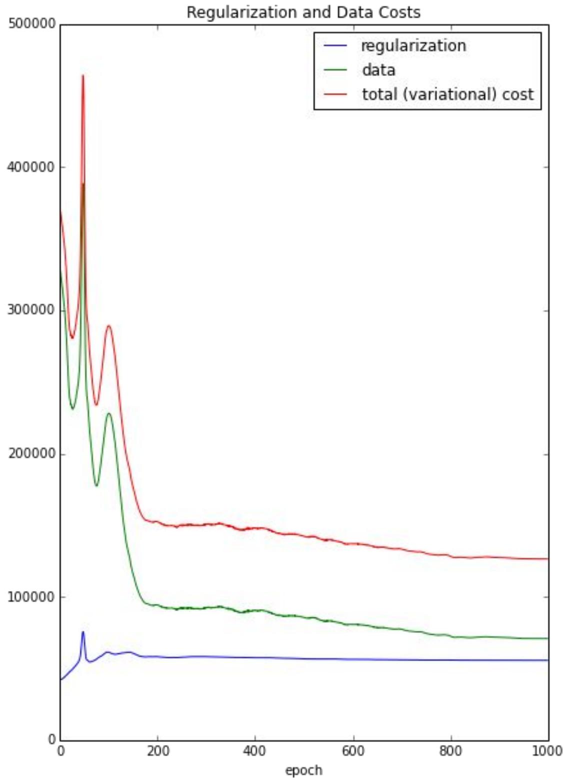
Results

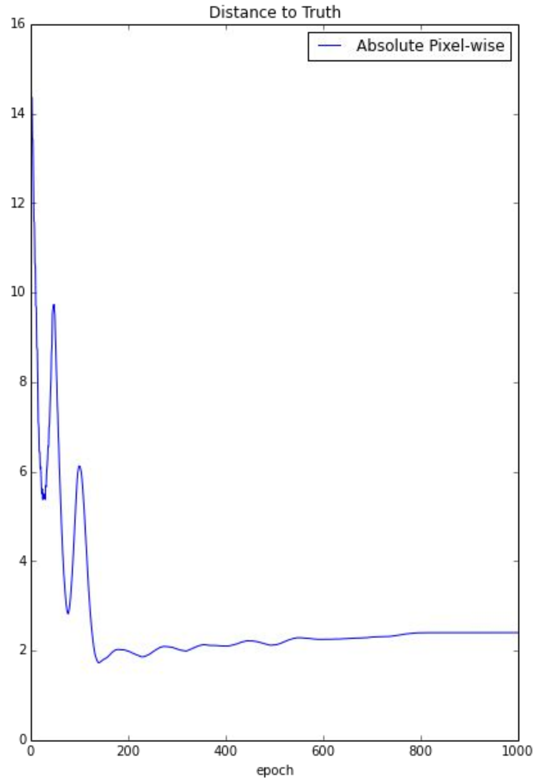
Left

Costs

Right

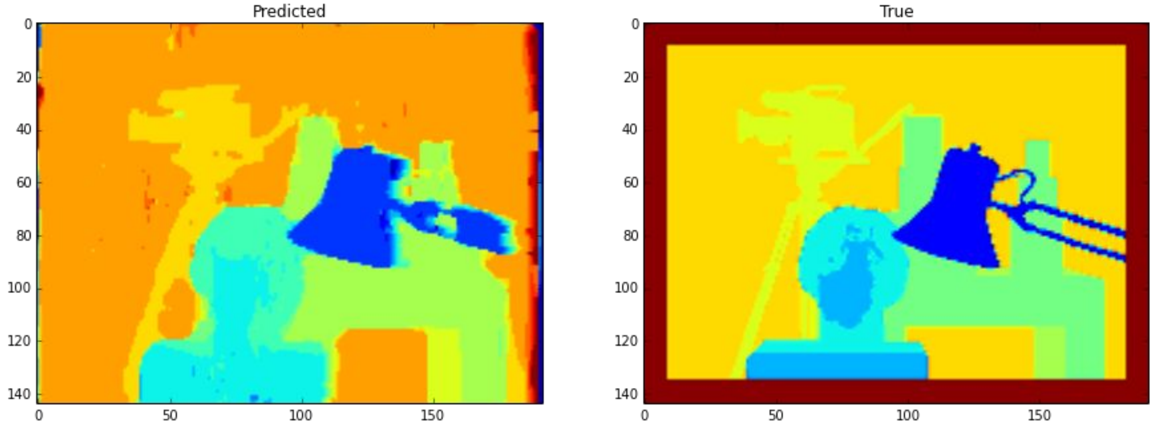
Distance to Truth





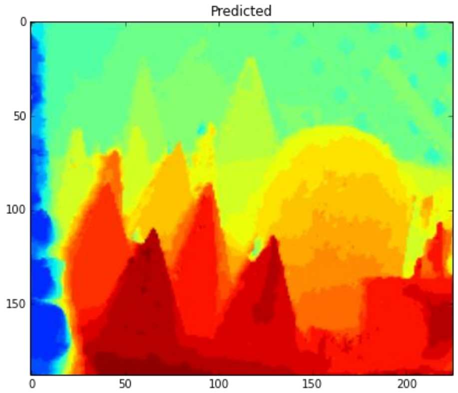
Results

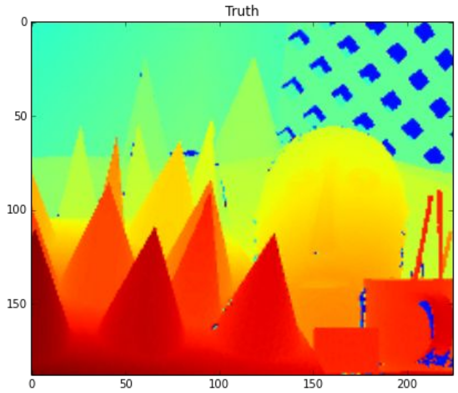
Tsukuba



Results

Cones





Results

Aloe