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EXACT FINITE RANGE DWBA CALCULATIONS FOR HEAVY-ION INDUCED NUCLEAR REACTIONS

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PROGRAM SUMMARY

Title of program: SATURN-1-FOR-EFR-DWBA

Catalogue number: ABPA

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue)

Computer: CDC 6600; *Installation*: Univ. of Texas Computation Center

Operating system: UT2D

Programming language used: FORTRAN IV

High speed store required: 32704 words

Number of bits per word: 60

Overlay structure: None

Number of magnetic tapes required: 3

Other peripherals used: Card reader, line printer, punch

Number of cards in combined program and test deck: 2509

Card punching code: CDC

Keywords: Nuclear, heavy ion, direct nuclear transfer reactions, exact finite-range DWBA (with recoil), no-recoil approximation, form factor, stripping, pickup, cross section, elastic scattering, Schrödinger equation, EFR–DWBA, NR–DWBA.

PROGRAM SUMMARY

Title of program: MARS-1-FOR-EFR-DWBA

Catalogue number: ABPB

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue)

Computer: CDC 6600; *Installation*: Univ. of Texas Computation Center

Operating system: UT2D

Programming language used: FORTRAN IV

High speed store required: 29632 words

Number of bits per word: 60

Overlay structure: None

Number of magnetic tapes required: 3

Other peripherals used: Card reader, line printer

Number of cards in combined program and test deck: 2175

* Supported in part by the US Atomic Energy Commission.

Card punching code: CDC

Keywords: Nuclear, heavy ion, direct nuclear transfer reactions, exact finite-range DWBA (with recoil), no-recoil approximation, form factor, stripping, pickup, cross section, elastic scattering, Schrödinger equation, EFR–DWBA, NR–DWBA.

Nature of physical problem

The package SATURN-MARS-1 consists of two programs SATURN and MARS for calculating cross sections of reactions transferring nucleon(s) primarily between two heavy ions. The calculations are made within the framework of the finite-range distorted wave Born approximation (DWBA). The first part, SATURN, prepares the form factor(s) either for exact finite-range (EFR) or for no-recoil (NR) approach. The prepared form factor is then used by the second part MARS to calculate either EFR–DWBA or NR–DWBA cross sections.

Method of solution

Either with EFR or NR approaches, a one-dimensional integral(s) is to be carried out in SATURN to evaluate certain kernel(s). There the gaussian quadrature is used, introducing

a specific technique [1] so as to minimize the number of quadrature points. Most of the basic techniques used in MARS are not very much different from those used in VENUS, a zero-range (ZR) DWBA program reported earlier [2]. Throughout SATURN and MARS, interpolation and other techniques [1] are used, so that EFR calculations can be performed with a speed which is not very much slower than the much simplified NR and/or ZR approximations.

Restrictions on the complexity of the program
In its present shape, SATURN-MARS-1 requires the use of the cluster approximation for treating multi-nucleon transfer reactions. Note, however, that in many applications the cluster approximation is sufficiently good, and also that removal of this restriction is made rather easily, by writing a booster program to feed results into SATURN. Presently set restrictions on the size of the calculations can be removed fairly easily without increasing the needed core very much.

Typical running time
The largest calculation so far made with SATURN-MARS-1 is the analysis of ^{208}Pb(^{16}O, ^{15}N)^{209}Bi reactions with $E(^{16}O) = 140$ MeV. With the CDC 6600 computer at the University of Texas, the running time ranged from 2 to 8 minutes when the spin of the final states in ^{209}Bi varied from 1/2 to 9/2. With the IBM360/195 computer at the Argonne National Laboratory, the running time of the same calculation was about 1/3 of the above figure. When lighter targets and lower E were used, most of the calculations were made within a minute, even with the CDC 6600.

References
[1] T. Tamura and K.S. Low, Phys. Rev. Letters 31 (1973) 1356;
K.S. Low and T. Tamura, Phys. Letters 48B (1974) 285;
T. Tamura, Phys. Rep., to be published.
[2] T. Tamura, F. Rybicki and W.R. Coker, Computer Phys. Commun. 2 (1971) 94.

LONG WRITE-UP

1. Introduction

The program package SATURN-MARS has been written having in mind to carry out, not only exact finite-range (EFR) distorted wave Born approximation (DWBA), but also EFR coupled-channel Born approximation (CCBA) calculations. However, the scope of this full-sized EFR–CCBA mode has still to be extended for general usage, and thus the version SATURN-MARS-1 to be presented here is restricted to the EFR–DWBA calculations, although it allows one to carry out also no-recoil (NR) DWBA calculations.

A large number of books and review articles concerning the formulation of DWBA calculations have appeared [1–4], covering both EFR and zero-range (ZR) calculations. However, practical applications of EFR–DWBA have been very scarce until recently, mainly because of its time-consuming nature. In this write-up we shall describe very briefly the various techniques we introduced in speeding up the calculations and in making the program size sufficiently small so that EFR–DWBA calculations can be carried out on a rather routine basis, and even with comparatively small size computers.

In section 2 we summarize formulas needed for EFR– and NR–DWBA calculations. As is seen, they are very close to those given in ref. [3]. Nevertheless, some reformulation has been made so that the relation between EFR and NR approaches becomes transparent, and also that the projectile–ejectile system is treated on an equal footing with the target–residual nucleus system. The latter reformulation is particularly convenient when an application is made to the analysis of heavy-ion induced reactions. For more details of the reformulation, as well as of the improved computational techniques which are described very briefly in section 3, the reader is referred to refs. [5] and [6].

The program, whose structure is described in section 4 is written in FORTRAN IV. The only subroutines needed which are not included are standard FORTRAN functions and standard input/output routines. I/O units 5, 6 and 7 are used for card input, printed output and card punch, respectively. I/O units 11 and 13 are used as scratch, while the unit 12 is used to store the form factor calculated in SATURN and then used in MARS. Since the same form factor is used by MARS repeatedly, the I/O unit 12 should have a semi-permanent nature.

Finally, we include in the Appendix some general comments regarding the optimum use of the programs. These are mainly directed to the case of EFR calculations where efficient use of the programs becomes important in maintaining numerical accuracy as well as speed in the numerical calculations.

2. Formulas for DWBA cross section

2.1. Form factors

The first part of our program, SATURN, constructs the form factors to be used later in the second part, MARS. We shall thus give in this sub-section the expression of such form factors for NR as well as for EFR calculations.

2.1.1. Form factor for EFR calculations

The form factor for the EFR calculation can be written as

$$F_{l_a;l_b}^{jls;\,\mathrm{AaBb}}(r_a, r_b) = \sum_{l_1 n_1 l_2 n_2} d_{l_1 n_1 l_2 n_2}^{jls;\,\mathrm{AaBb}} \times \sum_k \left[\sum_{\Lambda_{12}} r_a^{\Lambda_{12}} r_b^{l_1+l_2-\Lambda_{12}} \sum_{\Lambda_a \Lambda_b} \{\mathrm{CFAC}(\Lambda_a \Lambda_b; \Lambda_{12}; l_1 l_2 l) \cdot \mathrm{GEOK}(\Lambda_a \Lambda_b; l_a l_b; k; l_1 l_2 l)\} \right] G_k^{l_1 n_1 l_2 n_2}(r_a, r_b), \tag{1}$$

where the geometrical factors are given as

$$\mathrm{CFAC}(\Lambda_a \Lambda_b; \Lambda_{12}; l_1 l_2 l) = \sum_{\lambda_1, \lambda_2} \delta_{\lambda_1+\lambda_2;\Lambda_{12}} D_{l_1 \lambda_1 \lambda_1'} D_{l_2 \lambda_2 \lambda_2'} s_1^{\lambda_1} s_2^{\lambda_2} t_1^{\lambda_1'} t_2^{\lambda_2'}$$

$$\times \hat{\lambda}_1 \hat{\lambda}_2 \hat{\lambda}_1' \hat{\lambda}_2' \hat{l}_1 \hat{l}_2 \hat{\Lambda}_a \hat{\Lambda}_b (\lambda_1 0 \lambda_2 0 | \Lambda_a 0)(\lambda_1' 0 \lambda_2' 0 | \Lambda_b 0) \begin{Bmatrix} \lambda_1 & \lambda_2 & \Lambda_a \\ \lambda_1' & \lambda_2' & \Lambda_b \\ l_1 & l_2 & l \end{Bmatrix}, \tag{2}$$

with

$$D_{l_1 \lambda_1 \lambda_1'} = \delta_{\lambda_1+\lambda_1';l_1} [(2l_1+1)!/(2\lambda_1+1)!(2\lambda_1'+1)!]^{1/2}, \qquad \hat{\lambda}_1 = [2\lambda_1+1]^{1/2}, \quad \text{etc.,} \tag{2a}$$

and as

$$\mathrm{GEOK}(\Lambda_a \Lambda_b; l_a l_b; k; l_1 l_2 l) = \tfrac{1}{2} \mathrm{i}^{(l_1+l_2)-(l_a+l_b)} (-)^{k+l} (2k+1)(\Lambda_a 0 k 0 | l_a 0)(\Lambda_b 0 k 0 | l_b 0) W(l_a \Lambda_a l_b \Lambda_b; kl). \tag{3}$$

The first factor in eq. (1) is defined as

$$d_{l_1 n_1 l_2 n_2}^{jls;\,\mathrm{AaBb}} = C_{I_> I_< j l_1 n_1}^{(1)} C_{s_> s_< s l_2 n_2}^{(2)} (-)^{s+l_2-I_x} W(l_1 l_2 j s; l I_x), \tag{4}$$

while the kernel is defined as

$$G_k^{l_1 n_1 l_2 n_2}(r_a, r_b) = \int_{-1}^{1} w_{l_1 n_1}(r_1)\, w_{l_2 n_2}(r_2)\, P_k(\mu)\, \mathrm{d}\mu. \tag{5}$$

The reaction we have in mind is described as A(a, b)B, and we shall refer the set (a, b) as projectile–ejectile system, or as lighter system or simply as system 2, referring similarly the set (A, B) as target–residual system, or as heavier system or simply as system 1. When the reaction seen from the point of view of the lighter system is stripping (pick-up), we have the relation a = b + x (b = a + x) and consequently B = A + x (A = B + x), x being the transferred particle. The coordinate system we choose for the stripping (pick-up) reaction is given in fig. 1(a) (fig. 1(b)), where not only the vectors r_a and r_b, but also r_1 and r_2 are defined. The relation between these two pairs of independent coordinates is

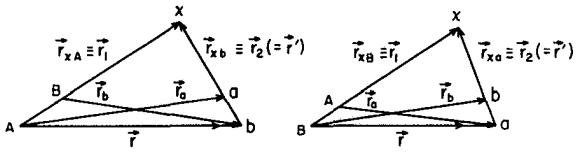

Fig. 1

$$r_1 = s_1 r_a + t_1 r_b \quad \text{and} \quad r_2 = s_2 r_a + t_2 r_b, \tag{6}$$

with

$$s_1 = aB/xT, \quad t_1 = -bB/xT, \quad s_2 = aA/xT \quad \text{and} \quad t_2 = -aB/xT \quad (T = a + A = b + B) \quad \text{(for stripping)},$$

$$s_1 = aA/xT, \quad t_1 = bA/xT, \quad s_2 = -bA/xT \quad \text{and} \quad t_2 = bB/xT \quad (T = a + A = b + B) \quad \text{(for pick-up)}. \tag{7}$$

Restricting for the moment to stripping reaction, the various quantum numbers that appear in eqs. (1) and (5) are explained as follows. The particle x has an intrinsic spin I_x, and when bound to the core A to form B has a total spin j and orbital angular momentum l_1, as well as n_1 nodes in its radial part of the wave function. When x is bound to form a, the quantum numbers (s, l_2, n_2) play the same role as did (j, l_1, n_1) in the heavier system. If the post form is employed, the function $w_{l_1 n_1}(r_1)$ that appears in (5) is nothing but the radial part, $U_{l_1 n_1}(r)$, of the wave function of the relative motion in B of x with respect to A, divided by $r_1^{l_1}$. The function $w_{l_2 n_2}(r_2)$ is defined similarly, but it is further multiplied by a factor $V(r_2)$, the potential used to bind x to b to form a. If the prior form is taken, $w_{l_2 n_2}(r_2)$ does not have this potential factor, but $w_{l_1 n_1}(r_1)$ includes a corresponding factor $V(r_1)$. In the case of pick-up reaction, the roles of the prior and post forms, as well as those of A and B and of a and b, are simply interchanged. We thus see in eq. (4) that

$$I_> = I_B, \quad I_< = I_A, \quad s_> = s_a \quad \text{and} \quad s_< = s_b \quad \text{(for stripping)},$$
$$I_> = I_A, \quad I_< = I_B, \quad s_> = s_b \quad \text{and} \quad s_< = s_a, \quad \text{(for pick-up)}, \tag{8}$$

which means that, for example, the coefficient $C^{(1)}$ in eq. (4), in the case of stripping, is nothing but the amplitude with which the eigenstate of B with spin I_B includes the state formed by vector coupling the motion of x to the eigenstate of A with spin I_A, further multiplied by an appropriate number which takes into account the exchange of the transferred nucleons in x with those that were in A from the beginning. The coefficient $C^{(2)}$ is interpreted similarly. Interpretation of $C^{(1)}$ and $C^{(2)}$ in the case of stripping will also be clear.

The quantum numbers l_a and l_b are orbital angular momenta, respectively, in the incident and exit channels of the partial distorted waves, while l is the orbital angular momentum transferred between them. Note that the selection rule for l in full is given by

$$l_1 + l_2 + l = 0 \quad \text{and} \quad j + s + l = 0. \tag{9}$$

The other quantum numbers $\lambda_1, \lambda_1', \lambda_2, \lambda_2', \Lambda_a, \Lambda_b$ and k are intermediary, and their role can be understood easily from their entry in various geometric factors. Note that $(jmj'm'|j''m'')$ is a Clebsch–Gordan coefficient, $W(abcd; ef)$ is a Racah coefficient and $\begin{Bmatrix} a & b & e \\ c & d & e' \\ f & f' & g \end{Bmatrix}$ is a 9–j symbol.

Finally in (5) the parameter μ is the cosine of the angle between two vectors r_a and r_b, and the dependence of r_1 and r_2 on this parameter can be found from the relation (6) together with (7).

2.1.2. Form factor for NR calculations

The form factor for the NR calculations can be expressed as

$$F_{\mathrm{NR}}^{jls;\,\mathrm{AaBb}}(r) = \sum_{l_1 n_1 l_2 n_2 \lambda_1 \lambda_1' k} \mathrm{i}^{l_1+l_2-l}(-)^k d_{l_1 n_1 l_2 n_2}^{jls;\,\mathrm{AaBb}} D_{l_1 \lambda_1 \lambda_1'} \hat{\lambda}_1 \hat{\lambda}_1' \hat{l}_1 \hat{l}_2 (\lambda_1 0 l 0 | k 0)(\lambda_1' 0 l_2 0 | k 0) W(l_1 l_2 \lambda_1 k; l \lambda_1') \tag{10}$$

$$\times r^{\lambda_1} \int G_k^{l_1 n_1}(r, r')\, \phi_{l_2 n_2}(r')\, r'\, \mathrm{d}r',$$

with the kernel

$$G_k^{l_1 n_1}(r, r') = \int_{-1}^{1} w_{l_1 n_1}(|r+r'|)\, P_k(\mu)\, \mathrm{d}\mu, \tag{11}$$

the parameter μ being the cosine of the angle between two vectors r and r' defined in fig. 1. The meaning of (10) is clear since every quantity that appears there has been explained previously, except that we introduced

$$\phi_{l_2 n_2}(r_2) = r_2^{l_2} w_{l_2 n_2}(r_2).$$

2.2. Cross sections

The expression for the cross section is given in the following very simple form, if the spin–orbit interaction is ignored in the distorting potential, as is customary the case for heavy-ion induced reactions;

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = C_1^2 S_1\, C_2^2 S_2\, \frac{\mu_a \mu_b}{(2\pi\hbar^2)^2}\, \frac{k_b}{k_a}\, \frac{(2I_>+1)(2s_>+1)}{(2I_A+1)(2s_a+1)} \left(\frac{4\pi}{k_a k_b}\right)^2 \tag{12}$$

$$\times \sum_{jls m_l} \left| \sum_{l_a l_b} (-)^{l_b+m_l} \hat{l}_a \hat{l}_b (l_a 0 l_b m_l | l m_l) I_{l_a l_b}^{l}\, G_{l_b - m_l} P_{l_b m_l}(\theta) \right|^2 ,$$

where, if EFR–DWBA calculation is considered,

$$I_{l_a l_b}^{jls} = J \iint \chi_{l_b}^{(-)*}(k_b, r_b)\, F_{l_a l_b}^{jls;\,\mathrm{AaBb}}(r_a, r_b)\, \chi_{l_a}^{(+)}(k_a, r_a)\, r_a r_b\, \mathrm{d}r_a \mathrm{d}r_b, \tag{13}$$

and $$G_{lm} = (-)^{(m-|m|)/2} [(l-|m|)!(l+|m|)!]^{1/2}.$$

In (12) the cross section is written by taking out explicitly the factor $C_1^2 S_1\, C_2^2 S_2$ so that the expression becomes as conventional as possible. If a one-nucleon transfer reaction is considered, or if a multi-nucleon transfer reaction is considered in the cluster approximation, we have a relation that $C_i^2 S_i = C^{(i)2}$ (i = 1 and 2), where $C^{(i)}$ was introduced in eq. (4). Therefore, when the cross section is written in the form of (12), the form factor $F^{jls;\,\mathrm{AaBb}}{}_{l_a l_b}(r_a, r_b)$ in (13) should be understood to be equal to that of (1) in which $C^{(1)} = C^{(2)} = 1$. In (12) and (13), k_a and k_b are wave numbers and μ_a and μ_b are reduced masses, respectively, in the incident and exit channels. The jacobian $J = (\mathrm{aB}/xT)^3$ for stripping and $= (\mathrm{bA}/xT)^3$ for pick-up reactions.

The cross section with the NR-approximation is again given by (12) but with

$$I_{l_a l_b}^{jls} = (\mathrm{B}/\mathrm{A})(\hat{l}_a \hat{l}_b / \hat{l})(l_a 0 l_b 0 | l 0) \int \chi_{l_b}(k_b, (\mathrm{A}/\mathrm{B})\, r)\, F_{\mathrm{NR}}^{jls;\,\mathrm{AaBb}}(r)\, \chi_{l_a}(k_a, r)\, \mathrm{d}r, \quad \text{(for stripping);}$$

$$I_{l_a l_b}^{jls} = (\mathrm{A}/\mathrm{B})(\hat{l}_a \hat{l}_b / \hat{l})(l_a 0 l_b 0 | l 0) \int \chi_{l_b}(k_b, r)\, F_{\mathrm{NR}}^{jls;\,\mathrm{AaBb}}(r)\, \chi_{l_a}(k_a, (\mathrm{B}/\mathrm{A})\, r)\, \mathrm{d}r, \quad \text{(for pick-up).} \tag{14b}$$

Because of the appearance of a new *CG* factor $(l_a 0 l_b 0 | l 0)$, a new selection rule which requires that

$$l_a + l_b + l = \text{even} \tag{15}$$

is imposed for the amplitude $I^{jls}_{l_a l_b}$ to be nonvanishing.

3. Remarks concerning numerical calculations with EFR approach

In EFR calculation, the complicated form factor (1) has to be evaluated at each (discretized) coordinates r_a and r_b mesh point in a huge, multidimensional space spanned by the quantum numbers j, l, s, l_a and l_b. The total number of quantities that are needed is thus huge, and its evaluation, storing and recovery require a lot of computer time.

We shall show that this number of mesh points, however, can be reduced greatly in practice. To see this, we first note that the integrand of (5) becomes negligibly small if r_1 and/or r_2 exceed their respective critical values R_{1c} and R_{2c}, which can be defined as the radius for which $W_{l_1 n_1}(r_1)$ or $w_{l_2 n_2}(r_2)$ are negligibly small compared with their peak values*. Therefore, it is necessary to retain in our calculation only pairs of (r_a, r_b) so that $r_i \lesssim R_{ic}$ ($i = 1$ and 2) for at least part of the values of μ between -1 and 1. Otherwise G_k and thus F become negligibly small.

The explicit dependence of r_1 and r_2 upon r_a, r_b and μ can best be seen in the following form, which is obtained (for stripping) from (6) and (7a);

$$r_1 = [(BA^{-1}\alpha\delta_{ab} + r_b)^2 + 2(b/a) r_a r_b \alpha^2 (1-\mu)]^{1/2}, \tag{16a}$$

$$r_2 = [(\alpha\delta_{ab})^2 + 2(b/a) r_a r_b \alpha^2 (1-\mu)]^{1/2}, \tag{16b}$$

where

$$\alpha = aB/xT \quad \text{and} \quad \delta_{ab} = (A/B) r_a - r_b. \tag{17}$$

In (17) we note that α is large being equal, e.g., to approximately 16 in the $^{208}Pb(^{16}O,^{15}N)^{209}Bi$ reaction [5], and that r_a and r_b are also large being at least equal to the lower cutoff radius $R_{\ell c}$ (about 8 fm in the reaction mentioned). Then, in order to keep $r_i < R_{ic}$, the following restrictions can be imposed upon r_a, r_b and μ: (*i*) $|\delta_{ab}|$ must be kept very small, e.g., $|\delta_{ab}| < 1$ fm and (*ii*) μ must be kept very close to unity in order to make $\alpha^2(1-\mu) \lesssim 1$.

What the restriction (*i*) implies is that the region in the (r_a, r_b) space in which F is to be calculated is confined within a very narrow band along the line defined by $\delta_{ab} = 0$, i.e., the (diagonal) line $r_b = (A/B) r_a \doteqdot r_a$. This reduces drastically the number of mesh points that must be calculated.

The restrictions (*i*) and (*ii*) together result in another important property that, for a given set of δ_{ab} and μ, both r_1 and r_2 depend very weakly on r_b. Therefore G_k and consequently F depends very weakly on r_b for a fixed value of δ_{ab}. Thus if we take the mesh size for δ_{ab} to be Δ and calculate F for r_b (for each fixed δ_{ab}) with a mesh size equal to $n\Delta$ ($n > 1$), the values of F at the intermediate $(n-1)$ points can be obtained by interpolation. Because of the weak dependence of F on r_b, the needed accuracy for F is not lost this way while its speed of evaluation is increased by a factor of n. Our experience showed that n can be taken as large as 10, although it depends on the value of Δ.

Further, the restriction (*ii*) can be used in speeding up the evaluation of the integral in (5). Note that $P_k(\mu)$ for a large k is a highly oscillatory function if it is considered in the whole range of μ from -1 to $+1$. However, if $P_k(\mu)$ is considered only in the range of μ from $\mu_{min} \doteqdot 1 - \alpha^{-2} \doteqdot 0.99$ to unity, it is not very oscillatory. Therefore, even when G_k has to be calculated up to a very large value of k, the number of mesh points (e.g., with the gaussian quadrature) to be taken for μ in performing the integral in (3.3) accurately can be much smaller than k, allowing this integration to be carried out very rapidly.

It should be noted that in (1) the summation over the quantum numbers λ_1, λ_2 $(= \Lambda_{12} - \lambda_1)$, Λ_a, Λ_b and Λ_{12} are taken before the summation over k is taken. The reason is that the quantity CFAC of (2) is larger than the quantity $\Sigma_{\Lambda_a \Lambda_b}$ CFAC × GEOK,

* The actual values of R_{1c} and R_{2c} vary largely from case to case. Very crudely, however, one may say that they range from 10 to 30 fm.

i.e., larger than the quantity in the curley bracket of (1) by a factor of about $\alpha^{l_1+l_2}$, where α was defined in (17). With the example of ^{208}Pb(^{16}O, ^{15}N)^{209}Bi reaction which has $\alpha \doteqdot 16$, we get $\alpha^{l_1+l_2} = 10^7$, if the final state considered in ^{209}Bi was $9/2^-$ ($l_1 = 5$ and $l_2 = 1$). This means that in carrying out the summation over Λ_a and Λ_b, about seven significant figures are lost. Since it is not difficult to obtain CFAC and GEOK with the accuracy of $14 \sim 15$ significant figures, this loss of accuracy is not serious. However, if summation over k is taken before those over Λ_a and Λ_b, it means that G_k has to be obtained with an accuracy of $14 \sim 15$ significant figures, which is a rather difficult task; and even if possible it is normally very time consuming. It is thus seen that a careful choice of the order of summation is very crucial in order to obtain the form factor with a sufficient accuracy. With the choice of the summation order made in (1), to obtain G_k with an accuracy of $6 \sim 7$ significant figures is normally sufficient.

We shall finally comment on the possibility of interpolating the amplitude $I^{l}_{l_a l_b}$ with respect to the (incident) orbital angular momentum l_a. Note that in heavy-ion induced reactions, one has to take into account l_a up to a rather high value in general, and this is one of the reasons why the calculation is more time consuming than it is for light-ion induced reactions. However, it should also be noted that the centrifugal potential is inversely proportional to the reduced mass which is much larger in heavy-ion reactions than it is in the light-ion reactions. Therefore, it is expected that in the heavy-ion reactions the variation of $I^{jls}_{l_a l_b}$ with a change of l_a by one unit (by keeping $l_b - l_a$ unchanged) is much weaker than it is in the light-ion reaction. If it is indeed the case, one may calculate, for example, $I^{jls}_{l_a l_b}$ only for even values of l_a and obtain those with odd l_a by interpolation, thereby reducing the computational time needed by about a factor of two. Our program has this provision and in many cases it was found that the error in this approximate treatment is less than 1%, though it is clear that the error incurred gets more significant if very few partial waves are involved in the reaction, as in the case of sub-Coulomb reactions. It should also be noted that to interpolate the real and imaginary parts of $I^{jls}_{l_a l_b}$ separately should be avoided, since these changes sign rather frequently resulting in poor interpolated values. However, to interpolate the magnitude and the phase angle separately is possible since these are fairly smooth functions of the angular momentum. This procedure is adopted in the program.

Another provision included in the program allows for a lower cut-off in the angular momentum space. This is fairly time saving for very high energy transfer reactions when many partial waves are involved and the lower partial waves contribute negligibly to the reaction cross section.

4. Structure and operation of the program

4.1. Form factor program SATURN

The main program SATURN first reads *five* cards which specify the type of reaction and the nature of nuclei that are involved in the reaction A(a,b)B, and also the specific way the calculation is to be made. It then calls BSAXON *twice* to calculate the bound-state wave function, i.e., $w_{l_i n_i}(r_i)$, for $i = 1$ and 2. Each time BSAXON is called, a set of *three* cards are read in, through which qunatum numbers and parameters needed to specify these wave functions are supplied. If the running is intended for calculating the EFR-form factor, SATURN then calls KERNEL which in turn calls GKRARB for each set of (r_a, r_b) and the integrand of the kernel of eq. (5) is calculated. The kernel is then calculated in KERNEL and stored on I/O unit 11. SATURN finally calls FRFMFC which first calls CFCCAL, to calculate the geometric factor CFAC of eq. (2), and then calls FFRARB to complete the calculation of the form factor of eq. (1). The form factor is stored on I/O unit 12, which is normally semi-permanent so that its content may be used by MARS (see section 4.2) repeatedly.

Just after BSAXON is called, another routine MONITR is called, and a check is made of whether or not the current forms of both SATURN and MARS have a sufficiently large allocation of the core for storing all the key quantities involved. If not, MONITR prints out a message suggesting possible changes in the input or else the necessary increment(s) of the core, and the calculation is terminated.

If SATURN is used to calculate the NR-form factor of eq. (10), it calls NRFMFC, after calling BSAXON in the same way as it did for EFR calculations, and the NR form factor is obtained. If interpo-

polation is required, it is done by calling INTERP from NRFMFC. The obtained NR form factor can be stored on I/O unit 12, or can be punched out in card form through I/O unit 7.

4.1.1. Input to SATURN

The read statements for the six basic cards are as follows:

READ(5.15) (KTRLST(N), N = 1,10), (KEXCST(N), N = 1, 6), (KOUTST(N), N = 1, 8)
READ(5,15) MASCA, NZCA, KPCA, ICATW, MASCB, NZCB, KPCB, ICBTW, MASSA, NZSA, KPSA, ISATW, MASSB, NZSB, KPSB, ISBTW
READ(5,15) JLSMAX, (JJTWR(N), LLTWR(N), ISTWR(N), N = 1, JLSMAX)
IF(KTRLST(1).EQ.0) READ(5,10) NATOTL, NBMIN, NBMAX, MESFCA, MESFCB, MGAUS, LDWMAX
IF(KTRLST(1).EQ.1) READ(5,10) NBMIN, NBMAX, MESFCB, MGAUS, KPUNCH
READ(5,20) XMESB, XMESA

The FORMATS 10, 15 and 20 are, respectively, (14I5), (24I3) and (10F7.2). The meaning of the quantities that appear in these cards are as follows:

CARD 1 In this card only the following quantities are used:

If KTRLST(1)=0(=1), EFR– (NR–) DWBA form factors are calculated.
If KEXCST(1)=0, lower cut off of l_a is not made.
If KEXCST(1)≠0, the form factors with $l_a+1<$ LAMIN=KEXCST(1) are not calculated.
If KEXCST(2)=0, form factors are calculated with $\Delta l_a = 1$.
If KEXCST(2)≠0, the calculation is made with the step of LASTEP=KEXCST(2). The present form of SATURN-MARS-1 allows only KEXCST(2)=0 or 2.

CARD 2 The sixteen quantities that are read in are $(M, Z, \pi, 2I)$, i.e., (mass, charge, parity, twice the spin) of nuclei A, B, a and b. Note that C stands for *capital* while S means *small.* Note also that $\pi = 0(1)$ for even (odd) parity.

CARD 3 JLSMAX = total number of different sets of the quantum numbers (j, l, s), that were defined in eqs. (9) and (14). JJTWR(N), LLTWR(N) and ISTWR(N) = $2j$, $2l$ and $2s$ of the Nth set. Note that LLTWR(N) should be in decreasing order.

CARD 4 This card is different for EFR and for NR calculations.

(i) EFR calculation:

NATOTL = Total number of mesh points of r_a, for a fixed value of r_b and with the mesh size XMESA (see CARD 5 below), at which the form factor is to be calculated. Namely r_a ranges from r_b–XMESA *(NATOTL-1)/2 to r_b+XMESA *(NATOTL-1)/2. Thus NATOTL must be an odd number.

NBMIN and NBMAX = smallest and largest mesh points for r_b, between which the form factor is to be calculated. Since XMESB is the mesh size for r_b (see CARD 5 below) NBMIN*XMESB and NBMAX *XMESB are lower and upper cut-off radii for the integral in eq. (13).

MESFCA and MESFCB = the factor by which the original mesh size of r_a and r_b are reduced after interpolation is made. These numbers and XMESA and XMESB are to be chosen so that XMESA/MESFCA = XMESB/MESFCB = mesh size with which the distorted waves are evaluated in MARS.

MGAUS = index of NGAUS for gaussian quadrature integration where NGAUS is the number of integration points. The correspondence of MGAUS and NGAUS is as follows:
MGAUS = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
NGAUS = 4, 8, 12, 16, 20, 24, 32, 40, 48, 64, 80, 96.
To check for convergence, MGAUS should be varied from 3 onwards although normally, MGAUS = 3 is sufficient.

LDWMAX = maximum value of l_a and l_b to be considered in DWBA calculation.

(ii) NR calculation:

NBMIN = 1 always.

NBMAX = number of mesh points of r, with the spacing of XMESB, at which NR form factor eq. (10) is to be calculated. NBMAX is to

be chosen so that NBMAX*XMESB is the upper cut off radius of the form factor.

MESFCB = the factor by which the original mesh size of r is reduced after interpolation so that the new mesh size equals that with which the distorted waves are obtained. Thus the total number of the mesh points eventually equals NBMAX*MESFCB.

MGAUS is the same as it was in EFR calculation.

If KPUNCH = 0 form factor is simply printed on standard I/O unit 6.
= 1 it is also punched on cards through I/O unit 7.
= 2 it is written on I/O unit 12.
= 3 it is punched through I/O unit 7 and written on I/O unit 12.

CARD 5 For the case of EFR calculation, the meaning of XMESA and XMESB was already explained. In the case of NR calculation the meaning of XMESB was also explained above. XMESA is not used.

4.1.2. Input to BSAXON

The READ statements for the three cards to be read in from BSAXON are

READ(5,51) KTRL2, KTRL3, KTRL4, KTRL8, KEX2, KEX4, NODER, LBTR, ISTW, JBTRTW, ITBEMX, KOPOT
READ(5,53) EGES, TMAS, PMAS, ZZT, ZZP, XMES2, ACURCY, AMUPMU
READ(5,53) VSX, VSOR, DFNR, DFNSO, RZR, RZSO, RZC

with the FORMAT 51 and 53 being (14I5) and (10F7.4) respectively. The quantities in these cards have the following meaning:

CARD 1 If KTRL2 = 0 (or 1) the binding energy (or VSX = depth of the Saxon potential) is searched. If KTRL3 = 0 the radius R in the Saxon potential is calculated as $R = r_0 A_1^{1/3}$, while if KTRL3 = 1 it is calculated as $R = r_0(A_1^{1/3}+A_2^{1/3})$. Here r_0 is the radius parameter (to be input from CARD-3 below), while A_1 and A_2 are respectively, the masses of the core and of the lighter particle that goes around it. If KTRL4 = 0, 1, 2 and 3, the quantity to be stored in URSAVE(N) in the blank COMMON, and is thus transferred to SATURN (as USINGL(N)) is $U_{l_i n_i}(r_i)$, $V(r_i)\,U_{l_i n_i}(r_i)$, $U_{l_i n_i}(r_i)/r^{l_i}$ and $V(r)\,U_{l_i n_i}(r_i)/r^{l_i}$, respectively. See eqs. (5) and (11), for the use of these different possibilities. If KTRL8 = 1, URSAVE(N) is punched out through I/O unit 7; if = 0, nothing takes place.

KEX2 = if non-zero, the total number of mesh points at which URSAVE is to be obtained.

KEX4 = Number of extra mesh points so that NXRA = KEX2+KEX4, and that NXRA*XMES2 gives the radius at which the external wave function begins to be calculated. Here XMES2 is the mesh size to be read in from CARD-2 below.

NODER = number of nodes which the bound state wave function should have. Note that the node at the origin and at infinity are not counted.

LBTR = l_i, where l_i was defined in eq. (5) and–or in eqs. (10) and (11).

ISTW = $2I_x$, where I_x was defined in eq. (4).

JBTRTW = $2j$ for systems 1, and = $2s$ for system 2.

ITBEMX = number of times iteration is allowed to be made (before the calculation is given up). If KOPOT $\neq$ 0, binding potential(s) is output with the interval of KOPOT.

CARD 2 EGES (> 0) = binding energy (to be equal to the experimental binding energy or to be guessed, depending on whether KTRL2 = 1 or = 0). TMAS = A_1 = mass of the core. PMAS = A_2 = mass of the outer particle. ZZT = charge number of the core. ZZP = charge number of the outer particle. XMES2 = mesh size with which URSAVE is to be calculated. ACURCY = accuracy. If the fractional change of the searched parameter, i.e., either EGES or VSX is less than ACURCY, at a stage of a current iteriaton, the calculation is completed.

CARD 3 These are all potential parameters: The first three are V, r_0 and a (i.e., depth, radius parameter and, diffuseness) for the central (Saxon) potential, while the next three are for spin-orbit interaction (of Saxon-derivative form). RZC is the radius parameter for the Coulomb potential.

Note that most of these data input from BSAXON, as well as the results of the calculation, are communicated to SATURN through the beginning part of the blank COMMON, and then eventually written in the beginning of I/O unit 12 so that they are further communicated to MARS. They are thus output by MARS together with its final result, i.e., the cross section. This precaution was taken because in practice the

form factor, once stored in I/O unit 12, is often used by MARS repeatedly, and thus re-confirmation of the input data in SATURN/BSAXON can become very significant.

4.1.3. Explanation of quantities in COMMON

In addition to the blank COMMON, SATURN has the following block COMMONs; CRAC, SATN, GAUS, RARB, ST12, GKAB. A brief explanation of the quantities that appear in these COMMON is as follows:

CRAC The quantities that appear here are used to calculate the geometrical factors. They are essentially the same as appeared in ref. [7].

SATN Most of the quantities that appear here have been explained already in the explanation of the input data. The rest has the following meaning.

KPCHAG = MOD ((KPCA+KPSA+KPCB+KPSB), 2) describes the possible change of parity between the incident and exit channels. It is equal to 0 or 1 depending on whether there is no change or there is change in parity.

L1TRTW, L2TRTW = $2l_1, 2l_2$ where l_1, l_2 are the LBTR input as the orbital angular momentum for the bound state calculation in BSAXON for the two systems 1 and 2, respectively.

KMAX = LDWMAX + $l_1 + l_2$ is the maximum k used in the calculation of G_k, the kernel function defined by eq. (5).

LLMAX = maximum of the transferred angular momentum l as defined in LLTWR(N) (= $2l$).

LBSTTL = total number of sets of (l, l_b) for a given l_a.

DCOE is the quantity d defined in eq. (4) for unit spectroscopic amplitude, i.e., $C^{(1)}C^{(2)} = 1.0$.

GAUS This block common is used differently at two different stages of the form factor calculation.

(i) At the time of loading of the program, it contains the preset values of *ABSCIS* and *WEIGHT* as defined in the block data GAUS. These are the roots and weight factors for a variety of different NGAUS for gaussian quadrature as explained in the input of MGAUS.

WWL1 and WWL2 are the bound-state wave functions $w_{l_1 n_1}(r_1)$ and $w_{l_2 n_2}(r_2)$ of eq. (5).

BSPAR and IBSPR are the bound state parameters used in BSAXON and to be communicated to program MARS via I/O unit 12.

(ii) After the kernel calculation is completed, the quantities defined in (i) above can be cleared. This block COMMON is thus used to store *CFAC*, i.e., the geometrical factors defined in eq. (2).

RARB Besides the various input quantities already explained, *RARB* contains various quantities pertinent to the radial range of the form factor to be constructed.

NBREAD = Number of r_b mesh points to form a block so that form factor quantities belonging to a given block are written or read, on or from I/O unit 12, as a group. Note that NBREAD must be at least 4.

NBTAIL is used when no recoil form factor is calculated. This defines how many of the NB points in the tail region are to be interpolated taking its logarithm. This gives better accuracy since the form factor is exponential at the tail region.

XMES(1) and N1MAX are the mesh size and the total number of mesh points at which the bound state wave function is calculated in BSAXON for system 1. They were input as XMES2 and KEX2 in BSAXON. XMES(2) and N2MAX play the same role for system 2.

ST12 As the name implies, this contains the coefficients in the coordinate transformation as defined by eq. (6).

GKAB This is used to communicate from subroutine KERNEL to GKRARB. It contains the current value of (r_a, r_b) whose kernel value is to be calculated, and the roots and weight factors for the appropriate NGAUS. XMAX(1) and (2) are defined as (N1MAX (or N2MAX) − 2) times XMES(1) or (2). These define the upper limits of the bound state wave functions that have been calculated and stored in WWL1 and WWL2. Note that this is necessary since the 4 point Lagrangian interpolation method is used to obtain the wave function at the needed radial point. Finally, COSAB2 is the lower limit in the evaluation of the kernel function.

4.2. DWBA program MARS

Since MARS is a DWBA program, and the methods of DWBA calculation are well known, our explanation of MARS will be very brief.

When MARS is used for an EFR calculation only one card is input from the main routine MARS. The rest of the information is supplied from I/O unit 12, prepared by SATURN. When a NR calculation is made, the same is true if KTAP12 = 1, where KTAP12 is one of the quantities read in from the one card mentioned above. If KTAP12 = 0, however, all the information for NR calculations has to be supplied in card form to be read in by MARS.

After making initial output of the basic data thus supplied, MARS calls JP1MAR twice; first for the incident channel with KINEX = 1 and then for the exit channel with KINEX = 2. When JP1MAR is called with KINEX = 1, it reads in one card for various controlling numbers and then one additional card to supply optical parameters. When JP1MAR is called with KINEX = 2, it reads in only one card for the optical parameters of the exit channel. In either case, JP1MAR calculates additional parameters like the c.m. energy, wave number, Sommerfeld parameter and so forth. JP1MAR next calls FLGLCH to calculate Coulomb wave functions at the matching radius and also POTEMS to produce the optical potential. Finally JP1MAR calls INEXCM by setting the parameter KWR = 1, and of course KINEX = 1 or 2, so that various parameters in the incident or exit channel are stores in separate locations. Note that in later use, INEXCM is called repeatedly with KWR = 2, and again with KINEX = 1 or 2, so that the parameters for the appropriate channel are retrieved for current use.

After these preparations are made, MARS calls CBCTRL and the DWBA calculation begins. As is seen, CBCTRL has a DO-loop for LAP1 = 1 ~ LAP1MX, where LAP1 = $l_a + 1$, l_a being defined, e.g., in eq. (13). For a given value of LAP1, CBCTRL first sets KINEX = 1 and calculates the distorted wave $\chi_{l_a}(k_a, r_a)$ in the incident channel. This is done by calling OVLAP, which then calls INTGMS, in which the stepwise solution of the differential equation is carried out. The obtained solution is then normalized to a unit flux incident wave and is stored in URISV(N), N = 1 ~ MSTOT, with the mesh size equal to XMES2. Note that N = 1 corresponds to first mesh point beyond the lower cutoff radius introduced in SATURN and MSTOT = (NBMAX-NBMIN-3)*MESFCB.

CBCTRL next sets KINEX = 2 and calls OVLAP which again calls INTGMS to calculate $\chi_{l_b}(k_b, r_b)$, for the set of l_b decided by subroutine LBSELK. This time, however, INTGMS calls INTPOL at appropriate values of r_b, and the form factor quantities are read in from I/O unit 12 for a block of NBREAD points of r_b. Since these form factor quantities have been calculated with mesh size equal to XMES2*MESFCA and XMES2*MESFCB, respectively, for r_a and r_b, an appropriate interpolation is made. Then by using URISV(N) as obtained previously, integration in (13) over r_a is carried out. The resultant quantities which are functions of r_b, as well as quantum numbers l and l_b are stored in URISV(N), N > MSTOT. The control is then returned to INTGMS, and the solution of the differential equation proceeds, while at the same time the integration in (13) over r_b is carried out. The obtained results is finally normalized in OVLAP, and the resultant $I^{jls}_{l_a l_b}$ of (13) are stored in I/O unit 11.

In the process of integration, INTGMS calls at each mesh point another subroutine URENOM, which in its present form guarantees that at any stage of integration the solution of the differential equation stays within the bound of $10^{-20} \sim 10^{20}$. This precaution is needed when handling higher partial waves using a computer whose range of OVER/UNDER FLOW is narrower than $10^{\pm 70}$.

After all the needed $I^{jls}_{l_a l_b}$ are calculated, CBCTRL returns the control to MARS, which calls ELCROS to calculate the elastic scattering cross sections in both channels, provided no lower cut-off or interpolation is being made in the angular momentum space. MARS then calls DWAMP to calculate the DWBA amplitude and finally calls DWCROS to calculate the DWBA cross section and the calculation is completed.

The above explanation of the general structure of MARS was given having in mind its use for EFR–DWBA calculation. Its use for NR–DWBA calculation is very much the same as above. The only difference is that there is no need to call INTPOL. The form factor is given, either from I/O unit 12 or from card, having from the beginning the mesh size equal to XMES2. The overlap integral is performed straight off at INTGMS as each exit channel wave function is being obtained.

4.2.1. Input to MARS

The read statement for one card to be input to MARS is found in the very beginning of the main program MARS, and it is

READ(5, 701), KTRLCB(5), KTRLCB(7), KTRLCB(8), KOUTCB(3), KTAP12, KRALSJ, NTHETA, NONE, TETADG(1), DTETAD, ELABI

with the FORMAT 701 being (8I5, 3F10.5). The meaning of the quantities in this card and what values are to be assigned to them is explained as follows.

If KTRLCB(5) = 0; MARS is used as a DWBA program. If KTRLCB(5) = 1 it is used as a simple optical model program.

If NR–DWBA is to be made, set KTRLCB(7) = 1 and KTRLCB(8) = 0.

If EFR–DWBA is to be made, set KTRLCB(7) = 0 and KTRLCB(8) = 1.

If KOUTCB(3) ≠ 0, some intermediate output in calculating the DWBA amplitude and cross sections can be taken. See DWAMP and DWCROS for detail.

KTAP12 has been explained above.

If KRALSJ < 5, ALSJF(N), N = 1, KRALSJ can be input with FORMAT(7F10.5). In many cases $C^{(2)}$ is known, and in such cases it is recommendable to

If KRALSJ < 5, ALSJF(N), N = 1, KRALSJ can be input with FORMAT (7F10.5). In many cases $C^{(2)}$ is known, and in such cases it is recommendable to set ALSFJ(N) = $C^{(2)}$. Then, by normalizing the theoretical cross section to data, one can extract the value of $C^{(1)}$. This input card must follow the one MARS card we are discussing.

NTHETA is the number of angles at which the cross sections are to be calculated. This number should not exceed 100. NONE is just for separating the input quantities in the card. TETADG(1) is the smallest angle in degree, while DTETAD is the increment of the angle also in degree.

ELABI is the incident energy in MeV of the projectile in the incident channel.

4.2.2. Input to JP1MAR

The read statements for two cards to be input to JP1MAR are found in the very beginning of this subroutine and read as follows:

READ(5, 701), KTRL(4), KTRL(9), KEXCOM(1), (KEXCOM(N), N = 12, 14), KTLOUT(3), KTLOUT(7), KTLOUT(10)

READ(5, 702), VSX, WSX, WSF, DFN, DFNW, DFNS, RZERO, RZEROW, RZEROS, RZEROC

with FORMAT 701 and 702, respectively, being (14I5) and (10F7.2).

CARD-1 is read in only when KINEX = 1, and supplies the following controlling quantities:

If KTRL(4) = 0, the radius R of the Saxon potential(s) is related to the corresponding radius parameter r_0 as $R = r_0 A^{1/3}$. If KTRL(4) = 1 the relation is replaced by $R = r_0(A^{1/3} + a^{1/3})$. Here A and a are masses of the target and the projectile.

If KTRL(9) = 0, a normal production run takes place. If KTRL(9) = 1, calculation of the overlap integral is executed only for l_a = KEXCOM(13) + 1. This option is useful to understand the details of the dynamics, because by setting KTLOUT(10) ≠ 0 (see below) a large amount of intermediate output can be obtained.

If KEXCOM(1) = 0, the differential equation for the distorted wave is solved from the origin, i.e., from $r = 0$. If KEXCOM(1) = N ≠ 0, the equation is solved beginning at r = N*XMES2. This option, if used with care, helps to speed up the calculation. Note, however, that r = KEXTCB(1)*XMES2 is the radius beyond which the form factor was calculated. (KEXTCB(1) is known from I/O unit 12.) Therefore, KEXCOM(1) should be smaller than KEXTCB(1) by at least 20, say.

If KEXCOM(12) = 0, the overlap integral $I^{jls}_{l_a l_b}$ are calculated in the standard way for all the values of l_a. If KEXCOM(12) = 2, calculation is made only for l_a values whose parity equals $\pi_A \pi_B \pi_a \pi_b$, and those with l_a of opposite parity are obtained by interpolation. With this option the calculation needs only half the machine time than otherwise, and in many cases accuracy is not lost significantly.

If KEXCOM(13) = L ≠ 0, the first L values of l_a are completely ignored in the calculation (lower l_a cut-off).

If KEXCOM(14) = 0, $(l_a)_{max}$, i.e., the maximum value of l_a to be considered equals LDWMAX-1, where LDWMAX is supplied from I/O unit 12. If KEXCOM(14) = L ≠ 0, $(l_a)_{max}$ = KEXCOM(14) – 1.

This option is very convenient, because the form factor prepared for l_a up to LDWMAX can be used also for the case with lower ELABI, in which it is sufficient to consider up to l_a whose value is much smaller than LDWMAX.

If KTLOUT(3) $\neq 0$, the values of l_a and l_b are output each time new values are assigned to these quantities.

If KTLOUT(7) = 0, no output from FLGLCH and POTEMS. If $\neq 0$, output from FLGLCH, and further if = 1, output of optical potentials at every 10 mesh points, while if > 1, output of optical potentials at every mesh point.

In the production running, set KTLOUT(10) = 0. As is seen in subroutines OVLAP, INTGMS, INTPOL and URENOM, an increased amount of output is obtained by increasing KTLOUT(10) from 1 to 5. This option can very conveniently be used in combination with KTRL(9) = 1.

CARD-2 is read with both KINEX = 1 and 2, and gives optical parameters for the incident and exit channels, respectively. The first three quantities VSX, WSX and WSF are depths of real, volume-imaginary and surface-(derivative)-imaginary parts of the Saxon potential, respectively. Note that the derivative parts is defined so that its peak value equals –i.WSF rather than –i.WSF/4. The next three quantities in CARD-2 are corresponding diffuseness and the other three quantities that follow are radius parameters. Finally RZEROC is the radius parameter for the Coulomb potential. See POTEMS if a more detailed understanding is needed.

4.2.3. *Quantities in COMMON*

The meaning of the quantities that appear in COMMON is often identified easily from their name alone. Also a large fraction of the quantities in COMMON are transferred from SATURN through I/O unit 12, and no further explanation is needed. Some others were already explained in the discussion of the input data in MARS and JP1MAR. We shall thus explain in the following a few relevant quantities.

In the block COMMON/DWBA/, TETARD(N) is the angle in radians that corresponds to TETADG(N) given in degrees, AOVERB = A/B(= B/A) for stripping (pick up); cf. eq. (14) for NR calculation. AOVERB is set to unity for EFR calculations. WNI = k_a and WNE = k_b. QVALGR is the Q-value in the transfer reaction concerned. LMXPAR = l_{max}, the maximum value of the transferred orbital angular momentum l, or LMXPAR = $l_{max} - 1$, depending on whether $(-)^{l_a+l_b+l_{max}} \doteq 1$ or -1. NBREDT, NBLMCM, NBSMCM and NTCOM are used only between INTGMS and INTPOL, and serve to relate the current mesh point, appearing in solving the differential equation, with that appearing to define the form factor being read in from I/O unit 12. Finally XMESA and XMESB are given from I/O unit 12 and are used to assign values to XMES1 and XMES2, where XMES2 was defined above, while XMES1 will be explained soon below.

Contrary to COMMON/DWBA/, which contains quantities needed mostly to bridge the incident and exit channels, the block COMMON/JP1/ contains quantities that are used in carrying out optical model calculations in each channel separately. As is seen in the beginning of JP1MAR, the quantities KTRL(N), KEXCOM(N) and KTLOUT(N), N = 1 ~ 15, are first set to zero, and possible non-zero values are supplied from CARD-1 in JP1MAR. In addition to those, explained referring to CARD-1, KEXCOM(11) and KEXCOM(15) are used in MARS in order to carry the values of l_a and LMXPAR, respectively.

LLROW(N), N = 1 ~ MXROW carry the value of $2l_b$, where l_b satisfies the triangular condition with the current value of l_a and LMXPAR. (This table is created by calling subroutine LBSELC). PMAS = a (= b), TMAS = A (= B) and RMAS = μ are, resspectively, projectile, target and reduced masses. CHARGE = ZZ′ is the product of charges of the projectile and target, CE = η is the Sommerfeld parameter and SGMAZZ = σ_0 is the Coulomb phase-shift of the s-wave. ECM and ELAB are c.m. and lab. energies, WN = k is the wave number of the relative motion, WNINI is the wave number at the origin, i.e., that which corresponds to the energy ECM+VSX and DR = XMES2*WN. Further NXMAX = KEXTCB(1) + MSTOT, where NXMAX*XMES2 is the radius at which the internal solution is matched to the asymptotic solution, i.e., the Coulomb function. NXPOT = NXMAX if NXMAX $\leqslant$ 480 and NXPOT = 480 otherwise. Here NXPOT*XMES2 is the radius up to which the optical potentials are to be prepared, beyond which only the Coulomb potential being considered.

The quantity NDFMES, standing for number of

different meshes, is set to 4 in the present form of MARS. When KEXCOM(1) = 0, and thus the differentail equation is to be solved beginning at $r = 0$, a small mesh XMES1 = 0.125*XMES2 is used. After going eight steps this way, the mesh size is doubled. Going four more steps the mesh size is again doubled. Repeating this last procedure once again the mesh size becomes XMES2, and this constant mesh is used up to the matching radius. This precaution is taken to avoid a loss of accuracy which can take place more easily for $r \doteqdot 0$ than for larger r. In heavy-ion reaction calculations, such precautions are less significant, and use of KEXCOM(1) $\neq 0$ may be made. In that case the constant mesh XMES2 is used throughout.

The block COMMON/COU/ carries Coulomb wave functions for the incident channel. Those for the exit channel appear at the beginning of the blank COMMON.

The contents of the blank COMMON vary from a block of related subroutines to another. In other words, they are used rather locally. Because of this reason, and because the way they are used can be understood rather easily from a large number of COMMENT cards inserted in various subroutines, it will not be necessary to explain in the text the contents of the blank COMMON as well as of the block COMMONS other than those explained above.

5. Test data and test run

Included in the program decks of SATURN and MARS are test decks for a EFR and NR–DWBA calculations. Both the EFR and NR test runs calculate the reaction ^{64}Ni(^{16}O, ^{15}N)^{65}Cu at $E_{lab}(^{16}O) =$ 56 MeV leading to the $2p_{1/2}$ state in ^{65}Cu. Note that in the NR calculation, only one set of l ($l = 0$) transfer is allowed whereas there are two sets ($l = 0$ and 1) for the EFR calculation.

References

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[5] K.S. Low and T. Tamura, Argonne National Lab., Physics Inf. Rep. No. PHY-1973B (1973) 655;
T. Tamura and K.S. Low, Phys. Rev. Lett. 31 (1973) 1356;
K.S. Low and T. Tamura, Phys. Lett. 48B (1974) 285.
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K.S. Low and T. Tamura, Phys. Rev., to be published.
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Appendix

General remarks concerning the use of the program

The following remarks are directed mainly for performing EFR calculation, because then the optimized use of the program becomes more significant than when NR calculation is to be made.

1. The user should accumulate experience by first calculating the kernel function by varying MGAUS from 3 onwards until a consistent form factor is obtained to 3 or 4 significant figures. Use that value of MGAUS for production running.

2. There are two criteria for the choice of the mesh size (equal to XMES2 in MARS and equal to XMESA/MESFCA and XMESB/MESFCB in SATURN) to be used in the calculation. The first is to make certain that the distorted waves are generated with sufficient accuracy. This is energy dependent and a simple estimate is to set XMES2 $< 0.2\,\pi/k$ where k is the asymptotic wave number in the incident or exit channels. Another equally important criterion, especially for heavy ion reactions is related to the argument advanced in section 3 regarding the sharp r_a dependence of the form factor for a given r_b. In order to perform the integration over r_a accurately, there must be a sufficient number of radial points within the narrow band of the form factor defined around the diagonal $r_b \doteqdot (A/B)r_a$. The width of this band may be given by

$$\text{XMESA*NATOTL} \doteqdot 5*(r_0*\text{MASSA}**0.33)/\alpha,$$

where α was defined in eq. (7). Since NATOTL should be at least 15 or so, the above equation imposes another condition on the choice of XMESA.

Anyway, the user had better confirm that NATOTL *XMESA is sufficiently large so that the form factor at both ends of the r_a range for a given r_b is at least 3 or 4 orders of magnitude smaller than its value at the peak.

3. The choice of NBMIN and NBMAX can be determined after various trial runs. This is best done using the no-recoil option of the program.

4. The proper use of lower cut-off, upper limit and the interpolation in the angular momentum space can best be tested

by running with KOUTCB(3) = 1 in MARS which will allow the output of the square of the DWBA transition amplitude. This will indicate fairly accurately the proper range of important partial waves as well as the accuracy of the interpolated transition amplitudes.

5. At an installation where a large core size can be used, NGTPMX and NBUFER should be increased for a most efficient performance of the kernel calculation, since then the use of the scratch tape 13 and the repeated calculations of Legendre function can be avoided.

6. At an installation with a smaller computer, the user may overlay the programs, which will save 4 or 5K words both in SATURN and in MARS. If the core space available is found still to small the user may reduce the dimensions of a few quantities which are taking large core spaces. How to make such a reduction, without causing error in the running, may be figured out by looking at the various procedures in subroutine MONITR, which checks that the core sizes assigned to various quantities are sufficiently large.

OUTPUT FROM SATURN TEST RUN

```
**********

FORM FACTOR CALCULATION ON PROGRAM SATURN (EFR)

FOR NI( 64)( 0( 16), N( 15))CU( 65)
     0  (+)( 0  (+), 1/2(-)) 1/2(-)

**********

KTRLST= -0 -0 -0 -0 -0 -0 -0 -0 -0 -0
KEXCST= -0 -0 -0 -0 -0 -0
KOUTST= -0 -0 -0 -0 -0 -0 -0 -0

(2J,2L,2S)= ( 1, 2, 1) , ( 1, 0, 1) , (

NATOTL,NBMIN,NBMAX          =   15    4   21
MESFCA,MESFCB,MGAUS,LDWMAX  =    1   10    3   61
XMESA=     .100      XMESB=   1.000

                    BSAXON IS CALLED

KTRL2,3,4,8=  1  0  2  0   KEX2,4=  300  100

MOTION BETWEEN NUCLEI WITH (A,Z)=(   1.00   1.00) AND (  64.00  28.00)
WITH EGES=  6.679  LTR=1  2S=1  2J= 1  NODER=1

       VSX       VSOR       DFNR       DFNSO        RZR       RZSO        RZC
    60.000      7.000       .650        .650      1.200      1.200      1.200

AMUPMU=-0.0  ACCURACY= .000100
NXRAWF=300   NXRA=400   NXRM= 47  XMES2=  .1000  XBAR= 4.8000

FINAL VALUE OF THE PARAMETERS, B.E.=   6.67900 VSX=  64.28856
VSX  HAS VARIED AS  60.0000 62.5266 63.4869 63.9074 64.1048 64.2005 64.2477 64.2711 64.2827

WAVE FUNCTIONS AT   1.00 FERMI INTERVAL
 -3.5160E-02  -2.9713E-01  -2.4437E-01  -5.6286E-04   1.2889E-01   9.7988E-02   4.5740E-02   1.8995E-02   7.9018E-03   3.3745E-03
  1.4804E-03   6.6479E-04   3.0449E-04   1.4183E-04   6.7017E-05   3.2065E-05   1.5510E-05   7.5754E-06   3.7317E-06   1.8525E-06
  9.2595E-07   4.6574E-07   2.3560E-07   1.1980E-07   6.1207E-08   3.1410E-08   1.6184E-08   8.3700E-09   4.3440E-09   2.2618E-09

WAVE FUNCTION TIMES R**(-LBTR)
 -3.5160E-01  -2.7012E-01  -1.1637E-01  -1.8157E-04   3.1437E-02   1.9213E-02   7.4984E-03   2.6753E-03   9.7554E-04   3.7082E-04
  1.4657E-04   5.9891E-05   2.5165E-05   1.0826E-05   4.7530E-06   2.1235E-06   9.6338E-07   4.4301E-07   2.0617E-07   9.6987E-08
  4.6067E-08   2.2073E-08   1.0660E-08   5.1861E-09   2.5397E-09   1.2514E-09   6.2007E-10   3.0886E-10   1.5459E-10   7.7726E-11

                    BSAXON IS CALLED
```

```
                                        BSAXON IS CALLED

      KTRL2,3,4,8=  1  0  3  0   KEX2,4=  300  100

      MOTION BETWEEN NUCLEI WITH (A,Z)=(   1.00   1.00) AND (  15.00   7.00)
      WITH EGES= 12.126  LTR=1  2S=1  2J= 1  NODER=0

            VSX       VSOR       DFNR       DFNSO        RZR       RZSO        RZC
         60.000      7.000       .650        .650      1.200      1.200      1.200

      AMUPMU=-0.0  ACCURACY= .000100
      NXRAWF=300   NXRA=400   NXRM= 29  XMES2=  .1000  XBAR= 2.9595

      FINAL VALUE OF THE PARAMETERS. B.E.=  12.12600 VSX=  61.23842
      VSX  HAS VARIED AS  60.0000 60.6604 60.9602 61.1035 61.1736 61.2083 61.2255 61.2341


WAVE FUNCTIONS AT   1.00 FERMI INTERVAL
  3.6129E-02   3.1752E-01   3.2577E-01   1.7786E-01   7.0869E-02   2.6112E-02   9.7640E-03   3.7671E-03   1.4950E-03   6.0685E-04
  2.5081E-04   1.0518E-04   4.4635E-05   1.9132E-05   8.2702E-06   3.6009E-06   1.5777E-06   6.9509E-07   3.0772E-07   1.3682E-07
  6.1069E-08   2.7352E-08   1.2290E-08   5.5376E-09   2.5017E-09   1.1329E-09   5.1416E-10   2.3383E-10   1.0654E-10   4.8626E-11


WAVE FUNCTION TIMES VCENTR+VSPIN
 -1.9990E+00  -1.7755E+01  -1.4641E+01  -4.2486E+00  -5.4634E-01  -4.9642E-02  -4.1889E-03  -3.5528E-04  -3.0720E-05  -2.7062E-06
 -2.4215E-07  -2.1950E-08  -2.0113E-09  -1.8598E-10  -1.7330E-11  -1.6258E-12  -1.5341E-13  -1.4551E-14  -1.3864E-15  -1.3263E-16
 -1.2735E-17  -1.2268E-18  -1.1854E-19  -1.1484E-20  -1.1154E-21  -1.0858E-22  -1.0593E-23  -1.0354E-24  -1.0139E-25  -9.9444E-27


WAVE FUNCTION TIMES VCENTR+VSPIN TIMES R**(-LBTR)
 -1.9990E+01  -1.6141E+01  -6.9718E+00  -1.3705E+00  -1.3325E-01  -9.7338E-03  -6.8670E-04  -5.0040E-05  -3.7926E-06  -2.9738E-07
 -2.3975E-08  -1.9775E-09  -1.6622E-10  -1.4197E-11  -1.2291E-12  -1.0767E-13  -9.5287E-15  -8.5092E-16  -7.6596E-17  -6.9441E-18
 -6.3358E-19  -5.8142E-20  -5.3636E-21  -4.9715E-22  -4.6283E-23  -4.3261E-24  -4.0586E-25  -3.8206E-26  -3.6080E-27  -3.4173E-28

                    THE FOLLOWING STATEMENTS ARE ISSUED FROM MONITR


********** THE FOLLOWING QUANTITIES ARE WRITTEN ON TAPE-12


MASCA,NZCA,KPCA,ICATW,MASCB,NZCB,KPCB,ICBTW=   64   28    0    0   65   29    1    1
MASCA,NZSA,KPSA,ISATW,MASSB,NZSB,KPSB,ISBTW=   16    8    0    0   15    7    1    1
JLSMAX,LDWMAX,NBREAD,LBSTTL,KEX1,KEX2      =    2   61   18    2   -0   -0


JJTWR(N),LLTWR(N),ISTWR(N)=    1    2    1    1    0    1

NATOTL,NBMIN,NBMAX,MESFCA,MESFCB,XMESA,XMESB,QVALGR=   15    4   21    1   10       .10000      1.00000     -5.44700
ENEPT,VNEPT,VSOR,DFNR,DFNSO,RZR,RZSO,RZC,XMES=   6.68  64.29   7.00   .65   .65  1.20  1.20  1.20   .10
ENEPT,VNEPT,VSOR,DFNR,DFNSO,RZR,RZSO,RZC,XMES=  12.13  61.24   7.00   .65   .65  1.20  1.20  1.20   .10
                              **********      FINITE RANGE KERNEL IS CALLED      **********


                    NBMIN,NBMAX,NBREAD,NBTOTL,NATOTL,MESFCA,MESFCB=    4   21   18   18   15    1   10
                    KMAX,N1MAX,N2MAX,NGAUS,MGAUS,NGTPMX,NBUFER    =   63  300  300   12    3 8192  4096
                    XMESA,XMESB,XMES(1),XMES(2)=     .1000   1.0000    .1000    .1000
                    S1,S2,T1,T2,XBARSB         =  13.0000  12.8000 -12.1875 -13.0000   2.9595

                    ITAP13,     NBREPT,NBSTEP,NBSTPF=    0    1   18   18
                    KREPT,KSTEP,KSTPF=    5   15    3
K=  0  RB= 10.0  6.0407E-14  1.9730E-11 -4.9599E-09 -2.5744E-07 -4.9183E-07 -2.1757E-08 -1.8066E-11 -4.1021E-15
K= 10  RB= 10.0  6.0351E-14  1.9734E-11 -4.9287E-09 -2.5631E-07 -4.8946E-07 -2.1656E-08 -1.7966E-11 -4.0687E-15
K=  0  RB= 20.0 -2.5292E-18 -1.1020E-16 -6.0722E-15 -4.1772E-13 -1.9111E-11 -2.4770E-11 -6.8631E-13 -4.0925E-16
K= 10  RB= 20.0 -2.5220E-18 -1.0990E-16 -6.0582E-15 -4.1701E-13 -1.9088E-11 -2.4737E-11 -6.8548E-13 -4.0858E-16
```

```
                    ********************  FRFMFC IS CALLED  ********************

 NBMIN,NBMAX,NBREAD,NBTOTL,NATOTL,MESFCA,MESFCB=    4   21   18   18   15    1   10
 NFBLOK,NGBLOK,KRANGE,NGAB,NGUSE,KMAX          =  540   63    5  270 1350   63
 LBSTTL                XMESA,XMESB             =    2  .100  1.000


DCOE= -3.333333E-01  4.082483E-01
                OUTPUT OF FORM FACTOR FOR A GIVEN RB AND (LB,L) FROM NA=1 TO NATOTL AT STEPS OF  2

        LA=    0        (LB,L)= (  0, 0)(
 RB=  4.0   0.          0.          0.          0.         -5.457E-04 -1.505E-05 -6.722E-09 -2.180E-12
 RB=  4.0   0.          0.          0.          0.          0.          0.          0.
 RB=  9.0   2.889E-13 -1.536E-10  5.535E-08  6.153E-06  7.886E-07 -3.841E-07 -6.189E-10 -2.482E-13
 RB=  9.0   0.          0.          0.          0.          0.          0.          0.
 RB= 14.0  -4.822E-15  5.663E-12  3.058E-10  1.300E-08  5.234E-08 -1.027E-08 -1.182E-10 -6.332E-14
 RB= 14.0   0.          0.          0.          0.          0.          0.          0.
 RB= 19.0   2.630E-16  1.240E-14  6.388E-13  3.501E-11  7.687E-10 -6.996E-11 -2.097E-11 -2.257E-14
 RB= 19.0   0.          0.          0.          0.          0.          0.          0.

        LA=   15        (LB,L)= ( 15, 1)( 15, 0)(
 RB=  4.0   0.          0.          0.          0.         -5.685E-04 -2.987E-06 -8.123E-10 -2.029E-13
 RB=  4.0   0.          0.          0.          0.         -2.771E-03 -7.880E-05 -3.439E-08 -1.082E-11
 RB=  9.0  -8.644E-14  7.301E-11 -3.278E-08 -1.877E-06 -2.740E-06 -8.151E-08 -6.367E-11 -2.069E-14
 RB=  9.0   1.669E-12 -8.425E-10  3.033E-07  3.393E-05  4.425E-06 -2.110E-06 -3.391E-09 -1.350E-12
 RB= 14.0  -6.275E-15 -1.308E-12 -5.433E-11 -2.085E-09 -1.824E-08 -3.247E-09 -1.079E-11 -4.521E-15
 RB= 14.0  -2.716E-14  3.128E-11  1.691E-09  7.198E-08  2.901E-07 -5.684E-08 -6.542E-10 -3.495E-13
 RB= 19.0  -4.241E-17 -1.739E-15 -7.914E-14 -3.900E-12 -1.111E-10 -1.116E-10 -2.028E-12 -1.407E-15
 RB= 19.0   1.456E-15  6.863E-14  3.539E-12  1.942E-10  4.269E-09 -3.874E-10 -1.164E-10 -1.251E-13

        LA=   30        (LB,L)= ( 30, 1)( 30, 0)(
 RB=  4.0   0.          0.          0.          0.         -1.329E-03 -6.999E-06 -1.769E-09 -4.060E-13
 RB=  4.0   0.          0.          0.          0.         -2.958E-03 -9.276E-05 -3.791E-08 -1.093E-11
 RB=  9.0  -1.897E-13  2.095E-10 -8.571E-08 -5.009E-06 -7.301E-06 -2.173E-07 -1.678E-10 -5.345E-14
 RB=  9.0   2.557E-12 -1.134E-09  4.061E-07  4.631E-05  6.335E-06 -2.847E-06 -4.542E-09 -1.771E-12
 RB= 14.0  -1.646E-14 -3.516E-12 -1.465E-10 -5.656E-09 -4.961E-08 -8.828E-09 -2.927E-11 -1.217E-14
 RB= 14.0  -3.932E-14  4.287E-11  2.324E-09  9.944E-08  4.018E-07 -7.829E-08 -9.021E-10 -4.779E-13
 RB= 19.0  -1.149E-16 -4.713E-15 -2.150E-13 -1.064E-11 -3.041E-10 -3.055E-10 -5.552E-12 -3.836E-15
 RB= 19.0   2.008E-15  9.467E-14  4.893E-12  2.695E-10  5.942E-09 -5.348E-10 -1.619E-10 -1.734E-13

        LA=   45        (LB,L)= ( 45, 1)( 45, 0)(
 RB=  4.0   0.          0.          0.          0.         -1.834E-03 -9.756E-06 -2.193E-09 -4.372E-13
 RB=  4.0   0.          0.          0.          0.         -2.220E-03 -8.504E-05 -3.130E-08 -7.822E-12
 RB=  9.0  -2.173E-13  4.004E-10 -1.427E-07 -8.627E-06 -1.254E-05 -3.736E-07 -2.833E-10 -8.727E-14
 RB=  9.0   3.460E-12 -1.300E-09  4.589E-07  5.406E-05  7.950E-06 -3.260E-06 -5.141E-09 -1.939E-12
 RB= 14.0  -2.750E-14 -6.127E-12 -2.567E-10 -1.001E-08 -8.816E-08 -1.568E-08 -5.183E-11 -2.125E-14
 RB= 14.0  -5.038E-14  5.040E-11  2.744E-09  1.184E-07  4.804E-07 -9.275E-08 -1.071E-09 -5.598E-13
 RB= 19.0  -2.027E-16 -8.326E-15 -3.815E-13 -1.901E-11 -5.463E-10 -5.487E-10 -9.972E-12 -6.848E-15
 RB= 19.0   2.385E-15  1.125E-13  5.835E-12  3.235E-10  7.165E-09 -6.363E-10 -1.948E-10 -2.076E-13

        LA=   60        (LB,L)= ( 60, 1)( 60, 0)(
 RB=  4.0   0.          0.          0.          0.         -1.901E-03 -1.035E-05 -1.989E-09 -3.271E-13
 RB=  4.0   0.          0.          0.          0.         -1.146E-03 -6.603E-05 -2.121E-08 -4.365E-12
 RB=  9.0  -1.110E-13  6.400E-10 -1.928E-07 -1.223E-05 -1.772E-05 -5.282E-07 -3.906E-10 -1.149E-13
 RB=  9.0   4.334E-12 -1.385E-09  4.739E-07  5.851E-05  9.412E-06 -3.437E-06 -5.336E-09 -1.920E-12
 RB= 14.0  -3.723E-14 -8.829E-12 -3.727E-10 -1.473E-08 -1.305E-07 -2.320E-08 -7.638E-11 -3.071E-14
 RB= 14.0  -6.156E-14  5.509E-11  3.019E-09  1.319E-07  5.377E-07 -1.025E-07 -1.188E-09 -6.091E-13
 RB= 19.0  -2.973E-16 -1.222E-14 -5.637E-13 -2.835E-11 -8.211E-10 -8.245E-10 -1.499E-11 -1.020E-14
 RB= 19.0   2.645E-15  1.247E-13  6.509E-12  3.641E-10  8.117E-09 -7.071E-10 -2.202E-10 -2.329E-13
```

```
**********

FORM FACTOR CALCULATION ON PROGRAM SATURN (NR)

FOR NI( 64)( 0( 16), N( 15))CU( 65)
     0  (+)( 0  (+), 1/2(-)) 1/2(-)

**********

KTRLST=  1 -0 -0 -0 -0 -0 -0 -0 -0
KEXCST= -0 -0 -0 -0 -0 -0
KOUTST= -0 -0 -0 -0 -0 -0 -0 -0

(2J,2L,2S)= ( 1, 0, 1) , (

NBMIN,NBMAX,MESFCB,MGAUS,KPUNCH =    1   40    5    3    2
XMESB                           =    .500

                                              BSAXON IS CALLED

      KTRL2,3,4,8=  1  0  2  0   KEX2,4=  300  100

      MOTION BETWEEN NUCLEI WITH (A,Z)=(   1.00   1.00) AND (  64.00  28.00)
      WITH EGES=  6.679  LTR=1  2S=1  2J= 1  NODER=1

            VSX       VSOR       DFNR      DFNSO        RZR       RZSO        RZC
         60.000      7.000       .650       .650      1.200      1.200      1.200

      AMUPMU=-0.0  ACCURACY= .000100
      NXRAWF=300   NXRA=400   NXRM= 47  XMES2=  .1000  XBAR= 4.8000

      FINAL VALUE OF THE PARAMETERS. B.E.=   6.67900 VSX=  64.28856
      VSX  HAS VARIED AS  60.0000 62.5266 63.4869 63.9074 64.1048 64.2005 64.2477 64.2711 64.2827

WAVE FUNCTIONS AT   1.00 FERMI INTERVAL
 -3.5160E-02  -2.9713E-01  -2.4437E-01  -5.6286E-04   1.2889E-01   9.7988E-02   4.5740E-02   1.8995E-02   7.9018E-03   3.3745E-03
  1.4804E-03   6.6479E-04   3.0449E-04   1.4183E-04   6.7017E-05   3.2065E-05   1.5510E-05   7.5754E-06   3.7317E-06   1.8525E-06
  9.2595E-07   4.6574E-07   2.3560E-07   1.1980E-07   6.1207E-08   3.1410E-08   1.6184E-08   8.3700E-09   4.3440E-09   2.2618E-09

WAVE FUNCTION TIMES R**(-LBTR)
 -3.5160E-01  -2.7012E-01  -1.1637E-01  -1.8157E-04   3.1437E-02   1.9213E-02   7.4984E-03   2.6753E-03   9.7554E-04   3.7082E-04
  1.4657E-04   5.9891E-05   2.5165E-05   1.0826E-05   4.7530E-06   2.1235E-06   9.6338E-07   4.4301E-07   2.0617E-07   9.6987E-08
  4.6067E-08   2.2073E-08   1.0660E-08   5.1861E-09   2.5397E-09   1.2514E-09   6.2007E-10   3.0886E-10   1.5459E-10   7.7726E-11
```

```
                                        BSAXON IS CALLED

        KTRL2,3,4,8=  1  0  1  0   KEX2,4=  300  100

        MOTION BETWEEN NUCLEI WITH (A,Z)=(   1.00   1.00) AND (  15.00   7.00)
        WITH EGES= 12.126  LTR=1  2S=1  2J= 1  NODER=0

              VSX       VSOR       DFNR      DFNSO       RZR       RZSO       RZC
           60.000      7.000       .650       .650     1.200     1.200     1.200

        AMUPMU=-0.0  ACCURACY= .000100
        NXRAWF=300   NXRA=400   NXRM= 29  XMES2=  .1000  XBAR= 2.9595

        FINAL VALUE OF THE PARAMETERS. B.E.=  12.12600 VSX=  61.23842
        VSX  HAS VARIED AS  60.0000 60.6604 60.9602 61.1035 61.1736 61.2083 61.2255 61.2341


WAVE FUNCTIONS AT   1.00 FERMI INTERVAL
  3.6129E-02   3.1752E-01   3.2577E-01   1.7786E-01   7.0869E-02   2.6112E-02   9.7640E-03   3.7671E-03   1.4950E-03   6.0685E-04
  2.5081E-04   1.0518E-04   4.4635E-05   1.9132E-05   8.2702E-06   3.6009E-06   1.5777E-06   6.9509E-07   3.0772E-07   1.3682E-07
  6.1069E-08   2.7352E-08   1.2290E-08   5.5376E-09   2.5017E-09   1.1329E-09   5.1416E-10   2.3383E-10   1.0654E-10   4.8626E-11


WAVE FUNCTION TIMES VCENTR+VSPIN
 -1.9990E+00  -1.7755E+01  -1.4641E+01  -4.2486E+00  -5.4634E-01  -4.9642E-02  -4.1889E-03  -3.5528E-04  -3.0720E-05  -2.7062E-06
 -2.4215E-07  -2.1950E-08  -2.0113E-09  -1.8598E-10  -1.7330E-11  -1.6258E-12  -1.5341E-13  -1.4551E-14  -1.3864E-15  -1.3263E-16
 -1.2735E-17  -1.2268E-18  -1.1854E-19  -1.1484E-20  -1.1154E-21  -1.0858E-22  -1.0593E-23  -1.0354E-24  -1.0139E-25  -9.9444E-27


                    THE FOLLOWING STATEMENTS ARE ISSUED FROM MONITR


********************* THE FOLLOWING QUANTITIES ARE WRITTEN ON TAPE-12


MASCA,NZCA,KPCA,ICATW,MASCB,NZCB,KPCB,ICBTW=   64   28    0    0   65   29    1    1
MASCA,NZSA,KPSA,ISATW,MASSB,NZSB,KPSB,ISBTW=   16    8    0    0   15    7    1    1
JLSMAX,LDWMAX,NBREAD,LBSTTL,KEX1,KEX2      =    1    0    0    0   -0   -0


JJTWR(N),LLTWR(N),ISTWR(N)=    1    0    1

NATOTL,NBMIN,NBMAX,MESFCA,MESFCB,XMESA,XMESB,QVALGR=    0    1   40    0    5     -0.00000        .50000     -5.44700
ENEPT,VNEPT,VSOR,DFNR,DFNSO,RZR,RZSO,RZC,XMES=   6.68  64.29   7.00   .65   .65   1.20   1.20   1.20    .10
ENEPT,VNEPT,VSOR,DFNR,DFNSO,RZR,RZSO,RZC,XMES=  12.13  61.24   7.00   .65   .65   1.20   1.20   1.20    .10
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                                NRFMFC IS CALLED.

                    N8MIN.N8MAX.N8INT.N8TAIL          =    1   40   23   20
                    MGAUS.NGAUS.KPUNCH.N1MAX.N2MAX    =    3   12    2  300  300
                    XMES8.XMES1.XMES2                 =    .5000    .1000    .1000

                    JLS=    1      N8MIN.N8MAX=    1   40      XMESB=    .5000

 1.34372E+01  9.94660E+00  5.23281E+00  5.28537E-01 -3.09950E+00 -5.06418E+00 -5.35522E+00 -4.41554E+00 -2.88693E+00 -1.35298E+00
-1.79279E-01  5.16706E-01  7.98105E-01  8.07084E-01  6.82824E-01  5.21971E-01  3.74916E-01  2.59289E-01  1.75507E-01  1.17537E-01
 7.84198E-02  5.23398E-02  3.50251E-02  2.35261E-02  1.58684E-02  1.07487E-02  7.31077E-03  4.99194E-03  3.42115E-03  2.35270E-03
 1.62314E-03  1.12318E-03  7.79407E-04  5.42279E-04  3.78229E-04  2.64423E-04  1.85266E-04  1.30075E-04  9.15043E-05  6.44906E-05
 4.55320E-05


NO RECOIL FORM FACTOR FOR JLS =    1       AT   .100 FERMI MESH

 1.53489E+01  1.49443E+01  1.44909E+01  1.39885E+01  1.34372E+01  1.28369E+01  1.21877E+01  1.14896E+01  1.07426E+01  9.94660E+00
 9.06226E+00  8.13884E+00  7.18622E+00  6.21426E+00  5.23281E+00  4.25706E+00  3.29022E+00  2.34083E+00  1.41743E+00  5.28537E-01
-3.01958E-01 -1.08471E+00 -1.81501E+00 -2.48818E+00 -3.09950E+00 -3.62584E+00 -4.08555E+00 -4.47857E+00 -4.80480E+00 -5.06418E+00
-5.24210E+00 -5.35662E+00 -5.41129E+00 -5.40964E+00 -5.35522E+00 -5.24520E+00 -5.09109E+00 -4.89802E+00 -4.67112E+00 -4.41554E+00
-4.13825E+00 -3.84208E+00 -3.53169E+00 -3.21175E+00 -2.88693E+00 -2.56887E+00 -2.25352E+00 -1.94381E+00 -1.64265E+00 -1.35298E+00
-1.08567E+00 -8.33695E-01 -5.98014E-01 -3.79562E-01 -1.79279E-01 -3.88483E-03  1.52906E-01  2.91598E-01  4.12696E-01  5.16706E-01
 6.01603E-01  6.71055E-01  7.26197E-01  7.68168E-01  7.98105E-01  8.17240E-01  8.26593E-01  8.27275E-01  8.20401E-01  8.07084E-01
 7.89799E-01  7.67957E-01  7.42331E-01  7.13696E-01  6.82824E-01  6.51968E-01  6.20052E-01  5.87478E-01  5.54650E-01  5.21971E-01
 4.90892E-01  4.60505E-01  4.30953E-01  4.02376E-01  3.74916E-01  3.49263E-01  3.24870E-01  3.01742E-01  2.79880E-01  2.59289E-01
 2.40178E-01  2.22293E-01  2.05584E-01  1.90005E-01  1.75507E-01  1.62071E-01  1.49611E-01  1.38073E-01  1.27400E-01  1.17537E-01
 1.08392E-01  9.99536E-02  9.21766E-02  8.50142E-02  7.84198E-02  7.22975E-02  6.66625E-02  6.14807E-02  5.67178E-02  5.23398E-02
 4.82700E-02  4.45273E-02  4.10879E-02  3.79284E-02  3.50251E-02  3.23354E-02  2.98567E-02  2.75724E-02  2.54670E-02  2.35261E-02
 2.17370E-02  2.00874E-02  1.85661E-02  1.71629E-02  1.58684E-02  1.46741E-02  1.35720E-02  1.25547E-02  1.16157E-02  1.07487E-02
 9.94800E-03  9.20849E-03  8.52532E-03  7.89410E-03  7.31077E-03  6.77161E-03  6.27318E-03  5.81232E-03  5.38614E-03  4.99194E-03
 4.62728E-03  4.28988E-03  3.97764E-03  3.68866E-03  3.42115E-03  3.17347E-03  2.94412E-03  2.73172E-03  2.53497E-03  2.35270E-03
 2.18382E-03  2.02731E-03  1.88226E-03  1.74780E-03  1.62314E-03  1.50756E-03  1.40037E-03  1.30095E-03  1.20874E-03  1.12318E-03
 1.04380E-03  9.70138E-04  9.01773E-04  8.38316E-04  7.79407E-04  7.24714E-04  6.73929E-04  6.26768E-04  5.82965E-04  5.42279E-04
 5.04481E-04  4.69365E-04  4.36734E-04  4.06411E-04  3.78229E-04  3.52034E-04  3.27684E-04  3.05045E-04  2.83996E-04  2.64423E-04
 2.46220E-04  2.29291E-04  2.13544E-04  1.98895E-04  1.85266E-04  1.72586E-04  1.60787E-04  1.49806E-04  1.39587E-04  1.30075E-04
 1.21221E-04  1.12978E-04  1.05304E-04  9.81581E-05  9.15043E-05  8.53079E-05  7.95370E-05  7.41618E-05  6.91549E-05  6.44906E-05
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