Chapter 8. Two-body NN Interactions

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This chapter is mainly summarized in the book by S. S. M. Wong, *Introductory Nuclear Physics* (Prentice-Hall, 1990, Englewood).

1 Two-nucleon system

1.1 Two-Fermion Systems

Pauli Principle says that

"Two identical fermion wave functions have to be **Antisymmetric** with respect to the simultaneous exchange of their space, spin, and isospin coordinates".

$$\begin{array}{lrcl} \textbf{Space part} & \pi_L & = & (-)^L \\ \textbf{Spin} & \pi_S & = & -(-)^S \\ \textbf{Isospin} & \pi_T & = & -(-)^T \end{array}$$

$$\begin{array}{rcl} \pi \;=\; \pi_L\,\pi_S\,\pi_T & = & (-)^{L+S+T} \;=\; -1 \\ L + S + T & = & \text{odd} \end{array}$$

$$\begin{array}{lll} T=0, & \text{L + S = odd}, & S=0, \quad L=1,3,... \\ & & S=1, \quad L=0,2,... \\ T=1, & \text{L + S = even}, & S=0, \quad L=0,2,... \\ & & S=1, \quad L=1,3,... \end{array}$$

In summary, the states of the two-nucleon system (Bound or scattering states) allowed by the Pauli principle are

Isospin	Spin	Ang. Mom.	notation
$T=1$	$S=0$	L even	SE
$T=1$	$S=1$	L odd	TO
$T=0$	$S=0$	L odd	SO
$T=0$	$S=1$	L even	TE

1.2 Two-Nucleon Bound System - Deuteron

(a) Possible states

Bound State	# of states	
$n + n$	0	
$p + n$	1	$\rightarrow$ Not the same T state
$p + p$	0	$\rightarrow$ Coulomb Repulsion
Scattering	$p-p$, or $p-n$	

(b) Experimental Facts

$$\begin{array}{rl} B.E. & = 2.2246 \text{ MeV} \\ J^\pi & = 1^+ \\ Q & = 0.00286 \text{ barns} \\ \mu_d & = 0.857\ \mu_p \\ r_d & = 1.963 \text{ fm} \end{array}$$

(c) Implications of Experimental Facts

(1) Weak Binding Energy of 2.2246 MeV

Thermal neutron Experiment $H(n, \gamma)d$

B.E. $= E_\gamma$ because of $E_n \approx 0$.

** Exceptionally weak interaction,
even though P.E. and K.E. are large.

** Ready to separate each other

** Hard to have excited states

(2) $J^\pi \;=\; 1^+$

$+ \;\rightarrow\; L \;= 0, 2, 4, ..$ (even)

Lowest $L = 0$. (No repulsive centrifugal force)

$J = L + S = 1 \quad \rightarrow S = 1$ (Spin triplet state)

$L + S + T =$odd $\;\rightarrow T = 0$.

Ground State of Deuteron ${}^3S_1$ $({}^{2S+1}L_J)$

(3) Small Q $\;\rightarrow\;$ Spherical but not perfectly.

Spherical $\qquad \rightarrow L = 0$ dominance

Not perfectly $\rightarrow$ Some admixture of other states,
possibly $L = 2$, Some admixture of ${}^3D_1$

The Deuteron wave function would be

$$\psi_d \;=\; a|{}^3S_1 \;+\; b|{}^3D_1 >$$

1.3 Deuteron Magnetic Dipole Moment

(a) Magnetic dipole moment

$$\vec{\mu}_d \;=\; \vec{\mu}_d^{orbital} + \vec{\mu}_d^{spin}$$

$$\vec{\mu}_d^{orbital} \;=\; \sum_i \frac{e\hbar}{2m_p}\vec{\ell}_i \;=\; \sum_i g_{\ell_i}\vec{\ell}_i$$

$$\begin{aligned} g_{\ell_i} \;&=\; \mu_N \quad \text{for a proton} \\ &=\; 0 \quad \text{for a neutron.} \end{aligned}$$

$$\vec{\mu}_d^{spin} \;=\; \sum_i \frac{e\hbar}{2m_p}\vec{S}_i \;=\; \sum_i g_S\vec{S}_i$$

$$\begin{aligned} g_S \;&=\; 2\mu_p \;=\; 5.59\ \mu_N \quad \text{for a proton} \\ &=\; 2\mu_n \;=\; \text{-}3.83\ \mu_N \quad \text{for a neutron.} \end{aligned}$$

$$\begin{aligned} \vec{\mu}_d \;&=\; g_p\vec{S}_p \;+\; g_n\vec{S}_n \;+\; \vec{\ell}_p \\ &\approx\; g_p\vec{S}_p \;+\; g_n\vec{S}_n \;+\; \tfrac{1}{2}\vec{L} \end{aligned}$$

Here we assume that each nucleon carries $\frac{1}{2}\vec{L}$.

(b) Experimental Value

$$\begin{aligned} &\mu_d({}^3S_1) = \mu_p + \mu_n &&= 0.880\ \mu_N \\ &\mu_d(\text{Exp}) &&= 0.857\ \mu_N \end{aligned}$$

Possible Explanation
1) Change of internal structures of p and n.
→ Not possible because of small B.E.
(Weak interaction).
2) Effect of Meson exchange current
→ Possible but small.
3) Admixture of $^3D_1$ state
→ Possible

(c) Admixture of $^3D_1$ state

$$\begin{aligned}\mu_d(^3S_1,\ \text{J=S=1,L=0}) &= \mu_p + \mu_n \\ &= 0.880\ \mu_N\end{aligned}$$

$$\begin{aligned}\mu_d(^3D_1,\ \text{J=S=1,L=2}) &= \tfrac{1}{8}[(g_p + g_n)(-2) + 6]\mu_N \\ &= 0.310\ \mu_N\end{aligned}$$

With admixture of $^3D_1$ state, the Deuteron wave ft can be written as,

$$\begin{aligned}|\psi_d> &= a\ |^3S_1\ +\ b\ |^3D_1>, \quad \text{with} \quad a^2\ +\ b^2\ =\ 1 \\ \mu_d &= a^2\ \mu_d(^3S_1)\ +\ b^2\ \mu_d(^3D_1)\ =\ 0.857\mu_N\end{aligned}$$

We obtain $b^2\ \sim\ 0.04$ and conclude that 4% admixture of $^3D_1$ state in the ground state of Deuteron exists.

1.4 Deuteron Electric Quadrupole Moment

(a) Charge Quadrupole Operator

For a spherical nucleus,

$$\begin{aligned} < x^2 > &= < y^2 > = < z^2 > \\ < r^2 > &= 3 < z^2 > \end{aligned}$$

For a nucleus departed from spherical shape
in the lowest order,the charge quadrupole operator

$$Q_0 = e(3z^2 - r^2)$$

$$\begin{aligned} Q_0 &< 0 \quad : \text{ Oblate, Pancake} \\ &= 0 \quad : \text{ Spherical} \\ &> 0 \quad : \text{ Prolate, Cigar-shaped} \\ &\qquad\qquad \text{(Elongated along z-axis)} \end{aligned}$$

Remember that Q_0 of Deuteron is positive,
but small ($+0.29\ c\ \text{fm}^2$).

(b) Expectation value of the quadrupole Operator

The operator is a spherical tensor of rank two, and carries two units of angular momentum.

$$\begin{aligned} Q_0 &= e(3z^2 - r^2) = er^2(3cos^2\theta - 1) \\ &= \sqrt{\frac{16\pi}{5}} er^2 Y_{20}(\theta, \phi) \end{aligned}$$

The expectation value of the quadrupole operator is defined as

$$Q_A = < J,\ M = J|\ Q_0\ |J,\ M = J >$$

in the substate of maximum M.

In consideration of the angular momentum addition rule

$$\vec{J} = \vec{2} + \vec{J} \Rightarrow J \geq 1$$

We see that $< Q_0 > = 0$ for $J = 0, \frac{1}{2}$ and $< Q_0 > \neq 0$ for the deuteron is the direct evidence of the presence of $^3D_1$ component.

The expectation value

$$\begin{aligned} Q_d = & \quad a^2 <^3 S_1, M = 1|Q_0|^3S_1, M = 1 > && \text{equals } 0 \\ & +2ab <^3 S_1, M = 1|Q_0|^3D_1, M = 1 > && \text{calculate} \\ & +b^2 <^3 D_1, M = 1|Q_0|^3D_1, M = 1 > && \text{negligible} \\ \approx & \quad 2ab \int_0^\infty R_s^*(r) r^2 R_D(r) r^2 dr \int Y_{00}^* Y_{20} Y_{20} d\Omega \end{aligned}$$

Most of the estimates gives $|b|^2 \sim 4\%$.

1.5 Tensor force and the deuteron D-state

(a) Deuteron D-state

$$|\psi_d> \;=\; a\psi_S \;+\; b\psi_D \;=\; a\;|^3S_1 \;+\; b\;|^3D_1>,$$

Is it possible with just a central force?

$$V \;=\; V_0 \;+\; V_s\vec{\sigma_1}\cdot\vec{\sigma_2}$$

Answer is **No.**

Let us forget about V_s,
because we have same triplet state in S and D.

$$(\nabla^2 - \frac{2\mu}{\hbar^2}V_0 - \frac{\ell(\ell+1)}{r^2})(a\psi_S \;+\; b\psi_D) = \frac{2\mu}{\hbar^2}\epsilon(a\psi_S \;+\; b\psi_D)$$

$$\begin{aligned}
\{\nabla^2 + \frac{2\mu}{\hbar^2}(\epsilon - V_0)\}\psi_S &= 0\\
\{\nabla^2 + \frac{2\mu}{\hbar^2}(\epsilon - V_0) - \frac{6}{r^2}\}\psi_D &= 0
\end{aligned}$$

No Unique central potential, because of the repulsive centrifugal potential.

(b) Tensor force

Introduce the tensor operator

$$\begin{aligned}
S_{12} &= \frac{3}{r^2}(\vec{\sigma_1}\cdot\vec{r})(\vec{\sigma_2}\cdot\vec{r}) - (\vec{\sigma_1}\cdot\vec{\sigma_2})\\
&\equiv (\frac{24\pi}{5})^{1/2}\sum_m(\vec{\sigma_1}\vec{\sigma_2})_{2m}Y^*_{2m}(\hat{r})
\end{aligned}$$

$$\begin{aligned}
(\nabla^2 + \frac{2\mu}{\hbar^2}(\epsilon - V_0))\psi_S &= a\frac{2\mu}{\hbar^2}V_T \underbrace{< \psi_S|S_{12}|\psi_S >}_{\text{zero}} \psi_S\\
&+\; b\frac{2\mu}{\hbar^2}V_T \underbrace{< \psi_S|S_{12}|\psi_D >}_{\text{nonzero}} \psi_D\\
(\nabla^2 + \frac{2\mu}{\hbar^2}(\epsilon - V_0) - \frac{6}{r^2})\psi_D &= a\frac{2\mu}{\hbar^2}V_T \underbrace{< \psi_D|S_{12}|\psi_S >}_{\text{nonzero}} \psi_S\\
&+\; b\frac{2\mu}{\hbar^2}V_T \underbrace{< \psi_D|S_{12}|\psi_D >}_{\text{nonzero}} \psi_D
\end{aligned}$$

2 Symmetry and nuclear force

2.1 Nuclear force from the exp. facts

In the last century, we have worked to understand nuclear force with various exp techniques, mainly through scattering exp, *i.e.* $p-p$, $n-p$ scattering. Unfortunately what we know so far is very limited,

1) Nuclear force depends on everything it can depend on.
2) Nuclear force is of short range.
3) Nucleon-Nucleon(NN) cross sections are *small*
   $\sigma \sim \pi R^2 \sim 250$ millibarns
   $\sigma_{exp} \sim 40$ mb
4) Nuclear force is repulsive at short distances.
   Sign change of phase angle occurs at $E \sim 250$ MeV.
   (Remember that $\delta > 0$ for attractive force,
   and $\delta < 0$ for repulsive.)
   $$\begin{aligned} k &= \sqrt{mE}/\hbar \sim 1.7 \text{ fm}^{-1} \\ R &= 1/k \sim 0.6 \text{ fm}. \end{aligned}$$

2.2 Nuclear force from symmetries

Symmetry Properties $\rightarrow$ Invariance Principle

Symmetry	Conserved Quantities
1) Space Translation	Total linear momentum
	$\Rightarrow \ V(\vec{r})$
	Galilean invariance
	$\Rightarrow \ V(\vec{p})$
2) Space Rotation	Angular momentum
	$\Rightarrow$ V should be scalar
3) Space Reflection	Parity
	$\Rightarrow$ V not $\vec{\sigma_1} \times \vec{\sigma_2}$
4) Time Reversal	
	$\Rightarrow$ V not $\vec{r} \cdot \vec{p}$
5) Permutation of Identical particles	
6) Charge Independence	$(p,p) = (n,n)$
	$[H, T_z] = 0$
	$T_z \vert\psi> = \frac{1}{2}(Z-N)\vert\psi>$
7) Isospin space rot	$[H, T^2] = 0$
	$T^2\vert\psi> = T(T+1)\vert\psi>$
	Mirror Nuclei
	(Isobaric analog states)

2.3 Isospin part

Possible to have isospin dependent component
in nuclear force, since

a bound state for $T = 0$
no bound state for $T = 1$

$$
\begin{aligned}
T &= \frac{1}{2}\sum_{i=1}^{A}\tau_i = \frac{1}{2}(\vec{\tau}_1 + \vec{\tau}_2) \\
T^2 &= \frac{1}{4}(\vec{\tau}_1^2 + \vec{\tau}_2^2 + 2\vec{\tau}_1\cdot\vec{\tau}_2) \\
\vec{\tau}_1\cdot\vec{\tau}_2 &= 2T^2 - \frac{1}{2}\vec{\tau}_1^2 - \frac{1}{2}\vec{\tau}_2^2
\end{aligned}
$$

$$
\begin{aligned}
< \psi_N|\vec{\tau}_1\cdot\vec{\tau}_2|\psi_N > &= -3 \quad \text{for } T = 0 \text{ (Attractive)} \\
&= 1 \quad \text{for } T = 1 \text{ (Repulsive)}
\end{aligned}
$$

where we use

$$
\tau^2|\psi_N > = \tau_x^2 + \tau_y^2 + \tau_z^2|\psi_N > = 3|\psi_N >
$$

2.4 Most General Form of Nuclear Force

$$
\begin{aligned}
V(r, \vec{\sigma}_1, \vec{\sigma}_2, \vec{\tau}_1, \vec{\tau}_2) = \; & V_0(r) \\
& + V_\sigma(r)(\vec{\sigma}_1\cdot\vec{\sigma}_2) \\
& + V_\tau(r)(\vec{\tau}_1\cdot\vec{\tau}_2) \\
& + V_{\sigma\tau}(r)(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2) \\
& + V_{LS}(r)(\vec{L}\cdot\vec{S}) \\
& + V_{LS\tau}(r)(\vec{L}\cdot\vec{S})(\vec{\tau}_1\cdot\vec{\tau}_2) \\
& + V_T(r)S_{12} \\
& + V_{T\tau}(r)S_{12}(\vec{\tau}_1\cdot\vec{\tau}_2) \\
& + V_Q(r)Q_{12} \\
& + V_{Q\tau}(r)Q_{12}(\vec{\tau}_1\cdot\vec{\tau}_2) \\
& + V_{pp}(r)(\vec{\sigma}_1\cdot\vec{p})(\vec{\sigma}_2\cdot\vec{p}) \\
& + V_{pp\tau}(r)(\vec{\sigma}_1\cdot\vec{p})(\vec{\sigma}_2\cdot\vec{p})(\vec{\tau}_1\cdot\vec{\tau}_2)
\end{aligned}
$$

where

$$
\begin{aligned}
S_{12} &= \frac{3}{r^2}(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r}) - (\vec{\sigma}_1\cdot\vec{\sigma}_2) \\
Q_{12} &= \frac{1}{2}\left[(\vec{\sigma}_1\cdot\vec{L})(\vec{\sigma}_2\cdot\vec{L}) + (\vec{\sigma}_2\cdot\vec{L})(\vec{\sigma}_1\cdot\vec{L})\right] \\
\vec{L}\cdot\vec{S} &= \frac{1}{2}(\vec{\ell}_1 + \vec{\ell}_2)\cdot(\vec{\sigma}_1 + \vec{\sigma}_2)
\end{aligned}
$$

3 Yukawa Theory of nuclear interaction

3.1 Scalar field

For **Photon source**

$$\rho \; = \; e\delta(\vec{r})$$

It obeys Poisson's equation,

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi \; = \; -4\pi e\delta(\vec{r})$$

LHS can be obtained from $E^2 - p^2c^2$ by plugging $E = -\frac{\hbar}{i}\frac{\partial}{\partial t}$, and $\vec{p} = \frac{\hbar}{i}\nabla$.

The static solution would be

$$\phi_{static} \; = \; \frac{e}{r}$$

because

$$\nabla^2\frac{1}{r} = -4\pi\delta(r)$$

Interaction energy with a second charge at r,

$$V(r) \; = \; \frac{e^2}{r}$$

For **Pion source**

$$\rho \; = \; g\delta(\vec{r})$$

It obeys Klein-Gordon equation,

$$\nabla^2\phi - \mu^2\phi - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\phi \; = \; -4\pi g\delta(\vec{r})$$

where $\mu = \frac{m_x c}{\hbar}$.

LHS can be obtained from $E^2 - p^2c^2 - m_x^2c^4$. The static solution would be

$$\phi_{static} \; = \; g\frac{e^{-\mu r}}{r}$$

Interaction energy with a second charge at r,

$$V(r) \; = \; -g\phi_{static} \; = \; -g^2\frac{e^{-\mu r}}{r}.$$

The force range becomes,

$$R \sim \mu^{-1} \sim \frac{\hbar c}{m_x c^2} \sim \frac{197.3}{135} \sim 1.5 \text{ fm}.$$

3.2 Pseudo scalar field

The simple scalar theory met some difficulties,

1) Not spin-dependent
2) Not negative parity

Why negative? Consider the reaction

$$\begin{array}{cccccc} & \pi^- & + & d & \rightarrow & 2n \\ J & 0 & & 1 & & 1 \\ \pi & ? & & + & & - \end{array}$$

$2n$ system should be $T = 1$.

$\rightarrow$ $L + S$ =even

$(S = 0, L = 0)$ or $(S = 1, L = 1)$.

J of deuteron is 1.

$\rightarrow$ So is $2n$. $\vec{L} + \vec{S} = 1$

$(S = 0, L = 1)$, $(S = 1, L = 0)$ or $(S = 1, L = 1)$.

Thus $(S = 1, L = 1)$ is possible.

$\rightarrow$ The parity of $2n$ should be $-$.

Therefore parity of pion should have a **negative parity**.

$$\phi(-\vec{r}) = -\phi(\vec{r}).$$

We have to find pseudo scalar field.

$$\begin{array}{lcll} \text{Pseudo scalar} & = & \text{Pseudo vector} \cdot & \text{Polar vector} \\ & & \vec{\sigma} = \vec{r} \times \vec{p} & \vec{p} \end{array}$$

The field equation becomes,

$$(\nabla^2 - \mu^2)\phi(\vec{r}) = -4\pi\frac{g}{\mu}\vec{\sigma} \cdot \vec{\nabla}\rho(\vec{r})$$

where $\vec{\sigma}$ is the spin of nucleon, and $\vec{\nabla}$ is the momentum of pion acting on pion source. The solution is the well-known One Pion Exchange Potential **OPEP** with a form of Yukawa potential.

$$\begin{aligned} V(r) &= \frac{g^2}{3}[(\vec{\sigma_1} \cdot \vec{\sigma_2}) + (1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2})S_{12}]\frac{e^{-\mu r}}{r} \\ S_{12} &= 3(\vec{\sigma_1} \cdot \hat{r})(\vec{\sigma_2} \cdot \hat{r}) - (\vec{\sigma_1} \cdot \vec{\sigma_2}) \end{aligned}$$

Homework Set # 1.

1. If instead of the observed value of $J^\pi = 1^+$ the ground state of a deuteron is $J^\pi = 0^-$, what are now the possible values of orbital angular momentum L, sum of intrinsic spin S, and isospin T in this hypothetical state? What are the implication for nuclear force if this is true?

4 NN scattering

4.1 $p - n$ potential (Deuteron)

In the last semester, we studied the deuteron in the spherical well. (See Lecture II-5.) A more realistic estimate for the deuteron problem was obtained using a three-dimensional spherical well with characteristics

$$V(r) = \begin{cases} -|V|, & r < a \quad \text{region I,} \\ 0, & r \geq a \quad \text{region II} \end{cases}$$

The range of nuclear force is approximately 2.3 fm, while the binding energy is 2.23 MeV. We estimated the depth of the potential well as

$$|V| \approx 29.0 \text{ MeV} \tag{1}$$

Homework Set # 2.

1. Obtain the wave function from the above square well potential.

2. Obtain the binding energy and wave function by using the program "NEPTUNE". Use the Woods-Saxon potential with a radius parameter $R = 2.3$ fm and $a = 0.6$ fm. Compare the result with one obtained in the square well potential.

1) Deuteron

a. Hulthen wavefunction

$$\begin{aligned} \phi_d &= \frac{1}{\sqrt{4\pi}} \frac{u(r)}{r} \\ u(r) &= \sqrt{\frac{2\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2}}(e^{-\alpha r} - e^{-\beta r}) \end{aligned}$$

where $\alpha^{-1} = 4.3$ fm, and $\beta = 7\alpha$.

4.2 NN scattering experiments

(a) Possible experiments
(projectile, target) $= (p,p), (p,n), (n,p), (n,n)$

Nucleon	Projectile	Target
proton	Hydrogen	Hydrogen (easy source)
neutron	Thermal n	Colliding beam (unstable)

$T = 1$ $\quad (p,p), (n,n) \quad \rightarrow$ Charge Independence
(Only Coulomb difference)
$T = 0, 1$ $\quad (p,n), (n,p) \quad \rightarrow$ Time Reversal Invariance
$((p,n) = (n,p))$

(b) Experimental Phase Shifts

Reaction	T	S	L	J	State	Spec. Nota.
(p,p)	1	0	even(0,2)	L	SE	$^1S_0,^1D_2$
		1	odd(1)	0,1,2	TO	$^3P_0,^3P_1,^3P_3$
(n,p)	1	0	even(0,2)	L	SE	$^1S_0,^1D_2$
		1	odd(1)	0,1,2	TO	$^3P_0,^3P_1,^3P_3$
(n,p)	0	0	odd(1)	L	SO	$^1P_1$
		1	even(0)	1	TE	$^3S_1$
			even(2)	1,2,3	TE	$^3D_1,^3D_2,^3D_3$

Note that Fig. 3.3(a) $\approx$ Fig. 3.3(b). It is because of experimental errors or weak charge dependence.

(c) Inelastic Scattering

Pion Production in NN scattering can be obtained for the energy greater than $E_{cm} > m_\pi \sim 140$ MeV, or $E_{lab} >\sim 300$ MeV. (Fig. 3.4).

Inelastic process $\rightarrow$ Absorption

$$\begin{array}{lcl} p + p & \rightarrow & \Delta^{++} + n \\ p + p & \rightarrow & p + n + \pi \\ p + p & \rightarrow & p + p + p + \bar{p} \end{array}$$

No problems for $E < 300$ MeV.

It means that loss of flux from the incident channel occurs. (Prob. Amp. in the incident channel is not conserved. $\Rightarrow$ Complex potential scattering.

The scattering amplitude becomes

$$f(\theta) = \sqrt{4\pi} \sum_{\ell=0}^{\infty} \sqrt{2\ell+1}\; f_\ell \; Y_{\ell 0}(\theta)$$

For pure elastic scattering,

$$f_\ell = \frac{1}{k} e^{i\delta_\ell} \sin\delta_\ell = \frac{1}{2ik}(e^{2i\delta_\ell} - 1)$$

where δ_ℓ is **real**. In fact the phase shifts are real for $E_{lab} < 300$ MeV.

For Inelastic scattering,

$$f_\ell = \frac{1}{2ik}(\eta_\ell e^{2i\delta_\ell} - 1)$$

where δ_ℓ is **imaginary**, and η_ℓ (< 1) is the absorption coefficient. or inelastic parameter.

In summary so far, we have

	Type of nuclear force
Deuteron	Central
	Tensor
Yukawa theory	Tensor
Low energy NN scattering	spin-orbit
	spin-spin
	isospin-isospin
High energy NN scattering	complex

4.3 Effective NN potentials

1. Hamada-Johnston Potential - Low energy

The most detailed fits to nuclear phase shifts have been obtained by using the local potentials by considering the type of potentials we have discussed. In other words, these potentials was obtained from purely phenomenological considerations, and parameterized with the restriction that they tend to the one-pion exchange potential at large distances. The typical one of such potentials is Hamada-Johnston potential (Nucl. Phys. **37** (1962) 38a.), which is given by

$$
\begin{aligned}
V &= V_C + S_{12}V_T + \vec{S}\cdot\vec{L}V_S + L_{12}V_{SS} \\
S_{12} &= \frac{3}{r^2}(\vec{\sigma_1}\cdot\vec{r})(\vec{\sigma_2}\cdot\vec{r}) - (\vec{\sigma_1}\cdot\vec{\sigma_2}) \\
L_{12} &= (\vec{\sigma_1}\cdot\vec{\sigma_2})\vec{L}^2 - \frac{1}{2}\left[(\vec{\sigma}_1\cdot\vec{L})(\vec{\sigma}_2\cdot\vec{L}) + (\vec{\sigma}_2\cdot\vec{L})(\vec{\sigma}_1\cdot\vec{L})\right] \\
V_C &= \frac{0.08}{3\mu_\pi}(\vec{\tau_1}\cdot\vec{\tau_2})(\vec{\sigma_1}\cdot\vec{\sigma_2})Y(x)[1 + a_C Y_(x) + b_C Y^2(x)] \\
V_T &= \frac{0.08}{3\mu_\pi}(\vec{\tau_1}\cdot\vec{\tau_2})Z(x)[1 + a_T Y_(x) + b_T Y^2(x)] \\
V_S &= \mu_\pi G_S Y^2(x)[1 + b_S Y(x)] \\
V_{SS} &= \mu_\pi G_{SS} Z(x)[1 + a_{SS} Y_(x) + b_{SS} Y^2(x)] \\
& Y(x) = \frac{e^{-x}}{x} \\
& Z(x) = (1 + \frac{3}{x} + \frac{3}{x^2})Y(x) \\
& x = \mu_\pi r
\end{aligned}
$$

The units of potential strengths are $(\mu_\pi^2/\hbar^2)$ MeV. The same hard core with its radius of 0.485 fm is added in all states.

2. Love-Franey NN effective interaction - High energy

W. G. Love and M. A. Franey [Phys. Rev. **C24** (1981) 1073; **C27** (1983) 438(E); **C31** (1985) 488.] determined a local representation of the free NN effective interaction for several projectile bombarding energies between 100 and 1000 MeV/nucleon and presented in tabular form. The form of the interaction has been tailored for use in calculations of elastic and inelastic proton scattering in this energy range as a superposition of Yukawa types,

$$
\begin{aligned}
V_{12}(r) &= V^C(r) + V^{LS}\vec{S}\cdot\vec{L} + V^T(r)S_{12} \\
V^C(r) &= \sum_{i=1}^{n} V_i^C Y(r/R_i), \quad Y(x) = e^{-x}/x \\
V^{LS}(r) &= \sum_{i=1}^{n} V_i^{LS} Y(r/R_i) \\
V^T(r) &= \sum_{i=1}^{n} V_i^T r^2 Y(r/R_i) \\
S_{12} &= \frac{3}{r^2}(\vec{\sigma_1}\cdot\vec{r})(\vec{\sigma_2}\cdot\vec{r}) - (\vec{\sigma_1}\cdot\vec{\sigma_2})
\end{aligned}
$$

Of course, the strength parameters are complex.

4.4 Theoretical approaches

(a) One Pion Exchange Potential (OPEP)

OPEP fits exp. data only for inter-nucleon distances
greater than 2 fm $\Rightarrow$ $m_\pi \sim 140$ MeV$/c^2$, or
$R \sim \frac{\hbar c}{m_\pi c^2} \sim 1.4$ fm.

Difficulties
a) Hard core at short distances
  $E > 250$ MeV for ${}^1S_0$-channel $\rightarrow$ δ becomes negative.
  $E > 300$ MeV for ${}^3S_1$-channel
  **Possible explanation
  $\Rightarrow$ Apply Pauli Principle in the quark picture
  (6 Fermions cannot be in S-states $\rightarrow$ repulsive effect)
  But not calculable in QCD.
b) Strength of pion-nucleon vertex
  $g \neq g_{\pi N}$ (Real pion vertex)

(b) One Boson Exchange Potential (OBEP)

Range	Propagators
$r > 2$ fm	One-pion
$1 < r < 2$ fm	Scalar meson or two-pion
$r < 1$ fm	Heavy vector meson, multi-pion, QCD effect.

(c) Phenomenological NN Potentials

Long	OPEP
Intermediate	OBEP
Short	By hand
Shortfalls	Not consider the lifetime of mesons
	OBE range $\rightarrow$ adjustable.

(d) Hadronic NN potentials

Propagator	Paris	Bonn
One-pion	yes	yes
Two-pion	yes	yes
3,4-pion	phenomenological	Estimate
Δ particle	no	yes
Antinucleon	yes	yes

(e) QCD level

Quantitative connection between NN and $q - q$ force is still lacking.
a) At large distances, Force between two bags of 3 quarks = Meson exchange.
b) QCD in the low energy limit - NJL Model, Skyrme Model, Chiral perturbation approach, QCD Sum Rules etc.

8. NN INTERACTION – FIGURE

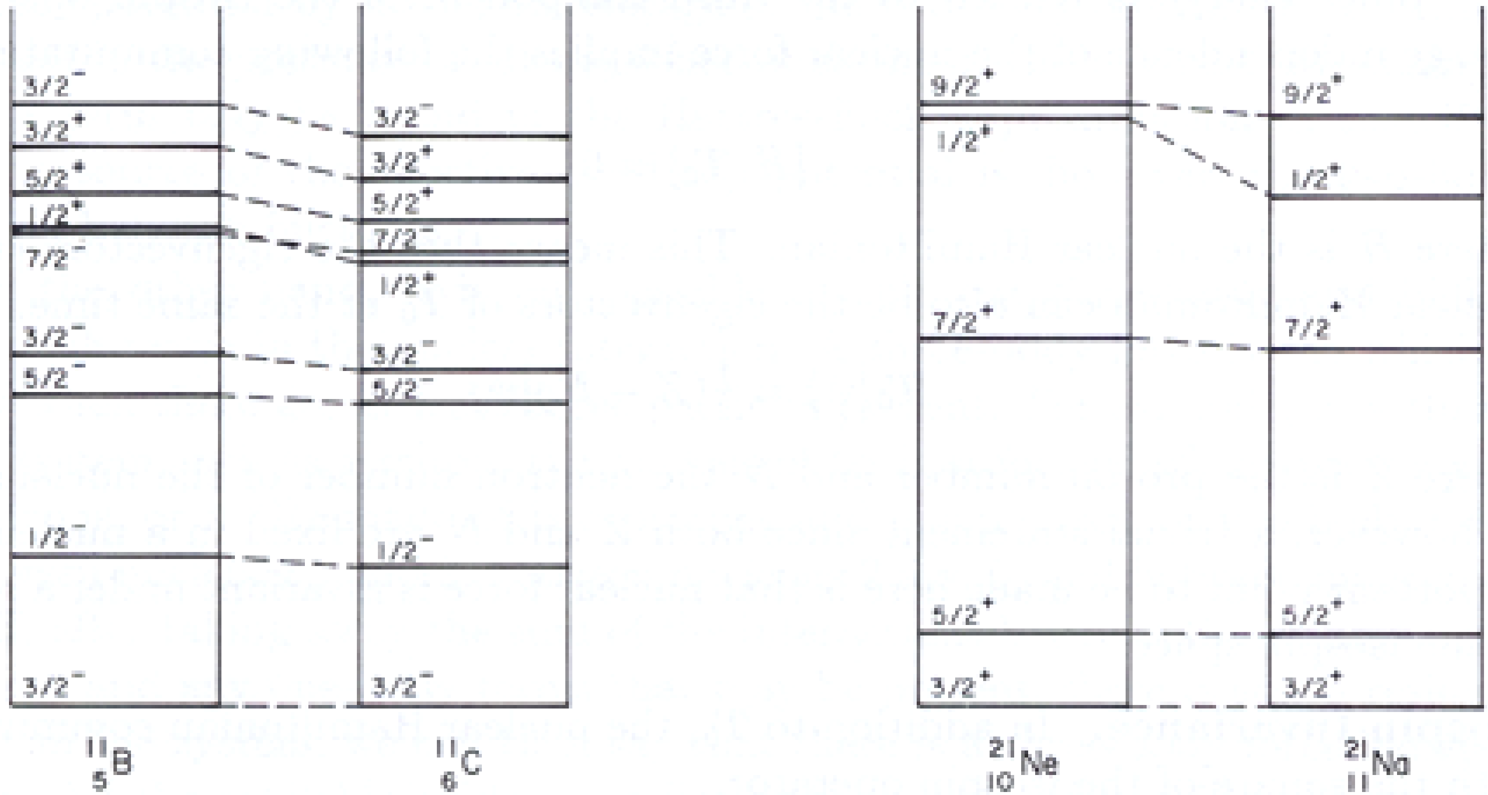
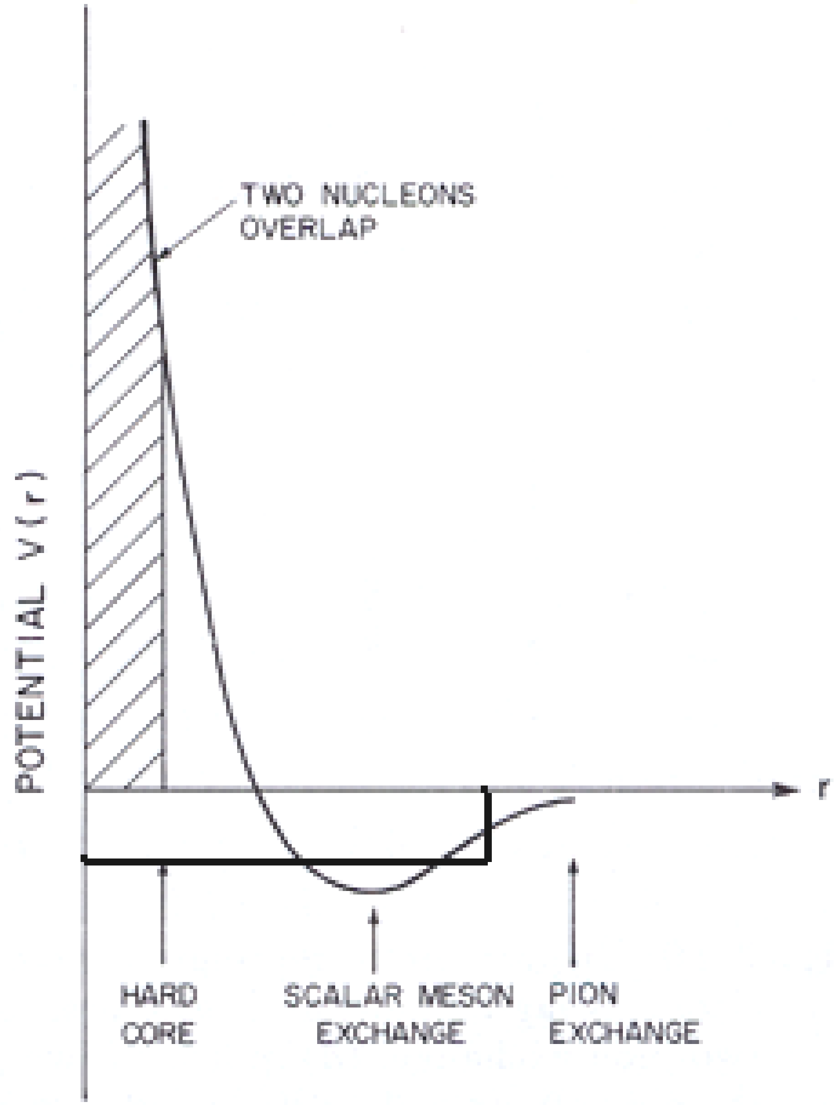
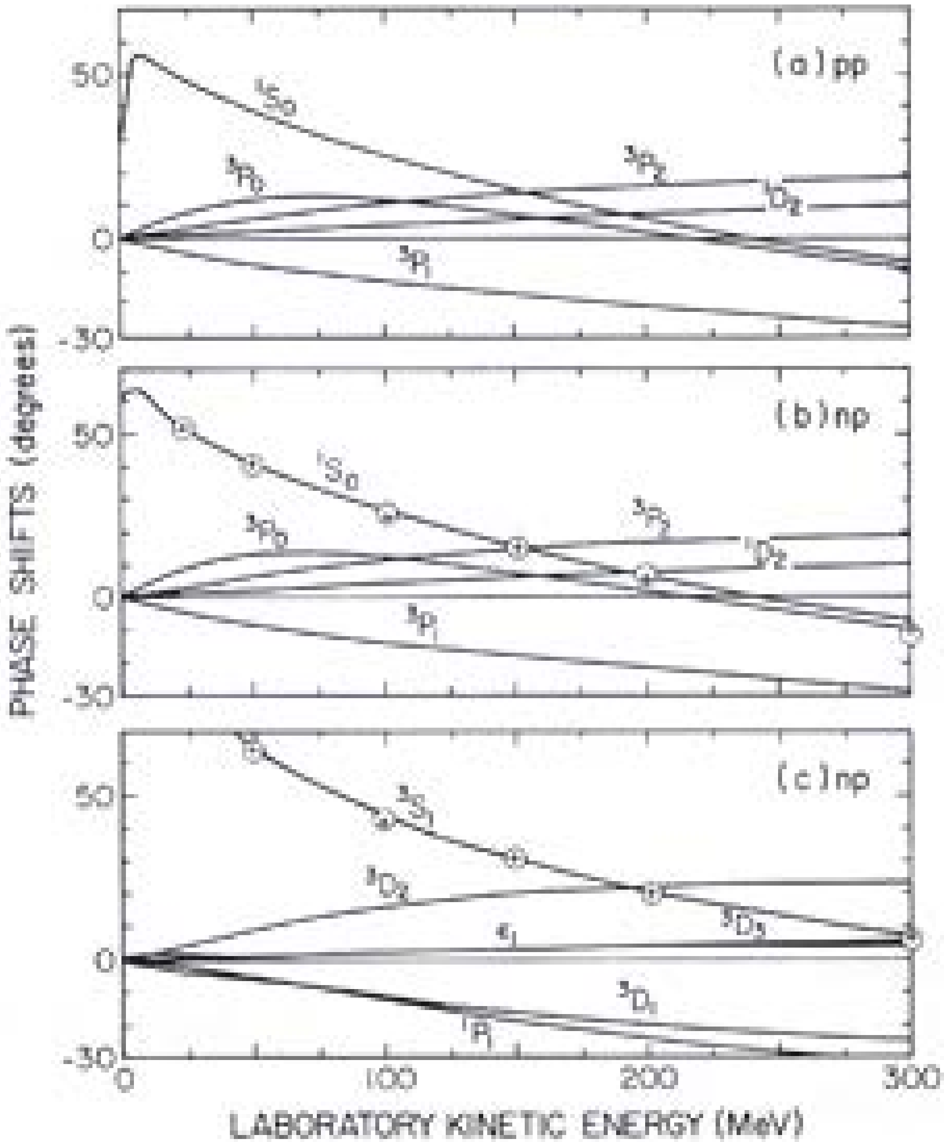


Fig. 3-1 A comparison of the low-lying spectra of members of the $A = 11$ and $A = 21$ isobars, showing the similarity of their level structure. The existence of isobaric analogue states, identical states in nuclei having the same A but different in N and Z, is a good indication of the isospin invariance of nuclear force.



POTENTIAL V(r)
TWO NUCLEONS
OVERLAP
r
HARD
CORE
SCALAR MESON
EXCHANGE
PION
EXCHANGE



(a)pp
(b)np
(c)np
$^1S_0$
$^3P_2$
$^3P_0$
$^1D_2$
$^3P_1$
$^3S_1$
$^3D_2$
ϵ_1
$^3D_3$
$^3D_1$
$^1P_1$
50
0
-30
100
200
300
PHASE SHIFTS (degrees)
LABORATORY KINETIC ENERGY (MeV)

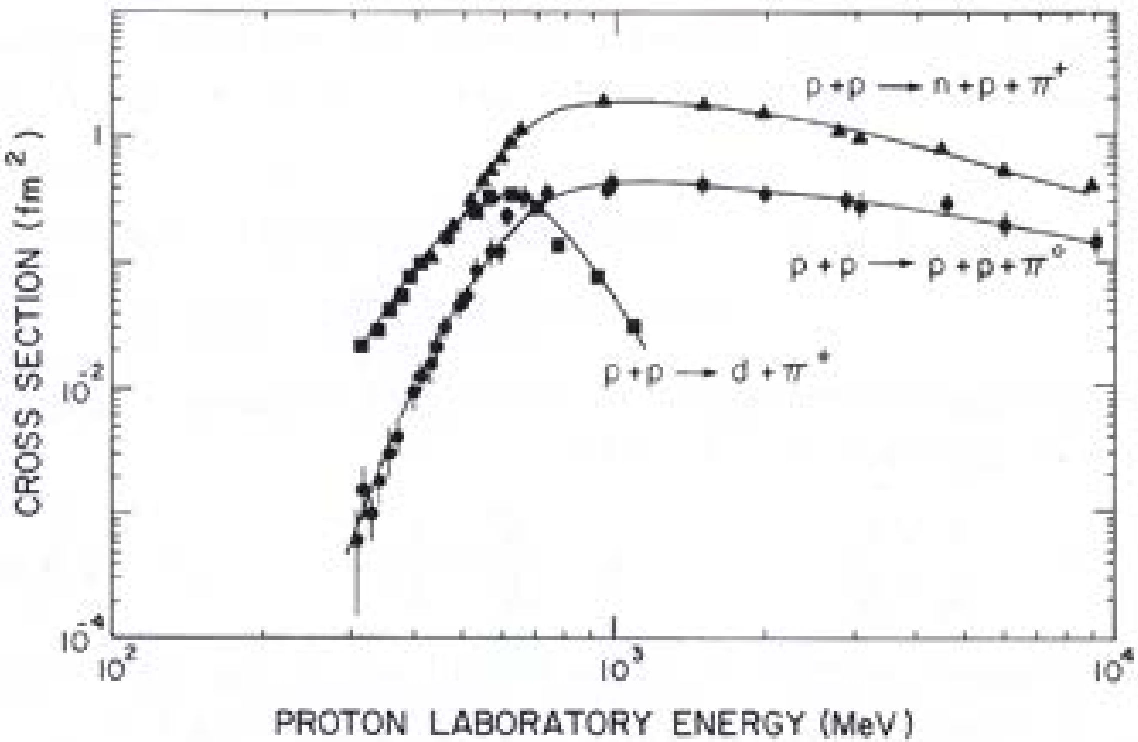


Fig. 3-4 Energy dependence of the total cross section for pion production in pp-scattering through reactions $p+p \to d+\pi^+$, $p+p \to p+n+\pi^+$ and $p+p \to p+p+\pi^0$. (Adapted from G. Jones, in *Pion Production and Absorption in Nuclei – 1981* ed. by R.D. Bent, AIP Conf. Proc. **79** [1982], Amer. Inst. Phys., New York, p. 15.)

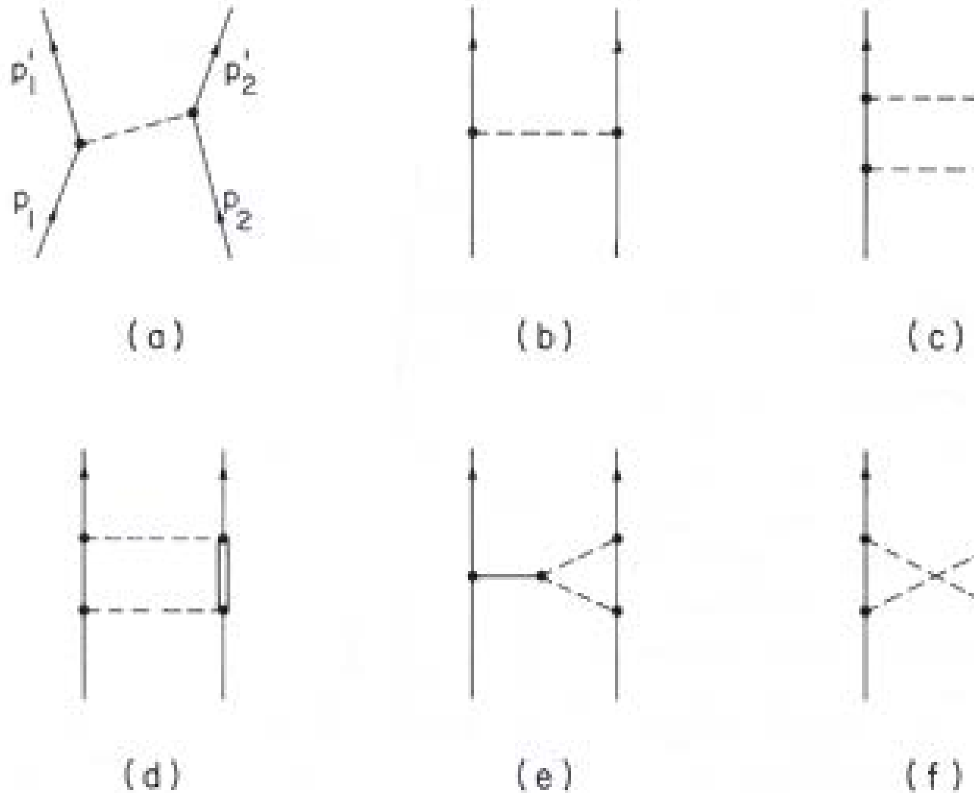
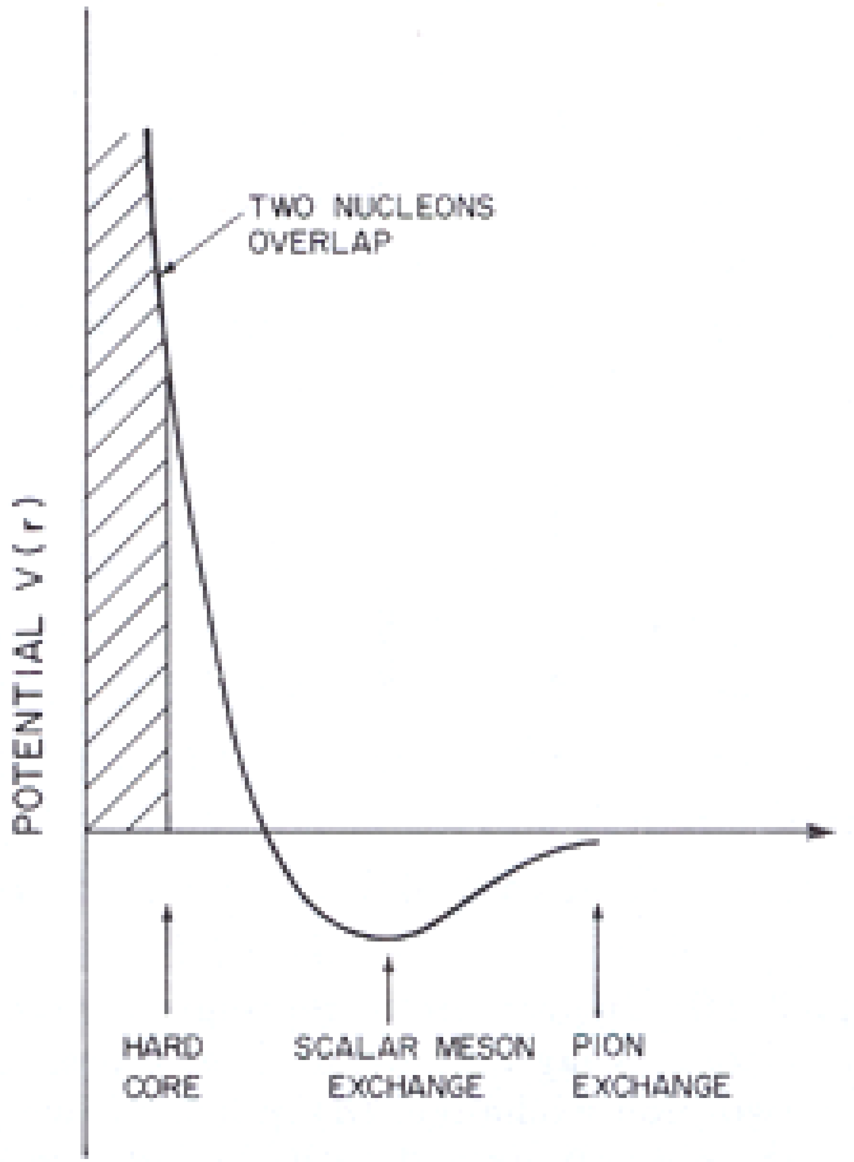
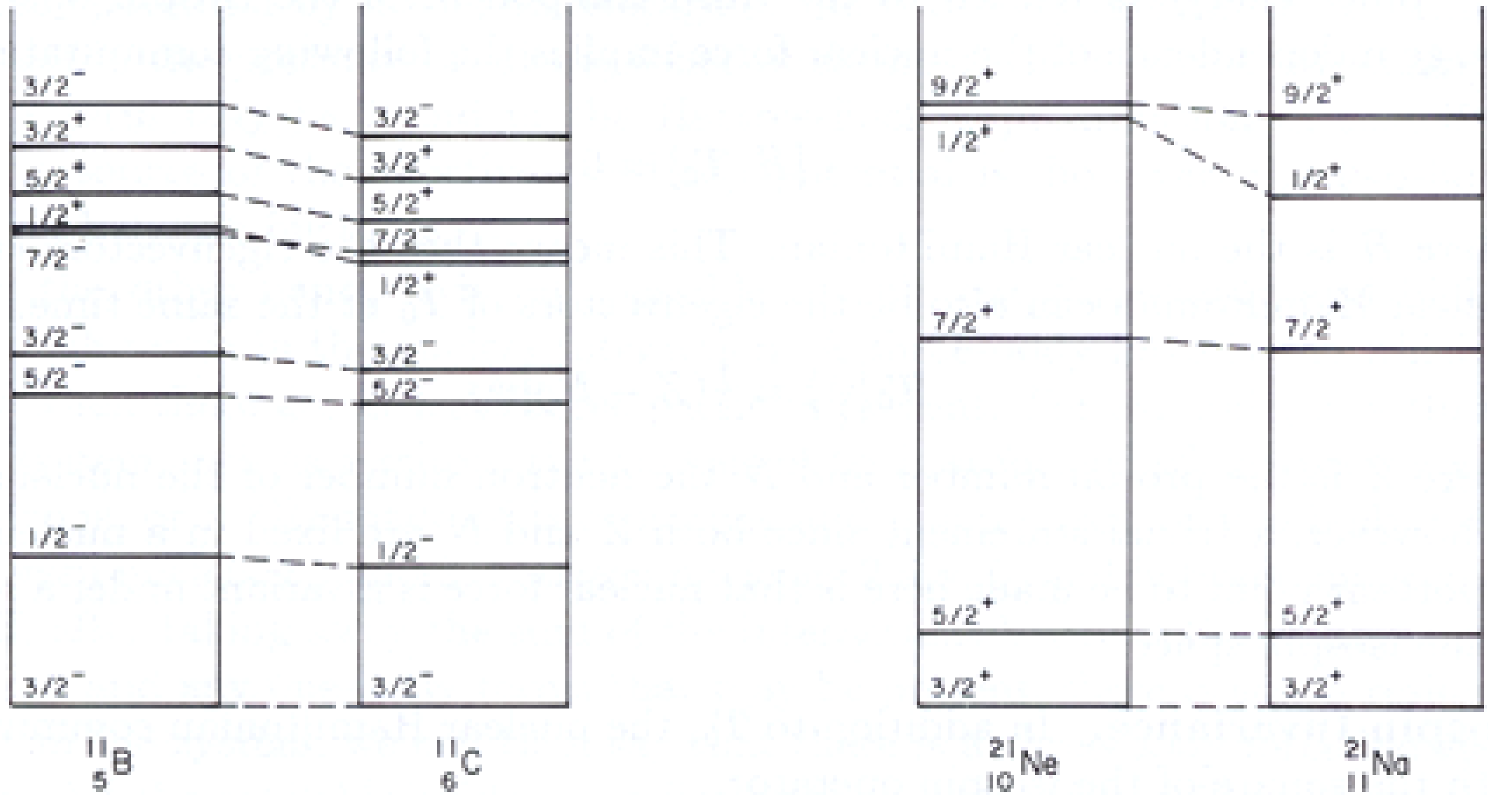


Fig. 3-10 Diagrammatic representation of meson exchange between two nucleons. Diagrams (a) and (b) represent one-pion exchange, (c) indicates two-pion exchange, (d) indicates two-pion exchange with intermediate state involving a Δ-particle, (e) shows a ρ-meson exchange and (f) is another type of two-pion exchange term where both pions are emitted before either one is absorbed.



TWO NUCLEONS OVERLAP
POTENTIAL V(r)
r
HARD CORE
SCALAR MESON EXCHANGE
PION EXCHANGE

8. NN INTERACTION – FIGURE



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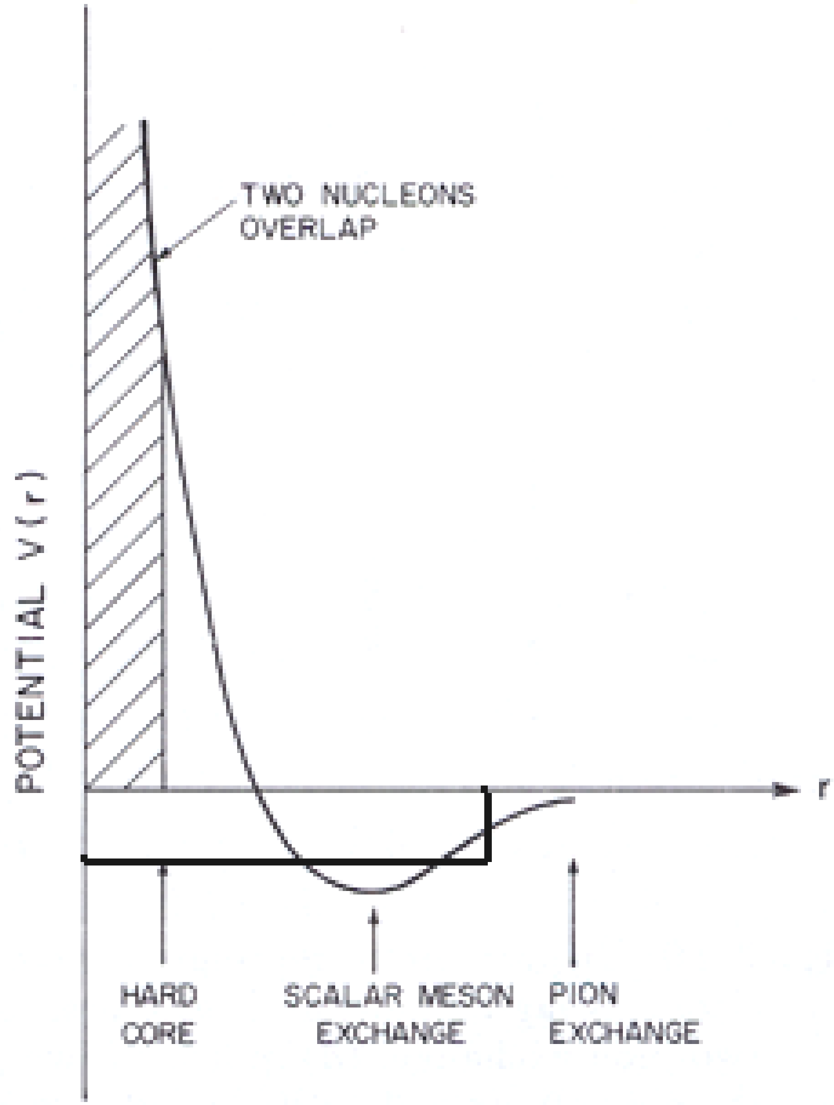
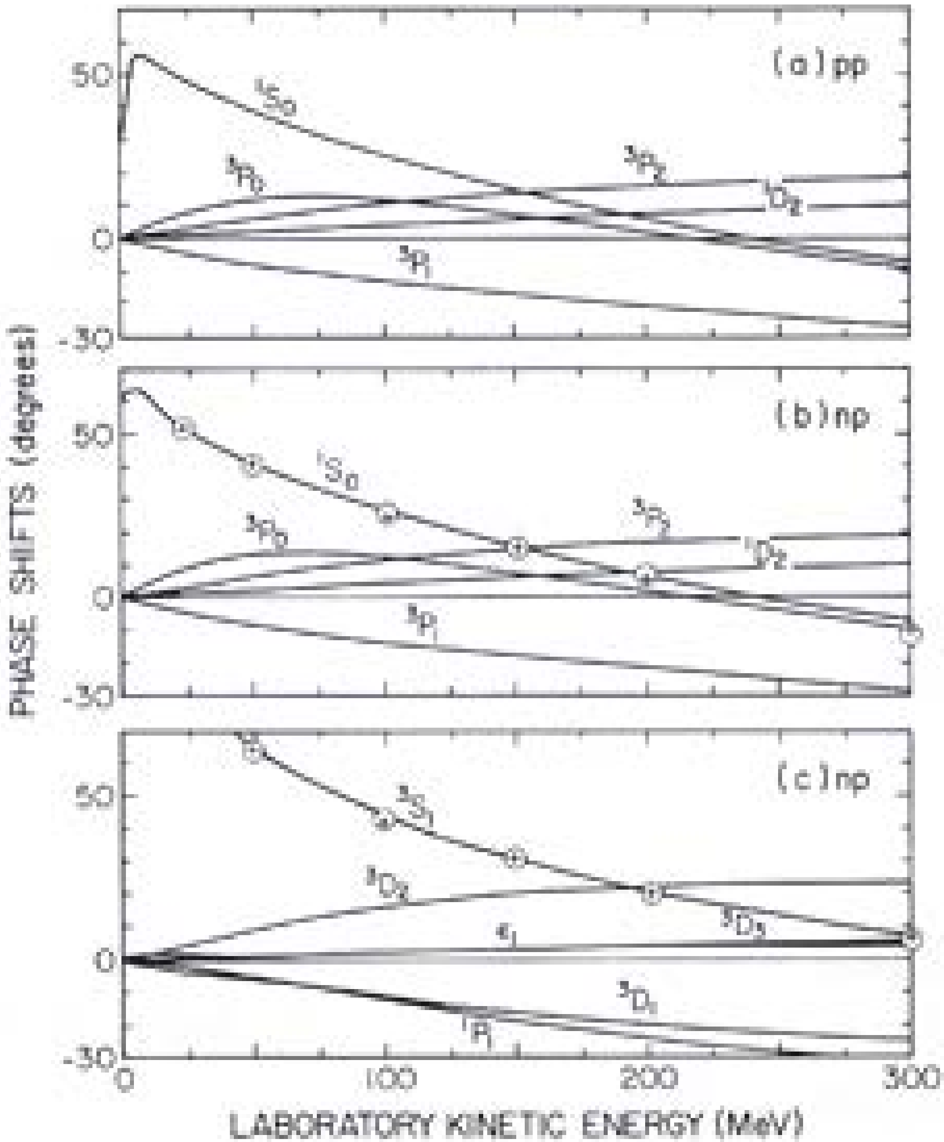
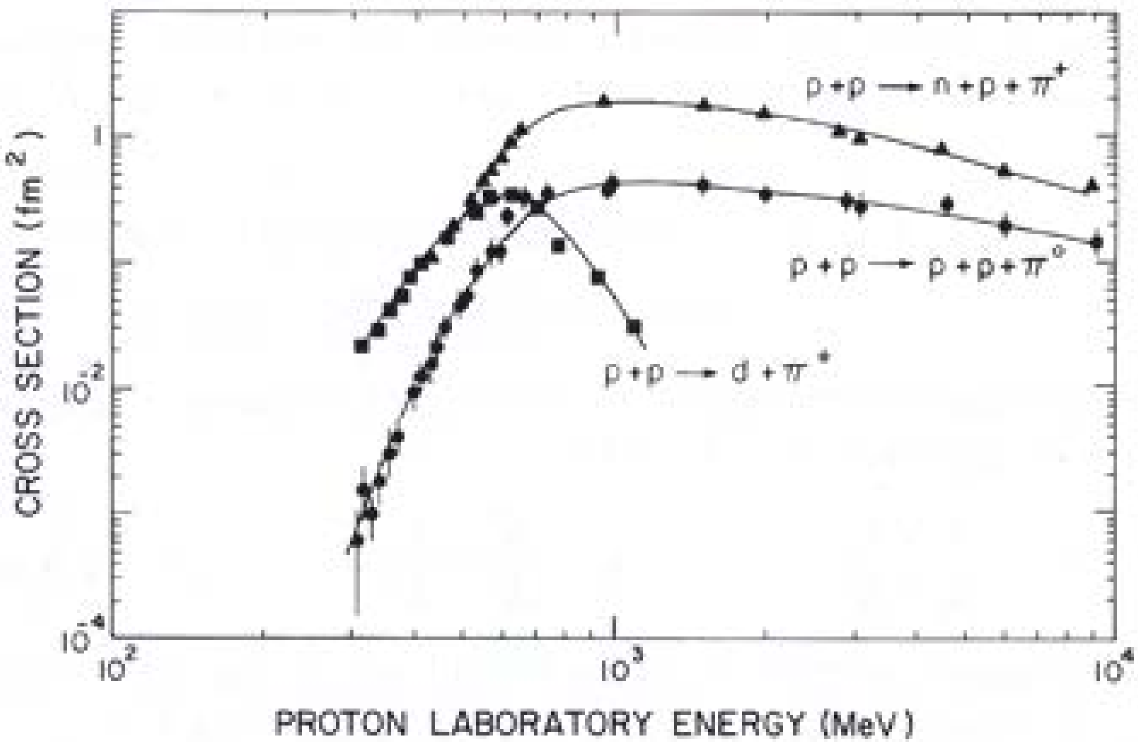

TWO NUCLEONS
OVERLAP
POTENTIAL V(r)
r
HARD
CORE
SCALAR MESON
EXCHANGE
PION
EXCHANGE


(a)pp
(b)np
(c)np
PHASE SHIFTS (degrees)
LABORATORY KINETIC ENERGY (MeV)
50
0
-30
100
200
300
$^1S_0$
$^3P_0$
$^3P_1$
$^3P_2$
$^1D_2$
$^3S_1$
$^3D_3$
$^3D_2$
ϵ_1
$^3D_1$
$^1P_1$



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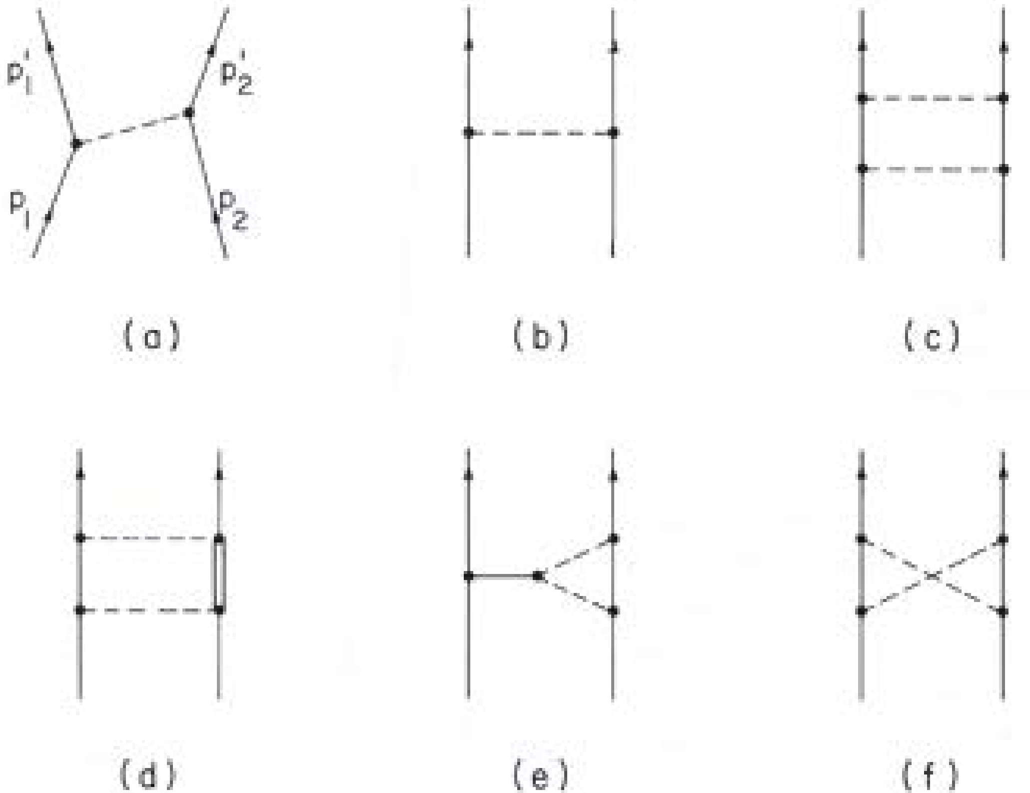

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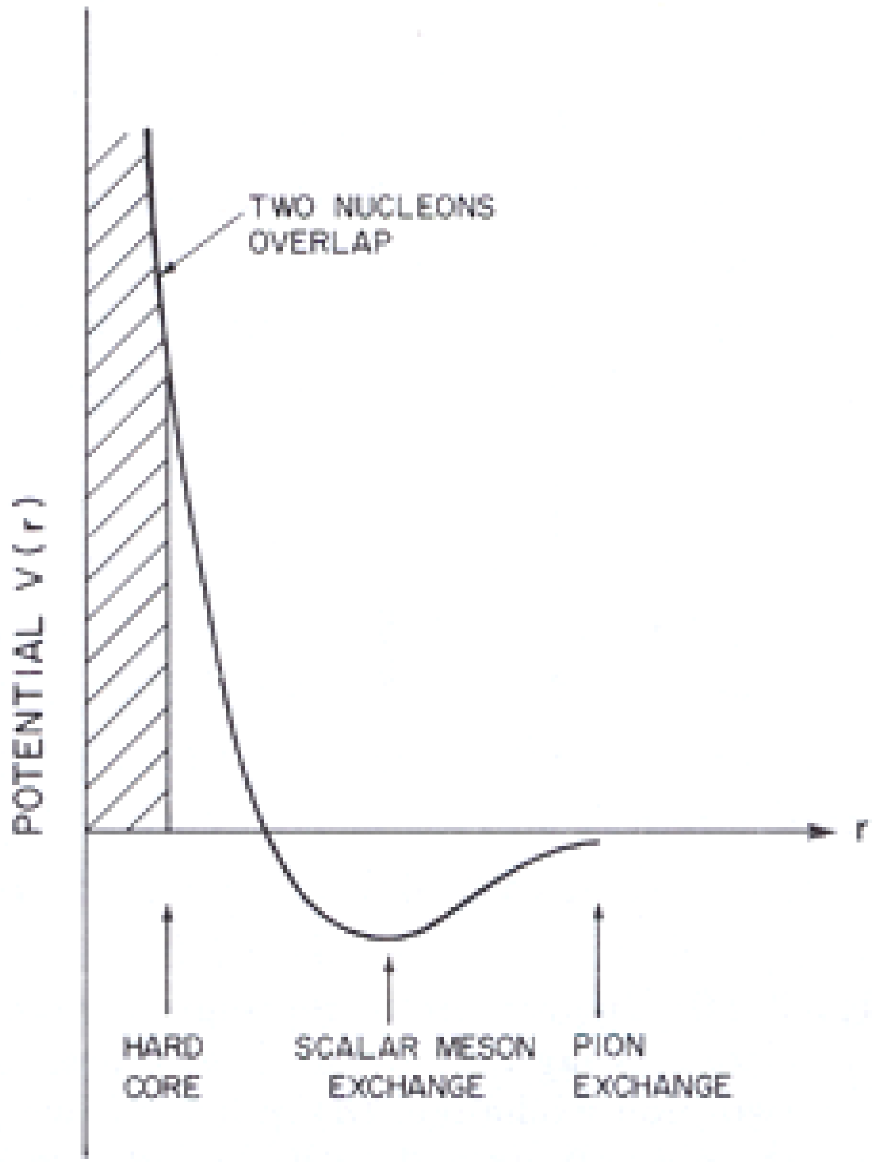
TWO NUCLEONS
OVERLAP
POTENTIAL V(r)
r
HARD
CORE
SCALAR MESON
EXCHANGE
PION
EXCHANGE