Microscopic DWBA calculations using DW81 *

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Abstract

The relationship between the one-body transition density (OBTD) coefficients in isospin representation and those in PN representation is given through Z-coefficients of respective representations. Also how the two-body interactions in terms of the singlet/triplet-even/odd components are transformed to those based on the V_{PP} and V_{PN} expressions are described. DW81 calculations using various combinations of the above representations (for both OBTD and two-body interaction) are performed to confirm an internal consistency. The $^{24}\mathrm{O}(p,p')^{24}\mathrm{O}(2^+)$ reaction at 70 MeV is chosen as an example for the test.

1 Relationship of OBTDs in isospin and PN representations

The one-body transition densities (OBTDs) and the associated Z coefficients in the PN representation are given as below:

$$\mathrm{OBTD}(\Delta J,\ pp \text{ or } nn) = \frac{1}{\sqrt{2\Delta J+1}}\left\langle f \parallel \left[a^+(k1)\otimes a(k2)\right]^{\Delta J} \parallel i\right\rangle, \quad (1)$$

$$Z(\Delta J,\ pp \text{ or } nn) = \frac{1}{\sqrt{2J_i+1}}\cdot \mathrm{OBTD}(\Delta J,\ pp \text{ or } nn). \quad (2)$$

The OBTDs and the Z coefficients in the isospin representation are given as below:

$$\mathrm{OBTD}(\Delta J,\Delta T) = \frac{1}{\sqrt{2\Delta J+1}\sqrt{2\Delta T+1}}\left\langle f ||| \left[a^+(k1)\otimes a(k2)\right]^{(\Delta J,\Delta T)} ||| i\right\rangle, \quad (3)$$

$$Z(\Delta J,\Delta T) = \frac{\sqrt{2\Delta T+1}}{\sqrt{2J_i+1}\sqrt{2T_f+1}}\langle T_i T_{iz},\Delta T\Delta T_z|T_f T_{fz}\rangle\cdot \mathrm{OBTD}(\Delta J,\Delta T). \quad (4)$$

The Z coefficients in the isospin and PN representations have the following transformation property:

$$Z(pp) = \frac{1}{\sqrt{2}}\left[Z(\Delta T=1)+Z(\Delta T=0)\right], \quad (5)$$

$$Z(nn) = \frac{1}{\sqrt{2}}\left[-Z(\Delta T=1)+Z(\Delta T=0)\right]. \quad (6)$$

*http://ribf.riken.jp/~ysatou/DWIA/dw81_doc.pdf

Table 1: OBTDs as calculated with NuShell in the two representations (using W and Wpn interactions) for the $\langle {}^{24}\mathrm{O}(2^+)|{}^{24}\mathrm{O}(0^+)\rangle$ transition.

Transition	Isospin representation		PN representation	
	$\Delta T = 0$	$\Delta T = 1$	pp	nn
$0d_{5/2}0d_{5/2}^{-1}$	-0.01595	-0.01030	0.00000	-0.00750
$0d_{5/2}0d_{3/2}^{-1}$	-0.05804	-0.03746	0.00000	-0.02732
$0d_{5/2}1s_{1/2}^{-1}$	-0.10142	-0.06546	0.00000	-0.04781
$0d_{3/2}0d_{5/2}^{-1}$	-0.13173	-0.08503	0.00000	-0.06220
$0d_{3/2}0d_{3/2}^{-1}$	-0.07525	-0.04857	0.00000	-0.03547
$0d_{3/2}1s_{1/2}^{-1}$	1.87328	1.20920	0.00000	0.88306
$1s_{1/2}0d_{5/2}^{-1}$	-0.09076	-0.05858	0.00000	-0.04278
$1s_{1/2}0d_{3/2}^{-1}$	-0.28904	-0.18657	0.00000	-0.13625

It is noted that the DW81 code [1] adopts the following convention with respect to the isospin z-component:

$$t_z = 1/2 \text{ for proton}, \tag{7}$$
$$t_z = -1/2 \text{ for neutron}. \tag{8}$$

The above mentioned relations can be confirmed for OBTDs given in Table 1 calculated with NuShell for the $\langle {}^{24}\mathrm{O}(2^+)|{}^{24}\mathrm{O}(0^+)\rangle$ transition, as an example, using the W and Wpn interactions.

2 Two-body interactions in terms of the singlet/triplet-even/odd components and those in terms of the V_{PP} and V_{PN} expressions

Program DW81 accepts the two-body interactions in two ways: (1) by specifying the "newrep" variable to be "1, 2, 3, or 4" and (2) by specifying it to be "0". For (1), the interaction variables (VP and VN in CARDs 3B and 3C, respectively) correspond to the components in terms of the singlet/triplet-even/odd representation as below:

$$VP(2) = V_{\mathrm{SE}}, \tag{9}$$
$$VP(3) = V_{\mathrm{TE}}, \tag{10}$$
$$VP(4) = V_{\mathrm{LSE}}, \tag{11}$$
$$VP(5) = V_{\mathrm{TNE}}, \tag{12}$$
$$VN(2) = V_{\mathrm{SO}}, \tag{13}$$
$$VN(3) = V_{\mathrm{TO}}, \tag{14}$$
$$VN(4) = V_{\mathrm{LSO}}, \tag{15}$$
$$VN(5) = V_{\mathrm{TNO}}. \tag{16}$$

For (2), and for the calculation of the (p,p') reaction,[1] the interaction coefficients required are those based on the V_{PP} and V_{PN} expressions as follows:

$$
\begin{aligned}
VP(2) &= V_0 + V_\tau, & (17)\\
VP(3) &= V_\sigma + V_{\sigma\tau}, & (18)\\
VP(4) &= \frac{1}{4}(V_{LS} + V_{LS\tau}), & (19)\\
VP(5) &= V_T + V_{T\tau}, & (20)\\
VN(2) &= V_0 - V_\tau, & (21)\\
VN(3) &= V_\sigma - V_{\sigma\tau}, & (22)\\
VN(4) &= \frac{1}{4}(V_{LS} - V_{LS\tau}), & (23)\\
VN(5) &= V_T - V_{T\tau}. & (24)
\end{aligned}
$$

Here the set (V_0, V_σ,V_τ,$V_{\sigma\tau}$, etc.) is in terms of transferred quanta and is given as below [2]:

$$
\begin{aligned}
V_0 &= \frac{1}{16}(3V_{TE} + 3V_{SE} + 9V_{TO} + 1V_{SO}), & (25)\\
V_\sigma &= \frac{1}{16}(1V_{TE} - 3V_{SE} + 3V_{TO} - 1V_{SO}), & (26)\\
V_\tau &= \frac{1}{16}(-3V_{TE} + 1V_{SE} + 3V_{TO} - 1V_{SO}), & (27)\\
V_{\sigma\tau} &= \frac{1}{16}(-1V_{TE} - 1V_{SE} + 1V_{TO} + 1V_{SO}), & (28)\\
V_{LS} &= \frac{1}{4}(3V_{LSO} + 1V_{LSE}), & (29)\\
V_{LS\tau} &= \frac{1}{4}(1V_{LSO} - 1V_{LSE}), & (30)\\
V_T &= \frac{1}{4}(3V_{TNO} + 1V_{TNE}), & (31)\\
V_{T\tau} &= \frac{1}{4}(1V_{TNO} - 1V_{TNE}). & (32)
\end{aligned}
$$

It is noted that the factor 1/4 in VP(4) [Eq.(19)] and in VN(4) [Eq.(23)] is due only to the convention of codes DWBA70 and DW81 (when newrep=0 is specified), which require the coefficient of $2\boldsymbol{L}\cdot\boldsymbol{\sigma}(=4\boldsymbol{L}\cdot\boldsymbol{S})$ [2].

The set of parameters used for a demonstration purpose in this report is a variant of the M3Y interaction [2], and is summarized in Table 2. They are given in the singlet/triplet-even/odd representation and can be directly fed into the interaction CARDs 3B and 3C (VP and VN variables) of the input file of DW81, when newrep$\neq$0.

The set of parameters based on the V_{PP} and V_{PN} expressions to be used in calculations with the newrep=0 option is summarized in Table 3.

3 Self-consistency check of DW81 input files

In DW81 calculations, nuclear structure part is taken care of by OBTDs (apart from the single particle wave function), which are given either with the isospin representation or

[1] For the (p,n) reaction with newrep=0, see the write-up of DW81.

Table 2: The M3Y two-body interaction parameters in the singlet/triplet-even/odd representation. The chosen components are those labeled 1, 4, 11, 14, 16, 17, and 20 in Table 1 of Ref. [2].

label	Channel	variable	R_1= 0.25 fm	R_2= 0.40 fm	R_3= 0.70 fm	R_3= 1.414 fm
1	SE	VP(2)	12454.0	−3835.0	—	−10.463
4	TE	VP(3)	21227.0	−6622.0	—	−10.463
16	LSE	VP(4)	0.0	−813.0	—	0.0
11	TNE	VP(5)	0.0	−1259.6	−28.41	—
20	SO	VN(2)	5018.0	1810.0	—	0.0
—	TO	VN(3)	0.0	0.0	—	0.0
17	LSO	VN(4)	−3733.0	−427.3	—	0.0
14	TNO	VN(5)	0.0	283.0	13.62	—

Table 3: The M3Y two-body interaction parameters in the V_{PP} and V_{PN} expressions. Derived from those in Table 2.

Channel	variable	R_1= 0.25 fm	R_2= 0.40 fm	R_3= 0.70 fm	R_3= 1.414 fm
$V_0 + V_\tau$	VP(2)	3113.5	−958.75	—	−2.616
$V_\sigma + V_{\sigma\tau}$	VP(3)	−3113.5	958.75	—	2.616
$\frac{1}{4}(V_{LS} + V_{LS\tau})$	VP(4)	−933.25	−106.825	—	0.0
$V_T + V_{T\tau}$	VP(5)	0.0	283.0	13.62	—
$V_0 - V_\tau$	VN(2)	10144.125	−2736.375	—	−5.231
$V_\sigma - V_{\sigma\tau}$	VN(3)	469.375	−574.625	—	0.0
$\frac{1}{4}(V_{LS} - V_{LS\tau})$	VN(4)	−466.625	−155.037	—	0.0
$V_T - V_{T\tau}$	VN(5)	0.0	−488.3	−7.395	—

Table 4: Representations (for both OBTD and two-body interaction) that each "newrep" specifier accepts.

		Two-body interaction specifier: newrep				
		0	1	2	3	4
		V_{PP}, V_{PN} rep.	singlet/triplet-even/odd rep.			
OBTD	Isospin rep.	×	×	○	○	○
	PN rep.	○	○	×	×	×

the PN representation, and the perturbative force responsible to the target excitation is given by the two-body interactions, which are given either with the singlet/triplet-even/odd representation or the V_{PP} and V_{PN} expressions.

Table 4 summarizes the relation between the value for the "newrep" variable, which specifies the input format for the two-body interaction, and the required representation for OBTD.

To check the consistency of the DW81 inputs, the differential cross section for the ^{24}O$(p,p')^{24}$O(2^+) reaction at 70 MeV was calculated in three different ways (newrep=0, 1, and 2&3). The calculated results are compared in Fig. 1. It is seen that the results are identical, and do not depend on the representation of OBTD and that of two-body interaction.

Some remarks on the DW81 inputs are given in Appendix A.

The input files used to obtain the curves in Fig. 1 are given in Appendices B, C, and D for the calculations with newrep=2&3, 1, and 0, respectively.

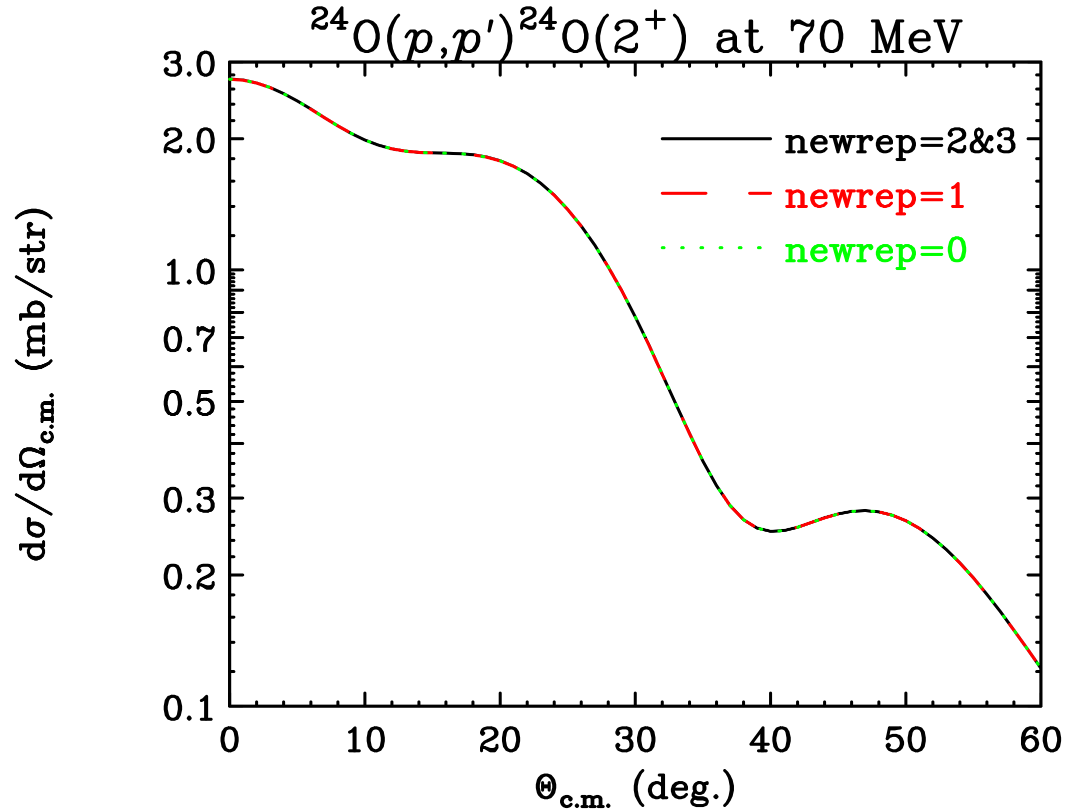

Figure 1: Differential cross sections for the ^{24}O$(p,p')^{24}$O(2^+) reaction at 70 MeV calculated with DW81 in three different choices for the newrep specifier: newrep=2&3 (black curve), 1 (red dashed curve), and 0 (green dotted curve).

Appendix A Some remarks on the DW81 inputs

1. The LS terms of the two-body interaction for the V_{PP} and V_{PN} expressions (newrep=0) must be obtained by multiplying a factor of 1/4 to the usual LS terms (as described in Section 2).

2. The wave functions of DW81 are normalized so that they become positive at infinity, [2] while those assumed in the shell-model calculations are normalized so that they are positive at origin. To reconcile the situation involving the two different phase conventions, appropriate phase factors have been taken into account in the OBTDs which appear in the sample input files in Appendices B, C, and D.

3. In the sample input files in Appendices B, C, and D, the factors in front of OBTDs in Eqs.(2) and (4) are separately provided as common multiplication factors to the amplitudes (CP and CN variables in CARD 7C).

4. The code adopts the isospin convention which assigns t_z=+1/2 for proton (as noted in Section 1).

[2] There seems to be an exception: some part of the external optical potential is normalized differently. For more details, see Ref. [3].

Appendix B DW81 input file with newrep=2&3

```
    1
   50  200 0.1
ECH   50
    2
181.        0.0       180.
                         3
    3
 4 2 0
     0.000 12454.000 21227.000     0.000     0.000
     0.000  5018.000     0.000 -3733.000     0.000
     0.250     0.250     0.250     0.250
     0.000 -3835.000 -6622.000  -813.000 -1259.600
     0.000  1810.000     0.000  -427.300   283.000
     0.400     0.400     0.400     0.400
     0.000     0.000     0.000     0.000   -28.410
     0.000     0.000     0.000     0.000    13.620
     0.700     0.700     0.700     0.700
     0.000   -10.463   -10.463     0.000     0.000
     0.000     0.000     0.000     0.000     0.000
     1.414     1.414     1.414     1.414
    4
 1 1
  70.0001.007276   1.000  24.000   8.000   1.347                  1.000   0.000
    107. -17.560   0.961   0.590           2.400   0.961   0.590
      2.                                 -15.240   1.297   0.531
     -1. -36.820   1.163   0.674          -7.700   1.163   0.674
    5
 1 1
-5.172  1.007276   1.000  24.000   8.000   1.347                  1.000   0.000
    107. -17.560   0.961   0.590           2.400   0.961   0.590
      2.                                 -15.240   1.297   0.531
     -1. -36.820   1.163   0.674          -7.700   1.163   0.674
    6
 3 0
SM
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 0.      2.      5.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 0.      2.      3.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
```

```
 -32.           1.8151
 1.      0.      1.      1.
    7
    2    1    8
NON RAY
  0.333333  0.333333
    2    2   0.07525
    2    1   0.13174
    2    3   1.87328
    1    2   0.05804
    1    1   0.01595
    1    3  -0.10142
    3    2  -0.28903
    3    1  -0.09075
    8
24O(p,p')24O DJ=2,DT=0
 0
    3
 4 3 0
     0.000 12454.000 21227.000     0.000     0.000
     0.000  5018.000     0.000 -3733.000     0.000
     0.250     0.250     0.250     0.250
     0.000 -3835.000 -6622.000  -813.000 -1259.600
     0.000  1810.000     0.000  -427.300   283.000
     0.400     0.400     0.400     0.400
     0.000     0.000     0.000     0.000   -28.410
     0.000     0.000     0.000     0.000    13.620
     0.700     0.700     0.700     0.700
     0.000   -10.463   -10.463     0.000     0.000
     0.000     0.000     0.000     0.000     0.000
     1.414     1.414     1.414     1.414
    7
    2    1    8
NON RAY
 -0.516398 -0.516398
    2    2   0.04857
    2    1   0.08503
    2    3   1.20920
    1    2   0.03746
    1    1   0.01030
    1    3  -0.06546
    3    2  -0.18657
    3    1  -0.05858
    8
24O(p,p')24O DJ=2,DT=1
 0
    9
```

```
    2 1.
    4 1.
-1
```

Appendix C DW81 input file with newrep=1

```
    1
   50  200 0.1
ECH   50
    2
181.       0.0      180.
                       3
    3
 4 1 0
     0.000 12454.000 21227.000     0.000     0.000
     0.000  5018.000     0.000 -3733.000     0.000
     0.250     0.250     0.250     0.250
     0.000 -3835.000 -6622.000  -813.000 -1259.600
     0.000  1810.000     0.000  -427.300   283.000
     0.400     0.400     0.400     0.400
     0.000     0.000     0.000     0.000   -28.410
     0.000     0.000     0.000     0.000    13.620
     0.700     0.700     0.700     0.700
     0.000   -10.463   -10.463     0.000     0.000
     0.000     0.000     0.000     0.000     0.000
     1.414     1.414     1.414     1.414
    4
 1 1
  70.0001.007276   1.000  24.000   8.000   1.347                  1.000   0.000
    107. -17.560   0.961   0.590           2.400   0.961   0.590
      2.                                 -15.240   1.297   0.531
     -1. -36.820   1.163   0.674          -7.700   1.163   0.674
    5
 1 1
-5.172  1.007276   1.000  24.000   8.000   1.347                  1.000   0.000
    107. -17.560   0.961   0.590           2.400   0.961   0.590
      2.                                 -15.240   1.297   0.531
     -1. -36.820   1.163   0.674          -7.700   1.163   0.674
    6
 6 3
SM
 0 1
 0.0     1.      1.      24.     7.      1.25                    1.
 -32.            1.8151
 0.      2.      5.      1.
 0 1
 0.0     1.      1.      24.     7.      1.25                    1.
 -32.            1.8151
 0.      2.      3.      1.
 0 1
 0.0     1.      1.      24.     7.      1.25                    1.
```

```
 -32.            1.8151
 1.      0.      1.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 0.      2.      5.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 0.      2.      3.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 1.      0.      1.      1.
    7
    2    1   16
NON RAY
  1.0       1.0
    1    1   0.00000
    1    2   0.00000
    1    3   0.00000
    2    1   0.00000
    2    2   0.00000
    2    3   0.00000
    3    1   0.00000
    3    2   0.00000
    4    4  -0.00750
    4    5  -0.02732
    4    6   0.04781
    5    4  -0.06220
    5    5  -0.03547
    5    6  -0.88306
    6    4   0.04278
    6    5   0.13625
    8
24O(p,p')24O DJ=2,V-PP,V-PN
 0
    9
    2 1.
-1
```

Appendix D DW81 input file with newrep=0

```
    1
   50  200 0.1
ECH   50
    2
181.       0.0      180.
                        3
    3
 4 0 0
     0.000  3113.500 -3113.500  -933.250     0.000
     0.000 10144.125   469.375  -466.625     0.000
     0.250     0.250     0.250     0.250
     0.000  -958.750   958.750  -106.825   283.000
     0.000 -2736.375  -574.625  -155.037  -488.300
     0.400     0.400     0.400     0.400
     0.000     0.000     0.000     0.000    13.620
     0.000     0.000     0.000     0.000    -7.395
     0.700     0.700     0.700     0.700
     0.000    -2.616     2.616     0.000     0.000
     0.000    -5.231     0.000     0.000     0.000
     1.414     1.414     1.414     1.414
    4
 1 1
  70.0001.007276   1.000  24.000   8.000   1.347                  1.000   0.000
    107. -17.560   0.961   0.590           2.400   0.961   0.590
      2.                                 -15.240   1.297   0.531
     -1. -36.820   1.163   0.674          -7.700   1.163   0.674
    5
 1 1
-5.172  1.007276   1.000  24.000   8.000   1.347                  1.000   0.000
    107. -17.560   0.961   0.590           2.400   0.961   0.590
      2.                                 -15.240   1.297   0.531
     -1. -36.820   1.163   0.674          -7.700   1.163   0.674
    6
 6 3
SM
 0 1
 0.0     1.      1.      24.     7.      1.25                    1.
 -32.            1.8151
 0.      2.      5.      1.
 0 1
 0.0     1.      1.      24.     7.      1.25                    1.
 -32.            1.8151
 0.      2.      3.      1.
 0 1
 0.0     1.      1.      24.     7.      1.25                    1.
```

```
 -32.            1.8151
 1.      0.      1.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 0.      2.      5.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 0.      2.      3.      1.
 0 1
 0.0     1.      0.      24.     8.      1.25                    1.
 -32.            1.8151
 1.      0.      1.      1.
    7
    2    1   16
NON RAY
  1.0       1.0
    1    1   0.00000
    1    2   0.00000
    1    3   0.00000
    2    1   0.00000
    2    2   0.00000
    2    3   0.00000
    3    1   0.00000
    3    2   0.00000
    4    4  -0.00750
    4    5  -0.02732
    4    6   0.04781
    5    4  -0.06220
    5    5  -0.03547
    5    6  -0.88306
    6    4   0.04278
    6    5   0.13625
    8
24O(p,p')24O DJ=2,V-PP,V-PN
 0
    9
    2 1.
-1
```

References

[1] Program DWBA70, R. Schaeffer and J. Raynal (unpublished); Extended version DW81, J. R. Comfort (unpublished).

[2] G. Bertsch, et al., Nucl. Phys, A284 (1977) 399.

[3] http://ribf.riken.jp/˜ysatou/r405n/omp.pdf