Light nuclei quasiparticle energy shifts in hot and dense nuclear matter

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Nuclei in dense matter are influenced by the medium. In the cluster mean-field approximation, an effective Schrödinger equation for the A-particle cluster is obtained accounting for the effects of the correlated medium such as self-energy, Pauli blocking, and Bose enhancement. Similar to the single-baryon states (free neutrons and protons), the light elements ($2 \leqslant A \leqslant 4$, internal quantum state ν) are treated as quasiparticles with energies $E_{A,\nu}(\mathbf{P}; T, n_n, n_p)$. These energies depend on the center-of-mass momentum $\mathbf{P}$, as well as temperature T and the total densities n_n, n_p of neutrons and protons, respectively. No β equilibrium is considered so that n_n, n_p (or the corresponding chemical potentials μ_n, μ_p) are fixed independently. For the single-nucleon quasiparticle energy shift, different approximate expressions such as Skyrme or relativistic mean-field approaches are well known. Treating the A-particle problem in appropriate approximations, results for the cluster quasiparticle shifts are given. Properties of dense nuclear matter at moderate temperatures in the subsaturation density region considered here are influenced by the composition. This in turn is determined by the cluster quasiparticle energies, in particular the formation of clusters at low densities when the temperature decreases and their dissolution due to Pauli blocking as the density increases. Our finite-temperature Green function approach covers different limiting cases: the low-density region where the model of nuclear statistical equilibrium and virial expansions can be applied and the saturation-density region where a mean-field approach is possible.

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I. INTRODUCTION

A well-established tool in treating many-particle systems is the quasiparticle concept. In contrast to free particles, the properties of quasiparticles, such as the dispersion relation, are modified due to the interaction with other particles. This relation between energy and momentum is often characterized by an energy shift and an effective mass that depends on temperature and density of the medium. A significant part of the interaction can be taken into account by introducing this approach. The quasiparticle concept is not restricted to elementary particles like nucleons only, but it can also be applied to composed particles, i.e., nuclei. (Note that the nucleons themselves are also composed particles.) In this work we evaluate these cluster quasiparticle energies within a microscopic approach.

Nuclear matter is a strongly interacting quantum fluid. To treat warm and dilute matter (i.e., at subsaturation baryon densities $n_B = n_n + n_p \lesssim 0.16$ fm^{-3} and temperatures $T \lesssim 20$ MeV) within a systematic quantum statistical approach, we start from a nonrelativistic Hamiltonian

$$H = \sum_1 E(1) a_1^\dagger a_1 + \frac{1}{2} \sum_{12,1'2'} V(12, 1'2') a_1^\dagger a_2^\dagger a_{2'} a_{1'}, \quad (1)$$

where $\{1\}$ denotes momentum $\hbar \mathbf{P}_1$, spin σ_1, and isospin τ_1 characterizing the neutron (n) or proton (p) state. The kinetic energy in fermion second quantization $a_1^\dagger, a_1$ contains $E(1) = \hbar^2 P_1^2 / 2m_1$, whereas the potential energy contains the matrix element $V(12, 1'2')$ of the nucleon-nucleon interaction.

Because there is no fundamental expression for the nucleon-nucleon interaction (like, e.g., the Coulomb interaction in charged particle systems [1]), it is taken to reproduce empirical data such as the nucleon-scattering phase shifts. Different parametrization are in use. Simple potentials as proposed by Yukawa, Yamaguchi, Mongan, Gogny, and others are based on two-nucleon phase shifts and can be used for exploratory calculations. For detailed calculations one can use more sophisticated potentials such as PARIS and BONN or their separable representations [2]. To obtain the empirical parameter values of nuclear matter at saturation density, three-body forces have been introduced in the Hamiltonian (1). In particular, the Argonne AV18/UIX potential [3] has been used to calculate light nuclei [4].

Near saturation density, nucleons can be treated as quasiparticles. Semiempirical approaches such as the Skyrme contact pseudopotential [5] and relativistic mean-field (RMF) approaches [6] parametrize the quasiparticle shift as a function of densities and temperature. Microscopic approaches such as Dirac-Brueckner Hartree Fock (DBHF) [7] give an appropriate description of the thermodynamic properties of warm and dense matter. For a recent review of the nuclear matter equation of state (EOS) see Ref. [8].

In the low-density limit, nuclear matter at finite temperature is a mixture of free nucleons and nuclei in chemical equilibrium as described by a mass action law. This chemical picture, where bound states are treated as new species, is also denoted as nuclear statistical equilibrium (NSE) and should be recovered as the low-density limit of a quantum statistical approach to nuclear matter at finite temperatures. The application of the NSE to expanding hot and dense matter, e.g., the statistical multifragmentation model in heavy-ion reactions, is very successful (for a recent review see Ref. [9]).

At increasing densities, the simple NSE approach becomes invalid because of interactions between the nucleons and nuclei. A systematic coherent description should include scattering states, avoid double counting of many-particle effects, and avoid semiempirical concepts such as excluded volume. This can be achieved within a physical picture where bound

states are produced by the interaction, investigating the few-particle propagator in the medium. The chemical picture can be used as a guideline to select important contributions within a perturbation approach and performing partial summations of the corresponding Feynman diagrams. In the case of charged particle systems, where the interaction is given by the Coulomb potential, the use of the chemical picture to introduce bound states in the partially ionized plasma has been extensively worked out; see Ref. [1].

The in-medium wave equation for a system of A nucleons is derived in Sec. II. Bound states describe nuclei with energy eigenvalues $E_{A,\nu}(P;T,n_p,n_n)$, where A denotes the mass number. The index ν specifies the internal state of the A-nucleon system such as spin, isospin, and excitation. In a homogeneous system, the center-of-mass momentum $\mathbf{P}$ of the cluster is conserved and can be used as quantum number. Assuming (local) thermal equilibrium, due to the influence of the surrounding matter these energy eigenvalues will depend on three parameters: the temperature T and the total number densities n_p, n_n of protons and neutrons, respectively [or the baryon density $n_B = n_n + n_p$ and the asymmetry parameter $\alpha = (n_n - n_p)/n_B$]. We do not consider β equilibrium due to weak interaction processes. Explicit results for the corresponding quasiparticle shifts are given for ^{2}H (deuteron d), ^{3}H (triton t), ^{3}He (helion h), and ^{4}He (α-particle) in Sec. IV. Concluding remarks will be given in Sec. V, where also the relation to the generalized Beth-Uhlenbeck formula [10–12] is outlined. A nuclear matter EOS is obtained that contains the quasiparticle energies of the single-nucleon states as well as of the light nuclei that fades with increasing density so that a smooth transition from low densities described by the NSE to saturation density is obtained.

II. MANY-PARTICLE APPROACH

A. Single-particle spectral function and quasiparticles

The basic equations of a Green function approach to the many-nucleon system can be found in different textbooks and articles; see Refs. [10,13]. Here we give only some final results.

The propagation of a single-nucleon excitation $\langle a_1^\dagger(t)a_{1'}\rangle$ can be expressed in terms of the single-particle spectral function $S_1(1,\omega)$ as

$$\langle a_1^\dagger(t)a_{1'}\rangle = \delta_{1,1'}\int\frac{d\omega}{2\pi}e^{i\omega t}f_{1,Z}(\omega)S_1(1,\omega), \tag{2}$$

where

$$f_{A,Z}(\omega) = [\exp\{\beta[\omega - Z\mu_p - (A-Z)\mu_n]\} - (-1)^A]^{-1} \tag{3}$$

is the Fermi or Bose distribution function that depends on the inverse temperature $\beta = 1/(k_B T)$ and the chemical potentials μ_p, μ_n (instead of the isospin quantum number τ_1 we use the charge number Z). A special case is the single-particle density matrix $\langle a_1^\dagger a_{1'}\rangle$ that can be used to evaluate the equation of state (EOS) for the nucleon density

$$n_\tau(\beta,\mu_p,\mu_n) = \frac{1}{\Omega}\sum_1\langle a_1^\dagger a_1\rangle\delta_{\tau,\tau_1} = 2\int\frac{d^3P_1}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}f_{1,Z}(\omega)S_1(1,\omega), \tag{4}$$

where Ω is the system volume and summation over spin direction is collected in the factor 2. Both the Fermi distribution function and the spectral function depend on the temperature and the chemical potentials μ_p, μ_n not given explicitly. We work with a grand canonical ensemble and have to use the EOS (4) to replace the chemical potentials by the densities n_p, n_n. For this equation of state, expressions such as the Beth-Uhlenbeck formula and its generalizations have been derived [10–12].

The spectral function is related to the self-energy according to

$$S_1(1,\omega) = \frac{2\mathrm{Im}\Sigma(1,\omega - i0)}{[\omega - E(1) - \mathrm{Re}\Sigma(1,\omega)]^2 + [\mathrm{Im}\Sigma(1,\omega - i0)]^2}, \tag{5}$$

where the imaginary part has to be taken for a small negative imaginary part in the frequency. The solution of the relation

$$E_1^{\mathrm{qu}}(1) = E(1) + \mathrm{Re}\Sigma\big[1, E_1^{\mathrm{qu}}(1)\big] \tag{6}$$

defines the single-nucleon quasiparticle energies $E_1^{\mathrm{qu}}(1) = E(1) + \Delta E^{\mathrm{SE}}(1)$. Expanding for small $\mathrm{Im}\Sigma(1,z)$, the spectral function yields a δ-like contribution, and the densities are calculated from Fermi distributions with the quasiparticle energies so that

$$n_\tau^{\mathrm{qu}}(\beta,\mu_p,\mu_n) = \frac{2}{\Omega}\sum_{P_1} f_{1,Z}\big[E_1^{\mathrm{qu}}(1)\big] \tag{7}$$

follows for the EOS in mean-field approximation. This result does not contain the contribution of bound states and therefore fails to be correct in the low-temperature, low-density limit where the NSE describes the nuclear matter EOS.

As shown in Refs. [10,11], the bound-state contributions are obtained from the poles of $\mathrm{Im}\Sigma(1,z)$ that cannot be neglected expanding the spectral function. A cluster decomposition of the self-energy has been proposed; see Fig. 1. The diagrams

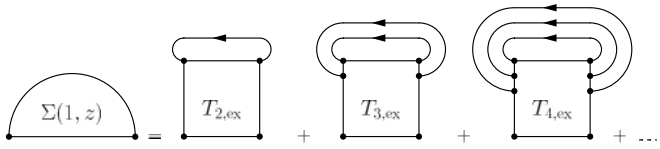

FIG. 1. Cluster decomposition of the single-nucleon self-energy. The index "ex" denotes full antisymmetrization, including all exchange diagrams. For bound states with $A \leqslant 4$ the direct ladder T matrices can be taken, without the full exchange at the end, as long as the particles have different internal quantum numbers σ, τ. The Fock term results from $T_{2,\mathrm{ex}}$ (first order in V, exchange term; see also Appendix B).

are calculated as

$$\Sigma(1, z_\nu) = \sum_{A>1} \sum_{\Omega_\lambda, 2\ldots A} G^{(0)}_{A-1}(2, \ldots, A, \Omega_\lambda - z_\nu) \times \mathrm{T}_A(1 \ldots A, 1' \ldots A', \Omega_\lambda). \quad (8)$$

The free $(A-1)$ quasiparticle propagator $G^{(0)}_{A-1}$ and the T_A matrix are given in Appendix A. The T_A matrices are related to the A-particle Green functions that read in bilinear expansion

$$G_A(1 \ldots A, 1' \ldots A', z_A) = \sum_{\nu P} \psi_{A\nu P}(1 \ldots A) \frac{1}{z_A - E^{\mathrm{qu}}_{A,\nu}(P)} \times \psi^*_{A\nu P}(1' \ldots A'). \quad (9)$$

The A-particle wave function $\psi_{A\nu P}(1 \ldots A)$ and the corresponding eigenvalues $E^{\mathrm{qu}}_{A,\nu}(P)$ result from solving the in-medium Schrödinger equation; see Sec. II B. In addition to the bound states, the summation over the internal quantum states ν also includes the scattering states.

The evaluation of the equation of state in the low-density limit is straightforward; see Appendix A. Considering only the bound-state contributions, we obtain the result

$$n_p(T, \mu_p, \mu_n) = \frac{1}{\Omega} \sum_{A,\nu,P} Z f_{A,Z}\left[E^{\mathrm{qu}}_{A,\nu}(P; T, \mu_p, \mu_n)\right],$$
$$n_n(T, \mu_p, \mu_n) = \frac{1}{\Omega} \sum_{A,\nu,P} (A-Z) f_{A,Z}\left[E^{\mathrm{qu}}_{A,\nu}(P; T, \mu_p, \mu_n)\right] \quad (10)$$

for the EOS describing a mixture of components (cluster quasiparticles) obeying Fermi or Bose statistics. The NSE is obtained in the low-density limit if the in-medium energies $E^{\mathrm{qu}}_{A,\nu}(P; T, \mu_p, \mu_n)$ can be replaced by the binding energies of the isolated nuclei $E^{(0)}_{A,\nu}(P)$. Note that at low temperatures Bose-Einstein condensation may occur.

We discuss the cluster decomposition of the single-nucleon self-energy (Fig. 1) in comparison with other approximations. Restricting the cluster decomposition only to the contribution of two-particle correlations, we obtain the so-called T_2G approximation. In this approximation, the Beth-Uhlenbeck formula is obtained for the EOS, as shown in Refs. [10,11]. The relation between this T-matrix approach and the Brueckner G-matrix approach was discussed in detail in Ref. [14]. Extended work has been performed using sophisticated interaction potentials to evaluate the quasiparticle energies in the DBHF approximation, for recent reviews see Refs. [7,15,16].

Replacing the T_2-matrix in Born approximation with the interaction potential V, we obtain the Hartree-Fock approximation

$$\Delta^{\mathrm{HF}}(1) = \sum_2 [V(12, 12) - V(12, 21)] f_{1,\tau_2}[E(2) + \Delta^{\mathrm{HF}}(2)]. \quad (11)$$

In this approximation, all correlations in the medium are neglected. The self-energy does not depend on frequency, i.e., it is instantaneous in time, with vanishing imaginary part.

B. Effective wave equation for the A-nucleon cluster

We consider the propagation of an A-nucleon cluster in warm, dense matter that is described by the A-particle Green function G_A. The solution will not only enter the cluster decomposition of the single-nucleon self-energy as considered in Sec. II A to calculate the contribution of bound states to the nuclear matter EOS in a systematic way but also determine the cluster decomposition of other quantities such as the polarization function, the dynamical structure factor, and so on. We can proceed as above in the single-nucleon case and investigate higher-order correlation functions. The A-nucleon spectral function S_A can be introduced that is related to G_A. Cluster quasiparticle excitations are determined by δ-like peaks in the spectral function S_A.

For the A-particle Green function, the perturbation expansion can be represented by Feynman diagrams. New elements such as the A-particle self-energy can be introduced; see Refs. [1,11,17]. In the low-density limit, the bound and scattering states of the A-particle cluster are obtained by performing the partial summation of ladder diagrams.

The interaction of the A-nucleon cluster with the surrounding nucleons can be considered in the cluster mean-field approximation given in Appendix B. Because nuclear matter can form clusters, the interaction of the A-nucleon cluster under consideration is taken with an arbitrary cluster B of the surrounding matter in first order of the nucleon-nucleon interaction but full antisymmetrization of all nucleons in the clusters A and B. As a result, the Hartree shift due to the interaction with the surrounding nucleons in free as well as in cluster states is obtained. The Pauli blocking terms due to the occupation of the phase space by free nucleons, as well as nucleons that are bound in clusters, are taken into account. In higher orders, the Bose enhancement or Pauli blocking of nucleonic clusters is obtained.

At present, the full self-consistent solution of the cluster mean-field approximation is out of reach. Only special cases can be solved, such as α cluster nuclei [18], where an α cluster is considered in a surroundings consisting of a few α clusters, with full antisymmetrization on the nucleonic level.

We calculate the modification of the A-nucleon cluster due to the surroundings considering only self-energy and Pauli blocking terms in the cluster mean-field approximation. Correlations in the medium are neglected. In momentum space, the distribution of free nucleons, forming a Fermi sphere, and the bound states that are characterized by a wave function are different functions, but the total amount of the occupied phase space is determined by the total density of the protons or neutrons. Considering the uncorrelated medium, the phase space occupation is described by a Fermi distribution function normalized to the total density of nucleons (see Appendix B):

$$\tilde{f}_1(1) = \frac{1}{\exp\left[E^{\mathrm{qu}}_1(1)/T - \tilde{\mu}_\tau/T\right] + 1} \approx \frac{n_\tau}{2} \left(\frac{2\pi\hbar^2}{mT}\right)^{3/2} \exp\left(-\frac{\hbar^2 P^2}{2mT}\right) \quad (12)$$

in the low-density, nondegenerate limit ($\tilde{\mu}_\tau < 0$). The effective chemical potential $\tilde{\mu}_\tau$ is determined by the normalization

condition $2\Omega^{-1}\sum_p \tilde{f}_1(p) = n_\tau$, where τ denotes isospin (neutron or proton), and has to be expressed in terms of these densities.

In ladder approximation, the A-particle Green function obeys a Bethe-Salpeter equation (BSE)

$$G_A(1\dots A, 1'\dots A', z_A) = G_A^{(0)}(1\dots A, z_A)\delta_{11'}\dots\delta_{AA'} + \sum_{1''\dots A''} G_A^{(0)}(1\dots A, z_A)V_A(1\dots A, 1''\dots A'') \times G_A(1''\dots A'', 1'\dots A', z_A), \quad (13)$$

where $V_A(1\dots A, 1'\dots A') = \sum_{i<j} V(ij, i'j')\prod_{k\neq i,j}\delta_{kk'}$ is the interaction within the A-particle cluster. z_A denotes a fermionic or bosonic Matsubara frequency. The free A-quasiparticle Green function results as

$$G_A^{(0)}(1\dots A, z_A) = \frac{[1-\tilde{f}_1(1)]\dots[1-\tilde{f}_1(A)] - \tilde{f}_1(1)\dots\tilde{f}_1(A)}{z_A - E_1^{\rm qu}(1) - \dots - E_1^{\rm qu}(A)}. \quad (14)$$

The solution of the BSE is given by the bilinear expansion (9). The A-particle wave function and the corresponding eigenvalues follow from solving the in-medium Schrödinger equation

$$\left[E_1^{\rm qu}(1) + \dots + E_1^{\rm qu}(A) - E_{A\nu}^{\rm qu}(P)\right]\psi_{A\nu P}(1\dots A) + \sum_{1'\dots A'}\sum_{i<j}[1-\tilde{f}_1(i)-\tilde{f}_1(j)]V(ij,i'j') \times \prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu P}(1'\dots A') = 0. \quad (15)$$

This equation contains the effects of the medium in the quasiparticle shift as well as in the Pauli blocking terms. Obviously the bound-state wave functions and energy eigenvalues as well as the scattering phase shifts become dependent on temperature and density. Two effects have to be considered: the single-nucleon quasiparticle energy shift and the Pauli blocking.

Detailed investigations of Eq. (15) have been performed for $A = 2$ (see Ref. [10]) describing interesting physics. From the solution of this in-medium two-particle Schrödinger equation or the corresponding T matrix, the medium-dependent scattering and possibly bound states are obtained, as well as the formation of a quantum condensate, including the crossover from Bose-Einstein condensation to Cooper pairing. Due to the self-energy shifts and the Pauli blocking, the energy of the deuteron, $E_d^{\rm qu}(P; T, \mu_p, \mu_n)$, as well as the scattering phase shifts, $\delta_\tau(E, P; T, \mu_p, \mu_n)$ (τ denoting the isospin singlet or triplet channel), will depend on the temperature and the chemical potentials. For a separable interaction $V(12, 1'2')$ like the PEST4 potential [2], solutions of the in-medium two-particle Schrödinger equation can be found, e.g., in Refs. [10,14,19]. We will evaluate the medium shift of the binding energy in the following section using perturbation theory.

III. EXPRESSIONS FOR THE LIGHT NUCLEI QUASIPARTICLE SHIFTS

A. Solution of the few-nucleon effective wave equation

The quasiparticle energies in warm and dense nuclear matter, $E_{A\nu}^{\rm qu}(P; T, n_p, n_n)$, are well defined functions of temperature and total proton and neutron densities, given by a peak in the A-particle spectral function. To evaluate these quantities for infinite matter from a first-principle quantum statistical approach, we have to perform some approximations. Note that the quasiparticle shifts can also be introduced by phenomenologically fitting empirical data. This is well known for the single-nucleon quasiparticle energy that can be adapted to reproduce the structure of nuclei [6].

The solution of the few-body in-medium Schrödinger equation (15) for separable interaction is simple in the case $A = 2$. For $A = \{3, 4\}$, a Faddeev approach can be used; see [20,21]. To obtain explicit expressions for the quasiparticle energy shifts, we will apply perturbation theory, which can be justified in the low-density region. Denoting the unperturbed wave function of the A-nucleon cluster with $\varphi_{A\nu P}(1\dots A)$, we have

$$E_{A\nu}^{\rm qu}(P) \approx \sum_{1\dots A, 1'\dots A'} \varphi_{A\nu P}(1\dots A)H^{\rm eff}(1\dots A, 1'\dots A') \times \varphi_{A\nu P}(1'\dots A'), \quad (16)$$

where the form of $H^{\rm eff}(1\dots A, 1'\dots A')$ is given by Eq. (15) after symmetrization.

Before calculating the in-medium quasiparticle energy eigenvalues (16), $A \leqslant 4$, we first consider the isolated A-nucleon problem to determine the unperturbed wave function $\varphi_{A\nu P}(1\dots A)$. Extended analysis has been carried out by Wiringa *et al.* using Green's function Monte Carlo calculations [4] with the AV18/UIX potential. Some properties are given in Table I. There is excellent agreement between theory and experimental data [22].

In contrast to the second virial coefficient of the EOS [10–12], which is determined by on-shell properties (binding energy and scattering phase shifts of the two-nucleon problem) determined from experiments, the nucleon interaction potential and the bound-state wave function must be known to evaluate the quasiparticle energies. This potential is derived by fitting empirical data that have to be reproduced. In previous work [10], the PARIS potential [2] has been used. Alternatively, the AV18/UIX potential [4] can be taken. Here,

TABLE I. Light cluster properties at zero density.

	Binding energy (MeV)	Mass (MeV/c^2)	Spin	rms radius (charge) (fm)	rms radius (point) (fm)
n	0	939.565	1/2	0.34	0
p (^{1}H)	0	938.783	1/2	0.87	0
d (^{2}H)	−2.225	1876.12	1	2.14	1.96
t (^{3}H)	−8.482	2809.43	1/2	1.77	1.59
h (^{3}He)	−7.718	2809.41	1/2	1.97	1.76
α (^{4}He)	−28.30	3728.40	0	1.68	1.45

we will use simple separable interaction potentials, which are fitted to the binding energy and the rms radius value of the respective cluster, to obtain analytical expressions for the quasiparticle shifts as function of P, T, n_p, n_n. Details are given in Appendix C.

1. Gaussian wave function approach

For the separable pair interaction in the A-nucleon cluster a Gaussian form is taken,

$$V_\nu(12,1'2') = \lambda_\nu e^{-\frac{(\mathbf{p}_2-\mathbf{p}_1)^2}{4\gamma_\nu^2}} e^{-\frac{(\mathbf{p}_2'-\mathbf{p}_1')^2}{4\gamma_\nu^2}} \times \delta_{p_1+p_2,p_1'+p_2'}\delta_{\sigma_1,\sigma_1'}\delta_{\sigma_2,\sigma_2'}\delta_{\tau_1,\tau_1'}\delta_{\tau_2,\tau_2'}, \quad (17)$$

where $\nu = \{A, Z\} := \{d, t, h, \alpha\}$ denotes the cluster under consideration.

To solve the A-particle Schrödinger equation, a variational approach is used. Two different classes of functions are considered. First, a Gaussian wave function is taken that allows for analytical expressions in evaluating the shifts. In the following Sec. III A2 a better, but more complex, Jastrow ansatz is considered that, however, allows in general only for numerical evaluation of the shifts.

For Gaussian wave functions, the center-of-mass motion can be easily separated. For vanishing center-of-mass motion, $P = 0$, we have

$$\varphi_\nu^{\text{Gauss}}(p_1 \ldots p_A) = \frac{1}{\text{norm}_\nu} e^{-(p_1^2+\cdots+p_A^2)/B_\nu^2}\delta_{\mathbf{p}_1+\cdots+\mathbf{p}_A,0}, \quad (18)$$

with the normalization $\sum_{p_1\ldots p_A} |\varphi_\nu^{\text{Gauss}}(p_1 \ldots p_A)|^2 = 1$. The parameter B_ν is fixed by the nucleonic point rms radius $\sqrt{\langle r^2\rangle_\nu}$, given in Table I,

$$B_\nu^2 = \frac{3(A-1)}{A\langle r^2\rangle_\nu}, \quad (19)$$

see Appendix C. Values for B_ν are presented in Table II.

Next we are interested in the parameter values $\tilde\lambda_\nu, \tilde\gamma_\nu$ of the potential (17) which yield the binding energy $E_\nu^{(0)}$ of the nucleus as well as the nucleonic point rms radius, given in Table I, using the variational ansatz (18). Calculating the kinetic energy as well as the potential energy for a Gaussian wave function with range parameter B_ν, we have

$$E_\nu^{(0)} = \frac{3(A-1)}{8}\frac{\hbar^2}{m}B_\nu^2 + \frac{A(A-1)}{2}\lambda_\nu \frac{\gamma_\nu^6 B_\nu^3}{\pi^{3/2}\left(B_\nu^2+2\gamma_\nu^2\right)^3}. \quad (20)$$

TABLE II. Light cluster wave-function parameter at zero density from the Gaussian approach.

ν	$\tilde\lambda_\nu$ (MeV fm^3)	$\tilde\gamma_\nu$ (fm^{-1})	B_ν (fm^{-1})
d (^{2}H)	−3677.2	0.753	0.625
t (^{3}H)	−1670.0	1.083	0.889
h (^{3}He)	−1957.5	0.960	0.804
α (^{4}He)	−1449.6	1.152	1.034

TABLE III. Light cluster wave-function parameter at zero density from the Jastrow approach.

ν	λ_ν (MeV fm^3)	γ_ν (fm^{-1})	a_ν (fm^{-1})	b_ν (fm^{-1})
d (^{2}H)	−1287.4	1.474	1.474	0.2317
t (^{3}H)	−1283.8	1.259	1.742	0.592
h (^{3}He)	−1527.0	1.105	1.528	0.542
α (^{4}He)	−1272.9	1.231	2.151	0.912

Varying B_ν, the potential parameters that have the minimum energy $E_\nu^{(0)}$ at B_ν, consistent with the rms radius value, are shown in Table II.

2. Jastrow wave function approach

Of course, the variational solution of the separable Gauss potential (17) by using Gaussians is not optimal, which is clearly seen for the two-nucleon system where the exact solution is known. A Jastrow ansatz that reproduces this exact solution for $A = 2$ is given by

$$\varphi_\nu^{\text{Jastrow}}(\mathbf{p}_1 \ldots \mathbf{p}_A) = \frac{1}{N_\nu}\prod_{i<j}\frac{e^{-\frac{(\mathbf{p}_j-\mathbf{p}_i)^2}{4a_\nu^2}}}{\frac{(\mathbf{p}_j-\mathbf{p}_i)^2}{4b_\nu^2}+1}. \quad (21)$$

The prefactor N_ν is determined by the normalization condition. The results for the parameter values λ_ν, γ_ν of the interaction potential, which reproduce the binding energies and rms values of the cluster (see Table I), as well as the parameter values a_ν, b_ν characterizing the wave function (21) within the variational approach, are given in Table III. For the calculations see Appendix C.

We shortly discuss the parameter values for the separable pair interaction (17) deduced from the empirical values for the binding energies and the rms radii of the cluster given in Table I. The Gaussian approach shown in Table II fails to characterize the wave function of weakly bound states such as in the deuteron case. This is the reason for the large differences in the potential parameters shown there. The potential parameters for the different clusters show less spread if the Jastrow ansatz (Table III) is used. It covers the exact solution for the two-nucleon problem so that the weak binding energy of the deuteron is no longer problematic. To discuss the different parameters for ^{3}H and ^{3}He, we have to consider in addition to the strong nucleon-nucleon interaction the Coulomb repulsion, which is responsible for the smaller value of the binding energy and the larger rms radius. The separable interaction used here is an effective interaction that cannot be transformed to a local interaction in coordinate space so that its interpretation is not obvious. To gain some insight we consider the value $\lambda_\nu\{\sum_p \exp[-p^2/(4\gamma_\nu^2)]\}^2 = V_\nu(0,0)$ that denotes the interaction potential at vanishing relative positions. Despite the parameter $-\lambda_h$ being larger than $-\lambda_t$ (see Table III) the value $-V_h(0,0) = 89.5$ MeV fm^{-3} is smaller than $-V_t(0,0) = 164.9$ MeV fm^{-3} that reflects the stronger repulsion of the protons in ^{3}He. The influence of the different potential parameters on the Pauli blocking shift will be discussed at the end of Sec. IV.

B. Expressions for the quasiparticle shifts

1. Nucleon quasiparticles

According to Eq. (15), the first contribution to the in-medium energy shifts of the cluster binding energies is given by the nucleon self-energies. Results for not only the energy shift of single-nucleon states but also the correction of the cluster binding energy that is calculated by integrating the nucleon quasiparticle shift over the cluster wave function are of interest.

The in-medium single-nucleon dispersion relation $E_1^{\rm qu}(P)$ can be expanded for small momenta P as

$$E_1^{\rm qu}(P) = \frac{\hbar^2}{2m_1}P^2 + \Delta E_1^{\rm SE}(P) = \Delta E_1^{\rm SE}(0) + \frac{\hbar^2}{2m_1^*}P^2 + \mathcal{O}(P^4), \tag{22}$$

where the quasiparticle energies are shifted by $\Delta E_1^{\rm SE}(0)$ and m_1^* denotes the effective mass of neutrons ($\tau_1 = n$) or protons ($\tau_1 = p$). Both quantities, $\Delta E_1^{\rm SE}(0)$ and m_1^*, are functions of T, n_p, n_n characterizing the surrounding matter. We expand with respect to the baryon density,

$$\Delta E_\tau(P;T,n_B,\alpha) = \delta E_\tau(P;T,\alpha)n_B + \mathcal{O}\left(n_B^2\right), \quad \frac{m_\tau^*}{m_\tau}(T,n_B,\alpha) = 1 + \delta m_\tau^{(0)}(T,\alpha)n_B + \mathcal{O}\left(n_B^2\right). \tag{23}$$

Different expressions are used to parametrize the nucleon quasiparticle shift at subsaturation density. For illustration we give the Skyrme I parametrization by Vautherin and Brink [5,23] that is used in a standard approach to the nuclear matter EOS by Lattimer and Swesty [23]. It represents an analytic expression for the quasiparticle shift of the single-nucleon states,

$$\Delta E_n^{\rm SE}(0) = \frac{t_0}{2}(1-x_0)n_n + \frac{t_0}{2}(2+x_0)n_p + \frac{t_3}{4}\left(2n_nn_p + n_p^2\right) + \left(\frac{t_1}{8} + \frac{3t_2}{8}\right)\tau_n + \left(\frac{t_1}{4} + \frac{t_2}{4}\right)\tau_p, \tag{24}$$

and the effective mass

$$\frac{m_n^*}{m_n} = \left\{1 + \left[\left(\frac{t_1}{4} + \frac{3t_2}{4}\right)n_n + \left(\frac{t_1}{2} + \frac{t_2}{2}\right)n_p\right]\frac{m_n}{\hbar^2}\right\}^{-1}, \tag{25}$$

with the parameter values $t_0 = -1057.3$ MeV fm^3, $t_1 = 235.9$ MeV fm^5, $t_2 = -100$ MeV fm^5, $t_3 = 14463.5$ MeV fm^6, $x_0 = 0.56$. Here, τ_n denotes the neutron kinetic energy per particle. Expressions for the protons are found by interchanging n and p. For symmetric matter we find the shift $\delta E_\tau^{\rm Skyrme}(T,0) = -792.97$ MeV fm^3.

Improvements have been made to optimize the calculation for various nuclei. Alternatively, RMF approaches have been developed starting from a model Lagrangian that couples the nucleons to mesons. The relativistic quasiparticle energy is given by

$$E_\tau^{\rm qu}(P) = \sqrt{\left[m_\tau c^2 - S(n_n,n_p,T)\right]^2 + \hbar^2c^2P^2} + V_\tau(n_n,n_p,T). \tag{26}$$

Expressions for S and V_τ can be given [24] determining the nucleon quasiparticle shift and the effective mass, which aim at obtaining the nuclear matter EOS for supernova collapses. The relativistic quasiparticle energy (26) in the nonrelativistic limit reads $\Delta E_\tau^{\rm SE}(0) = -S(n_n,n_p,T) + V_\tau(n_n,n_p,T)$ and $m_\tau^*/m_\tau = 1 - S(n_n,n_p,T)/(m_\tau c^2)$.

Recent work on RMF parametrization [6,7] intends to reproduce properties of nuclei but is also in agreement with microscopic DBHF calculations. The EOS of asymmetric nuclear matter has been investigated in the low-density region below the nuclear saturation density [16], and expansions of the quasiparticle shift in powers of the density have been given. The Dirac mass and the Landau mass are considered in Ref. [15]. A more detailed discussion of the nucleon quasiparticle approach, which is very successful in describing nuclear matter near saturation density, is beyond the scope of this article.

2. Deuteron quasiparticles

In the low-density limit, the shift of the deuteron binding energy can be calculated from the in-medium Schrödinger equation (15) taking the medium modifications as correction. Within perturbation theory, the shifts of the solution of this medium modified wave equation are given by the self-energy term and the Pauli blocking term,

$$E_d^{\rm qu}(P) = E_d^{(0)} + \frac{\hbar^2}{2m_d}P^2 + \Delta E_d^{\rm SE}(P) + \Delta E_d^{\rm Pauli}(P), \tag{27}$$

$m_d \approx 2m$ is the deuteron rest mass. After separation of the center-of-mass motion, the free deuteron wave function $\varphi_d(\mathbf{q}_1)$ is taken as a function of the relative momentum $\mathbf{q}_1 = (\mathbf{p}_2 - \mathbf{p}_1)/2$ (neglecting the proton-neutron mass difference). Perturbation theory gives for the self-energy term due to the single-nucleon self-energy shift $\Delta E_1^{\rm SE}(P) = E_1^{\rm qu}(P) - E(1)$ in the medium part of the effective Hamiltonian (15)

$$\Delta E_d^{\rm SE}(P) = \frac{1}{N_d}\sum_{\mathbf{q}_1}\varphi_d^*(\mathbf{q}_1)\left[\Delta E_n^{\rm SE}\left(\frac{\mathbf{P}}{2} + \mathbf{q}_1\right) + \Delta E_p^{\rm SE}\left(\frac{\mathbf{P}}{2} - \mathbf{q}_1\right)\right]\varphi_d(\mathbf{q}_1). \tag{28}$$

$N_d = \sum_{\mathbf{q}_1}|\varphi_d(\mathbf{q}_1)|^2$ is the normalization. For a separable interaction, the Pauli blocking term reads

$$\Delta E_d^{\rm Pauli}(P) = -\frac{1}{N_d}\sum_{\mathbf{q}_1,\mathbf{q}_1'}\varphi_d^*(\mathbf{q}_1)\left[\tilde{f}_n\left(\frac{\mathbf{P}}{2} + \mathbf{q}_1\right) + \tilde{f}_p\left(\frac{\mathbf{P}}{2} - \mathbf{q}_1\right)\right]V(\mathbf{q}_1,\mathbf{q}_1')\varphi_d(\mathbf{q}_1'), \tag{29}$$

where the distribution function $\tilde{f}_1(\mathbf{p})$ is given by Eq. (12). After applying the unperturbed Schrödinger equation, the interaction in Eq. (29) can be eliminated. With the reduced mass $m/2$ we obtain

$$\Delta E_d^{\rm Pauli}(P) = \frac{1}{N_d}\sum_{\mathbf{q}_1}|\varphi_d(\mathbf{q}_1)|^2\left[\tilde{f}_n\left(\frac{\mathbf{P}}{2} + \mathbf{q}_1\right) + \tilde{f}_p\left(\frac{\mathbf{P}}{2} - \mathbf{q}_1\right)\right]\left[\frac{\hbar^2}{m}q_1^2 - E_d^{(0)}\right]. \tag{30}$$

We give some more explicit expressions. Similar to the effective mass representation for the single-nucleon quasiparticle states, we can introduce the deuteron shift and effective mass according to

$$E_d^{\rm qu}(P) = E_d^{(0)} + \Delta E_d(0) + \frac{\hbar^2}{2m_d^*} P^2 + \mathcal{O}(P^4). \tag{31}$$

The self-energy contribution to the deuteron shift contains the contribution of the single-nucleon shift

$$\Delta E_d^{\rm rigid\ shift}(0) = \Delta E_n^{\rm SE}(0) + \Delta E_p^{\rm SE}(0). \tag{32}$$

The rigid shift of the nucleons is also present in the continuum states and will not change the binding energy of the bound states. It can be incorporated in the chemical potentials μ_n, μ_p, similar to the rest mass of the nucleons.

If we consider the effective mass of the nucleons $m_\tau^* \neq m_\tau$, we get the contribution

$$\Delta E_d^{\rm eff.mass}(P) = \frac{1}{N_d} \sum_{\mathbf{q}_1} |\varphi_d(\mathbf{q}_1)|^2 \frac{\hbar^2}{2m} \left(\frac{m_p}{m_p^*} + \frac{m_n}{m_n^*} - 2 \right) \times \left(q_1^2 + \frac{P^2}{4} \right). \tag{33}$$

The first term contributes to the deuteron shift $\Delta E_d^{\rm SE}(0)$ and the last term ($\propto P^2$) to the deuteron effective mass. With $\Delta m_d = m_d^* - m_d$ we find for the self-energy contribution

$$\frac{\Delta m_d^{\rm SE}}{m_d} = -\frac{1}{8} \left(\frac{m_p}{m_p^*} + \frac{m_n}{m_n^*} - 2 \right). \tag{34}$$

For the further evaluation, we need the free deuteron wave function. For the solution (21) of the Gaussian interaction (17), used in the Jastrow ansatz with parameter values given in Table III, we obtain for the Pauli blocking shift (30) the expression $\Delta E_d^{\rm Pauli,Jastrow}(P) = (n_p + n_n) \delta E_d^{\rm Pauli,J}(P)$ with

$$\begin{aligned} \delta E_d^{\rm Pauli,J}(P) &= \frac{1}{2} \left(\frac{2\pi\hbar^2}{mT} \right)^{3/2} \int dq q^2 \frac{e^{-2q^2/a_d^2}}{\left(q^2/b_d^2 + 1\right)^2} \\ &\times e^{-\frac{\hbar^2}{2mT}\left(\frac{P^2}{4} + q^2\right)} \frac{mT}{\hbar^2 Pq} \left(e^{\frac{\hbar^2 Pq}{2mT}} - e^{-\frac{\hbar^2 Pq}{2mT}} \right) \left(\frac{\hbar^2}{m} q_1^2 - E_d^{(0)} \right) \\ &\times \frac{\sqrt{2} a_d}{\sqrt{\pi} b_d^4 \left(-1 + e^{2\frac{b_d^2}{a_d^2}} \frac{\sqrt{\pi}}{2\sqrt{2}\frac{b_d}{a_d}} \left(1 + 4\frac{b_d^2}{a_d^2}\right) \mathrm{erfc}\left[\sqrt{2}\frac{b_d}{a_d}\right] \right)}, \end{aligned} \tag{35}$$

taking the nucleons in the medium as nondegenerate what, at finite temperatures, is justified in the low-density limit. (In the degenerate case, the Boltzmann distribution has to be replaced by the Fermi distribution.) In particular, we find the quasiparticle shift at $P = 0$

$$\delta E_d^{\rm Pauli,J}(0) = \frac{1}{2} \left(\frac{2\pi\hbar^2}{mT} \right)^{3/2} \frac{\hbar^2 a_d^2}{2m} \frac{\left[1 + \frac{\hbar^2 a_d^2}{4mT}\right]^{-1/2} - e^{2\frac{b_d^2}{a_d^2}\left(1 + \frac{\hbar^2 a_d^2}{4mT}\right)} \frac{\sqrt{2\pi} b_d}{a_d} \mathrm{erfc}\left[\sqrt{2}\frac{b_d}{a_d}\sqrt{1 + \frac{\hbar^2 a_d^2}{4mT}}\right]}{-1 + e^{2\frac{b_d^2}{a_d^2}} \frac{\sqrt{\pi} a_d}{2\sqrt{2} b_d} \left(1 + 4\frac{b_d^2}{a_d^2}\right) \mathrm{erfc}\left[\sqrt{2}\frac{b_d}{a_d}\right]}. \tag{36}$$

Values for the deuteron quasiparticle shift are given below in Table IV.

To obtain the Pauli blocking contribution to the deuteron effective mass, we can expand the Pauli blocking shift (35) for small values of P. We will not give the corresponding expressions here. The effective mass approximation for the deuterons in warm dense matter is limited to small values of P. For arbitrary P, after averaging over the direction between $\mathbf{q}_1$ and $\mathbf{P}$, we get the approximation

$$\Delta E_d^{\rm Pauli,Jastrow}(P) \approx \Delta E_d^{\rm Pauli,Jastrow}(0)\, e^{-\frac{\hbar^2}{8mT} P^2}. \tag{37}$$

Together with Eqs. (36), (32), and (33), we obtain the momentum-dependent quasiparticle energy (27) describing the deuteron in matter.

We give an estimate for the different contributions to the energy shift of the deuteron. Considering symmetric matter at $T = 10$ MeV and $n_B = 0.001$ fm^{-3}, the Pauli blocking shift for the deuteron is 0.35 MeV; see Table IV. According to Eq. (32), for the self-energy shift using the Skyrme parametrization (24) the value -1.59 MeV is obtained. Its absolute value is larger than the Pauli blocking shift, but the self-energy shift is also present in the free single nucleon states and will not influence the deuteron binding energy. The correction due to the change of the effective mass, Eq. (33), is 0.0038 MeV and can be neglected.

In contrast to the self-energy shift, the Pauli blocking shift depends strongly on temperature by the following reasons. The binding energy per nucleon characterizes the extension of the wave function in momentum space. The form of the Fermi distribution function is determined by the temperature T that we consider here to be of the same order as the binding energy per nucleon. Further, the Pauli blocking term is strongly dependent on the center-of-mass momentum P because of the overlap of the deuteron wave function in momentum space with the Fermi sphere. Therefore, the bound states with high momentum P are less modified by the Pauli blocking effects. The evaluation of the deuteron quasiparticle energy is further improved using a better wave function and an appropriate interaction potential and avoiding perturbation expansions. Comparison with other potentials such as the Yamaguchi Lorentzian form factor has been performed that gives only small changes (below 10%) to the deuteron quasiparticle shift.

TABLE IV. Temperature dependence of the first-order Pauli blocking shift for $A = 2, 3, 4$. Comparison between the Gauss (G) and the Jastrow (J) approach; T (in MeV), $\delta E_\nu^{\rm Pauli}$ (in MeV fm^3).

T	$\delta E_d^{\rm Pauli,G}$	$\delta E_d^{\rm Pauli,J}$	$\delta E_t^{\rm Pauli,G}$	$\delta E_t^{\rm Pauli,J}$	$\delta E_h^{\rm Pauli,G}$	$\delta E_h^{\rm Pauli,J}$	$\delta E_\alpha^{\rm Pauli,G}$	$\delta E_\alpha^{\rm Pauli,J}$
20	157.9	172.9	482.3	470.77	438.9	432.52	967.7	950.16
15	228.1	235.4	651.7	628.95	604.0	588.70	1263.8	1237.72
10	371.4	352.5	950.2	905.37	906.7	871.81	1749.1	1711.83
9	418.8	389.2	1037.9	986.54	998.4	957.23	1884.2	1844.90
8	477.1	433.3	1140.0	1081.19	1106.6	1058.08	2037.5	1996.84
7	550.3	487.7	1260.0	1192.82	1236.0	1178.72	2212.9	2171.86
6	644.4	556.2	1402.7	1326.29	1392.6	1325.31	2415.0	2375.56
5	768.8	645.2	1574.4	1488.50	1585.4	1506.83	2649.8	2615.75
4	939.0	766.2	1784.3	1689.66	1827.2	1736.97	2925.4	2903.94
3	1182.8	941.0	2045.4	1945.59	2137.3	2037.73	3252.1	3259.11
2	1553.9	1212.0	2377.0	2282.48	2546.2	2447.21	3644.7	3720.67
1	2169.8	1756.9	2808.9	2747.92	3104.5	3038.96	4123.0	4434.30

3. *Tritium and helium in matter*

We now consider the clusters with $A = 3$ (t, h) and $A = 4$ (α). Some of the relations given for the deuteron case in Sec. III B 2 can be generalized to higher values of A. In the low-density limit, the shift of the cluster binding energy can be calculated from the effective Schrödinger equation (15) taking the medium modifications as correction,

$$E_\nu^{\rm qu}(P) = E_\nu^{(0)} + \frac{\hbar^2}{2m_\nu} P^2 + \Delta E_\nu(P), \tag{38}$$

with $\nu = \{A, Z\} = \{t, h, \alpha\}$, and $m_\nu \approx Am$ denoting the rest mass of the cluster. Within perturbation theory, the shift $\Delta E_\nu(P) = \Delta E_\nu^{\rm SE}(P) + \Delta E_\nu^{\rm Pauli}(P)$ consists of the self-energy and Pauli blocking term that can be calculated from the effective wave equation (15) with the unperturbed wave function $\varphi_\nu(\mathbf{p}_1, \ldots, \mathbf{p}_A)$. After separation of the center-of-mass motion, $\varphi_\nu(\mathbf{q}_1, \ldots, \mathbf{q}_{A-1})$ is a function of the remaining Jacobian momenta $\mathbf{q}_i$; see Appendix C. We find for the energy shifts due to the single-nucleon self-energy shift in the wave equation (15)

$$\Delta E_\nu^{\rm SE}(P) = \frac{1}{N_\nu} \sum_{\mathbf{q}_i} |\varphi_\nu(\mathbf{q}_i)|^2 \left[\Delta E_n^{\rm SE}(\mathbf{p}_1) + \cdots + \Delta E_n^{\rm SE}(\mathbf{p}_A)\right] \tag{39}$$

and for the Pauli blocking term

$$\Delta E_\nu^{\rm Pauli}(P) = -\frac{1}{N_\nu} \sum_{\mathbf{q}_i, \mathbf{q}_{i'}} \varphi_\nu^*(\mathbf{q}_i) \sum_{i<j} \left[\tilde f_1(p_i) + \tilde f_1(p_j)\right] \times V(p_i p_j, p_i' p_j') \prod_{k \neq i,j} \delta_{p_k, p_k'} \varphi_\nu(\mathbf{q}_{i'}), \tag{40}$$

where $N_\nu = \sum_{\mathbf{q}_i} |\varphi_\nu(\mathbf{q}_i)|^2$ is the normalization. The single-nucleon momenta $\mathbf{p}_i$ have to be expressed by Jacobi momenta $\mathbf{q}_i$ including the center-of-mass momentum $\mathbf{P}$. The Pauli blocking term results as where also the momenta $\mathbf{p}_i$ have to be expressed in terms of the Jacobi momenta $\mathbf{q}_i$.

As for the single-nucleon and deuteron states, we can introduce the quasiparticle shift and effective mass according to

$$E_\nu^{\rm qu}(P) = E_\nu^{(0)} + \Delta E_\nu(0) + \frac{\hbar^2}{2m_\nu^*} P^2 + \mathcal{O}(P^4). \tag{41}$$

The self-energy contribution to the quasiparticle shift contains the contribution of the single-nucleon shift

$$\Delta E_\nu^{\rm rigid\,shift}(0) = (A - Z)\Delta E_n^{\rm SE}(0) + Z\Delta E_p^{\rm SE}(0). \tag{42}$$

Evaluating the distribution functions $f_{A,Z}[E_{A,\nu}^{\rm qu}(P)]$, Eq. (10), this contribution to the quasiparticle shift can be included renormalizing the chemical potentials μ_n, μ_p. The further discussion of the self-energy contribution to the quasiparticle shift and effective mass can be performed in analogy to the deuteron case and will not be repeated here.

Whereas in the deuteron case it was possible to eliminate the interaction potential calculating the Pauli blocking shift, Eq. (30), this is not longer possible in the case $A = 3, 4$, and we have to evaluate expression (40). Analytical expressions for the Pauli blocking shift will be given in the following Sec. IV, based on the more simple Gaussian ansatz for the unperturbed wave function of the cluster. For the Jastrow ansatz (21), results for $\delta E_\nu^{\rm Pauli,J}(0)$ are given below in Table IV.

As already discussed for the deuteron case, after angular averaging the Pauli blocking shift can be approximated as

$$\Delta E_\nu^{\rm Pauli}(P) \approx \Delta E_\nu^{\rm Pauli}(0) e^{-\frac{\hbar^2 P^2}{2A^2 mT}}. \tag{43}$$

It leads to the dissolution of the bound states below saturation density, starting at the Mott baryon density $n_B^{\nu,\rm Mott}(T, \alpha)$ depending on temperature T and asymmetry parameter α, where the A-nucleon bound state with $P = 0$ merges in the continuum of scattering states [11].

IV. RESULTS FOR THE CLUSTER QUASIPARTICLE SHIFTS AT LOW DENSITIES

We collect some results for the quasiparticle energy

$$E_\nu^{\rm qu}(P; T, n_n, n_p) = E_\nu^{(0)} + \frac{\hbar^2 P^2}{2Am} + \Delta E_\nu^{\rm SE}(P) + \Delta E_\nu^{\rm Pauli}(P) + \Delta E_\nu^{\rm Coul}(P) \tag{44}$$

for the light elements $\nu = \{d, t, h, \alpha\} = \{^2{\rm H}, ^3{\rm H}, ^3{\rm He}, ^4{\rm He}\}$ in warm, dense nuclear matter. In addition to the single-particle self-energy shift $\Delta E_\nu^{\rm SE}(P)$ and the Pauli blocking term $\Delta E_\nu^{\rm Pauli}(P)$, the quasiparticle energy shift contains also the

Coulomb shift $\Delta E_\nu^{\rm Coul}(P)$ that will not be elaborated on here. The remaining two contributions are calculated within perturbation theory.

The quasiparticle self-energy shift $\Delta E_\nu^{\rm SE}(P)$ is caused by the self-energy shift $\Delta E_1^{\rm SE}(P)$ of the single-nucleon energies. For the A-nucleon system, it contributes to the bound state energies (nuclei) as well as to the energy of scattering states, in particular the edge of the continuum of scattering states, see Ref. [1]. In addition to the rigid shift, the quasiparticle self-energy shift at zero momentum $\Delta E_\nu^{\rm SE}(0) = \Delta E_\nu^{\text{rigid shift}} + 3(A-1)\hbar^2 B_\nu^2(m-m^*)/(8m^2)$ contains the contribution of the effective nucleon masses, calculated for Gaussian wave functions where B_ν is given by Eq. (19). Similarly, the self-energy contribution to the cluster effective mass can be calculated.

The Pauli blocking contribution $\Delta E_\nu^{\rm Pauli}(P)$ may be taken in the approximation (43) so that we discuss only the cluster quasiparticle shift $\Delta E_\nu^{\rm Pauli}(0)$ here. The Hamiltonian describing the unperturbed cluster contains parameters for the interaction potential that are determined such that the correct binding energy and point rms radius of the nuclei, given in Table I, are reproduced. Two different approximations are considered.

(i) The variational function to solve the isolated cluster Schrödinger equation is taken as a Gaussian wave function with the parameter B_ν. A Gaussian potential (17) is constructed that reproduces the correct values for the binding energies and the cluster point rms radii. The corresponding values for $\tilde\lambda_\nu, \tilde\gamma_\nu$ are given in Table II. Now having the interaction and the wave function at our disposal, we can calculate the shift of the binding energies in the low-density region using perturbation theory. Expanding with respect to the densities n_n, n_p, we have

$$\Delta E_\nu^{\rm Pauli}(0) = [(1+\alpha_\nu)n_n + (1-\alpha_\nu)n_p]\delta E_\nu^{\rm Pauli}(T) + \mathcal{O}\left(n_B^2\right) \quad (45)$$

with $\alpha_d = 0, \alpha_t = 1/3, \alpha_h = -1/3, \alpha_\alpha = 0$. In the nondegenerate case ($\mu_\tau < 0$), expressions for $\delta E_\nu^{\rm Pauli}(T)$ are obtained in analytic form as

$$\delta E_\nu^{\rm Pauli,G}(T) = -\frac{A(A-1)}{4\pi^{3/2}}\left(\frac{2\pi\hbar^2}{mT}\right)^{3/2} \times \frac{\tilde\lambda_\nu B_\nu^3 \tilde\gamma_\nu^6}{\left(B_\nu^2+2\tilde\gamma_\nu^2\right)^{3/2}\left[B_\nu^2+2\tilde\gamma_\nu^2+\frac{\hbar^2 B_\nu^2}{c_\nu mT}\left(B_\nu^2+d_\nu\tilde\gamma_\nu^2\right)\right]^{3/2}}, \quad (46)$$

where $c_d = 2, c_t = c_h = 24, c_\alpha = 16; d_d = 0, d_t = d_h = 14, d_\alpha = 10$.

(ii) The same procedure as in (i), only the Jastrow function (21) is taken as variational ansatz for the wave function. The Gaussian potential (17) with parameter values λ_ν, γ_ν, see Table III, reproduces the correct values for the binding energies and the cluster point rms radii. This variational ansatz gives the exact solution (36) of the two-nucleon problem, so that it is expected to yield better results also for the higher clusters. However, no analytic expressions are obtained for $A > 2$.

In Table IV the comparison between the Gaussian ($\delta E_\nu^{\rm Pauli,G}$) and the Jastrow ansatz ($\delta E_\nu^{\rm Pauli,J}$) is shown for different temperatures. The differences are small (below 5%) for $A = 3, 4$ so that the analytical expression [4] can be used. The differences are larger for $A = 2$ because the deuteron wave function is not well approximated by a Gaussian. However, for $A = 2$ we can take the Jastrow result that is given analytically in Eq. (36).

The wave functions of the isolated clusters determine the quasiparticle shifts within perturbation theory, in particular the Pauli blocking shift. The correct reproduction of the rms radii of the nuclei is important to estimate the region in phase space, which is needed to form the bound state. This region, however, may already be occupied by nucleons of the medium, leading to the Pauli blocking contribution in the quasiparticle shift. Improvements in calculating the wave functions of the isolated nuclei are possible using more sophisticated potentials such as BONN, PARIS [2], or AV18/UIX [3] and applying advanced methods for the solution of the few-body problem such as the Faddeev-Yakubovski approach [20,21] or the Green's function Monte Carlo method [4]. For tritium, a comparison with calculations of the wave function by Wiringa has been performed, and reasonable agreement with the Jastrow ansatz used here has been found. Further improvements of our results for the quasiparticle shifts using more advanced approaches to the few-nucleon problem may be the subject of future considerations.

V. CONCLUSIONS

In the low-density limit, the NSE with binding energies of the isolated nuclei is obtained from the quantum statistical approach to nuclear matter. We consider only light clusters, $A \leqslant 4$, but heavier cluster may become more important as temperature goes down and density increases. Thus, focusing on only light elements in the EOS restricts the temperature and density parameter to the region where the mass fraction of heavy elements is small, but our approach may also be extended to heavier nuclei; see Ref. [25,26].

Deviation from the NSE are due to medium effects, which become relevant once the baryon density exceeds 10^{-4} fm^{-3}. The concept of the excluded volume [9,23,24] to mimic density effects cannot be rigorously derived in a quantum statistical approach. Correlation effects are described by the A-nucleon spectral functions that define the A-cluster quasiparticles. The dependence of the cluster quasiparticle energy on temperature and nucleon densities is approximated by analytical expressions, Eq. (36) for $A = 2$ and Eq. [4] for $A = 3, 4$, combined with the momentum dependence according to Eq. (43). Compared with more accurate numerical calculations, deviations are of the order of 5%. Analytical expressions for the cluster quasiparticle shifts are convenient for calculating the thermodynamic properties of nuclear matter in a large parameter range. The evaluation of the cluster-quasiparticle shifts is further improved considering more sophisticated potentials and wave functions as obtained, e.g., in Green's function Monte Carlo approaches. The values given

in the present work are approximate estimations, similarly to the Skyrme or RMF approaches for the single-nucleon quasiparticle case.

With the shift of the quasiparticle energies, properties such as the EOS (10) can be determined in the subsaturation region. It is possible to interpolate between the low-density limit where the NSE is valid and the saturation density where the single nucleon quasiparticle picture can be applied.

To go beyond the quasiparticle picture, the full A-nucleon spectral function S_A should be explored. Instead of δ-like quasiparticle structures, S_A accounts for weakly bound states as well as scattering phase shifts including resonances consistently. In the case of $A = 2$, from the spectral function the generalized Beth-Uhlenbeck formula [10,11] has been obtained. Recently, a cluster-virial expansion was discussed in Ref. [12], taking into account also cluster-cluster scattering phase shifts. To obtain rigorous results in a given order of density avoiding double counting, and to include medium effects, one has to pass over to a systematic quantum statistical approach. The full solution of the cluster mean-field approximation would be an important step in this direction. This refers also to the formation of quantum condensates due to the Bose distribution function occurring in the bound-state contribution as well as in the scattering contribution to the nucleon density [10,11,18,19]. The construction of a nuclear matter EOS remains a challenging topic not only in the high-density region (see Ref. [8]) but also in the low-density region where the concept of nuclear quasiparticles given here may be a valuable ingredient.

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APPENDIX A: T-MATRIX APPROACH TO THE SELF-ENERGY

A detailed derivation of the expressions for the self-energy and the EOS can be found in the literature, see, e.g., Refs. [1,10,11,13]. We give here only some relations, using the short notations $E_1 = E_1^{\rm qu}(1)$, $E_{A\nu P} = E_{A,\nu}^{\rm qu}(P)$.

In Eq. (8), the free $(A-1)$ quasiparticle propagator is

$$G^{(0)}_{A-1}(2,\ldots,A,z) = \frac{1}{z - E_2 - \cdots - E_A} \times \frac{f_{1,Z_2}(2)\ldots f_{1,Z_A}(A)}{f_{A-1,Z_{A-1}}(E_2+\cdots+E_A)}. \quad \text{(A1)}$$

The T_A matrices are related to the A-particle Green functions

$$T_A(1\ldots A,1'\ldots A',z) = V_A(1\ldots A,1'\ldots A') + V_A(1\ldots A,1''\ldots A'')G_A(1''\ldots A'',1'''\ldots A''',z) \times V_A(1'''\ldots A''',1'\ldots A') \quad \text{(A2)}$$

with the potential $V_A(1\ldots A,\ 1'\ldots A') = \sum_{i<j} V(ij, i'j') \prod_{k\neq i,j} \delta_{k,k'}$, and subtraction of double counting diagrams when inserting the T matrices into the self-energy. The A-particle propagator obeys a BSE (13) and is solved by the bilinear expansion (9).

The evaluation of the equation of state in the low-density limit is straightforward. With

$$T_A(1\ldots A,1'\ldots A',z) = \sum_{\nu,P} \frac{(z - E_1 - \cdots - E_A)\psi_{A\nu P}(1\ldots A)\psi^*_{A\nu P}(1'\ldots A')(E_{A\nu P} - E_{1'} - \cdots - E_{A'})}{z - E_{A\nu P}} \quad \text{(A3)}$$

we can perform the Ω_λ summation in Eq. (8). We obtain the result

$$\sum_{\Omega_\lambda} \frac{1}{\Omega_\lambda - z_\nu - E_2 - \cdots - E_A} \times \frac{(\Omega_\lambda - E_1 - \cdots - E_A)(E_{A\nu P} - E_1 - \cdots - E_A)}{\Omega_\lambda - E_{A\nu P}} = f_{A-1}(E_2+\cdots+E_A)\frac{z_\nu - E_1}{z_\nu + E_2 + \cdots + E_A - E_{A\nu P}} - f_A(E_{A\nu P})\frac{E_1 + \cdots + E_A - E_{A\nu P}}{z_\nu + E_2 + \cdots + E_A - E_{A\nu P}}. \quad \text{(A4)}$$

Taking Im $\Sigma(1,z)$ and integrating the δ function arising from the pole in the denominator, the leading term in density is $f_1(E_{A\nu P} - E_2 - \cdots - E_A) f_{A-1}(E_2 + \cdots + E_A) = f_A(E_{A\nu P})$. Neglecting the contribution of the scattering states, we obtain the generalized form (10) of the NSE.

APPENDIX B: THE CLUSTER MEAN-FIELD APPROXIMATION

The cluster mean-field approximation [17] is inspired by the chemical picture where bound states are considered as new species, to be treated on the same level as free particles. We consider the propagation of an A-particle cluster ($\{A, \nu, P\}$) in a correlated medium. The corresponding A-particle cluster self-energy is treated to first order in the interaction with the single particles as well as with the B-particle cluster states ($\{B, \bar{\nu}, \bar{P}\}$) in the medium. The B clusters in the

surrounding medium are distributed according to Eq. (3). Full antisymmetrization between both clusters A and B has to be performed, in analogy to the Fock term in the single-nucleon case.

For the A-particle problem, the effective wave equation reads

$$[E(1)+\cdots E(A)-E_{A\nu P}]\psi_{A\nu P}(1\ldots A) + \sum_{1'\ldots A'}\sum_{i<j}^{A} V_{ij}^{A}(1\ldots A,1'\ldots A')\psi_{A\nu P}(1'\ldots A') + \sum_{1'\ldots A'} V_{\text{matter}}^{A,\text{mf}}(1\ldots A,1'\ldots A')\psi_{A\nu P}(1'\ldots A')=0, \tag{B1}$$

with $V_{12}^{A}(1\ldots A,1'\ldots A')=V(12,1'2')\delta_{33'}\ldots\delta_{AA'}$. The effective potential $V_{\text{matter}}^{A,\text{mf}}(1\ldots A,1'\ldots A')$ describes the influence of the nuclear medium on the cluster bound states and has the form

$$V_{\text{matter}}^{A,\text{mf}}(1\ldots A,1'\ldots A')=\sum_{i}\Delta(i)\delta_{11'}\ldots\delta_{AA'} + {\sum_{i,j}}'\Delta V_{ij}^{A}(1\ldots A,1'\ldots A'). \tag{B2}$$

The self-energy like contribution

$$\Delta(1)=\sum_{2}V(12,12)_{\text{ex}}\tilde{f}(2)-\sum_{B=2}^{\infty}\sum_{\bar{\nu}\bar{P}}\sum_{2\ldots B}\sum_{1'\ldots B'}f_{B,\bar{Z}}(E_{B\bar{\nu}\bar{P}}) \times\sum_{i<j}^{B}V_{ij}^{B}(1\ldots B,1'\ldots B') \times\psi_{B\bar{\nu}\bar{P}}(1\ldots B)\psi^{*}_{B\bar{\nu}\bar{P}}(1'\ldots B')$$

contains the Hartree-Fock quasiparticle shift. The interaction-like contribution

$$\Delta V_{12}^{A}(1\ldots A,1'\ldots A') =-\delta_{33'}\ldots\delta_{AA'}\left\{\frac{1}{2}\left[\tilde{f}(1)+\tilde{f}(1')\right]\times V(12,1'2')+\sum_{B=2}^{\infty}\sum_{\bar{\nu}\bar{P}}\sum_{\bar{2}\ldots\bar{B}}\sum_{\bar{2}'\ldots\bar{B}'}f_{B,\bar{Z}}(E_{B\bar{\nu}\bar{P}}) \times\sum_{j}^{B}V_{1j}^{B}(1\bar{2}'\ldots\bar{B}',1'\bar{2}\ldots\bar{B}) \times\psi^{*}_{B\bar{\nu}\bar{P}}(2\bar{2}\ldots\bar{B})\psi_{B\bar{\nu}\bar{P}}(2'\bar{2}'\ldots\bar{B}')\right\} \tag{B3}$$

accounts for the Pauli blocking of the potential. The quantity

$$\tilde{f}(1)=f_1(1)+\sum_{B=2}^{\infty}\sum_{\bar{\nu}\bar{P}}\sum_{2\ldots B}f_{B,\bar{Z}}(E_{B\bar{\nu}\bar{P}})|\psi_{B\bar{\nu}\bar{P}}(1\ldots B)|^2 \tag{B4}$$

describes the effective occupation in momentum space. The bound states contribute according to their wave function and probability distribution. Note that within the mean-field approximation, the effective potential $V_{\text{matter}}^{A,\text{mf}}$ remains energy independent, i.e., instantaneous. In addition to the Hartree-Fock term and the Pauli blocking term, determined by the effective occupation $\tilde{f}(1)$, the additional terms in ΔV_{12}^{A} and $\Delta(1)$ account for antisymmetrization.

APPENDIX C: WAVE FUNCTIONS AND SHIFTS

A. Quasideuteron

In the deuteron case, we introduce Jacobi coordinates such that $q_1=(p_2-p_1)/2=p_{\text{rel}}$, $q_2=p_1+p_2=P$ or $p_1=-q_1+q_2/2$, $p_2=q_1+q_2/2$. The kinetic energy is $KE=\frac{\hbar^2}{2m}(2q_1^2+\frac{1}{2}q_2^2)$, the interaction is parametrized by the separable Gaussian (17)

$$V(q_1,q_2,q_1',q_2')=\lambda e^{-\frac{q_1^2}{\gamma^2}}e^{-\frac{q_1'^2}{\gamma^2}}\delta_{q_2,q_2'}. \tag{C1}$$

The deuteron wave function (21) results as $\varphi_d(q_1)\propto e^{-2q_1^2/a^2}/(q_1^2/b^2+1)$. The value of the rms radius follows as

$$\text{rms}^2=\frac{1}{2}\langle[(r_1-R)^2+(r_2-R)^2]\rangle =\frac{1}{4}\int d^3q_1\left[\frac{\partial}{\partial q_1}\varphi_d(q_1)\right]^2. \tag{C2}$$

As in the case of single nucleons, the quasiparticle shifts can be expanded as power series of the densities,

$$\Delta E_d(P;T,n_B,\alpha)=\delta E_d(P;T,\alpha)n_B+\mathcal{O}\big(n_B^2\big). \tag{C3}$$

The first-order term δE_d consists of the self-energy contribution and the Pauli blocking contribution, see Sec. III B2, in particular $\delta E_d^{\text{rigidshift}}=\delta E_p(0;T,\alpha)+\delta E_n(0;T,\alpha)$. Furthermore we have

$$\frac{m_d^*}{m_d}(T,n_B,\alpha)=1+\delta m_d(T,\alpha)n_B+\mathcal{O}\big(n_B^2\big). \tag{C4}$$

Values for $\delta E_d^{\text{Pauli}}(0;T)$ are given in Table IV. For $T=10$ MeV we have $\delta m_d(10,0)=21.3$ fm^3, whereas for $T=4$ MeV the value $\delta m_d(4,0)=87.1$ fm^3 results. Due to the Pauli blocking both quantities are strongly temperature dependent. At zero temperature and low densities, we find for the Gaussian interaction

$$\delta E_d^{\text{Pauli}}(P;0)=-\frac{1}{2}\lambda_d\psi_d(P/2)e^{-\frac{P^2}{4\gamma_d^2}}\frac{\int d^3q_1 e^{-q_1^2/\gamma_d^2}\psi_d(q_1)}{\int d^3q_1|\psi_d(q_1)|^2}. \tag{C5}$$

B. Quasitriton/helion

Next we consider $A=3\,(t,h)$. Jacobi coordinates are $q_1=\frac{1}{2}(p_2-p_1)$, $q_2=\frac{2}{3}(-\frac{1}{2}p_1-\frac{1}{2}p_2+p_3)$, $q_3=p_1+p_2+p_3$ or $p_1=-q_1-\frac{1}{2}q_2+\frac{1}{3}q_3$, $p_2=q_1-\frac{1}{2}q_2+\frac{1}{3}q_3$, $p_3=q_2+\frac{1}{3}q_3$. The kinetic energy is $KE=\frac{\hbar^2}{2m}(2q_1^2+\frac{3}{2}q_2^2+\frac{1}{3}q_3^2)$.

We start from a Gaussian pair interaction (17) that gives in Jacobian coordinates the three-nucleon interaction

$$V_3^{\rm pair}(q_1,q_2,q_3,q_1',q_2',q_3') = \lambda\delta_{q_3,q_3'}\Big\{\delta_{q_2,q_2'}e^{-\frac{q_1^2}{\gamma^2}}e^{-\frac{q_1'^2}{\gamma^2}} + \delta_{q_1',q_1+\frac{q_2'-q_2}{2}}e^{-\frac{(q_1+\frac{3}{2}q_2)^2}{4\gamma^2}}e^{-\frac{(q_1-\frac{1}{2}q_2+2q_2')^2}{4\gamma^2}} + \delta_{q_1',q_1-\frac{q_2'-q_2}{2}}e^{-\frac{(q_1-\frac{3}{2}q_2)^2}{4\gamma^2}}e^{-\frac{(q_1+\frac{1}{2}q_2-2q_2')^2}{4\gamma^2}}\Big\}. \quad \text{(C6)}$$

The Gaussian variational ansatz (18) reads after introducing Jacobians (indices in B, a, b are omitted)

$$\varphi_3^{\rm Gauss}(q_1,q_2,q_3) \propto e^{-\frac{2q_1^2}{B^2}}e^{-\frac{3q_2^2}{2B^2}}\delta_{q_3,P}. \quad \text{(C7)}$$

The Jastrow variational ansatz (21) motivated by the solution of the two-particle problem, reads after introduction of the reduced Jacobian coordinates $\vec{x}_i = \mathbf{q}_i/b$ and choosing the coordinates as $\vec{x}_1 = x_1\{(1-z^2)^{1/2},0,z\}$, $\vec{x}_2 = x_2\{0,0,1\}$

$$\varphi_3^{\rm Jastrow}(x_1,x_2,z) \propto \frac{e^{-\frac{3}{2}\frac{b^2}{a^2}x_1^2-\frac{9}{8}\frac{b^2}{a^2}x_2^2}}{(x_1^2+1)\left(\frac{1}{4}x_1^2+\frac{9}{16}x_2^2+\frac{3}{4}x_1x_2z+1\right)\left(\frac{1}{4}x_1^2+\frac{9}{16}x_2^2-\frac{3}{4}x_1x_2z+1\right)}. \quad \text{(C8)}$$

The kinetic energy follows as

$$KE_3 = \frac{\hbar^2}{m}\frac{b^2}{N_3}\int dx_2\,x_2^2\int dx_1\,x_1^2\left(x_1^2+\frac{3}{4}x_2^2\right)\times\int_{-1}^{1}dz\varphi_3^2(x_1,x_2,z) \quad \text{(C9)}$$

with the norm

$$N_3 = \int dx_2\,x_2^2\int dx_1\,x_1^2\int_{-1}^{1}dz\varphi_3^2(x_1,x_2,z). \quad \text{(C10)}$$

For the potential energy we obtain

$$PE_3 = 3\lambda_3\frac{b^3}{4\pi^2N_3}\int dx_2\,x_2^2\left[\int dx_1\,x_1^2e^{-x_1^2\frac{b^2}{\gamma^2}}\times\int_{-1}^{1}dz\varphi_3(x_1,x_2,z)\right]^2. \quad \text{(C11)}$$

The nucleonic point rms radius follows as

$$\mathrm{rms}_3^2 = \frac{1}{b^2N_3}\int dx_2\,x_2^2\int dx_1\,x_1^2\times\int_{-1}^{1}dz\left[\frac{1}{6}\left(\frac{\partial\varphi_3}{\partial\vec{x}_1}\right)^2+\frac{2}{9}\left(\frac{\partial\varphi_3}{\partial\vec{x}_2}\right)^2\right], \quad \text{(C12)}$$

in particular $\mathrm{rms}_3^2 = 2/B^2$ for the Gaussian ansatz (18).

At finite temperature, the Pauli blocking contribution to the quasiparticle shift is given by

$$\delta E_3^{\rm Pauli,J}(P) = \frac{3\lambda_3b^3}{8\pi^2N_3}\left(\frac{2\pi\hbar^2}{mT}\right)^{3/2}\int dx_2x_2^2\int dx_1x_1^2\times\int dz_1\varphi_3(x_1,x_2,z_1)e^{-x_1^2\frac{b^2}{\gamma^2}}e^{-\frac{\hbar^2b^2}{2mT}\left(\vec{x}_1+\frac{1}{2}\vec{x}_2-\frac{1}{3}\vec{x}_3\right)^2}\times\int dx_5x_5^2\int dz_5\varphi_3(x_5,x_2,z_5)e^{-x_5^2\frac{b^2}{\gamma^2}}. \quad \text{(C13)}$$

At zero temperature where $p_1 \approx 0$ ($\vec{x}_1 = \vec{x}_3/3 - \vec{x}_2/2; \vec{x}_3 = 0$), the shift is given by

$$\delta E_3^{\rm Pauli,J}(0) = \frac{3\lambda_3}{2N_3}\int dx_2x_2^2\frac{e^{-x_2^2(\frac{3b^2}{2a^2}+\frac{b^2}{4\gamma^2})}}{\left(x_2^2+1\right)\left(\frac{x_2^2}{4}+1\right)^2}\int dx_5x_5^2\times\int dz_5\varphi_3(x_5,x_2,z_5)e^{-x_5^2\frac{b^2}{\gamma^2}}. \quad \text{(C14)}$$

C. The α quasiparticle

To solve the four-nucleon Schrödinger equation in the zero-density limit, we separate the center-of-mass motion from the internal motion introducing Jacobian coordinates, $q_1 = -\frac{1}{2}p_1+\frac{1}{2}p_2$, $q_2 = -\frac{1}{3}p_1-\frac{1}{3}p_2+\frac{2}{3}p_3$, $q_3 = -\frac{1}{4}p_1-\frac{1}{4}p_2-\frac{1}{4}p_3+\frac{3}{4}p_4$, $q_4 = p_1+p_2+p_3+p_4$. The inverse transformation is $p_1 = \frac{1}{4}q_4-\frac{1}{3}q_3-\frac{1}{2}q_2-q_1$, $p_2 = \frac{1}{4}q_4-\frac{1}{3}q_3-\frac{1}{2}q_2+q_1$, $p_3 = \frac{1}{4}q_4-\frac{1}{3}q_3+q_2$, $p_4 = \frac{1}{4}q_4+q_3$.

The Schrödinger equation separates with $\varphi_{\alpha,P}(1,2,3,4) = \varphi_4(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3)\delta_{\mathbf{P},\mathbf{q}_4}$. The kinetic energy is $KE_4 = \frac{\hbar^2}{2m}(2q_1^2+\frac{3}{2}q_2^2+\frac{4}{3}q_3^2+\frac{1}{4}P^2)$. The potential energy follows as

$$\begin{aligned}V_4^{\rm pair}(q_1,q_2,q_3,q_4,q_1',q_2',q_3',q_4') &= \lambda_\alpha\delta_{q_4,q_4'}\Big\{\delta_{q_3,q_3'}\delta_{q_2,q_2'}e^{-\frac{q_1^2}{\gamma_\alpha^2}}e^{-\frac{q_1'^2}{\gamma_\alpha^2}}\\ &+\delta_{q_3,q_3'}\delta_{q_1',q_1+\frac{1}{2}(q_2'-q_2)}e^{-\frac{(q_1+\frac{3}{2}q_2)^2}{4\gamma_\alpha^2}}e^{-\frac{(q_1-\frac{1}{2}q_2+2q_2')^2}{4\gamma_\alpha^2}}\\ &+\delta_{q_3,q_3'}\delta_{q_1',q_1-\frac{1}{2}(q_2'-q_2)}e^{-\frac{(q_1-\frac{3}{2}q_2)^2}{4\gamma_\alpha^2}}e^{-\frac{(q_1+\frac{1}{2}q_2-2q_2')^2}{4\gamma_\alpha^2}}\\ &+\delta_{q_1,q_1'}\delta_{q_2',q_2+\frac{2}{3}(q_3-q_3')}e^{-\frac{(-q_2+\frac{4}{3}q_3)^2}{4\gamma_\alpha^2}}e^{-\frac{(-3q_2-2q_3+6q_3')^2}{36\gamma_\alpha^2}}\\ &+\delta_{q_1',q_1-\frac{1}{2}(q_3-q_3')}\delta_{q_2',q_2-\frac{1}{3}(q_3-q_3')}e^{-\frac{(q_1+\frac{1}{2}q_2+\frac{4}{3}q_3)^2}{4\gamma_\alpha^2}}\\ &\times e^{-\frac{(6q_1+3q_2-4q_3+12q_3')^2}{144\gamma_\alpha^2}}\end{aligned}$$

$$+\delta_{q_1',q_1+\frac{1}{2}(q_3-q_3')}\delta_{q_2',q_2-\frac{1}{3}(q_3-q_3')}e^{-\frac{(-q_1+\frac{1}{2}q_2+\frac{4}{3}q_3)^2}{4\gamma_\alpha^2}} \times e^{-\frac{(-6q_1+3q_2-4q_3+12q_3')^2}{144\gamma_\alpha^2}}\Bigg\}. \quad \text{(C15)}$$

To solve the internal motion we use a variational ansatz for the wave function. The Gaussian variational ansatz (18) reads after introducing Jacobians

$$\varphi_\alpha^{\text{Gauss}}(q_1,q_2,q_3,q_4) \propto e^{-\frac{2q_1^2}{B^2}}e^{-\frac{3q_2^2}{2B^2}}e^{-\frac{4q_3^2}{3B^2}}\delta_{q_4,P}\,. \quad \text{(C16)}$$

The Jastrow variational ansatz (21)

$$\varphi_{\alpha,P}^{\text{Jastrow}}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4) \propto \frac{e^{-\frac{1}{4a^2}(p_2-p_1)^2}}{\frac{(p_2-p_1)^2}{4b^2}+1}\cdots\frac{e^{-\frac{1}{4a^2}(p_4-p_3)^2}}{\frac{(p_4-p_3)^2}{4b^2}+1} \times \delta_{\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3+\mathbf{p}_4,\mathbf{P}}. \quad \text{(C17)}$$

reads after introduction of the reduced Jacobian momenta $\vec{x}_i = \mathbf{q}_i/b$, with $\vec{x}_1 = x_1\,\{\sqrt{1-z_1^2}\cos(\phi_1), \sqrt{1-z_1^2}\sin(\phi_1), z_1\}$; $\vec{x}_2 = x_2\{0,0,1\}$; $\vec{x}_3 = x_3\,\{\sqrt{1-z_3^2},0,z_3\}$,

$$\varphi_4(x_1,z_1,\phi_1,x_2,x_3,z_3) = \frac{e^{-\frac{b^2}{a^2}(2x_1^2+\frac{3}{2}x_2^2+\frac{4}{3}x_3^2)}}{(x_1^2+1)\left[\left(\frac{1}{4}x_1^2+\frac{9}{16}x_2^2+1\right)^2-\frac{9}{16}x_1^2x_2^2z_1^2\right]\left(\frac{1}{4}x_2^2+\frac{4}{9}x_3^2-\frac{2}{3}x_2x_3z_3+1\right)} \frac{1}{\left[\left(\frac{1}{4}x_1^2+\frac{1}{16}x_2^2+\frac{4}{9}x_3^2+\frac{1}{3}x_2x_3z_3+1\right)^2-\left(\frac{1}{4}x_1x_2z_1+\frac{2}{3}x_1x_3\left\{z_1z_3+\sqrt{1-z_1^2}\sqrt{1-z_3^2}\cos\phi_1\right\}\right)^2\right]}. \quad \text{(C18)}$$

We evaluate the norm as

$$N_4 = \int_0^\infty dx_1x_1^2\int_{-1}^1\frac{dz_1}{2}\int_0^{2\pi}\frac{d\phi_1}{2\pi} \times\int_0^\infty dx_2x_2^2\int_0^\infty dx_3x_3^2\int_{-1}^1\frac{dz_3}{2}\varphi_4^2. \quad \text{(C19)}$$

The kinetic energy of the internal motion is calculated from

$$\text{KE}_4 = \frac{\hbar^2b^2}{m\ N_4}\int_0^\infty dx_1x_1^2\int_{-1}^1\frac{dz_1}{2}\int_0^{2\pi}\frac{d\phi_1}{2\pi}\int_0^\infty dx_2x_2^2 \times\int_0^\infty dx_3x_3^2\int_{-1}^1\frac{dz_3}{2}\left(x_1^2+\frac{3}{4}x_2^2+\frac{2}{3}x_3^2\right)\varphi_4^2. \quad \text{(C20)}$$

The potential energy is (all six terms give the same contribution; we take the first one as representative)

$$\text{PE}_4 = \frac{6\lambda b^3}{2\pi^2N_4}\int_0^\infty dx_2x_2^2\int_0^\infty dx_3x_3^2\int_{-1}^1\frac{dz_3}{2} \times\left(\int_0^\infty dx_1x_1^2\int_{-1}^1\frac{dz_1}{2}\int_0^{2\pi}\frac{d\phi_1}{2\pi}e^{-\frac{b^2}{\gamma^2}x_1^2}\varphi_4\right)^2. \quad \text{(C21)}$$

The integrals over ϕ_1 can be performed analytically.

The nucleonic point rms radius follows as

$$\text{rms}_4^2 = \frac{1}{b^2N_4}\int_0^\infty dx_1x_1^2\int_{-1}^1\frac{dz_1}{2}\int_0^{2\pi}\frac{d\phi_1}{2\pi}\int_0^\infty dx_2x_2^2 \times\int_0^\infty dx_3x_3^2\int_{-1}^1\frac{dz_3}{2} \times\left[\frac{1}{8}\left(\frac{\partial\varphi_4}{\partial\vec{x}_1}\right)^2+\frac{1}{6}\left(\frac{\partial\varphi_4}{\partial\vec{x}_2}\right)^2+\frac{3}{16}\left(\frac{\partial\varphi_4}{\partial\vec{x}_3}\right)^2\right], \quad \text{(C22)}$$

in particular $\text{rms}_4^2 = 9/4B^2$ for the Gaussian ansatz (18).

Calculating the rms radius for the Jastrow ansatz, the three terms give the same contribution so that we take the last one as representative

$$\begin{aligned}\text{rms}_4^2 = {}& \frac{9}{16\ b^2\ N_4}\int_0^\infty dx_1x_1^2\int_{-1}^1\frac{dz_1}{2}\int_0^{2\pi}\frac{d\phi_1}{2\pi}\int_0^\infty dx_2x_2^2 \\ &\times\int_0^\infty dx_3x_3^2\int_{-1}^1\frac{dz_3}{2}\varphi_4^2\frac{1}{f_1^2\left(f_2^2-f_3^2\right)^2} \\ &\times\Bigg\{x_3^2\left[\frac{8b^2}{3a^2}f_1\left(f_2^2-f_3^2\right)+\frac{32}{9}\left(f_2^2-f_3^2\right)+\frac{64}{9}f_1f_2\right]^2 \\ &+x_2^2\left[-\frac{8}{3}\left(f_2^2-f_3^2\right)+\frac{8}{3}f_1f_2\right]^2+x_1^2\left[-\frac{16}{3}f_1f_3\right]^2 \\ &+2x_2x_3z_3\left[\frac{8b^2}{3a^2}f_1\left(f_2^2-f_3^2\right)+\frac{32}{9}\left(f_2^2-f_3^2\right)\right. \\ &\left.+\frac{64}{9}f_1f_2\right]\left[-\frac{8}{3}\left(f_2^2-f_3^2\right)+\frac{8}{3}f_1f_2\right] \\ &+2x_1x_3(z_1z_3+\sqrt{1-z_1^2}\sqrt{1-z_3^2}\cos\phi_1) \\ &\times\left[\frac{8}{3\beta^2}f_1\left(f_2^2-f_3^2\right)+\frac{32}{9}\left(f_2^2-f_3^2\right)+\frac{64}{9}f_1f_2\right] \\ &\times\left[-\frac{16}{3}f_1f_3\right]+2x_1x_2z_1 \\ &\times\left[-\frac{8}{3}\left(f_2^2-f_3^2\right)+\frac{8}{3}f_1f_2\right]\left[-\frac{16}{3}f_1f_3\right]\Bigg\}, \end{aligned} \quad \text{(C23)}$$

where we used the abbreviations $f_1 = (\frac{1}{4}x_2^2 + \frac{4}{9}x_3^2 - \frac{2}{3}x_2x_3z_3 + 1)$, $f_2 = (\frac{1}{4}x_1^2 + \frac{1}{16}x_2^2 + \frac{4}{9}x_3^2 + \frac{1}{3}x_2x_3z_3 + 1)$, $f_3 = (\frac{1}{4}x_1x_2z_1 + \frac{2}{3}x_1^2x_3^2\,\{z_1z_3 + \sqrt{1-z_1^2}\,\sqrt{1-z_3^2}\cos\phi_1\})$.

At finite densities, we take into account self-energies and Pauli blocking due to the medium. We obtain 12 terms that differ only by the isospin (p, n) dependence so that

$$\Delta E_\alpha^{\rm Pauli}(P) = -6\sum_{1234,1'2'}\varphi^*_{\alpha,P}(1,2,3,4)[f_p(1)+f_n(1)] \times V(1,2;1',2')\varphi_{\alpha,P}(1',2',3,4). \quad \text{(C24)}$$

First we consider $P = 0$. At $T = 0$, the Fermi distribution function is replaced by the step function. In the low-density limit where $\mathbf{p}_1 = 0$ or $\vec{x}_1 = -\frac{1}{2}\vec{x}_2 - \frac{1}{3}\vec{x}_3$, we have

$$\delta E_4^{\rm Pauli,J}(0;0) = -\frac{6\lambda}{2\,N_4}\int_{-1}^{1}\frac{dz_3}{2}\int_0^\infty dx_3x_3^2\int_0^\infty dx_2x_2^2 \times \frac{e^{-\frac{b^2}{a^2}(2x_2^2+\frac{14}{9}x_3^2+\frac{2}{3}x_2x_3z_3)}e^{-\frac{b^2}{\gamma^2}(\frac{1}{4}x_2^2+\frac{1}{9}x_3^2+\frac{1}{3}x_2x_3z_3)}}{\left(x_2^2+\frac{4}{9}x_3^2+\frac{4}{3}x_2x_3z_3+1\right)\left[\left(\frac{5}{2}x_2^2+\frac{1}{9}x_3^2+\frac{1}{3}x_2x_3z_3+1\right)^2-9\left(\frac{1}{2}x_2^2+\frac{1}{3}x_2x_3z_3\right)^2\right]} \times \frac{1}{\left(x_2^2+\frac{16}{9}x_3^2-\frac{8}{3}x_2x_3z_3+1\right)\left[\left(\frac{1}{2}x_2^2+\frac{17}{9}x_3^2+\frac{5}{3}x_2x_3z_3+1\right)^2-\left(\frac{1}{2}x_2^2+\frac{8}{9}x_3^2+\frac{5}{3}x_2x_3z_3\right)^2\right]} \times\left\{\int_0^\infty dx_5x_5^2\int_{-1}^{1}\frac{dz_5}{2}\int_0^{2\pi}\frac{d\phi_5}{2\pi}e^{-\frac{b^2}{\gamma^2}x_5^2}\varphi_4(x_5,x_2,x_3,z_5,z_3,\phi_5)\right\}. \quad \text{(C25)}$$

The integral over ϕ_5 can be performed analytically.

For arbitrary temperatures we find in the low-density limit, where we have $f_\tau(p) = \frac{1}{2}n_\tau(2\pi\hbar^2/mT)^{3/2}\exp(-\frac{\hbar^2}{2mT}p^2)$, the expression

$$\delta E_4^{\rm Pauli,J}(0;T) = -\frac{3\lambda b^3}{2\pi^2N_4}\left(\frac{2\pi\hbar^2}{m\,T}\right)^{3/2}\int_{-1}^{1}\frac{dz_3}{2}\int_0^\infty dx_3x_3^2\int_0^\infty dx_2x_2^2\left\{\int_{-1}^{1}\frac{dz_1}{2}\int_0^\infty dx_1x_1^2\int_0^{2\pi}\frac{d\phi_5}{2\pi} \times e^{-\frac{b^2}{\gamma^2}x_1^2}e^{-\frac{\hbar^2b^2}{2mT}(x_1^2+\frac{1}{4}x_2^2+\frac{1}{9}x_3^2+x_1x_2z_1+\frac{1}{3}x_2x_3z_3+\frac{2}{3}x_1x_3(z_1z_3+\sqrt{1-z_1^2}\sqrt{1-z_3^2}\cos(\phi_1))} \times\varphi_4(x_1,x_2,x_3,z_1,z_3,\phi_1)\}\left\{\int_0^\infty dx_5x_5^2\int_{-1}^{1}\frac{dz_5}{2}\int_0^{2\pi}\frac{d\phi_5}{2\pi}e^{-\frac{b^2}{\gamma^2}x_5^2}\varphi_4(x_5,x_2,x_3,z_5,z_3,\phi_5)\right\}. \quad \text{(C26)}$$

For finite P we have to introduce in the Fermi distribution $p_1/b = \frac{1}{4}\mathbf{P}/b - \frac{1}{3}\vec{x}_3 - \frac{1}{2}\vec{x}_2 - \vec{x}_1$. Expansion for small P gives the Pauli blocking contribution to the effective mass.

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