Antisymmetric distorted wave impulse approximation calculations of spin transfer cross sections for $(^3\vec{\mathrm{He}},\vec{t})$ reactions to the continuum

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A formalism is presented for the calculation of spin transfer cross section for intermediate energy charge exchange $(^3\vec{\mathrm{He}},\vec{t})$ [and $(\vec{p},\vec{n})$] reactions to the continuum. The nuclear structure part of the formalism is based on the continuum Tamm-Dancoff method and the nuclear reaction part is treated within the distorted wave impulse approximation. In the nuclear structure part, we thus include particle-hole correlations and continuum effects on the excited particle. The knockon-exchange effect and the damping of the particle through an imaginary potential added to the single particle real potential are also taken into account. Results of numerical calculations are presented for the inclusive cross sections $\sigma(0°)$ and the spin transfer coefficients $D_{nn}(0°)$ for forward scattering, for the intermediate energy $(^3\vec{\mathrm{He}},\vec{t})$ and $(\vec{p},\vec{n})$ reactions on ^{12}C and ^{90}Zr targets leading to Gamow-Teller resonances in the continuum. It is shown that $D_{nn}(0°)$ for this case takes a value of $-1/3$, regardless of the details of nuclear structure and reaction mechanisms, if the reaction is assumed to proceed as an orbital angular momentum transfer $l_t=0$ process, and that deviations from $-1/3$ come from admixture of $l_t=2$ components. A simple closed form of the expression is derived which can be used to understand a subtle dependence of the D_{nn} values on nuclear structure and effective interaction.

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I. INTRODUCTION

The charge-exchange (p,n) and $(^3\mathrm{He},t)$ reactions at intermediate energies have successfully been used to study spin-isospin excitation modes in nuclei [1]. The most extensive studies have been made of the Gamow-Teller (GT) resonance with quantum numbers $l_t=0,\ s_t=1,\ t_t=1,\ j_t^\pi=1^+$ as observed in $0°$ (p,n) spectra. The spectra at higher scattering angles $(\theta\geqslant 0°)$ show evidence for spin-flip dipole $(l_t=1,\ s_t=1,\ t_t=1,\ j_t^\pi=0^-,1^-,2^-)$ and spin-flip quadrupole $(l_t=2,\ s_t=1,\ t_t=1,\ j_t^\pi=1^+,2^+,3^+)$ resonances in the excitation energy region of $E_x=5\sim 40$ MeV. The resonances are generally broad and strongly overlapping. Therefore, one has to decompose the spectra into various multipole components in order to determine the strength functions of states with different j_t^π. With only inclusive spectra, however, it is often difficult to achieve this separation, particularly for various j_t^π components with the same l_t value.

Additional data that are sensitive not only to the l_t value but also to the j_t^π value are thus needed in order to achieve this decomposition. The spin (polarization) transfer cross sections or equivalently the spin (polarization) transfer coefficient data are useful for this purpose. These can be extracted by measuring the polarization of the outgoing particle from the reaction induced by the polarized incident beam. A large amount of effort has recently been put forth to measure such coefficients, particularly the $D_{nn}(\theta)$ value at $\theta=0°$, for the $(\vec{p},\vec{n})$ reactions. The measurements were made at LAMPF [2–13], using the NTOF facilities. A similar facility has been recently built at RCNP, Osaka and has started to produce data [13–15]. No measurement has been reported thus far of the $(^3\vec{\mathrm{He}},\vec{t})$ reaction; however, a facility for the measurement is now under construction and thus data will be taken in the near future [16]. Because of the strong absorptive nature of the projectile and ejectile involved in the reactions, the data are expected to reveal different features of the modes apart from those disclosed by the (p,n) reaction, and in this sense it is important to study both reactions simultaneously.

The primary aim of the present paper is to present a formulation of the spin transfer cross section (and coefficient) calculations for the $(^3\vec{\mathrm{He}},\vec{t})$ reaction. Such a formulation is presented in Sec. II. In formulating the expressions to be calculated, we keep in mind the application of the formalism to the $(\vec{p},\vec{n})$ reaction. For this purpose, we follow closely our previous work [17], in which a formulation was presented for fully microscopic calculations of the inclusive $(^3\mathrm{He},t)$ and (p,n) reaction cross sections including a knockon-exchange effect, within the usual framework of the distorted wave impulse approximation (DWIA). In the present work, we extend the formulation to handle the spin transfer cross sections. A significant improvement is made in this formulation, especially in the way the continuum effects are taken into account. For this purpose, we incorporate the continuum Tamm-Dancoff approximation method into the formalism. The Pauli blocking effects on the excited particle are also

included. In Ref. [17], the knockon-exchange effect was calculated exactly, but in the present study a plane wave approximation (PWA) is introduced, particularly in dealing with recoil effects. Further, we ignore, as was done in Ref. [17], the spin-orbit force in the distorting potentials. This is made since it has been shown [18,19] that the force gives rise almost no effect, particularly, on the D_{nn} value, in which we are primarily interested.

In Sec. III, we show results of numerical calculations of the exclusive cross sections $\sigma(0°)$ and the spin transfer coeficients $D_{nn}(0°)$, of the intermediate energy $(^3\vec{\mathrm{He}},\vec{t})$ and $(\vec{p},\vec{n})$ reactions on ^{12}C and ^{90}Zr targets, leading to GT 1^+ resonances in the continuum region. The results are compared with available experimental data. Details of some physical effects on the $D_{nn}(0°)$ values, particularly effects of the $l_t=2$ component in the transition, are investigated. It is shown that the $D_{nn}(0°)$ value for this case takes a value of $-1/3$, regardless of details of nuclear structure and reaction mechanisms, if the reaction is assumed to proceed as an orbital angular momentum transfer $l_t=0$ process. The deviations from $-1/3$ come from the admixture of an $l_t=2$ component. A simple closed form of the expression is derived, which can be used conveniently to understand the results of the complicated numerical calculations.

II. FORMULATION

The charge exchange reaction process considered in this paper may be symbolically written as $a+A\rightarrow b+B$ where $a(b)$ and $A(B)$ represent, respectively, the projectile (ejectile) and the target (residual) nuclei. The notations a, b, A, and B will also be used to stand collectively for all quantum numbers of the intrinsic degrees of freedom of these particles. We restrict our interest to the cases where the spin s_a (s_b) of the projectile (ejectile) is 1/2 $(s_a=s_b=1/2)$, such as $(^3\mathrm{He},t)$ and (p,n) reactions. We also assume that the target nucleus A is a doubly closed-shell nucleus. Thus the spin parity $I_A^{\pi_A}=0^+$.

A. Spin transfer coefficients and spin transfer cross sections

In the spin (polarization) transfer measurements, the incident particle is polarized and the spin polarization of the outgoing particle is measured. From such measurements, one first deduces the so-called spin (polarization) transfer coefficients D_{ii} which can then be used to decompose the inclusive cross sections σ into four component cross sections σ_0, σ_x, σ_y, and σ_z, where σ_0 is the cross section involving no spin transfer $(s_t=0)$ while σ_x, σ_y, and σ_z are the cross sections involving the spin transfer $(s_t=1)$ along the directions of the x, y, and z axes, respectively [20,21],

$$\sigma_0=\frac{\sigma}{4}(1+D_{xx}+D_{yy}+D_{zz}),$$

$$\sigma_x=\frac{\sigma}{4}(1+D_{xx}-D_{yy}-D_{zz}),$$

$$\sigma_y=\frac{\sigma}{4}(1-D_{xx}+D_{yy}-D_{zz}),$$

$$\sigma_z=\frac{\sigma}{4}(1-D_{xx}-D_{yy}+D_{zz}). \tag{2.1}$$

Here the y and z axes are chosen to be parallel to $\mathbf{p}_a\times\mathbf{p}_b$ and $\mathbf{p}_a+\mathbf{p}_b$, $\mathbf{p}_a$ and $\mathbf{p}_b$ being the momenta of the incident and outgoing particles. The x axis is then fixed from the y and z axes as an axis in the right-handed Cartesian coordinate system. σ_i with $i=x$, y, and z are usually called the spin transfer cross sections.

In the present formulation, σ_α $(\alpha=0,\ x,\ y,$ and $z)$ is calculated as follows. First of all, use is made of the general form of the transition operator $\hat{T}$ for the reaction, namely,

$$\hat{T}=\sum_{\alpha=0}^{3}\hat{T}_\alpha\hat{\sigma}_\alpha, \tag{2.2}$$

where $\hat{\sigma}_0=1$, and $\hat{\sigma}_x$, $\hat{\sigma}_y$, and $\hat{\sigma}_z$ are the Pauli spin operators for the nucleon in the projectile (^{3}He) interacting with another nucleon in the target, while $\hat{T}_\alpha$ is the operator operating on all other degrees of freedom associated with these two interacting nucleons. The details of this operator will be discussed in the next subsection. At this stage, we only note that the spin transfer coefficient D_{ii} can be given in terms of $\hat{T}_\alpha$ as

$$D_{ii}=\frac{\mathrm{Tr}\left\{\left[\hat{T}_0\hat{T}_0^\dagger+\sum_{j=1}^{3}(2\delta_{j,i}-1)\hat{T}_j\hat{T}_j^\dagger\right]\delta(H-E)\right\}}{\sum_\alpha \mathrm{Tr}[\hat{T}_\alpha\hat{T}_\alpha^\dagger\delta(H-E)]}, \tag{2.3}$$

where the trace is taken over final nuclear states, H is the Hamiltonian, and E the excitation energy.

The next step is to calculate the external operator $\hat{\rho}_\alpha$ as

$$\hat{\rho}_\alpha=(\phi_b\chi_b^{(-)}|\hat{T}_\alpha|\phi_a\chi_a^{(+)}), \tag{2.4}$$

where ϕ_a (ϕ_b) and $\chi_a^{(+)}$ $(\chi_b^{(-)})$ are, respectively, the intrinsic and distorted wave functions of the incident (outgoing) particle $a(b)$. As remarked in the Introduction, we ignore the spin-orbit force in generating $\chi_a^{(+)}$ and $\chi_b^{(-)}$. Also, we ignore the D-state admixture in the projectile intrinsic functions ϕ_a and ϕ_b. $\hat{\rho}_\alpha$ defined by Eq. (2.4) serves as an external operator which produces excitation of the target nucleus. Further we define the strength function S_α as

$$S_\alpha=\mathrm{Im}[-\langle\rho_\alpha|\Psi_\alpha\rangle/\pi], \tag{2.5}$$

where

$$|\rho_\alpha\rangle=\hat{\rho}_\alpha|\Phi_0\rangle, \tag{2.6}$$

$$|\Psi_\alpha\rangle=G|\rho_\alpha\rangle. \tag{2.7}$$

In Eq. (2.6), $|\Phi_0\rangle$ is the target ground state, while $G=1/(E-H+i\epsilon)$ is the propagator of the final excited nucleus. $|\rho_\alpha\rangle$ is the excited state created by operating with $\hat{\rho}_\alpha$ on $|\Phi_0\rangle$ and is often referred to as a doorway state. The state is further propagated by the full propagator G of the system to develop into the final continuum state $|\Psi_\alpha\rangle$. Finally, the spin transfer cross section σ_α can be given in terms of the strength function S_α as

$$\sigma_\alpha\left(\equiv\frac{d^2\sigma_\alpha}{dE_b d\Omega_b}\right)=\frac{\mu_a\mu_b}{(2\pi\hbar^2)^2}\frac{k_a}{k_b}\frac{1}{2}S_\alpha, \tag{2.8}$$

where μ_a (μ_b) and k_a (k_b) are, respectively, the reduced mass and wave number of the projectile (ejectile).

The evaluation of $|\Psi_\alpha\rangle$ is performed by following the continuum Tamm-Dancoff method [22,23], taking into account Pauli blocking effects on the excited particles. The latter effects are treated by means of an orthogonalization method as employed earlier in Ref. [24]. Further, the external operator $\hat{\rho}_\alpha$ is calculated by following the method described in Ref. [17], the details of which will be discussed in the following subsections.

B. Transition operators

The transition operator we use in the present study is the effective interaction of Love and Franey (LF) [25] that includes a knockon-exchange contribution. In second quantized form, it is given as [17,26]

$$\hat{T}=\int dx_1dx_2[v^D(x_1,x_2)\hat{\rho}_T(x_1,x_1)\hat{\rho}_P(x_2,x_2) + v^E(x_1,x_2)\hat{\rho}_T(x_1,x_1')\hat{\rho}_P(x_2,x_2')], \tag{2.9}$$

where $x_i\equiv(\mathbf{r}_i,\sigma_i,\tau_i)$ stands for the spatial, spin, and isospin coordinates of the two interacting nucleons in the target ($i=1$) and the projectile ($i=2$). The primed coordinates are those used after the exchange of the nucleons in the target and projectile has taken place. $v^{D(E)}$ in Eq (2.9) denotes the direct (exchange) part of the effective two-body interaction and can explicitly be written as

$$v^{D(E)}(x_1,x_2)=\sum_{st}\sum_k\left(\sum_q S_{kq}Y^*_{kq}(\Omega)\right)V^{D(E)}_{stk}(r)P_{st}, \tag{2.10}$$

where the k sum is taken over $k=0$ (central) and $k=2$ (tensor) terms, while $S_{00}=1$ and S_{2q} is the usual tensor operator. P_{st} is the projection operator projecting onto the spin-isospin subspace with quantum numbers s and t. The corresponding radial parts of the effective interaction are denoted by $V^{D(E)}_{stk}(r)$. The quantities $\hat{\rho}_T(x_1,x_1')$ and $\hat{\rho}_P(x_2,x_2')$ in Eq. (2.9) are the nonlocal density operators for the target and projectile systems, respectively. [The corresponding local operators are $\hat{\rho}_T(x_1,x_1)$ and $\hat{\rho}_P(x_2,x_2)$, appearing in the direct term in Eq. (2.9).] These density operators can be expressed in terms of the nucleon field creation and annihilation operators, $\hat{\psi}^\dagger_{T(P)}(x)$ and $\hat{\psi}_{T(P)}(x)$, as

$$\hat{\rho}_{T(P)}(x_i,x_i')=\hat{\psi}^\dagger_{T(P)}(x_i)\hat{\psi}_{T(P)}(x_i'). \tag{2.11}$$

In order to handle the spin transfer of the projectile explicitly, we single out the spin part of the projectile density operator. For this purpose, we write the field operator for the projectile, $\hat{\psi}_P(x_2)$, as

$$\hat{\psi}_P(x_2)=\sum_{im\nu}c_ic_mc_\nu\phi_i(\mathbf{r}_2)\xi_m(2)\eta_\nu(2), \tag{2.12}$$

where $\phi(\mathbf{r})$, ξ, and η denote the spatial, spin, and isospin parts of the single particle wave function, respectively, and c_i ($c_i^\dagger$), c_m ($c_m^\dagger$), and c_ν ($c_\nu^\dagger$) are the corresponding parts of the single particle annihilation (creation) operators. Using Eq. (2.12), $\hat{\rho}_P(x_2,x_2')$ may be expressed as

$$\hat{\rho}_P(x_2,x_2')=\sum_{s_2m_2}\hat{\rho}_{s_2m_2}(x_2,x_2')\hat{\sigma}_{s_2m_2}, \tag{2.13}$$

where

$$\hat{\rho}_{s_2m_2}(x_2,x_2') = \frac{1}{\sqrt{2}}\sum[\phi_{l_2}(\mathbf{r}_2)\phi_{l_2}(\mathbf{r}_2')]_{00}[\xi(2)\tilde{\xi}(2')^*]_{s_2m_2} \times[\eta(2)\tilde{\eta}(2')^*]_{t_2\nu_2}[c^\dagger_{l_2}c_{\tilde{l}_2}]_{00}[c^\dagger\tilde{c}]_{t_2\nu_2} \tag{2.14}$$

and

$$\hat{\sigma}_{s_2m_2}\equiv\sqrt{2}[c^\dagger\tilde{c}]_{s_2m_2}. \tag{2.15}$$

In Eqs. (2.14) and (2.15) we have introduced the notation

$$[A_{j_1}\tilde{B}_{j_2}]_{jm}=\sum_{m_1m_2}\langle j_1m_1j_2m_2|jm\rangle A_{j_1m_1}\tilde{B}_{j_2m_2}, \tag{2.16}$$

where $\langle\cdots|\cdots\rangle$ stands for the Clebsch-Gordan coefficient and $\tilde{B}_{jm}=(-1)^{j+m}B_{j-m}$ (time reversal phase). As is clear, $\hat{\sigma}_{s_2m_2}$ is the operator describing the spin transfer of the projectile with the magnitude s_2 and its projection m_2. s_2 takes the values of 0 and 1.

Inserting Eq. (2.13) into (2.9), $\hat{T}$ may be rewritten as

$$\hat{T}=\sum_{s_2m_2}\hat{T}_{s_2m_2}\hat{\sigma}_{s_2m_2}, \tag{2.17}$$

where

$$\hat{T}_{s_2m_2}=\int dx_1dx_2[v^D(x_1,x_2)\hat{\rho}_T(x_1,x_1)\hat{\rho}_{s_2m_2}(x_2,x_2) + v^E(x_1,x_2)\hat{\rho}_T(x_1,x_1')\hat{\rho}_{s_2m_2}(x_2,x_2')]. \tag{2.18}$$

Note that $\hat{T}_{s_2m_2}$ given by the above equation is simply related to $\hat{T}_\alpha$ in Eq. (2.2) by

$$\hat{T}_\alpha = \sum_{s_2 m_2} F_{\alpha, s_2 m_2} \hat{T}_{s_2 m_2}, \tag{2.19}$$

where

$$F_{0,00}=1, \quad F_{0,11}=0, \quad F_{0,10}=0, \quad F_{0,1-1}=0,$$
$$F_{x,00}=0, \quad F_{x,11}=-1/\sqrt{2}, \quad F_{x,10}=0, \quad F_{x,1-1}=1/\sqrt{2},$$
$$F_{y,00}=0, \quad F_{y,11}=i/\sqrt{2}, \quad F_{y,10}=0, \quad F_{y,1-1}=i/\sqrt{2},$$
$$F_{z,00}=0, \quad F_{z,11}=0, \quad F_{z,10}=1, \quad F_{z,1-1}=0. \tag{2.20}$$

Further, $\hat{\sigma}_0=\hat{\sigma}_{00}$, and the relations between $(\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ and $\hat{\sigma}_{1m}$ are well known. Equation (2.2) with (2.19) now defines our transition operator $\hat{T}$ for the reaction. Note, however, that details of the target density operator $\hat{\rho}_T(x_1, x_1')$ have not yet been given. It will be described later in Sec. II D.

C. The correlated source function method

The most involved parts of our calculations are those of the doorway state $|\rho_\alpha\rangle$ and the continuum wave function $|\Psi_\alpha\rangle$ given by Eqs. (2.6) and (2.7), respectively. In this subsection, we describe the details of the calculation of $|\Psi_\alpha\rangle$. The calculation of $|\rho_\alpha\rangle$ will be discussed in the next subsection.

For the calculation of $|\Psi_\alpha\rangle$, use is made of the correlated source function method previously developed and used by us, e.g., in Refs. [22] and [23]. The method makes use of the fact that the Hamiltonian H of the target system is given by

$$H = H_0 + V_{ph}, \tag{2.21}$$

where H_0 is the single particle Hamiltonian and V_{ph} is the residual particle-hole (ph) interaction. As remarked before, we assume that the target is a double magic nucleus with spin parity $I_A^{\pi_A}=0^+$. Furthermore we approximate the target ground state wave function by a single Slater determinant in terms of the independent particle model. This means that we choose $|\Phi_A\rangle$ to be an eigenfunction of H_0 instead of H,

$$H_0|\Phi_A\rangle = E_A|\Phi_A\rangle \quad (E_A=0). \tag{2.22}$$

The above approximation neglects the effects of V_{ph} on the ground state wave function. The effects of V_{ph} are included, however, in the wave function of the excited states. This different treatment of V_{ph} for the ground state and the excited states is known as the Tamm-Dancoff approximation. To proceed we split H_0 into $H_0=H_h+H_p$ where H_h is the Hamiltonian of the hole nucleus and H_p is the Hamiltonian of the excited particle p. The hole state function $|\Phi_h\rangle$ is then defined by

$$H_h|\Phi_h\rangle = E_h|\Phi_h\rangle, \tag{2.23}$$

where E_h is the hole energy. Finally, the Hamiltonian of the excited particle p is given by

$$H_p = T_p + U_p, \tag{2.24}$$

where T_p is the kinetic energy operator and $U_p=V_p+iW_p$ is a complex one-body potential.

Using the Hamiltonian of Eq. (2.21), we can derive from Eq. (2.7) the following integral equation for the continuum wave function:

$$|\Psi_\alpha\rangle = G_0|\rho_\alpha\rangle + G_0 V_{ph}|\Psi_\alpha\rangle, \tag{2.25}$$

where G_0 is the unperturbed Green's function defined by

$$G_0 = \frac{1}{\omega - H_0 + i\epsilon}. \tag{2.26}$$

To solve Eq. (2.25) we first transform the integral equation into an equivalent integral equation [22,23]

$$|\Lambda_\alpha\rangle = |\rho_\alpha\rangle + V_{ph}G_0|\Lambda_\alpha\rangle, \tag{2.27}$$

so that

$$|\Psi_\alpha\rangle \equiv G_0|\Lambda_\alpha\rangle. \tag{2.28}$$

Note that $|\Lambda_\alpha\rangle$ thus defined in Eq. (2.28) plays a similar role as $|\rho_\alpha\rangle$ in Eq. (2.7). We call $|\Lambda_\alpha\rangle$ the correlated source function since it includes the correlations due to V_{ph}.

Equation (2.27) is now solved by first integrating over all coordinates, except the radial coordinate of particle p. In this way the equation is reduced to a set of coupled-channels (CCs) equations for the radial wave functions of particle p. In order to achieve this we first expand both $|\Lambda_\alpha\rangle$ and $|\rho_\alpha\rangle$ in terms of the channel wave functions

$$|[y_p\Phi_h]_{jm}\rangle = \Sigma_{m_p m_h}\langle j_p m_p j_h m_h|jm\rangle |y_{\nu_p j_p m_p}\Phi_{\nu_h j_h m_h}\rangle, \tag{2.29}$$

where y_p is the wave function of the single-particle state p and Φ_h is the hole nucleus wave function of Eq. (2.23). Note that (the quantum number) p excludes the quantum number of the radial motion; p thus represents the total angular momentum j_p, its projection m_p, and the isospin projection ν_p. The notation $y_{\nu_p j_p m_p}$ $(\Phi_{\nu_h j_h m_h})$, instead of y_p (Φ_h), is used in the right-hand side (RHS) of Eq. (2.29). This is done in order to show explicitly the dependence of y_p (Φ_h) on $(\nu_p j_p m_p)$ $[(\nu_h j_h m_h)]$. The channel expansions of $|\Lambda_\alpha\rangle$ and $|\rho_\alpha\rangle$ are then given by

$$|\Lambda_\alpha\rangle = \sum_{j_t m_{j_t}} \sum_{ph} \frac{\lambda_{\alpha,ph}(r)}{r} |[y_p\Phi_h]_{j_t m_{j_t}}\rangle,$$

$$|\rho_\alpha\rangle = \sum_{j_t m_{j_t}} \sum_{ph} \frac{\rho_{\alpha,ph}(r)}{r} |[y_p\Phi_h]_{j_t m_{j_t}}\rangle, \tag{2.30}$$

where j_t and m_{j_t} are the total angular momenta transferred in the reaction and its projection, respectively. The quantities $\lambda_{\alpha,ph}(r)$ and $\rho_{\alpha,ph}(r)$ are the correlated and uncorrelated radial source functions, respectively. Note that both source functions depend on the quantum numbers $j_t m_{j_t}$. These in-

dices are not shown explicitly. The sum involved in Eq. (2.30) is taken over all ph pairs.

Inserting the expansions of Eq. (2.30) into Eq. (2.27), we obtain a set of CC equations for $\lambda_{\alpha,ph}(r)$,

$$\lambda_{\alpha,ph}(r)=\rho_{\alpha,ph}(r)+\Sigma_{p'h'}\int dr'dr''V^{j_t}_{ph,p'h'}(r,r')g^{(+)}_{p'h'} \times(r',r'')\lambda_{\alpha,p'h'}(r''), \tag{2.31}$$

where the ph matrix elements $V^{j_t}_{ph,p'h'}(r,r')$ are defined by

$$V^{j_t}_{ph,p'h'}(r,r')=rr'([y_{p'}\Phi_{h'}]_{j_t}|V_{ph}|[y_p\Phi_h]_{j_t}). \tag{2.32}$$

Here the round brackets denote an integration over the channel variables. Introducing column vectors of dimension $1\times N_c$, where N_c is the number of ph pairs

$$|\lambda_\alpha\rangle=\begin{pmatrix}\lambda_{\alpha,p_1h_1}\\ \cdots\\ \cdots\end{pmatrix}, \tag{2.33}$$

$$|\rho_\alpha\rangle=\begin{pmatrix}\rho_{\alpha,p_1h_1}\\ \cdots\\ \cdots\end{pmatrix}, \tag{2.34}$$

we can write the CC equations in matrix form as

$$|\lambda_\alpha\rangle=|\rho_\alpha\rangle+\mathcal{V}G_0|\lambda_\alpha\rangle, \tag{2.35}$$

where $\mathcal{V}$ is the interaction matrix

$$(\mathcal{V})_{ph,p'h'}(r,r')=V^{j_t}_{ph,p'h'}(r,r') \tag{2.36}$$

of dimension $N_c\times N_c$ and $\mathcal{G}_0$ is the diagonal Green's function matrix defined by

$$(\mathcal{G}_0)_{ph,p'h'}=g^{(+)}_{ph}\delta_{ph,p'h'}. \tag{2.37}$$

In the last equation, $g^{(+)}_{ph}$ is the radial optical model Green's function. Note that the operation of $\mathcal{G}_0$ (and $\mathcal{V}$) onto $|\lambda_\alpha\rangle$ in Eq. (2.35) involves a radial integration in addition to the matrix multiplication. Further we note that in practical calculations, we replace the simple free Green's function $\mathcal{G}_0$ by $\mathcal{G}^p_0$, which includes the effects of the Pauli blocking due to the occupied particles. The explicit expression for $\mathcal{G}^p_0$ may be found in Ref. [24]. Equation (2.35) is the final equation to be solved. The source functions $\rho_{\alpha,ph}(r)$ can be calculated in a straightforward manner. Since the details of the calculation have already been described [22,23], we shall not reproduce them here.

Once $|\lambda_\alpha\rangle$ is obtained, it is easy to calculate $|\Psi_\alpha\rangle$. In partial wave expansion $|\Psi_\alpha\rangle$ is given by

$$|\Psi_\alpha\rangle=\sum_{j_tm_{j_t}}\sum_{ph}\frac{\psi_{\alpha,ph}(r)}{r}|[y_p\Phi_h]_{j_tm_{j_t}}\rangle, \tag{2.38}$$

and is represented by a column vector

$$|\psi_\alpha\rangle=\begin{pmatrix}\psi_{\alpha,ph}\\ \cdots\\ \cdots\end{pmatrix}. \tag{2.39}$$

It is then easy to see that $|\psi_\alpha\rangle$ can be obtained as

$$|\psi_\alpha\rangle=\mathcal{G}_0|\lambda_\alpha\rangle. \tag{2.40}$$

D. Source functions

The essential ingredients in our basic Eq. (2.35) to be solved are the source functions $\rho_{\alpha,ph}(r)$ which have been introduced in the channel expansion of the doorway state $|\rho_\alpha\rangle$ [see Eq. (2.30)]. In the present subsection, we evaluate $\rho_{\alpha,ph}(r)$. Using Eq. (2.30), together with Eqs. (2.4) and (2.6), $\rho_{\alpha,ph}(r)$ can be given as

$$\rho_{\alpha,ph}(r)=r\langle[a^\dagger_p(r)a_{\tilde{h}}]_{j_tm_t}\phi_b\chi^{(-)}_b|\hat{T}_\alpha|\phi_a\chi^{(+)}_a\Phi_0\rangle, \tag{2.41}$$

where $a^\dagger_p(r)$ is the creation operator that creates a nucleon in the spin-isospin-angular state y_p at the radial distance r, and a_h is the annihilation operator for the single hole state h. $\tilde{h}$ is the time reversed state corresponding to h. Note that the quantum number label h includes all those numbers necessary for specifying the single hole state, including the radial quantum number. This is not the case, however, for the particle state p, which specifies the spin, isospin, and angular parts of the single particle state as remarked above. $[a^\dagger_p(r)a_{\tilde{h}}]_{j_tm_t}$ creates a ph state with total angular momentum j_t and projection m_{j_t}.

In order to calculate the RHS of Eq. (2.41), we now express $\hat{\rho}_T(x_1,x_1')$ in $\hat{T}_\alpha$ in terms of $a^\dagger_p(r)$ and a_h. For this purpose, $\hat{\psi}^\dagger_T$ and $\hat{\psi}_T$ are expanded in terms of $\{a^\dagger_p(r)\}$ and $\{a_h\}$ as

$$\hat{\psi}^\dagger_T(x_1)=\sum_p y^*_p a^\dagger_p(r),$$

$$\hat{\psi}_T(x_1)=\sum_h y_h u_h(r)a_h, \tag{2.42}$$

where $u_h(r)$ is the radial wave function of the single hole state h. Use of the above expansions now leads to

$$\hat{\rho}_T(x_1,x_1')=\sum_{phl_1m_{l_1}s_1m_1t_1\nu_1j_tm_{j_t}}\langle l_1m_{l_1}s_1m_1|j_tm_{j_t}\rangle \times\left\langle\frac{1}{2}\nu_p\frac{1}{2}\nu_h\middle|t_1\nu_1\right\rangle \times\hat{j}_p\hat{j}_h\hat{l}_1\hat{s}_1\begin{pmatrix} l_p & 1/2 & j_p\\ l_h & 1/2 & j_h\\ l_1 & s_1 & j_t\end{pmatrix} \times u_h(r)[Y_p(\hat{\mathbf{r}}_1)Y_h(\hat{\mathbf{r}}_1')]_{l_1m_{l_1}}[\xi(1)\tilde{\xi}(1')^*]_{s_1m_1} \times[\eta(1)\tilde{\eta}(1')^*]_{t_1\nu_1}[a_p^\dagger(r)a_{\tilde{h}}]_{j_tm_{j_t}}, \tag{2.43}$$

where $Y_{l_pm_p}$ are spherical harmonics and the large parenthesis denotes the 9-j symbol. Using Eq. (2.43), the evaluation of $\rho_{\alpha,ph}(r)$ is now straightforward. In carrying out the calculations, however, we introduce the plane wave approximation (PWA) in dealing with the recoil effect of the knockon-exchange process, as will be discussed later. The result of the calculation can be written in the following form:

$$\rho_{\alpha,ph}(r)=\sum_{s_2m_2l_tm_{l_t}}F_{\alpha,s_2m_2} \times\langle s_2m_2j_tm_{j_t}|l_tm_{l_t}\rangle\sum_{t_1s_1l_1k}\alpha^{j_ts_t\nu_1}_{t_1s_1l_1kl_t} \times[t^D_{pht_1s_1l_1kl_tm_{l_t}}(r)+t^E_{pht_1s_1l_1kl_tm_{l_t}}(r)], \tag{2.44}$$

where $t^D_{pht_1s_1l_1kl_tm_{l_t}}(r)$ and $t^E_{pht_1s_1l_1kl_tm_{l_t}}(r)$ are the *radial dependent reduced* transition amplitudes while

$$\alpha^{j_ts_t\nu_1}_{t_1s_1l_1kl_t}=W(s_1l_ts_tl_1;j_tk)\hat{t}_1^{-1}\langle t_b\nu_b|[c^\dagger\tilde{c}]_{t_1\nu_1}|t_a\nu_a\rangle. \tag{2.45}$$

The Racah coefficient W involved in Eq. (2.45) describes the coupling of various angular momenta, while $\langle t_b\nu_b|[c^\dagger\tilde{c}]_{t_1\nu_1}|t_a\nu_a\rangle$ is a projectile isospin matrix element.

Before writing down the explicit expressions for T^D and T^E, two remarks are given here: The first is that t^D and t^E are the portions of the direct and exchange DWIA transition amplitudes $T^D_{t_1s_1l_1kl_tm_{l_t}}$ and $T^E_{t_1s_1l_1kl_tm_{l_t}}$ already evaluated in Ref. [17]. In fact $t^{D(E)}$ is related to $T^{D(E)}$ by

$$T^{D(E)}_{t_1s_1l_1kl_tm_{l_t}}=\sum_{ph}C_{ph}\int dr\,r^2u_{np}(r)t^{D(E)}_{pht_1s_1l_1kl_tm_{l_t}}(r), \tag{2.46}$$

where $u_{np}(r)$ is the radial wave function of the single particle state np, n being the radial quantum number. In Ref. [17], continuum effects were neglected so that the excited particle is assumed to occupy a bound single particle state (np). C_{ph} is the spectroscopic amplitude of a ph configuration in the final state. Equation (2.46) indicates that the reduced amplitude $t^{D(E)}$ can be obtained by following the method presented in Ref. [17] for calculating $T^{D(E)}$.

The second remark is that in the present formalism, we introduce an approximation in dealing with the recoil effect in the exchange term t^E, as remarked above. The approximation may be explained by considering the exchange form factor $F^E_{t_1s_1l_1kltm_{l_t}}(\mathbf{r},\boldsymbol{\rho})$ defined by Eq. (15b) of Ref. [17], which is a portion of t^E. The well known no-recoil (NR) approximation of F^E that neglects completely the recoil effect is

$$F^{E;\mathrm{NR}}_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r},\boldsymbol{\rho})\approx f^{\mathrm{NR}}_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r})\,\delta(\boldsymbol{\rho}),$$

$$f^{\mathrm{NR}}_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r})=\int d\boldsymbol{\rho}F^E_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r},\boldsymbol{\rho}). \tag{2.47}$$

In the PWA, the recoil effect is described by a (recoil) factor $e^{-i\alpha\mathbf{k}_a\cdot\boldsymbol{\rho}/a}$. A possible improvement of the above NR approximation is then to replace f^{NR} by the following f^{PW} that takes into account the recoil factor within the PWA:

$$F^{E;\mathrm{PW}}_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r},\boldsymbol{\rho})\approx f^{\mathrm{PW}}_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r})\,\delta(\boldsymbol{\rho}),$$

$$f^{\mathrm{PW}}_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r})=\int d\boldsymbol{\rho}F^E_{t_1s_1l_1kl_tm_{l_t}}(\mathbf{r},\boldsymbol{\rho})e^{-i\alpha\mathbf{k}_a\cdot\boldsymbol{\rho}/a}. \tag{2.48}$$

Here we introduced a parameter α in the recoil factor. We treat it as an adjustable parameter and fit it such that the resultant approximate cross section reproduces the exact cross section σ^E as closely as possible. It has turned out that a close fit is obtained with $\alpha\approx1$.

With such an approximation, $t^{D(E)}$ can be given as

$$t^D_{t_1s_1l_1kl_tm_{l_t}}(r) =(-1)^{l_1}\hat{l}_t^{-1}\rho^D_{l_1}(r)\int d\rho\,\rho^2V^D_{t_1s_1k}(\rho)Y^D_{kl_tl_1,l_tm_{l_t}}(r,\rho), \tag{2.49}$$

$$t^E_{t_1s_1l_1kl_tm_{l_t}}(r) =\sqrt{4\pi}\hat{l}_1\hat{l}_t^{-1}\sum_{l_r\lambda}\sum_{\lambda_1\lambda_2l_p'}(-1)^l\hat{\lambda}_1\hat{\lambda}_2 \times\langle\lambda_10\lambda_20|\lambda0\rangle W(\lambda_1\lambda_2l_1l;\lambda l_p') \times(-1)^{l_1+k-l_t}\langle k0\lambda0|l_r0\rangle\hat{k}\hat{\lambda}\hat{l}_1\hat{l}_rW(l\lambda l_tk;l_1l_r) \times\langle lm_ll_r0|l_tm_t\rangle\int d\rho\,\rho^2V^E_{t_1s_1k}(\rho)j_{l_r}(r,\rho) \times Y^E_{\lambda_2ll_p',l_tm_{l_t}}(r,\rho), \tag{2.50}$$

where

$$Y^{D(E)}_{\lambda_2 l l'_p, l_t m_{l_t}}(r,\rho) = \int dr' X_{l_t m_{l_t}}(r') d^{D(E)}_{\lambda_2 l l'_p}(r r' \rho), \tag{2.51}$$

$$X_{l_t m_{l_t}}(r') = \int \chi_b^{(-)*}(\mathbf{r}') \chi_a^{(+)}(\mathbf{r}') i^{l_t} Y_{l_t m_{l_t}}(\hat{\mathbf{r}}') d\Omega_{r'}. \tag{2.52}$$

X_{lm} is the distortion factor, while $d^{D(E)}_{\lambda_2 l l'_p}(r r' \rho)$ originates from the density of the projectile and is given as

$$d^{D(E)}_{\lambda_2 l l'_p}(r r' \rho) = \frac{2\pi}{2\lambda_2 + 1} \sum_m \hat{l} \langle l 0 l'_p m | \lambda_2 m \rangle \times \int d^{D(E)}_{\lambda}(r'_2 \rho) Y_{\lambda_2 m}(\mu') Y_{l'_p m}(\mu) d\mu, \tag{2.53}$$

where $\mu \equiv \cos\theta = \hat{\mathbf{r}}' \cdot \hat{\mathbf{r}}$ and $\mu' \equiv \cos\theta' = \hat{\mathbf{r}}' \cdot \hat{\mathbf{r}}'_2$. Note that $\mathbf{r}'$ (relative coordinate between the projectile and the target), $\mathbf{r}$ and $\mathbf{r}_2$ are related by $\mathbf{r}'_2 = \mathbf{r} - \mathbf{r}'$. Further $d^D_\lambda(r'_2 \rho)$ and $d^E_\lambda(r'_2 \rho)$ are defined by

$$\phi(\mathbf{r}_2, \mathbf{r}_2) \equiv [\phi_{l_2}(\mathbf{r}_2) \phi_{l_2}(\mathbf{r}_2)]_{00} = \sum_{\lambda_2} d^D_{\lambda_2}(r'_2, \rho)(-1)^{\lambda_2} [Y_{\lambda_2}(\hat{\mathbf{r}}'_2) Y_{\lambda_2}(\hat{\rho})]_{00}, \tag{2.54}$$

$$\phi(\mathbf{r}_2, \mathbf{r}'_2) \equiv [\phi_{l_2}(\mathbf{r}_2) \phi_{l_2}(\mathbf{r}'_2)]_{00} = \sum_{\lambda_2} d^E_{\lambda_2}(r'_2, \rho)(-1)^{\lambda_2} [Y_{\lambda_2}(\hat{\mathbf{r}}'_2) Y_{\lambda_2}(\hat{\rho})]_{00}, \tag{2.55}$$

where $\phi(\mathbf{r}_2, \mathbf{r}'_2 = \mathbf{r}_2 + \boldsymbol{\rho})$ $[\phi(\mathbf{r}_2, \mathbf{r}_2)]$ is the spatial nonlocal (local) density of the projectile that was already introduced before in Eq. (2.14). The details were presented in Ref. [17] of the method of the calculations of $d^{D(E)}_{\lambda_2}$ and $d^{D(E)}_{\lambda_2 l l'_p}$.

We have already developed as described in Ref. [17] a computer code DCP1 in order to carry out the calculation of $T^{D(E)}$. The code has recently been modified for the calculation of $t^{D(E)}$ and $\rho_{\alpha,ph}(r)$. The resultant $\rho_{\alpha,ph}(r)$ are then used as inputs for the continuum TDA calculations.

The formalism presented here for $(\vec{^3\mathrm{He}}, \vec{t})$ can be applied to the $(\vec{p}, \vec{n})$ reaction as well, if a certain limit is taken for the projectile density. In fact, if we adopt the following limits:

$$\phi(\mathbf{r}_2, \mathbf{r}_2) \to \delta(\mathbf{r}_2), \qquad \phi(\mathbf{r}_2, \mathbf{r}'_2) \to \delta(\mathbf{r}_2), \tag{2.56}$$

the resultant source functions are reduced to the corresponding source functions for the $(\vec{p}, \vec{n})$ reaction.

TABLE I. Calculated $\sigma(0°)$ values in comparison with the experimental data.

Target	(a,b)	E/a(MeV)	Cal.	Exp.	Ref.
^{12}C	(p,n)	200	7.5	7 ± 1	[27]
		300	6.2	6 ± 1	[14]
	(h,t)	200	28	23 ± 5	[28]
		300	21	25	[29]
^{90}Zr	(p,n)	200	83	79	[2]
		300	67	68 ± 1	[13]
	(h,t)	200	143	149	[28]
		300	140	130	[30]

III. RESULTS OF NUMERICAL CALCULATIONS AND DISCUSSIONS

Using the formalism and also the computer code described above, numerical calculations of the inclusive cross section, $\sigma(0°)$, and the spin transfer coefficient, $D_{nn}(0°)$, were performed for both $(\vec{p}, \vec{n})$ and $(\vec{^3\mathrm{He}}, \vec{t})$ reactions at incident energies of $E_a = 200$ and 300 MeV/nucleon. Specific calculations were made for ^{12}C and ^{90}Zr targets leading to the GT 1^+ resonance state. In what follows, we first present the results for $\sigma(0°)$, together with some details and a discussion of various aspects of the calculations. Finally, we present the results for $D_{nn}(0°)$ values and discuss some characteristic features of the results.

A. Inclusive cross sections

Table I presents the calculated cross sections in comparison to experimental data [2,13,14,27–30]. The reaction part of the calculations was carried out using the LF interaction [25] (210 MeV parameters for $E_a = 200$ MeV case and 325 MeV parameters for the $E_a = 300$ MeV case) which includes the knockon-exchange contribution. The distorted waves for the ^{12}C target were generated by using the optical potentials obtained in Ref. [31] for the 200 MeV case and in Ref. [32] for the 300 MeV case. We ignored, however, both real and imaginary spin orbit terms included in the potentials of Refs. [31,32]. Such neglect of the spin-orbit part of the potential was made for all the calculations performed in this work. Because of this we slightly modified the strength of the imaginary part of the potential; particularly, we reduced the strength by 20% for the 300 MeV case. The potential parameters for the ^{90}Zr target case were taken from Ref. [33] for the 200 MeV case. Those for the 300 MeV were not known experimentally and thus generated from those of 200 MeV by taking into account the systematics of the energy dependence of the potential suggested, e.g., by Hama *et al.* [34], i.e., by reducing the strength of the real potential to 1/3 and increasing the strength of the imaginary potential by 5%.

The potentials for the $(^3\mathrm{He}, t)$ reactions were generated by using the single folding procedure based on those used for the (p,n) calculations, except for the case of the 200 MeV ^{90}Zr$(^3\mathrm{He}, t)^{90}$Nb reaction. In that case, the straightforward single folding procedure does not yield satisfactory results,

as demonstrated by Yamagata *et al.* [35]. The simple folding procedure introduces too strong an absorption. In fact, in our previous study of ^{90}Zr(^{3}He,t) reaction at 600 MeV in Ref. [17] use was made of the single folding procedure, and we found that the resultant (^{3}He,t) reaction cross sections were underestimated by about 48% relative to the (p,n) cross sections at 200 MeV. In order to avoid this difficulty, we decided to reduce the strength of the imaginary potentials for ^{3}He and t by 30%.

In the structure part of the calculations, use was made of a residual particle-hole interaction, which is a simple zero-range interaction of the $(\sigma\cdot\sigma)(\tau\cdot\tau)$ type. The strength of the interaction was adjusted so that the calculated resonance energy reproduces the experimental value. The single particle potentials for p and the hole state h were taken from our previous studies of Refs. [22,36]. Finally, we remark that in obtaining the final calculated cross sections, we introduced overall normalization factors of 0.20 and 0.84, respectively, for the ^{12}C and ^{90}Zr target cases. The significant reduction of 0.20 is needed for the ^{12}C target case, since our calculation assumes ^{12}C to be a $p_{3/2}$ closed shell nucleus [37]. The normalization constant 0.84 for ^{90}Zr may be ascribed to the spreading of the GT strength to higher excitation energies [14]. We note that this introduction of rather artificial overall normalization factors fortunately does not affect the D_{nn} values to be presented in the next subsection: In fact we confirmed that the use of the more realistic Cohen-Kurath wave functions [38] for the initial and final states for the ^{12}C target case leads to a cross section which in fact agrees well with the experimental data, and also predicts essentially the same D_{nn} value as obtained in the present study.

The cross sections thus calculated are tabulated in Table I together with the experimental data. As is seen, with the above choice of parameters, the experimental cross sections are fairly well reproduced by the calculations. We note, however, that there are still fairly large uncertainties involved in the experimental cross sections; cross sections reported in other references than those cited there (the references cited in Table I are the most recent ones) do not necessarily agree, even within the experimental uncertainties given in the table.

As is well known, there are two orbital angular momentum transfers $l_t=0$ and 2 possible for the $j_t^\pi=1^+$ transition and these two contribute incoherently to the cross section. It is remarkable that the contribution from the $l_t=2$ component is almost negligible in the (p,n) cross section, but definitely non-negligible in the (^{3}He,t) cross section. This enhanced contribution from the $l_t=2$ transition in the (^{3}He,t) reaction was understood as arising from the stronger absorptive nature of the (^{3}He,t) reaction compared to the (p,n) reaction [17]. As will be discussed later, the $l_t=2$ component plays a crucial role in the deviation of the $D_{nn}(0°)$ value from the $-1/3$ that is obtained when only the $l_t=0$ component is taken into account in the calculation.

B. $D_{nn}(0°)$ values

Table II summarizes the calculated $D_{nn}(0°)$ values. The calculated values are shown for three different cases, where both $l_t=0$ and 2 are taken into account (full calculation), both are taken into account but the tensor exchange effect is neglected, and only $l_t=0$ is taken into account in the calculation. For the purpose of later discussions, we shall denote the calculated $D_{nn}(0°)$ values for these three different cases as $D_{nn}^{l_t=0,2}$, $D_{nn}^{l_t=0,2,\ \text{no TE}}$, and $D_{nn}^{l_t=0}$, respectively. The experimental data [2,13,14] are available for the $(\vec{p},\vec{n})$ reactions for both targets and energies and are included in Table II. The data are, however, not available at present for the $(^3\vec{\text{He}},\vec{t})$ reaction. It is seen that the calculated final $D_{nn}(0°)$ values $[D_{nn}^{l_t=0,2}(0°)]$ for the $(\vec{p},\vec{n})$ reactions fit the data very well for both targets and incident energies. A similar agreement has also been reported earlier by Wakasa *et al.* [13,14]. We now discuss some of characteristic features of the results.

TABLE II. Calculated $D_{nn}(0°)$ values in comparison with the experimental values.

Target	E (MeV)		$(\vec{p},\vec{n})$ Cal	$(\vec{p},\vec{n})$ Exp	Ref.	$(^3\vec{\text{He}},\vec{t})$ Cal
^{12}C	200	$D_{nn}^{l=0}$	-0.33			-0.33
		$D_{nn}^{l=0,2\text{no TE}}$	-0.27			-0.23
		$D_{nn}^{l=0,2}$	-0.25	-0.26 ± 0.07	[2]	-0.20
	300	$D_{nn}^{l=0}$	-0.33			-0.33
		$D_{nn}^{l=0,2\text{no TE}}$	-0.27			-0.24
		$D_{nn}^{l=0,2}$	-0.23	-0.22 ± 0.02	[13]	-0.19
^{90}Zr	200	$D_{nn}^{l=0}$	-0.33			-0.33
		$D_{nn}^{l=0,2\text{no TE}}$	-0.28			-0.21
		$D_{nn}^{l=0,2}$	-0.30	-0.30 ± 0.06	[2]	-0.24
	300	$D_{nn}^{l=0}$	-0.33			-0.33
		$D_{nn}^{l=0,2\text{no TE}}$	-0.26			-0.16
		$D_{nn}^{l=0,2}$	-0.23	-0.20 ± 0.03	[14]	-0.13

First of all, we note that the $D_{nn}^{l_t=0}(0°)$ values obtained by taking into account only the $l_t=0$ contribution takes always the value of $-1/3$ $(=-0.33)$, the value previously predicted by PWIA results [39,40]. As the present results show, the value remains the same even if distortion is included in the calculation. We shall show in what follows that the result is a rather general consequence of the assumption that the reaction proceeds as a single $l_t=0$ process. The result is thus independent of the details of the reaction mechanism and nuclear structure.

In order to see this, we first reduce the expression for D_{nn} given by Eq. (2.3) to a more usual form of DWIA. This is achieved, if one retains only the contribution from one particular final state involved in the sum (trace) in Eq. (2.3). The resultant D_{nn} may be given as [41]

$$D_{nn}=-\frac{|T_z|^2}{|T_x|^2+|T_y|^2+|T_z|^2}, \tag{3.1}$$

where T_i is the transition amplitude and is explicitly given as $T_i=\langle\Phi_{j_tm_t}\phi_b\chi_b^-|\hat{T}_i|\Phi_0\phi_a\chi_a^+\rangle$. Here $\Phi_{j_tm_t}$ describes the final resonance state. Note that in obtaining the above expres-

sion, use was made of the fact that the final state $\Phi_{j_t m_t}$ is an unnatural parity state and hence the transition amplitude T_0 vanishes.

The transition amplitude T_i can be expressed in terms of a more conventional DWIA transition amplitude $T_{l_t m_{l_t}}$ specified by the transferred orbital angular momenta (l_t, m_{l_t}) [see Eq. (2.46)] as

$$T_x = \frac{1}{\sqrt{2}} \sum_{l_t, m_{l_t}} [(1-1 j_t m_{j_t} | l_t m_{l_t}) T_{l_t m_{l_t}} - (11 j_t m_{j_t} | l_t m_{l_t}) T_{l_t m_{l_t}}],$$

$$T_y = \frac{-i}{\sqrt{2}} \sum_{l_t, m_{l_t}} [(1-1 j_t m_{j_t} | l_t m_{l_t}) T_{l_t m_{l_t}} + (11 j_t m_{j_t} | l_t m_{l_t}) T_{l_t m_{l_t}}],$$

$$T_z = \sum_{l_t, m_{l_t}} (10 j_t m_{j_t} | l_t m_{l_t}) T_{l_t m_{l_t}}. \tag{3.2}$$

At $\theta = 0°$, only the component with $m_{l_t} = 0$ is nonvanishing. The expressions for T_i are then simplified and their resultant squares can be expressed as

$$|T_x|^2 = |T_y|^2 = \left| \sum_{l_t} (1-1 j_t 1 | l_t 0) T_{l_t 0} \right|^2,$$

$$|T_z|^2 = \left| \sum_{l_t} (10 j_t 0 | l_t 0) T_{l_t 0} \right|^2. \tag{3.3}$$

Inserting the above expression into Eq. (3.1), $D_{nn}(0°)$ is rewritten as

$$D_{nn}(0°) = - \frac{\left| \sum_{l_t} (10 j_t 0 | l_t 0) T_{l_t 0} \right|^2}{2 \left| \sum_{l_t} (1-1 j_t 1 | l_t 0) T_{l_t 0} \right|^2 + \left| \sum_{l_t} (10 j_t 0 | l_t 0) T_{l_t 0} \right|^2}. \tag{3.4}$$

If one then keeps only the contribution from either $l_t = j_t - 1$ or $l_t = j_t + 1$, the above equation is reduced to

$$D_{nn}(0°) = - \frac{j_t}{2 j_t + 1}, \quad j_t = l_t + 1,$$

$$= - \frac{j_t + 1}{2 j_t + 1}, \quad j_t = l_t - 1, \tag{3.5}$$

from which it follows for the GT transition with $j_t = 1$ and $l_t = 0$ case that

$$D_{nn}(0°) = -1/3. \tag{3.6}$$

Equation (3.5) is exactly the same as that derived before, based on a PWIA [39,40] and also predicts the D_{nn} values for other spin flip, e.g., $(l_t j_t^{\pi}) = (10^-)$, (12^-), and (23^+) transitions as -1, $-2/5$, and $-3/7$, respectively. Any deviation from these values is then ascribed to admixture of other l_t components. Note that for the 0^- state, there exists only one possible $l_t = 1$. Therefore in this case, the $D_{nn}(0°)$ value always takes the value of -1, indicating that the spin flip probability is unity.

Let us now turn to $D_{nn}^{l_t=0,2}$ (full calculation) and $D_{nn}^{l_t=0,2 \text{ no TE}}$, both including the contributions from $l_t = 2$ processes. It is seen that both deviate significantly from the $-1/3$ $l_t = 0$ value; the absolute magnitudes are smaller by 10–60 % (though the sign remains negative). It is also remarkable that the deviation is much larger for the $(^3\vec{\text{He}}, \vec{t})$ reaction than in $(\vec{p}, \vec{n})$. Using Eq. (3.4), we now derive a theoretical expression for $D_{nn}(0°)$ that includes the contribution from the $l_t = 2$ component.

In doing this, use may be made of the fact that the ratio $|T_{20}|/|T_{00}|$ is much smaller than unity and thus T_{20}/T_{00} can be treated as a small parameter. It is easy then to find that up to the first order in T_{20}/T_{00}, $D_{nn}(0°)$ is given as

$$D_{nn}(0^+ \to 1^+; \theta = 0^o) = -\frac{1}{3} + \Delta D_{nn}, \tag{3.7}$$

where

$$\Delta D_{nn}(0°) \approx \frac{2\sqrt{2}}{3} \text{Re}\left[\frac{T_{20}}{T_{00}}\right] = \frac{2\sqrt{2}}{3} \frac{|T_{20}|}{|T_{00}|} \cos\vartheta = 0.94 \frac{|T_{20}|}{|T_{00}|} \cos\vartheta. \tag{3.8}$$

Here ϑ is the phase angle between T_{00} and T_{20}. Equations (3.7) and (3.8) tell us indeed that the deviation of the $D_{nn}(0°)$ value from $-1/3$ is caused by the $l_t = 2$ component and that $\Delta D_{nn}(0°)$ is a very good signature of the $l_t = 2$ component in the excitation of the GT resonance.

It is worthwhile to remark here that the above results are valid if one includes the spin-orbit force in the distorting potentials. As has been shown [18,19], the $D_{nn}(0°)$ value is little affected by the spin-orbit force. We may understand the reason for this in the results obtained above that the value is essentially determined by the relative value of the $l_t = 0$ and 2 components of the transition amplitudes, which are expected to be rather insensitive to the spin-orbit force.

Before proceeding, we note that the direct (T_{20}^D) and exchange (knockon-exchange) (T_{20}^E) terms often contribute comparatively to T_{20}. We may thus write T_{20} as $T_{20} = T_{20}^D + T_{20}^E$. Further these T_{20}^D and T_{20}^E include contributions coming from the central and tensor parts of the effective interaction. For the later purpose of discussion, we denote here the contribution from the tensor interaction to T_{20}^E as T_{20}^{TE}.

Sometime ago Wakasa *et al.* [13,14] and others [7] pointed out that the tensor exchange amplitude T_{20}^{TE} plays a crucial role in explaining the observed $D_{nn}(0°)$ values. We

TABLE III. Calculated $\Delta D_{nn}^{\mathrm{TE}}$, $\Delta D_{nn}^{D+\mathrm{CE}}$, and ΔD_{nn} values.

Target	E/a(MeV)	$(\vec{p},\vec{n})$			$(^3\vec{\mathrm{He}},\vec{t})$		
		$\Delta D_{nn}^{\mathrm{TE}}$	$\Delta D_{nn}^{D+\mathrm{CE}}$	ΔD_{nn}	$\Delta D_{nn}^{\mathrm{TE}}$	$\Delta D_{nn}^{D+\mathrm{CE}}$	ΔD_{nn}
^{12}C	200	0.02	0.06	0.08	0.03	0.10	0.13
	300	0.04	0.06	0.10	0.05	0.09	0.14
^{90}Zr	200	−0.02	0.05	0.03	−0.03	0.12	0.09
	300	0.03	0.07	0.10	0.03	0.17	0.20

may extract the contribution $\Delta D_{nn}^{\mathrm{TE}}$, from this tensor exchange amplitude T_{20}^{TE} to the total $\Delta D_{nn}(0°)$ by taking the difference between $D_{nn}^{l=0,2}$ and $D_{nn}^{l=0,2\ \mathrm{no\ TE}}$ in Table II, i.e., as $\Delta D_{nn}^{\mathrm{TE}}=D_{nn}^{l=0,2}-D_{nn}^{l=0,2\ \mathrm{no\ TE}}$. The contribution from the rest of the amplitudes, the direct and central exchange amplitudes, which we shall denote as $\Delta D_{nn}^{D+CE}(0°)$, may also be estimated as $\Delta D_{nn}^{D+\mathrm{CE}}=D_{nn}^{l=0,2\ \mathrm{no\ TE}}-D_{nn}^{l=0}$. To see the size of the contributions from the tensor exchange and the direct + central exchange, we summarize in Table III the extracted $\Delta D_{nn}^{\mathrm{TE}}(0°)$ and $\Delta D_{nn}^{D+\mathrm{CE}}(0°)$ values. We also present the sum, i.e., the total deviation $\Delta D_{nn}(0°)$. As seen from Table III, $\Delta D_{nn}^{\mathrm{TE}}(0°)$ explains a large portion (15–40 %) of the total $\Delta D_{nn}(0°)$, but always contributes less than 50% for the reactions considered in the present study. In our previous study [41], we investigated the relative contributions from the direct and exchange processes, particularly the Q dependence. It was observed that the direct contribution depends very strongly on the Q value; it is small for small Q, but almost linearly increases with Q. Therefore in the small Q region, the exchange contribution dominates, but the situation is reversed in the large Q region.

Another remark that we wish to make here is that the deviations $\Delta D_{nn}^{\mathrm{TE}}$ and $\Delta D_{nn}^{D+\mathrm{CE}}$ are positive for most of the cases considered, thus contributing to a reduction of the absolute magnitude of the resultant $D_{nn}(0°)$ value. The positive value implies that the phase difference ϑ of the amplitudes T_{00} and T_{20} is less than 90°. The nonvanishing phase difference comes from the complex character of the LF effective interaction and the distortion effect. In Ref. [41] we have also studied the complex character resulting from the effective interaction and distortion: it was found that the effective interaction is dominated by its real part, so that effective interactions do not result in a large imaginary part, except for the tensor exchange interaction, which is dominated by its imaginary part. The distortion certainly introduces an imaginary part, but it makes equal contributions to T_{00} and T_{20}, so that the effect is cancelled out in the ratio. Therefore, T_{20}/T_{00} tends to a real number, either positive or negative. As seen in Table III, both $\Delta D_{nn}^{\mathrm{TE}}(0°)$ and $\Delta D_{nn}^{D+\mathrm{CE}}(0°)$ are positive, except for $\Delta D_{nn}^{\mathrm{TE}}$ for the 200 MeV ^{90}Zr target case where we find negative values of −0.02 and −0.03, respectively for both (p,n) and $(^3\mathrm{He},t)$ reactions. The sign is, however, reversed at 300 MeV, implying that there is a subtle dependence of $\Delta D_{nn}^{\mathrm{TE}}(0°)$ on some nuclear structure and reaction factors. For the reaction factor, Wakasa *et al.* [13] found a rather subtle sensitivity of $\Delta D_{nn}^{\mathrm{TE}}(0°)$ on the effective interaction; the LF interactions at 270 and 325 MeV lead to $D_{nn}(0°)$ values that are notably different from one another. In Ref. [41], we present a detailed study of T_{20}^{TE} values, demonstrating that the subtle dependence on the nuclear structure and effective interaction results in as a consequence of a cancellation involved in a radial integration of essentially a product of the hole wave function and the effective interaction.

Another remarkable feature seen in Table II is that the ΔD_{nn} values for the $(^3\vec{\mathrm{He}},t)$ reactions are considerably larger than for the $(\vec{p},\vec{n})$ reaction. This feature may be understood from the fact that the $l_t=2$ contribution becomes larger in $(^3\vec{\mathrm{He}},t)$ than in $(\vec{p},\vec{n})$, due to the stronger absorption involved in the $(^3\vec{\mathrm{He}},t)$ reaction as already remarked earlier. A tendency of increase of ΔD_{nn} values is also seen as the incident energy per nucleon increases. This tendency may be understood similarly due to the increasing absorption with increasing incident energy, which in turn enhances the contribution from the $l_t=2$ process.

IV. CONCLUDING REMARKS

A formalism is presented for the calculation of the spin transfer cross section, or equivalently the spin transfer coefficient, of intermediate energy charge exchange $(^3\vec{\mathrm{He}},\vec{t})$ and $(\vec{p},\vec{n})$ reactions to the continuum, based on the continuum Tamm-Dancoff method. Explicit account is taken of the knockon-exchange effect and damping of the particle through the imaginary part of the complex optical potential for the excited particle.

Numerical calculations were performed of the exclusive cross section, $\sigma(0°)$, and the spin transfer coefficient, $D_{nn}(0°)$, for the $(\vec{p},\vec{n})$ and $(^3\vec{\mathrm{He}},\vec{t})$ reactions on ^{12}C and ^{90}Zr targets, leading to the GT 1^+ resonance states. The $D_{nn}(0°)$ values calculated by using the parameters that are determined by fitting the calculated $\sigma(0°)$ to experiment are found to agree with the experimental data very well. Some of the characteristic features of the D_{nn}-values were studied. It was shown that $D_{nn}(0°)$ takes a value of $-1/3$, regardless of the details of nuclear structure and reaction mechanisms, if the reaction is assumed to proceed as an orbital angular momentum transfer $l_t=0$ process and that the deviation, $\Delta D_{nn}(0°)$, from $-1/3$ comes from the admixture of $l_t=2$ component. A simple closed form of the expression was derived, which clearly and directly shows that the deviation of the $D_{nn}(0°)$ value from $-1/3$ comes from the effect of the

$l=2$ component of the transition. The deviation $\Delta D_{nn}(0°)$ may thus represent a good signature of the $l=2$ component in the GT transition.

Studies were also made of effects of the knockon-exchange processes mediated by the tensor component in the effective interaction on $\Delta D_{nn}(0°)$, and we found that the effect generally reduces the absolute magnitude of $D_{nn}(0°)$ by 15–40 %. The studies also revealed that the deviation $\Delta D_{nn}(0°)$ is significantly larger for the $(^3\vec{\mathrm{He}},\vec{t})$ reaction than for the $(\vec{p},\vec{n})$ reaction. Unfortunately, experimental data for $(^3\vec{\mathrm{He}},\vec{t})$ are not available at this moment for testing this theoretical prediction; however, it is expected that data will be available in the near future [16].

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