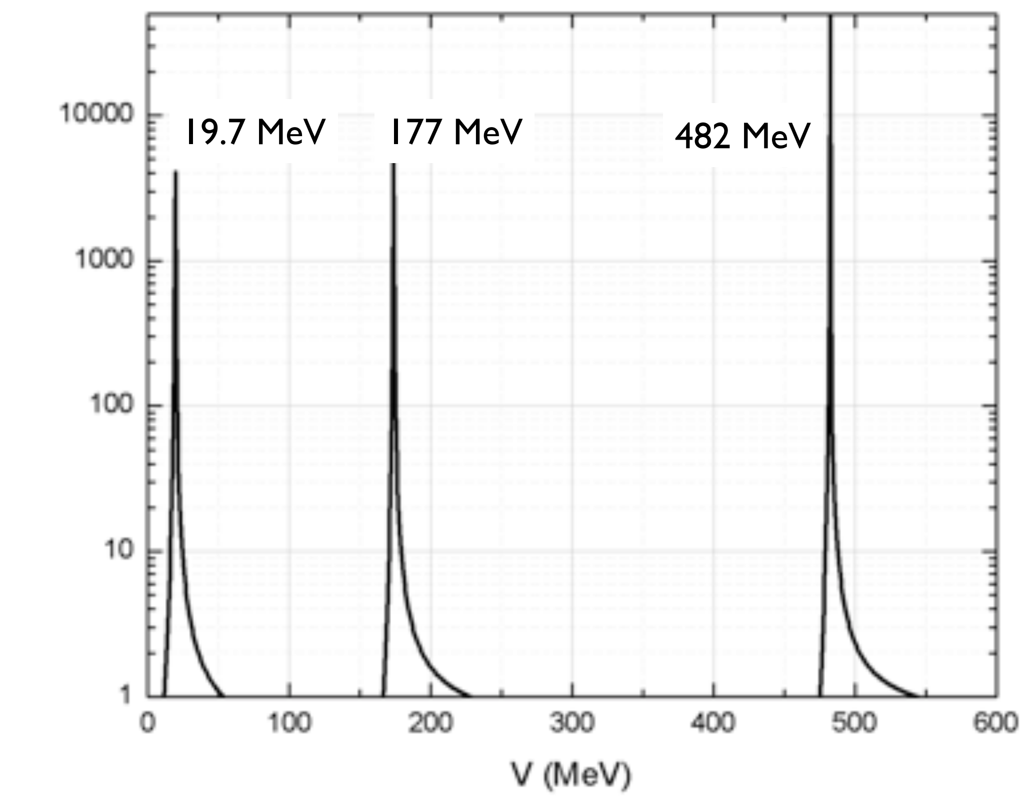
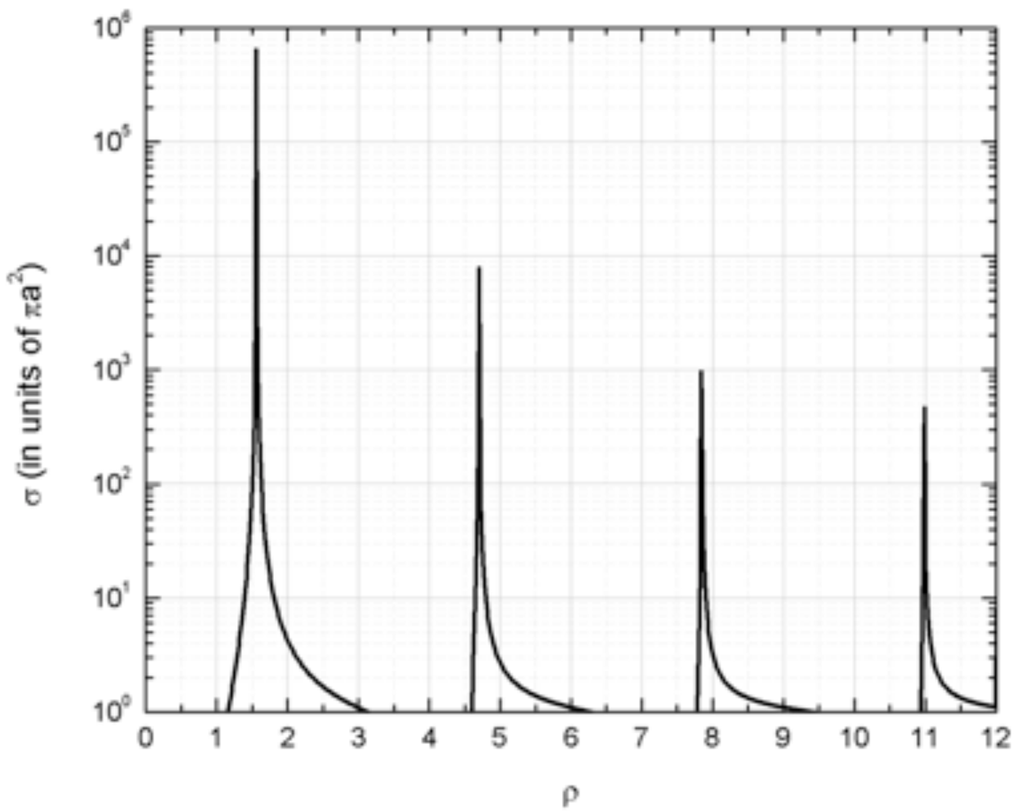
Homework #3.1

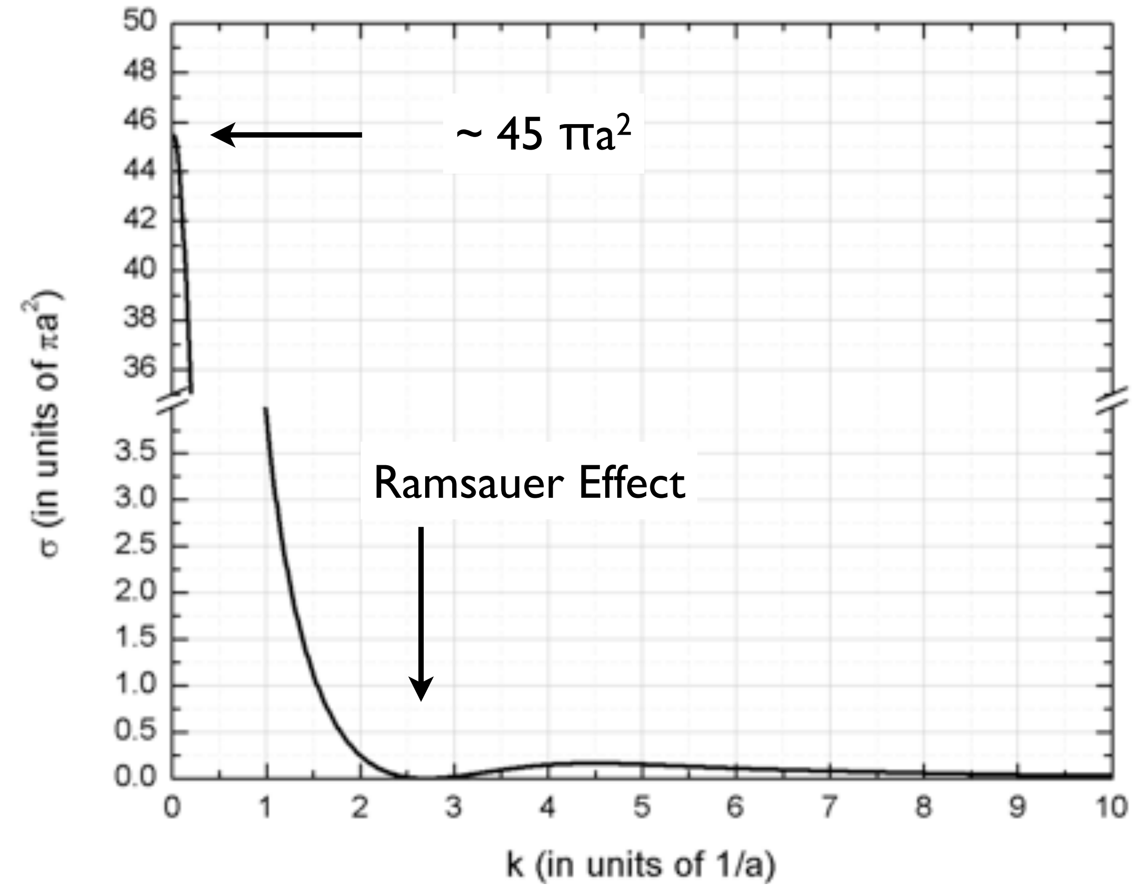
Draw σ vs. V_0 at low energy limit. Confirm that, if we set $\rho = 4.8 \sim Ka$, $\sigma(0) \sim 45\pi a^2$ with two bound states.
Discuss physics of the graph.





Homework #3.2

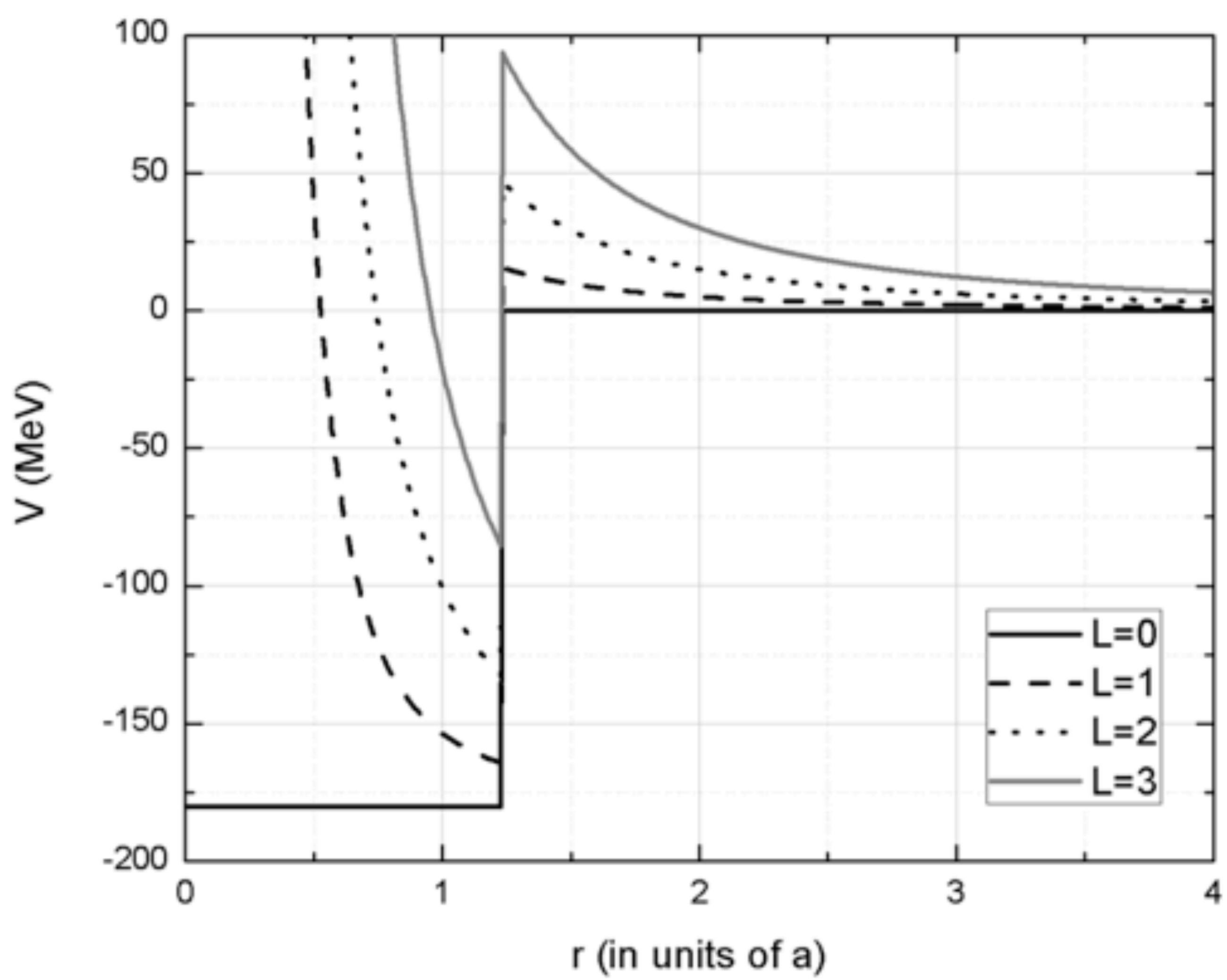
Draw σ vs. hbk for a square well potential, ρ = 4.8 and confirm a transparency.



Homework #3.3

Draw V^{eff} (l=0, 1, 2, 3) for a square well with ρ = 4.8. Obtain the bound states with l=0, 1, 2, 3.

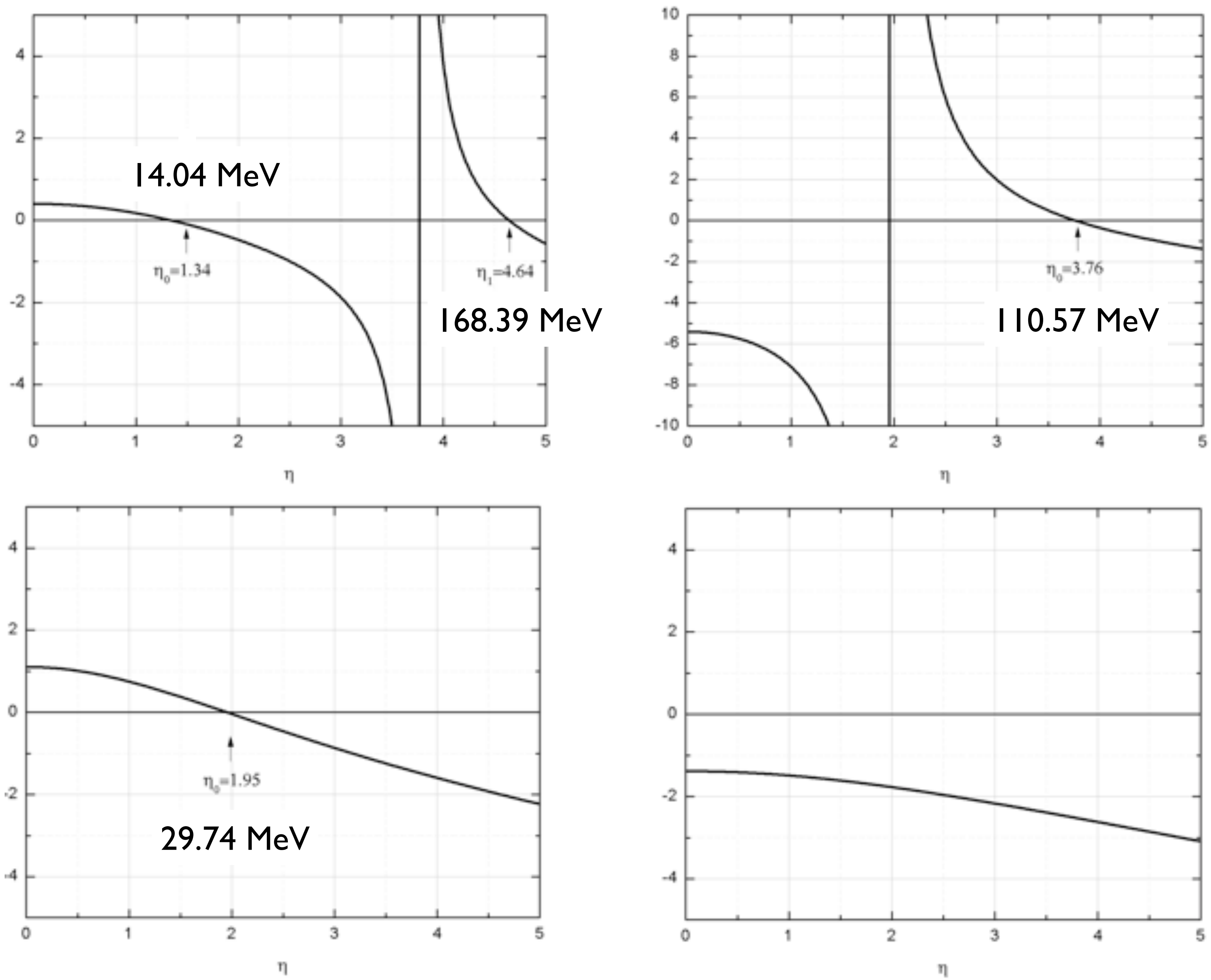
$$V_{eff} = -V_0 + \frac{\hbar^2\, l\,(l+1)}{2\,\mu\, r^2}$$



Homework #3.3

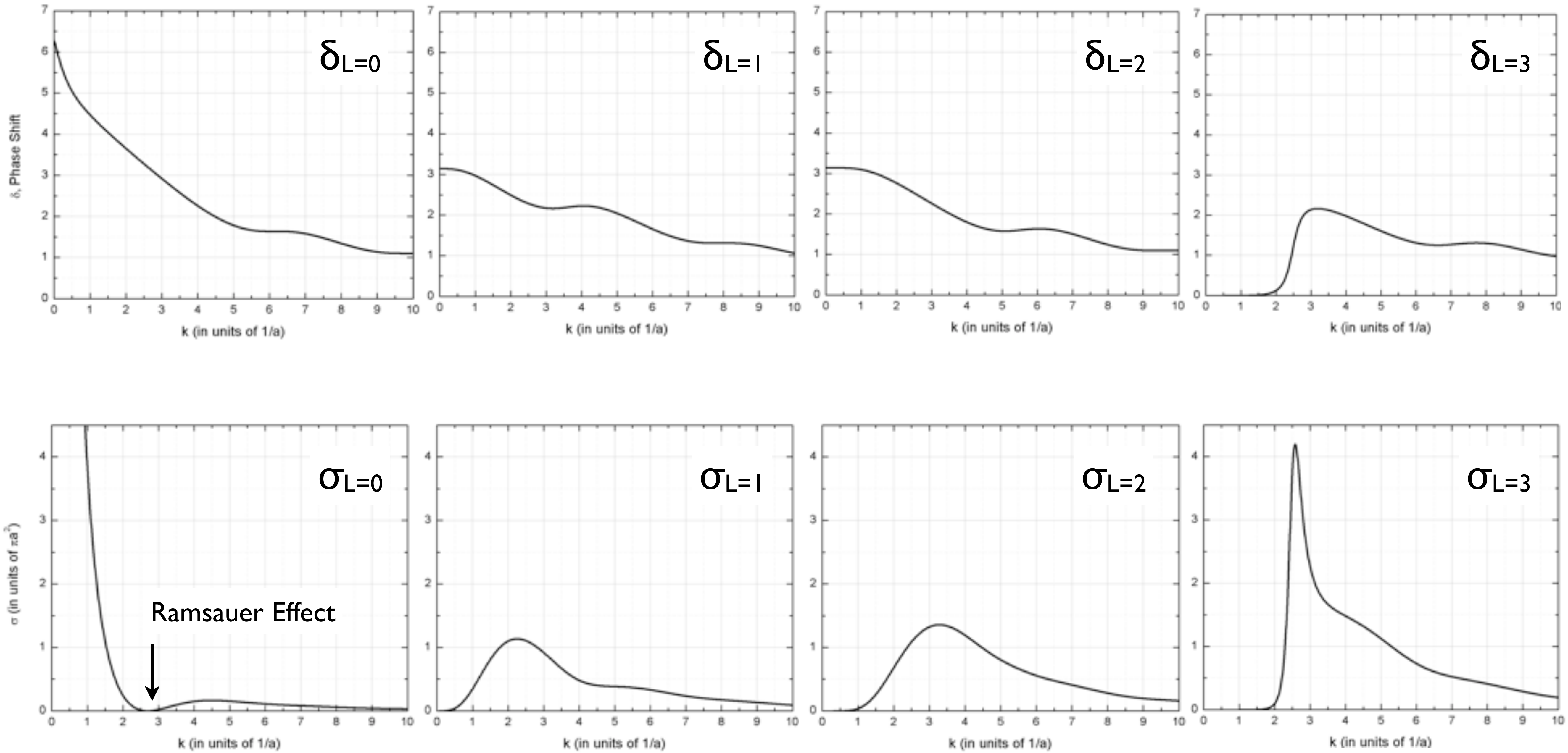
Draw V^{eff} (l=0, 1, 2, 3) for a square well with ρ = 4.8. Obtain the bound states with l=0, 1, 2, 3.

$$\frac{1}{h_L^{(+)}[i\kappa r]}\frac{d}{dr}h_L^{(+)}[i\kappa r]\,|_{r=a} = \frac{1}{j_L\,(Kr)}\frac{d}{dr}j_L[Kr]\,|_{r=a}$$



Homework #3.4

Obtain the partial cross sections with l= 0, 1, 2, 3 for a square well potential, $\rho = 4.8$ in terms of hka.



HOMEWORK #3

```
a0 = 2.3 × 10^-15 ; (* Radius : m *)
ℏc = 197. × 10^-15; (* MeV m *)
mE = 938. / 2; (* Reduced mass : MeV *)
E0 = 1 × 10^-9; (* Energy : MeV *)
ρ = 4.8;
```

$$\texttt{KvsV[V\_]} = \sqrt{\frac{2\,m_E\,(E_0 + V)}{\hbar c^2}};$$

$$\texttt{Kvsk[k\_]} = \sqrt{\frac{\rho^2}{a_0{}^2} + k^2};$$

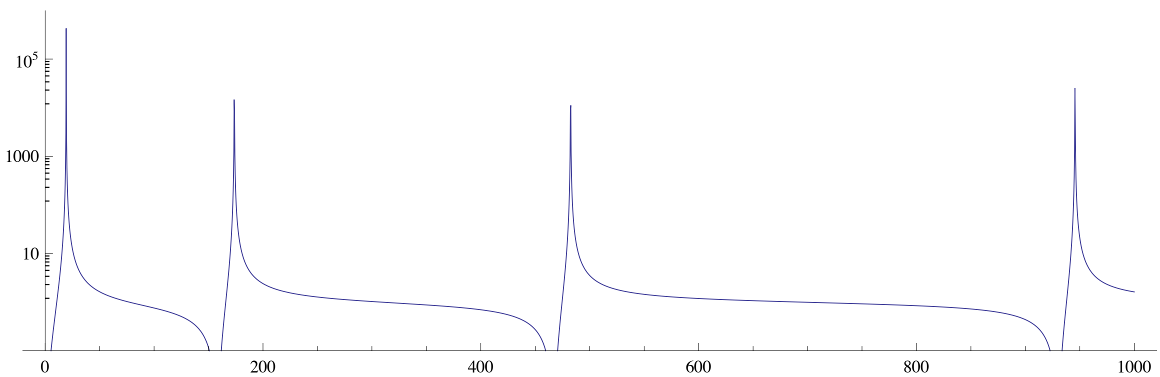
Homework #3.1

Draw σ vs. V_0 at low energy limit. confirm that, if we set $\rho = 4.8 \sim Ka$, $\sigma(0) \sim 45\,\pi a^2$ with two bound states. Discuss physics of the graph.

"Cross section as function of potential depth in units of πa^2";

$$\sigma_{low}[\texttt{V\_}] = \left(\frac{\texttt{Tan[KvsV[V]}\ a_0\texttt{]}}{\texttt{KvsV[V]}\ a_0} - 1\right)^2;$$

```
LogPlot[σlow[V], {V, 0, 1000}, PlotRange → {10^-1, 10^6},
 ImageSize → {600, 200}, AspectRatio → 0.3]
```

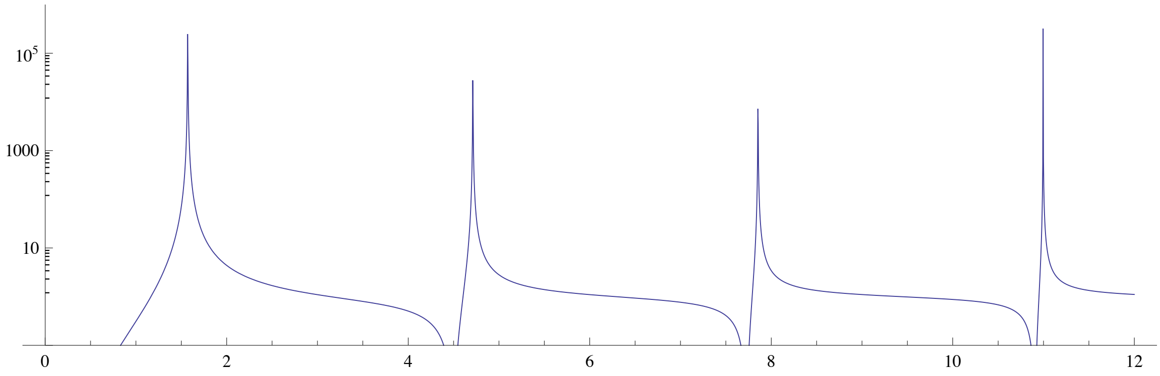


```
"Cross section as a function of ρ in units of πa²"
```

$\sigma\rho_{low}[\rho\_] = \left(\frac{\mathrm{Tan}[\rho]}{\rho} - 1\right)^2;$

```
LogPlot[σρlow[ρ], {ρ, 0, 12}, PlotRange → {10^-1, 10^6},
 ImageSize → {600, 200}, AspectRatio → 0.3]
```

Cross section as a function of ρ in units of πa^2



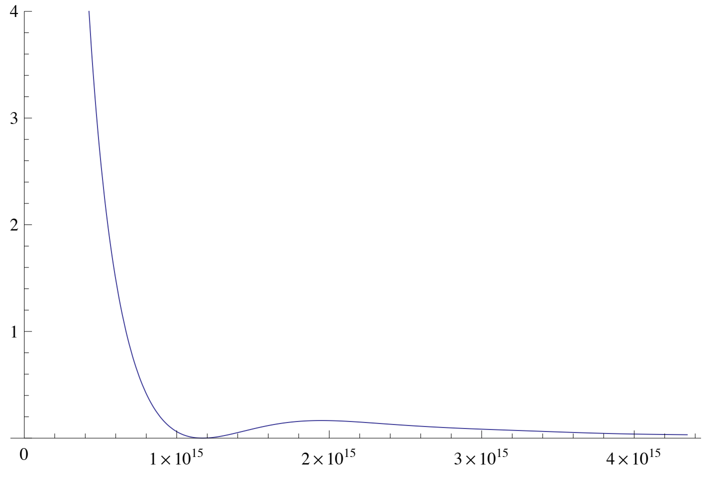
Homework #3.2

Draw σ vs. $\hbar$k for a square well potential, ρ = 4.8 and confirm a transparency.

$\delta[k\_] = \mathrm{ArcTan}\left[\frac{(k / \mathrm{Kvsk}[k])\,\mathrm{Tan}[\mathrm{Kvsk}[k]\,a_0] - \mathrm{Tan}[k\,a_0]}{1 + (k / \mathrm{Kvsk}[k])\,\mathrm{Tan}[\mathrm{Kvsk}[k]\,a_0]\,\mathrm{Tan}[k\,a_0]}\right];$

$\sigma k[k\_] = \frac{1}{\pi\,a_0{}^2}\,\frac{4\,\pi}{k^2}\,\mathrm{Sin}[\delta[k]]^2;$

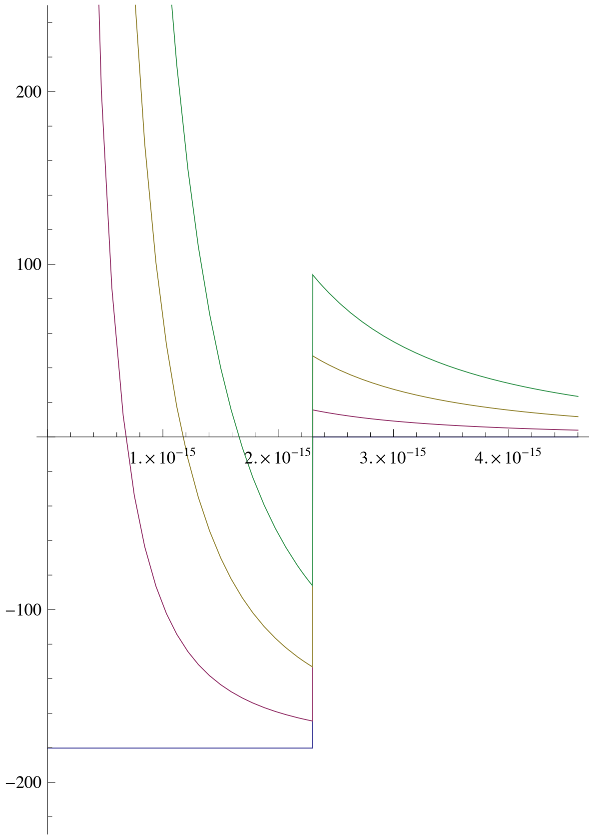
$\mathrm{Plot}\left[\sigma k[k], \left\{k, 0, \frac{10}{a_0}\right\}, \mathrm{PlotRange} \to \{0, 4\}\right]$



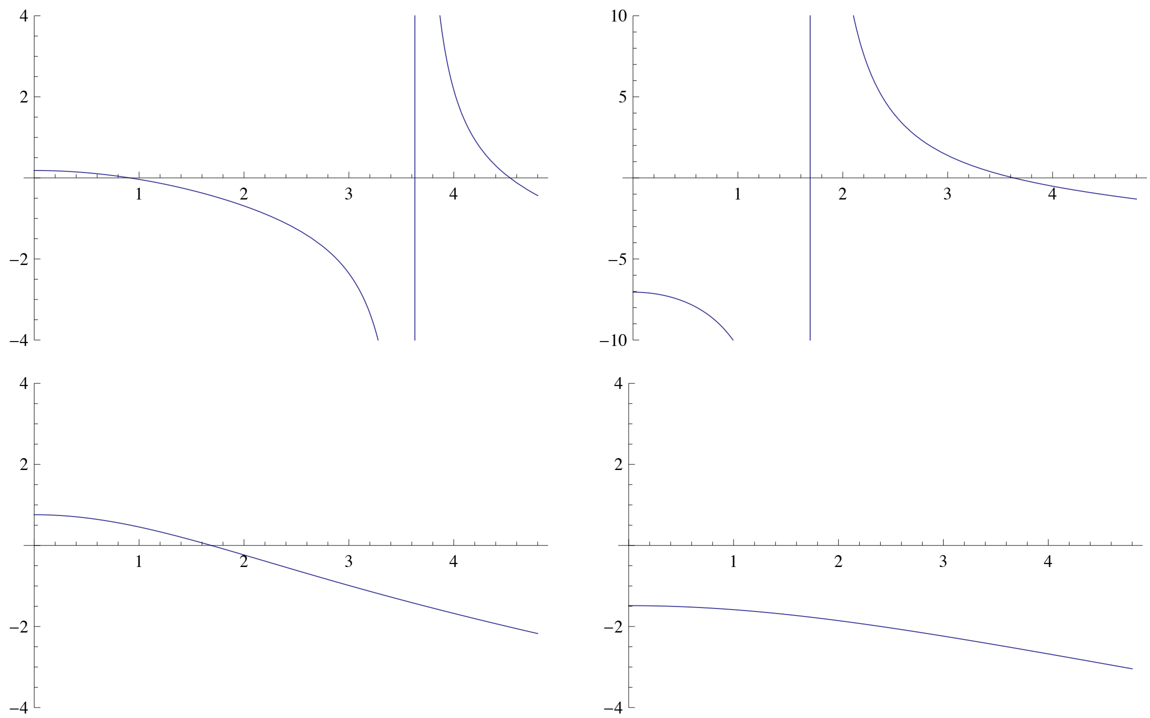
Homework #3.3

Draw V_{eff} (L = 0, 1, 2, 3) for a square well with ρ = 4.8. Obtain the bound states with L = 0, 1, 2, 3.

```
V0 = ρ^2 ħc^2 / (2 mE a0^2);

Veff[L_, r_] := If[r < a0, -V0 + ħc^2 L (L + 1) / (2 mE r^2), ħc^2 L (L + 1) / (2 mE r^2)]

Plot[{Veff[0, r], Veff[1, r], Veff[2, r], Veff[3, r]},
 {r, 0, 2 * a0}, PlotRange → {-V0 - 50, 250}, AspectRatio → 1.5]
```



```
"Regular function - Spherical Bessel function";
jL[l_, k_, x_] = SphericalBesselJ[l, x k];
"It's derivative at a boundary r=a0";
dj[l_, k_] := D[jL[l, k, x], x] /. x → a0;

"Spherical Hankel function hL(x) = yL(x) + i nL(x)";
hL[l_, k_ , x_] = SphericalHankelH1[l, I k x];
"It's derivative at a boundary r=a0";
dh[l_, k_] := D[hL[l, k, x], x] /. x → a0;

"Outer solution";
outer[l_, k_] := dh[l, k] / hL[l, k, a0];
"Inner solution";
inner[l_, K_] := dj[l, K] / jL[l, K, a0];
F0[l_, η_] := Re[outer[l, η] - inner[l, (1/a0) Sqrt[ρ^2 - η^2]]] (1/10^15)
GraphicsGrid[{{Plot[F0[0, η], {η, 0, ρ}, PlotRange → {-4, 4}],
    Plot[F0[1, η], {η, 0, ρ}, PlotRange → {-10, 10}]},
   {Plot[F0[2, η], {η, 0, ρ}, PlotRange → {-4, 4}],
    Plot[F0[3, η], {η, 0, ρ}, PlotRange → {-4, 4}]}}, ImageSize → {600, 400}]
```



Homework #3.4

Obtain the partial cross sections with L = 0, 1, 2, 3 for a square well potentiia, $\rho = 4.8$ in terms of $\hbar ka$

```
jL[l_, x_] := SphericalBesselJ[l, x];
dj[l_, X_] := D[jL[l, x], x] /. x → X;
nL[l_, x_] := SphericalBesselY[l, x];
dn[l_, X_] := D[nL[l, x], x] /. x → X;
g[l_, k_] := Kvsk[k]/k dj[l, Kvsk[k] a0]/jL[l, Kvsk[k] a0];
δL[l_, k_] := ArcTan[(dj[l, k a0] - g[l, k] jL[l, k a0])/(dn[l, k a0] - g[l, k] nL[l, k a0])];
δm[l_, k_] = If[δL[l, k] < 0, δL[l, k] + π, δL[l, k]];
δm2[k_] = If[k < FindRoot[δm[0, k] == 0, {k, 10^15}][[1, 2]], δm[0, k] + π, δm[0, k]];
σL[l_, k_] := 4 π/k^2 (2 l + 1) Sin[δL[l, k]]^2;
GraphicsGrid[{{Plot[δm2[k], {k, 0, 10/a0}, PlotRange → {0, 2 π}], Plot[δm[1, k], {k, 0, 10/a0},
    PlotRange → {0, 2 π}], Plot[δm[2, k], {k, 0, 10/a0}, PlotRange → {0, 2 π}],
   Plot[δm[3, k], {k, 0, 10/a0}, PlotRange → {0, 2 π}]}}, ImageSize → {600, 150}]
GraphicsGrid[{{Plot[σL[0, k]/(π a0^2), {k, 0, 10/a0}, PlotRange → {0, 5}],
   Plot[σL[1, k]/(π a0^2), {k, 0, 10/a0}, PlotRange → {0, 5}],
   Plot[σL[2, k]/(π a0^2), {k, 0, 10/a0}, PlotRange → {0, 5}],
   Plot[σL[3, k]/(π a0^2), {k, 0, 10/a0}, PlotRange → {0, 5}]}}, ImageSize → {600, 150}]
```