

# GLAUBER MODEL FOR HEAVY ION COLLISIONS FROM LOW ENERGIES TO HIGH ENERGIES \*

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The Glauber model is extensively applied to heavy ion collision for describing a number of interaction processes over a wide range of energies from near the Coulomb barrier to higher energies. The model gives the nucleus-nucleus interaction in terms of interaction between the constituent nucleons with a given density distribution. The model is a semiclassical model picturing the nuclear collision in the impact parameter representation where the nuclei move along the collision direction in a straight path. In these lectures we derive this model and discuss its applications in variety of problems in nuclear and high energy physics.

## I. CROSS SECTIONS AND IMPACT PARAMETER REPRESENTATION

The total reaction cross section in the collision of two nuclei as per the partial wave analysis is given by

$$\sigma_R = \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)(1 - |S_l|^2). \quad (1)$$

Here  $S_l = \exp(-2i\delta_l)$  is called the scattering matrix and  $\delta_l$  is the nuclear phase shift. The factor  $(1 - |S_l|^2)$  is called the transmission coefficient and  $|S_l|^2$  is referred to as transparency function or the probability that the projectile undergoes no interaction at a given  $l$ . In a semiclassical approximation, one can write angular momentum  $l$  in terms of momentum  $k$  and impact parameter  $b$  as

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$$l + \frac{1}{2} = kb. \quad (2)$$

Thus, using  $(2l + 1) = 2kb$  and  $\sum_l = k \int db$  we can get the reaction cross section in impact parameter representation as

$$\sigma_R = 2\pi \int b db (1 - |S(b)|^2). \quad (3)$$

The factor  $Tr(b) = 1 - |S(b)|^2$  is nothing but the transmission coeff. If the two nuclei are assumed to be sharp spheres with radii  $R_1$  and  $R_2$  then

$$\begin{aligned} Tr(b) &= 1 \quad \text{for } b \leq R_1 + R_2 \\ &= 0 \quad \text{for } b > R_1 + R_2. \end{aligned} \quad (4)$$

The total reaction cross section in this case is given by

$$\sigma_R = 2\pi \int_0^{R_1+R_2} b db = \pi(R_1 + R_2)^2. \quad (5)$$

This is the well-known geometric formula for the cross section.

The nucleus-nucleus differential cross section as a function of center of mass scattering angle  $\theta$  is described as

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2, \quad (6)$$

where the scattering amplitude  $f(\theta)$  for the non identical spinless nuclei is given by partial wave analysis which can be written as a sum of Coulomb and nuclear scattering amplitudes as

$$\begin{aligned} f(\theta) &= f_C(\theta) + f_N(\theta) \\ &= f_C(\theta) + \frac{1}{2ik} \sum (2l + 1)(e^{2i\sigma_l})(S_l - 1)P_l(\cos \theta). \end{aligned} \quad (7)$$

Here  $f_C(\theta)$ , the Coulomb scattering amplitude and  $\sigma_l$ , the Coulomb phase shift are given by the following expressions:

$$f_C(\theta) = -\frac{\eta}{2k} \text{cosec}^2 \frac{\theta}{2} \exp \left[ 2i\sigma_0 - 2i\eta \ln \sin \frac{\theta}{2} \right], \quad (8)$$

$$\sigma_{l+1}(\eta) = \sigma_l(\eta) + \tan^{-1} \left( \frac{\eta}{l+1} \right). \quad (9)$$

The S wave Coulomb phase shift  $\sigma_0$  can be set equal to zero without any loss of generality.

## II. THE GLAUBER MODEL

The Glauber model of multiple collision processes provides a quantitative consideration of the geometrical configuration of the nuclei when they collide. The Glauber model basically describe the nucleus-nucleus interaction in terms of elementary nucleon-nucleon interaction. It is based on the assumption that the nucleus travels in a straight line path. At high energies this approximation is very good. At low energies the nucleus is deflected from straight line path due to Coulomb repulsion. The so called Coulomb modified Glauber model (discussed in the next section) also gives very good description of heavy ion scattering at low energies.

Consider the collision of a projectile nucleus  $A$  on a target nucleus  $B$ . Define  $t(\mathbf{b})d\mathbf{b}$  as the probability for having a nucleon-nucleon collision within the transverse area element  $d\mathbf{b}$  when one nucleon is situated at an impact parameter  $\mathbf{b}$  relative to another nucleon which is normalized according to

$$\int t(\mathbf{b})d\mathbf{b} = 1. \quad (10)$$

We define the probability of finding a nucleon in the volume element  $d\mathbf{b}_A dz_A$  in the nucleus  $A$  at the position  $(\mathbf{b}_A, z_A)$  is  $\rho_A(\mathbf{b}_A, z_A)d\mathbf{b}_A dz_A$  which is normalized as

$$\int \rho_A(\mathbf{b}_A, z_A)d\mathbf{b}_A dz_A = 1. \quad (11)$$

Similarly, the probability of finding a nucleon in the volume element  $d\mathbf{b}_B dz_B$  in the nucleus  $B$  at the position  $(\mathbf{b}_B, z_B)$  is  $\rho_B(\mathbf{b}_B, z_B)d\mathbf{b}_B dz_B$  which is normalized as

$$\int \rho_B(\mathbf{b}_B, z_B)d\mathbf{b}_B dz_B = 1. \quad (12)$$

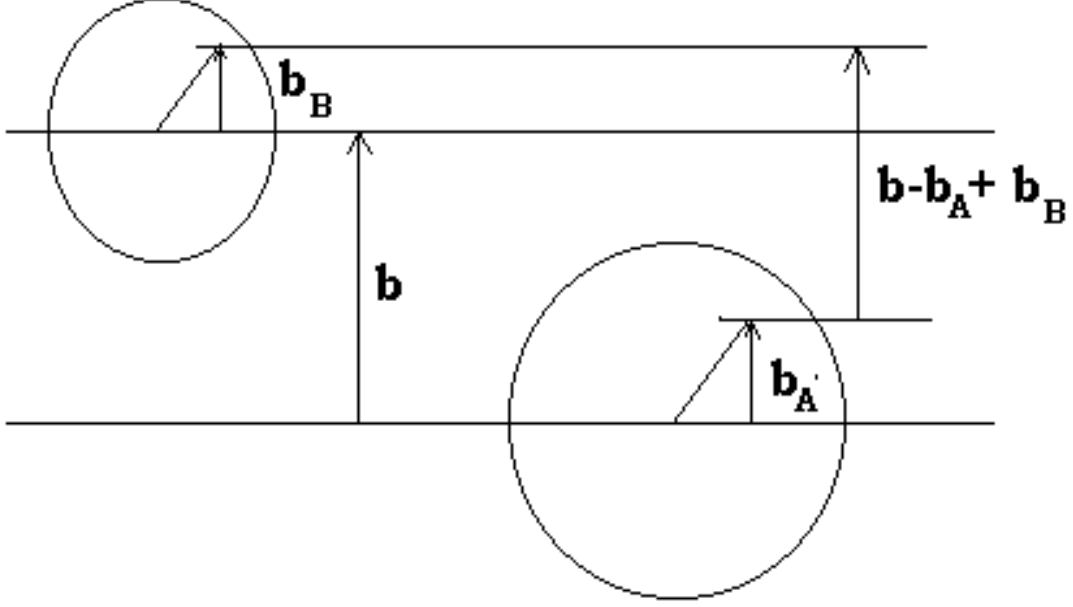


FIG. 1. Collision of two nuclei at an impact parameter  $b$

The probability for occurrence of a nucleon-nucleon collision [see Fig. (1)] when the nuclei  $A$  and  $B$  are situated at an impact parameter  $\mathbf{b}$  relative to each other is given by

$$T(b)\sigma_{NN} = \int \rho_A(\mathbf{b}_A, z_A) d\mathbf{b}_A dz_A \rho_B(\mathbf{b}_B, z_B) d\mathbf{b}_B dz_B t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B) \sigma_{NN}. \quad (13)$$

This can be written in terms of z-integrated densities as

$$T(b)\sigma_{NN} = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B) \sigma_{NN}. \quad (14)$$

Here  $\sigma_{NN}$  is the inelastic nucleon nucleon cross section. Thus, the collision probability we are talking about is for an inelastic collision. There can be upto  $A \times B$  collision. The probability of occurrence of  $n$  collisions will be

$$P(n, b) = \binom{AB}{n} (1-s)^n (s)^{AB-n}. \quad (15)$$

Here,  $s = 1 - T(b)\sigma_{NN}$ . The total probability for the occurrence of an inelastic event in the collision of  $A$  and  $B$  at an impact parameter  $\mathbf{b}$  is

$$\frac{d\sigma_{in}^{AB}}{db} = \sum_{n=1}^{AB} P(n, b) = 1 - s^{AB}. \quad (16)$$

The total inelastic cross section can be written as

$$\sigma_{in}^{AB} = 2\pi \int b db (1 - s^{AB}). \quad (17)$$

From here one can read the scattering matrix as

$$|S(b)|^2 = s^{AB} = (1 - T(b)\sigma_{NN})^{AB}. \quad (18)$$

In the optical limit, where a nucleon of projectile undergoes only one collision in the target nucleus

$$|S(b)|^2 \simeq \exp(-T(b)\sigma_{NN}AB). \quad (19)$$

The scattering matrix can be defined in terms of eikonal phase shift  $\chi(b)$  as

$$S(b) = \exp(-i\chi(b)). \quad (20)$$

Comparing Eq. (19) with Eq. (20), the imaginary part of eikonal phase shift is given by

$$\text{Im}\chi(b) = T(b)\sigma_{NN}AB/2. \quad (21)$$

If the ratio of real to imaginary part of NN scattering amplitude is  $\alpha_{NN}$  then real part of  $\chi(b)$  is

$$\text{Re}\chi(b) = T(b)\alpha_{NN}\sigma_{NN}AB/2. \quad (22)$$

Once we know the phase shift and thus the scattering matrix, we can calculate the reaction cross section and the angular distribution.

### III. CALCULATION OF $T(B)$ IN MOMENTUM SPACE

In the co-ordinate space  $T(b)$  is derived as

$$T(b) = \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B t(\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B). \quad (23)$$

It is a four dimensional integration: two over  $\mathbf{b}_A$  and two over  $\mathbf{b}_B$ . It is convenient to write it in momentum space as

$$T(b) = \frac{1}{(2\pi)^2} \int \rho_A^z(\mathbf{b}_A) d\mathbf{b}_A \rho_B^z(\mathbf{b}_B) d\mathbf{b}_B \exp(-i\mathbf{q} \cdot (\mathbf{b} - \mathbf{b}_A + \mathbf{b}_B)) f_{NN}(q) d^2q. \quad (24)$$

Here  $f_{NN}(q)$  is the  $q$  dependence of NN scattering amplitude given by

$$t(\mathbf{b}) = \frac{1}{(2\pi)^2} \int e^{-i\mathbf{q} \cdot \mathbf{b}} f_{NN}(q) d^2q \quad (25)$$

$$\begin{aligned} T(b) &= \frac{1}{(2\pi)^2} \int \exp(-i\mathbf{q} \cdot \mathbf{b}) \rho_A^z(\mathbf{b}_A) \exp(i\mathbf{q} \cdot \mathbf{b}_A) d\mathbf{b}_A \rho_B^z(\mathbf{b}_B) \exp(-i\mathbf{q} \cdot \mathbf{b}_B) d\mathbf{b}_B f_{NN}(q) d^2q \\ &= \frac{1}{(2\pi)^2} \int e^{-i\mathbf{q} \cdot \mathbf{b}} S_A(\mathbf{q}) S_B(-\mathbf{q}) f_{NN}(q) d^2q \\ &= \frac{1}{2\pi} \int J_0(qb) S_A(\mathbf{q}) S_B(-\mathbf{q}) f_{NN}(q) q dq. \end{aligned} \quad (26)$$

Here  $S_A(q)$  and  $S_B(-q)$  are the fourier transforms of the nuclear densities. The function  $f_{NN}(q)$  is the fourier transform of the profile function  $t(\mathbf{b})$ . The profile function for the NN scattering can be taken as delta function if the nucleons are point particles. In general it is taken as a gaussian function of width  $r_0$  as

$$t(\mathbf{b}) = \frac{\exp(-b^2/r_0^2)}{\pi r_0^2}. \quad (27)$$

Thus,

$$\begin{aligned} f_{NN}(q) &= \int e^{i\mathbf{q} \cdot \mathbf{b}} t(\mathbf{b}) d\mathbf{b} \\ &= \frac{1}{\pi r_0^2} \int e^{i\mathbf{q} \cdot \mathbf{b}} \exp(-b^2/r_0^2) d\mathbf{b} \\ &= \exp(-r_0^2 q^2/4). \end{aligned} \quad (28)$$

Here,  $r_0^2 = 0.439 \text{ fm}^2$  is the range parameter and  $\sigma_{NN}$  is the nucleon-nucleon inelastic cross section which is taken as  $3.2 \text{ fm}^2$  at high energies.

### A. Calculation of $T(b)$ using Gaussian densities

If the nuclear densities are assumed to be of the Gaussian shape given by

$$\rho_i(r) = \rho_i(0) \exp(-r^2/a_i^2) \quad (i = 1, 2). \quad (29)$$

Here the parameters  $\rho_i(0)$  and  $a_i$  are adjusted to reproduce the experimentally determined surface texture of the nucleus. The  $z$ -integrated density will be

$$\rho_i^z(r) = \rho_i(0) \sqrt{\pi} a_i \exp(-b^2/a_i^2). \quad (30)$$

This one can write in the momentum representation as

$$S_i(q) = \rho_i(0) (\sqrt{\pi} a_i)^3 \exp(-q^2 a_i^2/4). \quad (31)$$

The overlap integral  $T(b)$  can be written as

$$\begin{aligned} T(b) &= \frac{1}{(2\pi)^2} \int e^{-i\mathbf{q}\cdot\mathbf{b}} S_1(\mathbf{q}) S_2(-\mathbf{q}) f_{NN}(q) d^2q \\ &= \frac{1}{(2\pi)^2} \pi^3 \rho_1(0) \rho_2(0) a_1^3 a_2^3 \int e^{-i\mathbf{q}\cdot\mathbf{b}} \exp(-a^2 q^2/4) d^2q, \end{aligned} \quad (32)$$

where  $a^2 = a_1^2 + a_2^2 + r_0^2$ . Performing  $q$  integration we get

$$\begin{aligned} T(b) &= \frac{1}{(2\pi)^2} \pi^3 \rho_1(0) \rho_2(0) a_1^3 a_2^3 \left[ \frac{4\pi}{a^2} \exp(-b^2/a^2) \right] \\ &= \pi^2 \frac{\rho_1(0) \rho_2(0) a_1^3 a_2^3}{a_1^2 + a_2^2 + r_0^2} \exp(-b^2/(a_1^2 + a_2^2 + r_0^2)). \end{aligned} \quad (33)$$

## B. Calculation of $T(b)$ using 2pf densities

The two parameter fermi density is given by

$$\rho(r) = \frac{\rho_0}{1 + \exp(\frac{r-c}{d})}, \quad (34)$$

where  $\rho_0 = 3 / \left( 4\pi c^3 (1 + \frac{\pi^2 d^2}{c^2}) \right)$ . Thus, the momentum density can be derived as

$$S(q) = \frac{8\pi\rho_0}{q^3} \frac{ze^{-z}}{1 - e^{-2z}} \left( \sin x \frac{z(1 + e^{-2z})}{1 - e^{-2z}} - x \cos x \right). \quad (35)$$

Where  $z = \pi dq$  and  $x = cq$ . Here  $d$  is the diffuseness and  $c$ , the half value radius in terms of rms radius  $R$  for the 2pf distribution is calculated by  $c = (5/3R^2 - 7/3\pi^2 d^2)^{1/2}$ . The equation can be solved numerically for this density and the overlap integral can be extracted.

### C. NN scattering parameters $\sigma_{NN}$ and $\alpha_{NN}$

In this section, we give the expressions for  $\sigma_{NN}$  and  $\alpha_{NN}$  obtained from the parameterization of experimentally measured cross section. The average  $\sigma_{NN}$  in terms of proton numbers ( $Z_P$  and  $Z_T$ ) and neutron number ( $N_P$  and  $N_T$ ) of projectile and target nuclei is written as

$$\sigma_{NN} = \frac{N_P N_T \sigma_{nn} + Z_P Z_T \sigma_{pp} + (Z_P N_T + N_P Z_T) \sigma_{np}}{A_P A_T}. \quad (36)$$

Here, pp cross section  $\sigma_{pp}$  and nn cross section  $\sigma_{nn}$  are given in ( $\text{fm}^2$ ) by

$$\sigma_{pp} = \sigma_{nn} = 1.373 - 1.504/\beta + 0.876/\beta^2 + 6.867\beta^2. \quad (37)$$

Here  $\beta$  is the velocity of projectile nucleon. For np cross section  $\sigma_{np}$ , two expression are used. If the energy per nucleon  $E_N > 10$  MeV, then

$$\sigma_{np} = -7.067 - 1.818/\beta + 2.526/\beta^2 + 11.385\beta. \quad (38)$$

For  $E_N < 10$  MeV,

$$\sigma_{np} = \frac{273}{(1. - 0.0553E_N)^2 + 0.35E_N} + \frac{1763}{(1. + 0.334E_N)^2 + 6.8E_N}. \quad (39)$$

The average  $\alpha_{NN}$  is written as

$$\alpha_{NN} = \frac{N_P N_T \alpha_{nn} \sigma_{nn} + Z_P Z_T \alpha_{pp} \sigma_{pp} + (Z_P N_T + N_P Z_T) \alpha_{np} \sigma_{np}}{\sigma_{NN} A_P A_T}. \quad (40)$$

The parameterized forms of  $\alpha_{pp}$ ,  $\alpha_{nn}$  and  $\alpha_{np}$  are written as

$$\alpha_{pp} = \alpha_{nn} = 0.1810 + 4.0818p + 0.3327p^2 \quad (41)$$

and

$$\alpha_{np} = -.698 + 4.9762p - 1.277p^2, \quad (42)$$

where  $p$  is the momentum of projectile nucleon in GeV.



#### IV. COULOMB MODIFIED GLAUBER MODEL

In the nucleus-nucleus collision at low energies it is the surface region of the nucleus that contributes to the scattering amplitude in a non-trivial manner. Since  $V/E \ll 1$  even for low and intermediate energies in the surface region, the eikonal approximation can be extended to the low energy heavy ion collisions. The basic assumption of the Glauber model is the description of the relative motion of the two nuclei in terms of straight line trajectory. The density overlaps are evaluated along straight lines associated with each impact parameter  $b$ . The modification in the straight line trajectory due to the Coulomb field can not be ignored, especially in the case of heavily charged systems at relatively low bombarding energies.

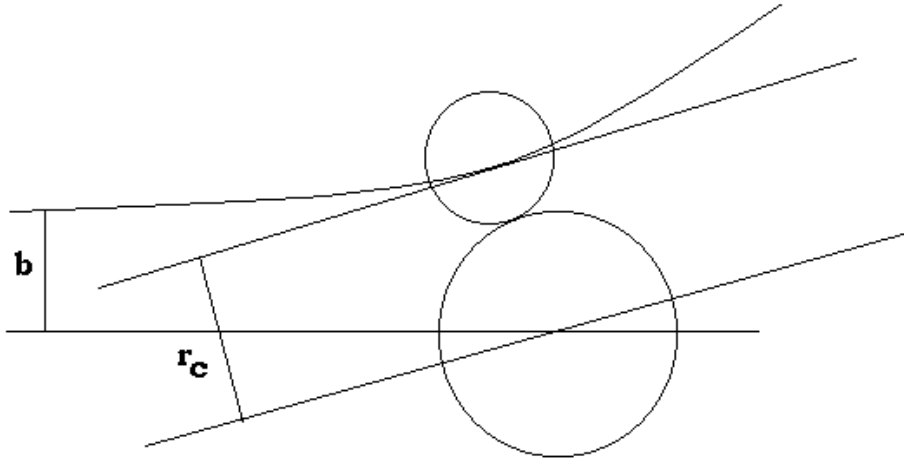


FIG. 2. The straight line trajectory is assumed at the distance of closest approach  $r_c$

For low energy heavy ion reactions the straight line trajectory is assumed at the distance of closest approach  $r_c$  [see Fig. (2)] calculated under the influence of the Coulomb potentials for each impact parameter  $b$  as given by,

$$r_c = (\eta + \sqrt{\eta^2 + b^2 k^2})/k, \quad (43)$$

which is a solution of the equation

$$E - \frac{Z_1 Z_2 e^2}{r} - \frac{\hbar^2 k^2 b^2}{2\mu r^2} = 0, \quad (44)$$

$$E - \frac{Z_1 Z_2 e^2}{r} - E \frac{b^2}{r^2} = 0, \quad (45)$$

$$kr^2 - 2\eta r - b^2 k = 0. \quad (46)$$

Here  $\eta$  is the dimensionless Sommerfeld parameter defined as

$$\eta = \frac{Z_1 Z_2 e^2}{\hbar v} = \frac{Z_1 Z_2 e^2 k}{E}. \quad (47)$$

Thus finally the reaction cross section will be

$$\sigma_{in}^{AB} = 2\pi \int b db \left(1 - (1 - T(r_c)\sigma_{NN})^{AB}\right). \quad (48)$$

Figure (3) shows the reaction cross section for  $^{16}\text{O} + ^{208}\text{Pb}$  system as a function of projectile energy per nucleon calculated with Glauber model (GM) and Coulomb modified

Glauber model (CMGM) along with the data. At higher energies both GM and CMGM result merge. But at lower energies one requires to take Coulomb modification in the trajectory.

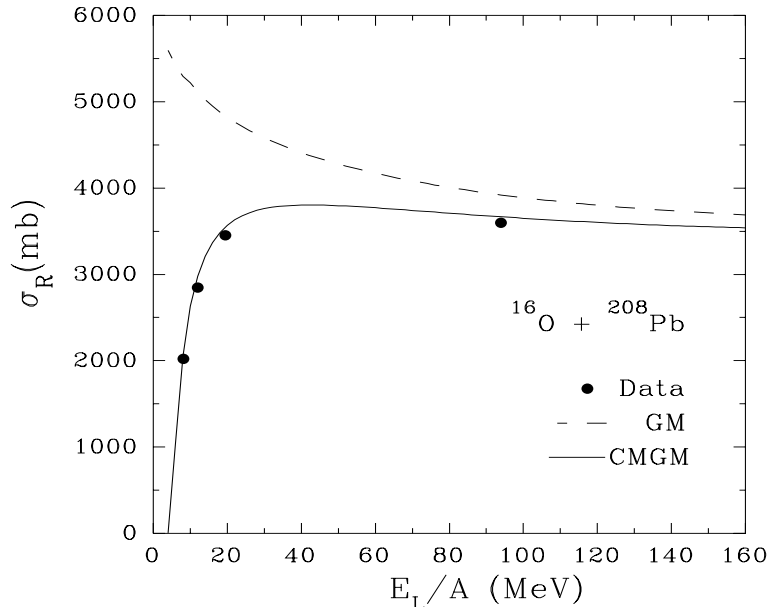


FIG. 3. Reaction cross section for  $^{16}\text{O} + ^{208}\text{Pb}$  system calculated with Glauber model (GM) and Coulomb modified Glauber model (CMGM) along with the data

## V. PARTICIPANT-SPECTATOR PICTURE IN HIGH ENERGY HEAVY ION COLLISIONS

Nucleus-nucleus collisions at ultrarelativistic energies are being explored in order to search for the formation of quark gluon plasma. Already several experimental results are available. One of the strategies is to describe these results as the superposition of the nucleon-nucleon collisions and look for possible departures from this prescription. When two nuclei collide at these energies, the nucleons which come in the overlap region depending on the impact parameter are called participant nucleons and those which do not participate are called spectators. [see Fig. (4)]

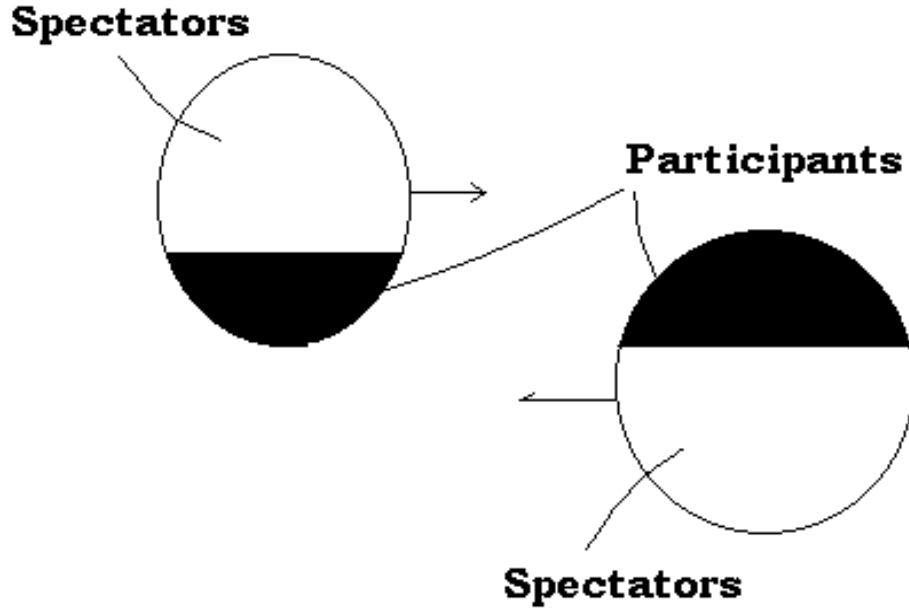


FIG. 4. Spectator participant picture of heavy ion collision

These spectators and participants nucleon decide how much energy is going in the forward and how much in the transverse direction which can be measured. From these measured energies one can know the impact parameter for a particular event. Let us write the total inelastic cross section for a nucleus-nucleus collision as

$$\sigma_{in}^{AB} = 2\pi \int b db \left(1 - (1 - T(b)\sigma_{NN})^{AB}\right). \quad (49)$$

The term  $(1 - T(b)\sigma_{NN})^{AB}$  gives the probability that in a nucleus nucleus collision none of the nucleons collided with each other. For nucleon-nucleon collision  $A = B = 1$  thus  $s = (1 - T(b)\sigma_{NN})$  gives the probability that two nucleon at an impact parameter  $b$  do not collide.

When two nuclei  $A$  and  $B$  collide the probability of nucleon remaining in the projectile will be

$$P_P = s(b)^B \quad (50)$$

and the probability of nucleon remaining in the target will be

$$P_T = s(b)^A. \quad (51)$$

The probability of having  $\alpha$  participant nucleons from the nucleus  $A$  is given by binomial distribution as

$$P(\alpha, b) = \binom{A}{\alpha} (1 - s^B)^\alpha (s^B)^{A-\alpha}. \quad (52)$$

Similarly,

$$P(\beta, b) = \binom{B}{\beta} (1 - s^A)^\beta (s^A)^{B-\beta}. \quad (53)$$

The number of collisions will be

$$P(n, b) = \binom{AB}{n} (1 - s)^n (s)^{AB-n}. \quad (54)$$

The average number of projectile participant and its standard deviation for each impact parameter  $b$  is given by

$$\langle \alpha \rangle = A[1 - s(b)^B], \quad (55)$$

$$\sigma_\alpha^2 = A[1 - s(b)^B]s(b)^B. \quad (56)$$

The average number of target participant and its standard deviation for each impact parameter  $b$  is given by

$$\langle \beta \rangle = B[1 - s(b)^A], \quad (57)$$

$$\sigma_\beta^2 = B[1 - s(b)^A]s(b)^A. \quad (58)$$

The average number of total participant from both projectile and target for each impact parameter is given by

$$N_{\text{participants}} = A[1 - s(b)^B] + B[1 - s(b)^A]. \quad (59)$$

The average number of N-N collisions is given by

$$N_{\text{collisions}} = AB[1 - s(b)]. \quad (60)$$

The Fig. (5) shows the number of participants and the number of N-N collisions as a function of impact parameter with the expressions given in the text along with the calculations with the Monte Carlo code FRITIOF and Geometric model of J. Gosset et al.

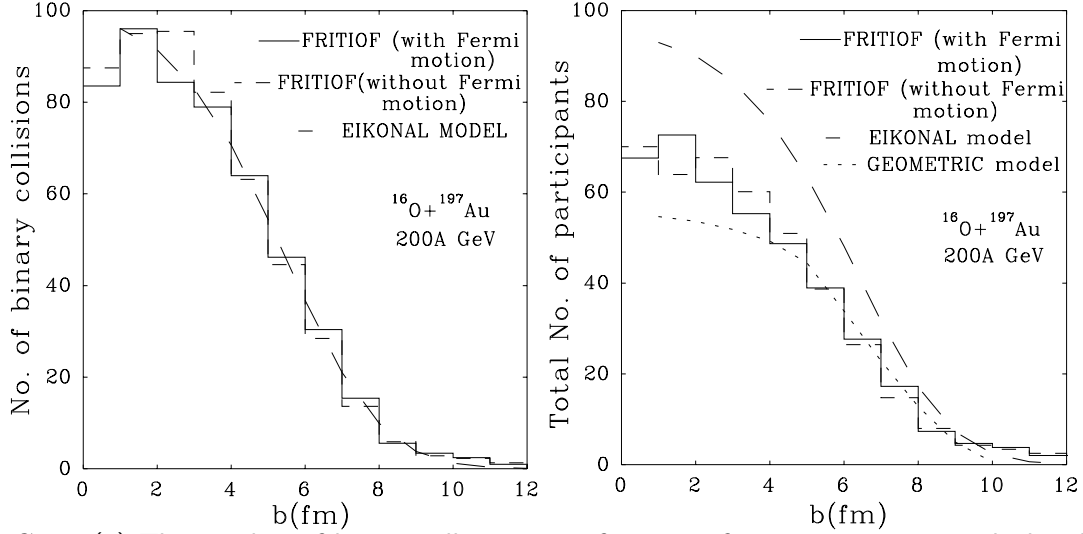


FIG. 5. (a) The number of binary collisions as a function of impact parameter calculated with the expressions given in the text along with the Monte Carlo code FRITIOF and Geometric model of J. Gosset et al. (b) The total number of participants as function of impact parameter.

### A. Forward energy

The forward energy is measured in the detector called zero degree calorimeter (ZDC). The energy measured in the ZDC is can be related to the number of spectators as

$$E_Z(\alpha) = (A - \alpha)E_0, \quad (61)$$

where  $E_0$  is the projectile energy per nucleon in the lab frame.

The cross section of  $\alpha$  nucleons participating from projectile is

$$\sigma_\alpha = 2\pi \int P(\alpha, b) b db. \quad (62)$$

The forward energy flow cross section then can be written as

$$\frac{d\sigma}{dE_Z} = \frac{\sigma_\alpha}{E_0}, \quad (63)$$

taking into account the discrete nature of the variable  $\alpha$ . The average forward energy for a collision is written as

$$E_Z(\alpha) = E_0(A - \langle \alpha \rangle) = E_0 A [1 - (1 - s^B)] = E_0 A s^B. \quad (64)$$

### B. Excitation energy

When the nucleons collide at relativistic energies they become excited nucleons which due to time dilation become long-lived and decay into secondary fragments outside the nuclei. This energy which goes in exciting the nucleons is called excitation energy which manifests in the measurement of transverse energy and multiplicity. The (maximum) excitation energy which is related to the number of participants is written as

$$E_{ex} = E_{cm} - m_N(\alpha + \beta). \quad (65)$$

The centre of mass energy for this case is given as

$$E_{cm}^2 = ((\alpha E_A + \beta E_B)^2 - (\alpha \mathbf{P}_A + \beta \mathbf{P}_B)^2), \quad (66)$$

where

$$P_A = \sqrt{E_A^2 - m_N^2} \quad \text{and} \quad P_B = \sqrt{E_B^2 - m_N^2}. \quad (67)$$

The cross section for having  $\alpha$  participants from  $A$  and  $\beta$  participants from  $B$  is

$$\sigma_{\alpha\beta} = 2\pi \int P(A, \alpha, b) P(B, \beta, b) b db. \quad (68)$$

Here  $E_A$  and  $\mathbf{P}_A$  are the total energy and momentum of the nucleus  $A$  per nucleon and  $m_N$  is nucleon mass. The cross sections for all the possible combinations of  $\alpha$  and  $\beta$  are calculated and the results are binned to obtain the cross section for each of the excitation energy bin.

### C. Multiplicity

The total charged particle multiplicity in pp collision for a center of mass energy  $\sqrt{s}$  is given by

$$\langle n_{ch} \rangle = 0.88 + 0.44 \ln(s) + 0.118(\ln(s))^2. \quad (69)$$

The center of mass energy for pair of participants from nucleus  $A$  and nucleus  $B$  can be written as

$$s_{\alpha\beta} = \left( \frac{2E_{cm}}{\alpha + \beta} \right)^2. \quad (70)$$

Thus, the total multiplicity in nucleus-nucleus collision in the case of  $\alpha$  nucleons participating from nucleus  $A$  and  $\beta$  participating from nucleus  $B$  can be written as

$$\langle M \rangle = \left( 0.88 + 0.44 \ln(s_{\alpha\beta}) + 0.118(\ln(s_{\alpha\beta}))^2 \right) \left( \frac{\alpha + \beta}{2} \right). \quad (71)$$

The cross sections and the multiplicity for all the possible combinations of projectile and target participants are calculated and the results are binned to obtain the cross section for each of the multiplicity bin.

### VI. $J/\psi$ PRODUCTION CROSS SECTION

The probability for  $j/\psi$  production in a nucleus nucleus collision can be found out if we know the  $j/\psi$  production cross section in a nucleon nucleon collision,

$$P_{j/\psi}^{AB}(b) = AB\sigma_{j/\psi}^{NN}T(b), \quad (72)$$

The cross section is

$$\sigma_{j/\psi}^{AB} = AB\sigma_{j/\psi}^{NN} \int d\mathbf{b} T(b). \quad (73)$$

The integral is 1 as per the normalization condition



$$\frac{\sigma_{j/\psi}^{AB}}{\sigma_{j/\psi}^{NN}} = AB. \quad (74)$$

The  $j/\psi$  once produced can collide with the other nucleons present in the nucleus and thus it is possible that it is absorbed (converted into  $D(c\bar{u})$  and  $\bar{D}(\bar{c}u)$ ) during the path it travels in the nucleus. This nuclear absorption can be taken into account by writing

$$\frac{\sigma_{j/\psi}^{AB}}{\sigma_{j/\psi}^{NN}} = AB \exp(-L\sigma_{abs}\rho_0). \quad (75)$$

Here,  $\rho_0 = 0.14 \text{ fm}^{-3}$  is the equilibrium nuclear matter density,  $\sigma_{abs} = 6.2 \text{ mb}$  is the absorption cross section and  $L$  is the average path length given by

$$L = L_A + L_B = \frac{3}{4} \left( \frac{A-1}{A} R_A + \frac{B-1}{B} R_B \right), \quad (76)$$

where,  $R_A = r_0 A^{1/3}$  and  $R_B = r_0 B^{1/3}$ .

Figure (6) shows the suppression of  $J/\Psi$  over Drell-Yan pairs as a function of the average nuclear path length  $L$  of the  $c\bar{c}$  pre-resonance, for NA38, NA50 and NA51 data.

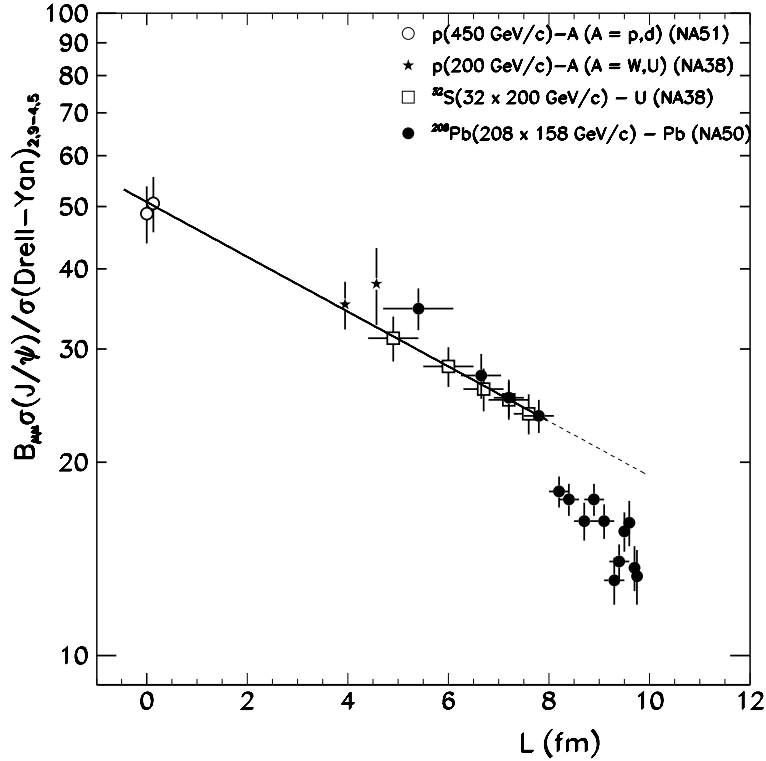


FIG. 6. The suppression of  $J/\Psi$  over Drell-Yan pairs in  $2.9 \text{ GeV} < M < 4.5 \text{ GeV}$  as a function of the average nuclear path length  $L$  of the  $c\bar{c}$  pre-resonance, for NA38, NA50 and NA51 data. The NA50 Pb+Pb data (filled circles) is from the 1996 run.

## VII. THE MONTE CARLO EVENT GENERATORS

There are many Monte Carlo modes available to study the relativistic heavy ion collisions as superposition of nucleon-nucleon collisions such as FRITIOF and IRIS. In these codes the two nuclei are made to interact at a random impact parameter. A Woods-Saxon distributed nuclear density, giving the probability of finding a nucleon at a given distance from the centre of the nucleus, is used. Each nucleon is given space coordinates and the number of subcollisions is calculated letting a projectile hit the nucleus at a random impact parameter. Here a frozen straight line geometry is used and all nucleons inside a cylinder surrounding the path of the projectile are considered to participate. The number of binary collisions is recorded and the two partners involved in each subcollision are determined. In each subcollision, momentum is exchanged.

FRITIOF is based on the LUND string picture of hadron-hadron interactions. In this picture, each nucleon-nucleon collision results in excitation of the nucleon by the stretching of a string between the valence quark and diquark. In nucleus-nucleus interactions each nucleon can make several encounters, the objects can get further excited, thereby increasing their masses during the passage through the nucleus. A phenomenological excitation function determines the mass and momentum of the string after each interaction. Finally, all the excited objects hadronize independently, like massless relativistic strings, according to the Lund model for the jet fragmentation. The hadronization takes place outside the nuclei and thus no intranuclear cascading is considered.

Three other models based on string picture of hadron-hadron interactions are IRIS, MCFM and VENUS all of which are colour exchange models based on the Dual Parton Model (DPM) of Capella et al. Here the basic mechanism of string formation is colour exchange between the quarks of the colliding nucleons. In these models the string properties can be calculated from structure functions. HIJET is an extension of the ISAJET model of hadron interactions and MARCO is based upon a phenomenological parameterization of nucleon-nucleon collisions. ATTILA has the possibility of rope formation from

overlapping strings and MCFM and HIJET which allow cascading of produced particles when the assumed particle formation time is short. RQMD is an extension to relativistic energies of the Quantum Molecular Dynamics which has been applied to nucleus-nucleus collisions at much lower energies.

The nucleus-nucleus collision geometry should be the same for all of the Monte-Carlo codes since nuclear density distributions are well-known from nuclear physics and should not be treated as free parameters. It should be noted that deviations as small as 5-10 % in the treatment of the nuclear geometry are significant since one hopes to draw conclusions about deviations from the measurements which are of similar magnitude. It is clear that before firm conclusions can be drawn upon the significance of the differences in the physics of the models at the nucleon-nucleon level, or on whether there exists experimental evidence for rescattering of the produced particles, or even for QGP formation, it will be necessary to ensure that the various models treat the nuclear geometry correctly and consistently.

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