Programa de doctorado de Fsica Nuclear

Reacciones Nucleares

http://atomix.us.es/institucional/doctorado/

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Santiago de Compostela 23-27 de marzo de 2009

Direct versus compound reactions

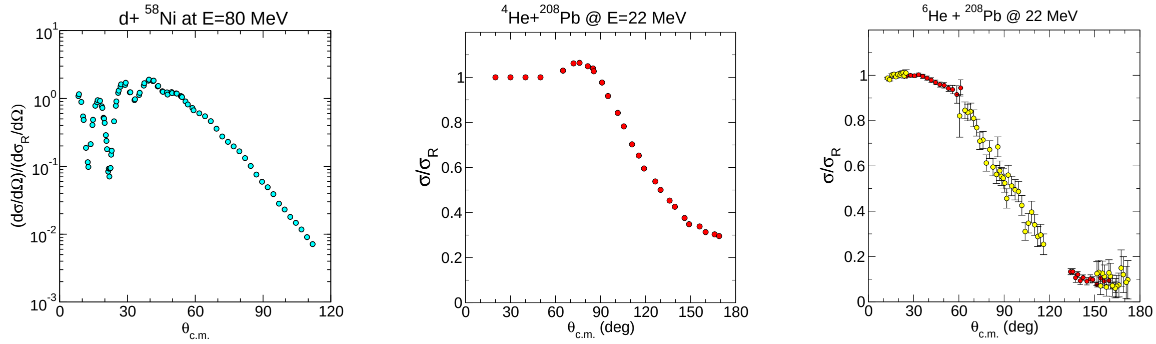
DIRECT: elastic, inelastic, transfer,...

- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about $\pi/2$ (peaked forward)

COMPOUND: complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- "lose of memory" $\Rightarrow$ almost symmetric distributions forward/backward

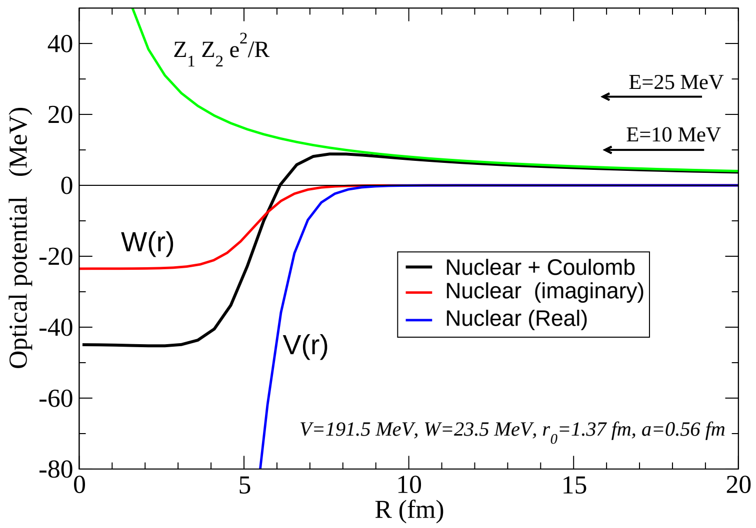
Elastic scattering



What can we learn from an optical model analysis of the elastic cross section?

Elastic scattering: phenomenology

EFFECTIVE PROJECTILE-TARGET INTERACTION:



Elastic scattering: phenomenology

☞ Depending on the bombarding energy E and the charges of the interacting nuclei, we observe different types of elastic scattering.

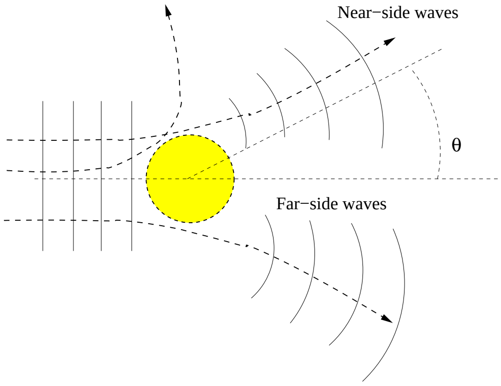
☞ This can be characterized in terms of the Coulomb (or Sommerfeld) parameter:

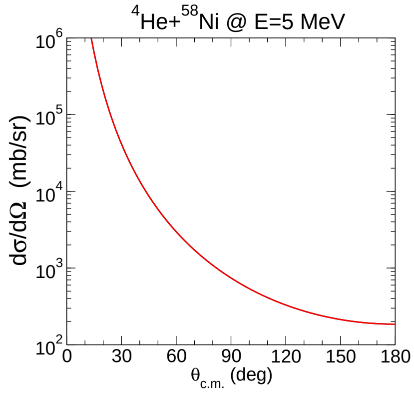
$$\eta = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \hbar v}$$

- E well above the Coulomb barrier ($\eta \lesssim 1$) $\Rightarrow$ Fraunhofer scattering
- E around the Coulomb barrier ($\eta \gg 1$) $\Rightarrow$ Fresnel scattering
- E well below the Coulomb barrier ($\eta \ggg 1$) $\Rightarrow$ Rutherford scattering

Elastic scattering: phenomenology

RUTHERFORD SCATTERING





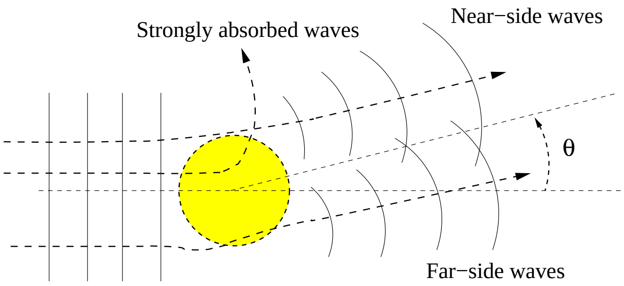
- Purely Coulomb potential ($\eta \ggg 1$)
- Bombarding energy well below the Coulomb barrier
- Obeys Rutherford law:

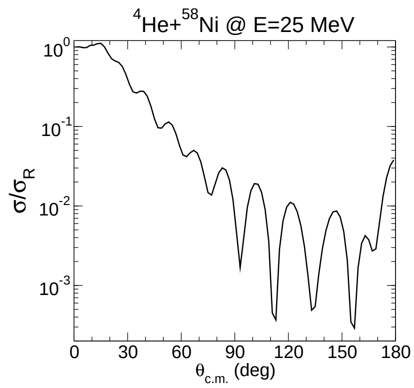
$$\frac{d\sigma}{d\Omega} = \frac{zZe^2}{4E}\frac{1}{\sin^4(\theta/2)}$$

.

Elastic scattering: phenomenology

FRAUNHOFER SCATTERING:

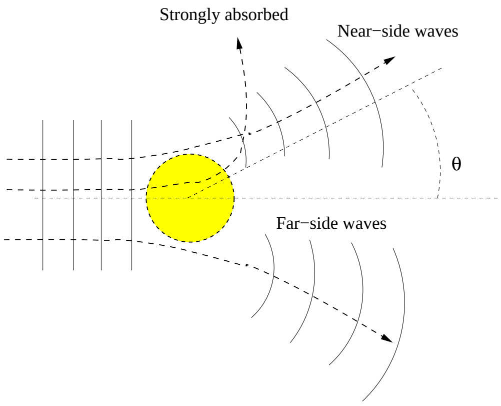


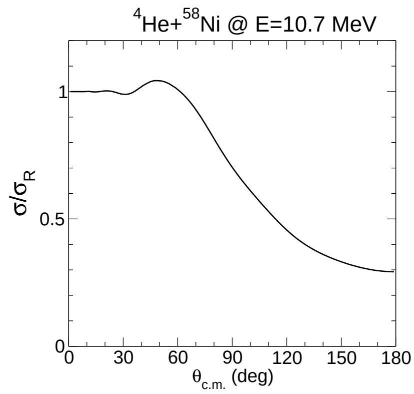


- Bombarding energy well above Coulomb barrier
- Coulomb weak ($\eta \lesssim 1$)
- Nearside/farside interference pattern (difracction)

Elastic scattering: phenomenology

FRESNEL SCATTERING:





- Bombarding energy around or near the Coulomb barrier
- Coulomb strong ($\eta \gg 1$)
- 'Illuminated' region $\Rightarrow$ interference pattern (near-side/far-side)

  'Shadow' region $\Rightarrow$ strong absorption

Elastic scattering: optical model

How does one describe the motion of a particle in quantum mechanics?

- Hamiltonian: $H = T_R + U(R)$

  $U(R)$: optical model $\Rightarrow$ effective projectile-target interaction

- Schrodinger equation: $[H - E]\Psi(\mathbf{R}) = 0$
- Partial wave expansion of the model wavefunction:

$$\Psi(\mathbf{R}) = \sum_{LM} C^{LM} \frac{f^L(R)}{R} Y_{LM}(\hat{R})$$

- $f^L(R)$ obtained as solution of:

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2} + \frac{\hbar^2 L(L+1)}{2\mu R^2} + U(R) - E\right] f^L(R) = 0$$

Elastic scattering: optical model

Numerical procedure:

1. Fix a *matching radius*, R_m, such that $V_{\rm nuc}(R_m) \ll$
2. Integrate $f(R)$ from $R = 0$ up to R_m, starting with the condition:

$$\lim_{R\to 0} f^L(R) = 0$$

3. At $R = R_m$ impose the boundary condition:

$$f^L(R) \to I_L(R) - S_L O_L(R)$$

☞ S_L=*scattering matrix*

☞ I_L *and* O_L *are the so called* *incoming* *and* *outgoing* *waves:*

$$\begin{aligned} I_L(R) &= \frac{1}{\sqrt{v}}(KR)\, h_L^*(KR) \propto e^{-i(KR-\eta\log 2KR)} \\ O_L(R) &= \frac{1}{\sqrt{v}}(KR)\, h_L(KR) \propto e^{i(KR-\eta\log 2KR)} \end{aligned}$$

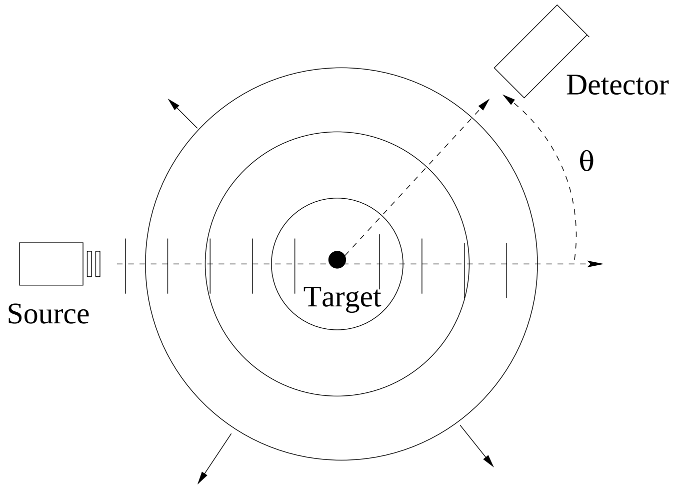
The S-matrix

- S_L =coefficient of the outgoing wave for partial wave L.
- Phase-shifts: $S_L = e^{2i\delta_L}$
- $U(R) = 0 \Rightarrow$ No scattering $\Rightarrow S_L = 1 \Rightarrow \delta_L = 0$
- U real $\Rightarrow |S_L| = 1 \Rightarrow \delta_L$ real

  U complex $\Rightarrow |S_L| < 1 \Rightarrow \delta_L$ complex
- For $L \gg \Rightarrow S_L \to 1$

Elastic scattering: the scattering amplitude

Which one of the many solutions of Schrödinger equation is the one that correspond to a scattering experiment?



$$\boxed{\Psi_{\mathbf{K}_i}(\mathbf{R}) = e^{i\mathbf{K}_i\cdot\mathbf{R}} + \chi^{(+)}_{\mathbf{K}_i}(\mathbf{R})}$$

- Scattering amplitude: $A(\theta)$

$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = e^{i\mathbf{K}_i\cdot\mathbf{R}} + \chi^{(+)}_{\mathbf{K}_i}(\mathbf{R}) \to e^{i\mathbf{K}_i\cdot\mathbf{R}} + A(\theta)\frac{e^{iK_iR}}{R}$$

- Partial wave decomposition:

$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = \frac{1}{R}\sum_{LM} C^{LM} f^L(R) Y_{LM}(\hat{R}) \to \frac{1}{R}\sum_{LM} C^{LM}\left[I_L(R) - S_L O_L(R)\right] Y_{LM}(\hat{R})$$

- Incident plane wave:

$$\begin{aligned} e^{i\mathbf{K}_i\cdot\mathbf{R}} &= \sum_{LM} 4\pi Y^*_{LM}(\hat{K}_i) i^L Y_{LM}(\hat{R}) j_L(K\ R) \\ &= \sum_{LM} \frac{2\pi i\sqrt{v}}{KR} Y^*_{LM}(\hat{K}_i) i^L Y_{LM}(\hat{R})\left[I_L(R) - O_L(R)\right] \end{aligned}$$

- Outgoing spherical waves:

$$\chi^{(+)}_{\mathbf{K}_i}(\mathbf{R}) \to \sum_{LM} \frac{2\pi i\sqrt{v}}{KR} Y^*_{LM}(\hat{K}_i)(1 - S^L) Y_{LM}(\hat{R}) O_L(R)$$

Scattering amplitude and cross sections

- Scattering amplitude:
  - Nuclear potential alone:

$$A(\theta) = \frac{i}{2K}\sum_L (2L+1)P_L(\cos\theta)(1-S^L)$$

  - Nuclear+Coulomb: $A(\theta) = A_C(\theta) + A'(\theta)$

$$A_C(\theta) = \frac{i}{2K}\sum_L (2L+1)(1-e^{2i\sigma_L})P_L(\cos\theta)$$

$$A'(\theta) = \frac{i}{2K}\sum_L (2L+1)e^{2i\sigma_L}(1-S^L)P_L(\cos\theta)$$

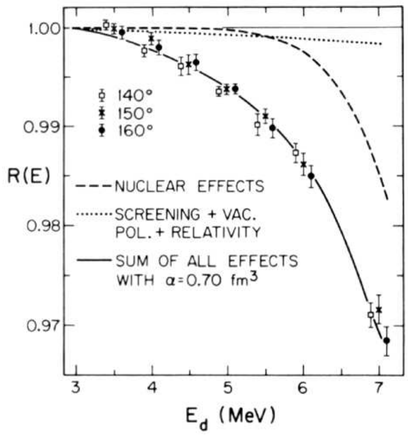
- Differential cross section:

$$\frac{d\sigma}{d\Omega} = |A(\theta)|^2$$

Extracting structure information from elastic scattering measurements

Eg: deuteron polarizability from d+^{208}Pb:

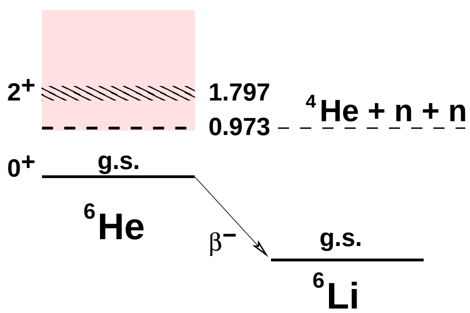
- Deuteron polarizability: $\mathbf{P} = \alpha \mathbf{E}$
- For $E < V_b$, the main deviation from Rutherford scattering comes from dipole polarizability.
- In the adiabatic limit ($E_x \gg$): $V_{\rm dip} = -\alpha \frac{Z_1 Z_2 e^2}{2R^4}$

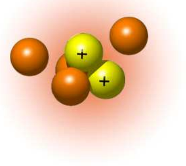


Rodning, Knutson, Lynch and Tsang, PRL49, 909 (1982)
$\alpha = 0.70 \pm 0.05$ fm^3

Halo and Borromean nuclei: the ^{6}He case

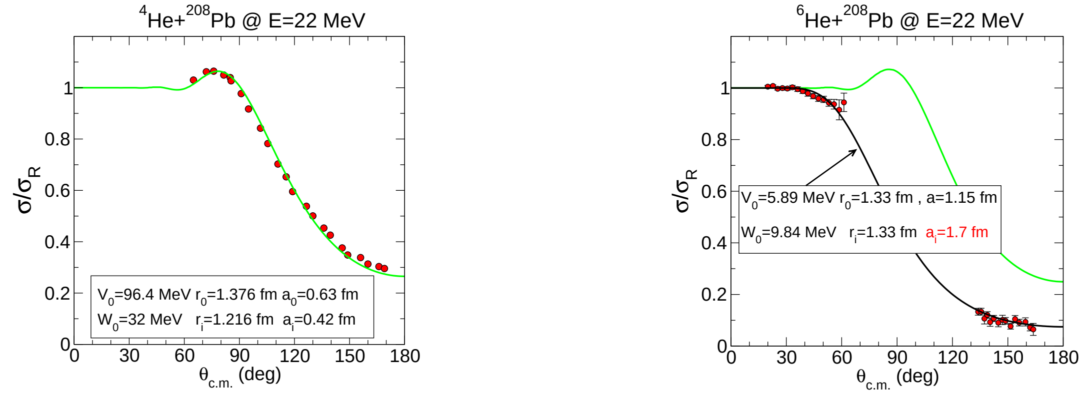
- Radioactive:
  ^{6}He $\xrightarrow{\beta^-}$ ^{6}Li ($t_{1/2}$ ≃807 ms)
- Weakly bound:
  $\epsilon_b = -0.973$ MeV
- Neutron halo
- Borromean system:
  n-n and α-n unbound
- $\sim$ 3 body system:
  α almost inert





Normal versus halo nuclei

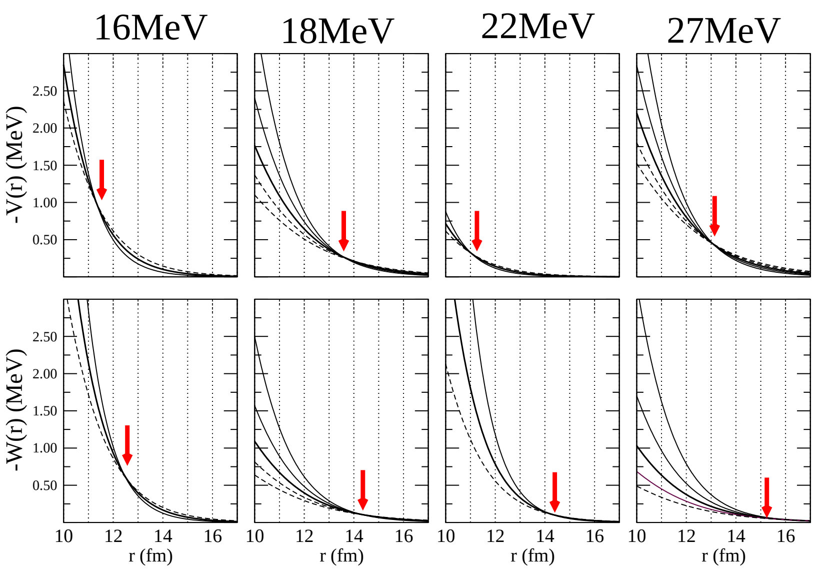
How does the halo structure affect the elastic scatterig?



- ^{4}He+^{208}Pb shows typical Fresnel pattern $\to$ *strong absorption*
- ^{6}He+^{208}Pb shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
- ^{6}He+^{208}Pb requires a large imaginary diffuseness $\to$ *long-range absorption*

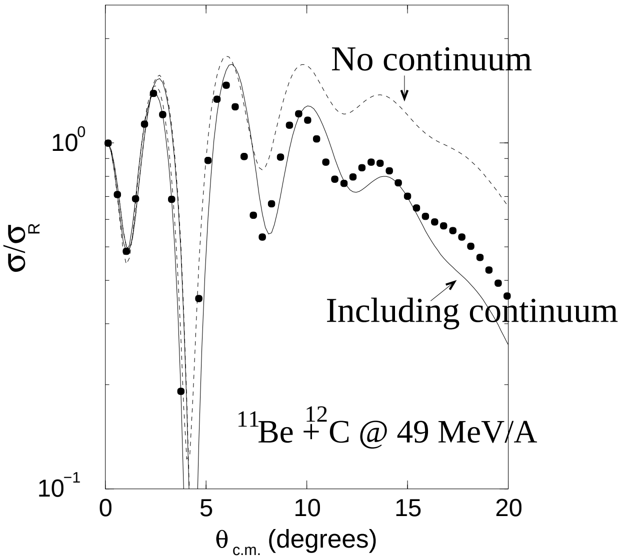
Optical model calculations for ^{6}He+^{208}Pb

RADIUS OF SENSITIVITY OF V(R) AND W(R)



Imaginary part $\Rightarrow$ long range compared to strong absorption radius

Normal versus halo nuclei: Fraunhofer



☞ In Fraunhofer scattering the presence of the continuum produces a reduction of the elastic cross section

`Fresco`, `Xfresco` and `Sfresco`

- What is FRESCO?

  Program developed by Ian Thompson since 1983, to perform coupled-reaction channels calculations in nuclear physics.

- Some general features:
  - Multi-platform (Windows, Linux, Unix, VAX)
  - Treats many direct reaction models: elastic scattering (optical model), transfer, inelastic excitation to bound and unbound states, etc
  - Can be run in text mode and graphical mode (`XFRESCO` interface)
  - FRESCO and XFRESCO can be freely downloaded at `http://www.fresco.org.uk/`
  - `SFRESCO`: Extension of Fresco, to provide χ^2 searches of potential and coupling parameters.

Optical model calculations with Fresco

Essential ingredients of an OM calculation:

- **Physical:**
  - Identify projectile and target (mass, spin, etc)
  - Incident energy
  - Parametrization of the optical potential
- **Numerical:**
  - Radial step for numerical integration (HCM in fresco)
  - Maximum radius R for integration (RMATCH)
  - Maximum angular momentum L. (JTMAX)

RMATCH and JTMAX are linked by: $kR_g\,(1 - 2\eta/kR_g) \approx L_g + 1/2$
(L_g=grazing angular momentum)

Elastic scattering: optical model

Effective potential: $U(R) = U_{\rm nuc}(R) + U_{\rm coul}(R)$

- Coulomb potential: charge sphere distribution

$$\boxed{U_c(R) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_c}\left(3 - \frac{R^2}{R_c^2}\right) & \text{if } R \leq R_c \\ \frac{Z_1 Z_2 e^2}{R} & \text{if } R \geq R_c \end{cases}}$$

- Nuclear potential (complex): Woods-Saxon parametrization

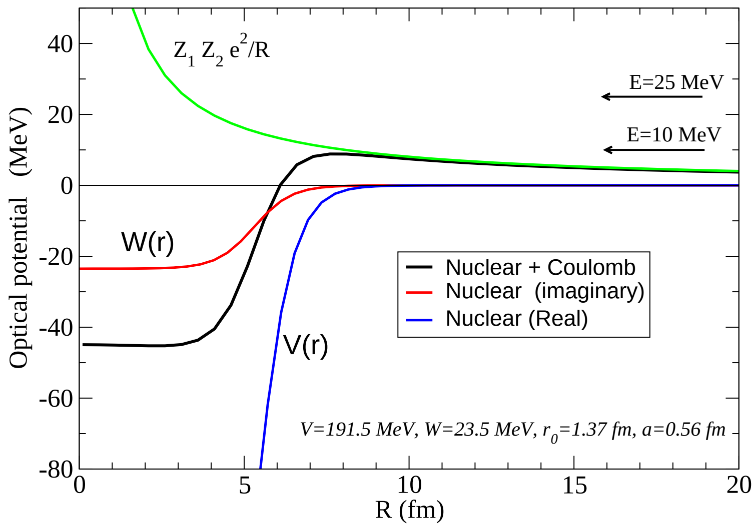
$$\boxed{U_{\rm nuc}(R) = V(r) + iW(r) = -\frac{V_0}{1 + \exp\left(\frac{R-R_0}{a_0}\right)} - i\,\frac{W_0}{1 + \exp\left(\frac{R-R_i}{a_i}\right)}}$$

Typically: $R_0 = r_0(A_p^{1/3} + A_t^{1/3})$

- r_0=reduced radius ($r_0 \sim 1.1 - 1.4$ fm)
- A_p,A_t: projectile, target masses (amu)

Elastic scattering: effective potential

Effective potential: $U(R) = U_{nuc}(R) + U_{coul}(R)$



OM example: ^{4}He+^{58}Ni

Input example: `4he58ni_e10.in`

```
4he58ni_e10.in: 4He + 58Ni elastic scattering Ecm=10.0 MeV
NAMELIST
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
      thmin=1.0 thmax=180.0 thinc=2.0
      smats=2 xstabl=1
      elab=10.7 /

&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28 nex=1 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
&partition /

&POT kp=1 at=58 rc=1.4 /
&POT kp=1 type=1
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /

&overlap /
&coupling /
```

Elastic scattering example

General variables

```
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
      thmin=1.00 thmax=180.00 thinc=2.00
      smats=2 xstabl=1
      elab=10.7 /
```

Mass partitions
& states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28
        nex=1 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
&partition /
```

Potentials

```
&POT kp=1 itt=F at=58 rc=1.4 /
&POT kp=1 type=1
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /
```


Elastic scattering example

Essential input variables: FRESCO namelist

```
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
      thmin=1.00 thmax=180.00 thinc=2.00
      smats=2 xstabl=1
      elab=10.7 /
```

- `hcm`: step for integration of radial equations.
- `rmatch`: matching radius (for `R > RMATCH` asymptotic behaviour is assumed)
- `elab`: laboratory energy
- `jtmax`: maximum total angular momentum (projectile+target+relative)
- `smats`: trace variable
  `smats=2` → print elastic S-matrix
- `xstbl`: trace variable
  `xstbl=1` → print cross sections

Elastic scattering with Fresco

Essential input variables: partitions and states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28
         nex=1 /
```

- namep / namet: projectile / target name
- massp / masst: projectile / target mass (amu)
- zp / zt: projectile / target charge
- nex: number of (pairs) of states in this partition

```
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- jp / jt: projectile / target spins
- bandp / bandt: projectile / target parities ($\pm$ 1)
- cpot: index of potential for this pair of states.

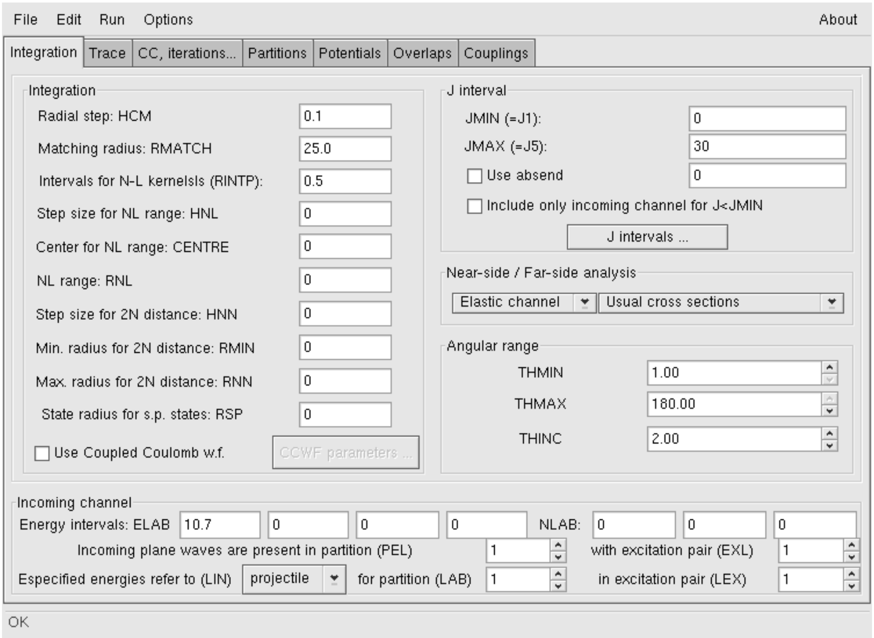
Elastic scattering with fresco

```
&POT kp=1 type=0 ap=0 at=58 rc=1.4 /
&POT kp=1 type=1 shape=0
    p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /
```

- kp: index to identify this potential
- ap, at: projectile and target mass, for conversion from reduced to physical radii: $R = r(\mathrm{ap}^{1/3} + \mathrm{at}^{1/3})$
- type, shape: potential cathegory and shape: $\Rightarrow$
  - type=0: Coulomb potential
    shape=0: uniform charge sphere
  - type=1: volume nuclear potential
    shape=0: Woods-Saxon shape
- rc: reduced radius for charge distribution
- p1,p2,p3: V_0, r_0, a_0 (real part)
- p4,p5,p6: W_0, r_i, a_i (imaginary part)

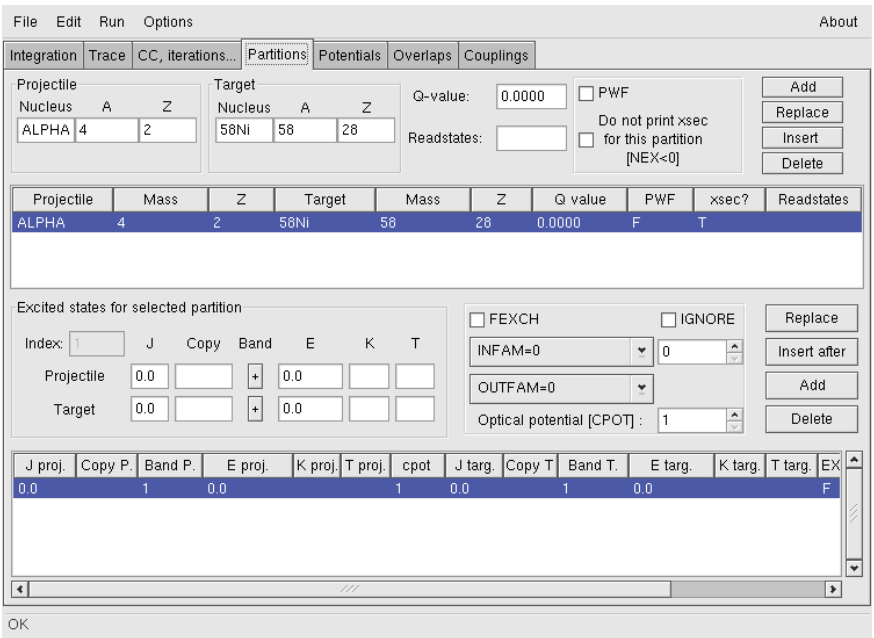
Xfresco interface

General variables:



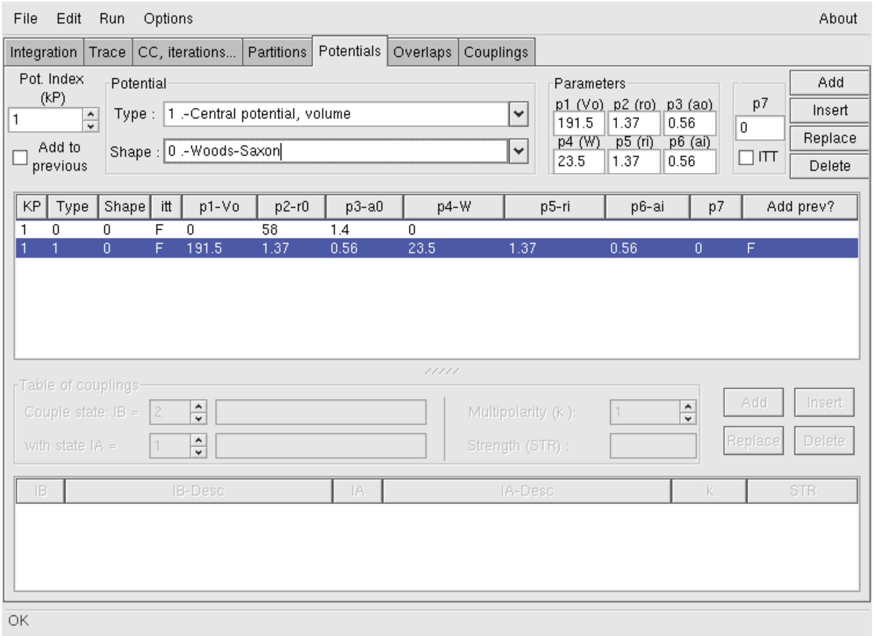
Optical model with XFRESCO

Partitions & states:



Optical model with XFRESCO

Potentials:



Useful output information in OM calculations

Useful output files:

- Main output file (stdout)
- `fort.201` : Elastic scattering angular distribution
  - `thmax > 0`: relative to Rutherford.
  - `thmax < 0`: absolute units (mb/sr).
- `fort.7`: Elastic S-matrix (real part, imaginary part, angular momentum)
- `fort.56`: Fusion (absorption), reaction and inelastic cross section for each angular momentum

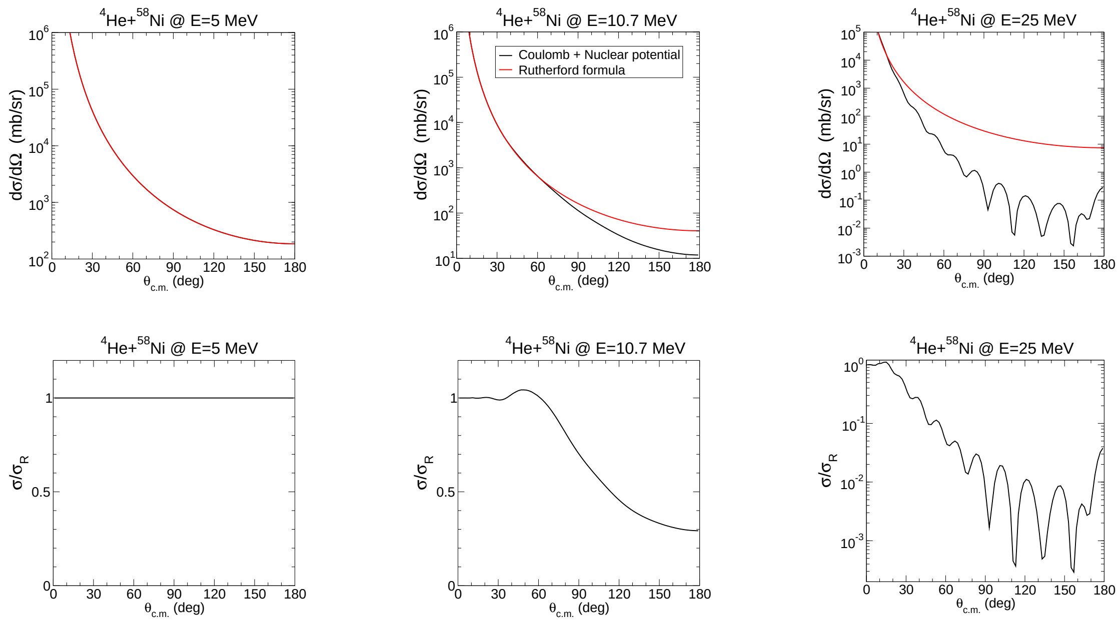
Elastic scattering: optical model

Dynamical effects: ^{4}**He+**^{58}**Ni at E=5, 10.7, 25 and 50 MeV**

$E_{\rm lab}$ (MeV)	η	k (fm^{-1})	$\lambda = 1/k$ (fm)	$2a_0$ (fm)
5	7.95	0.920	1.087	17.2
10.7	5.62	1.34	0.746	8.06
25	3.55	2.06	0.485	3.44
50	2.51	2.91	0.343	1.69

- $\eta \ggg 1$: Rutherford scattering: $\sigma(\theta) \propto 1/\sin^4(\theta/2)$
- $\eta \gg 1$: Fresnel scattering (rainbow)
- $\eta \leq 1$: Fraunhofer scattering (oscillatory behaviour):

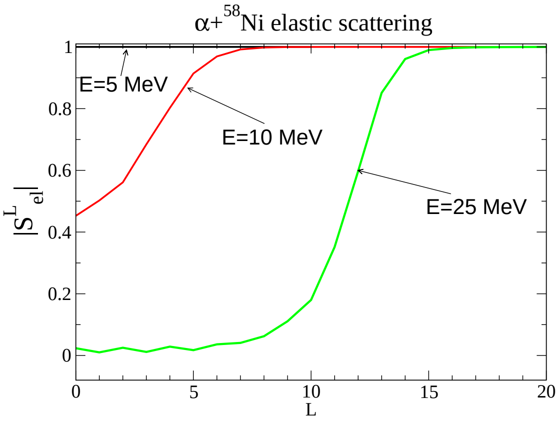
Elastic scattering: energy dependence

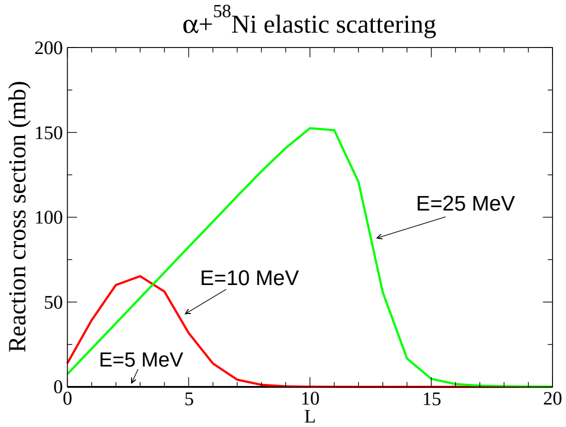


Rutherford scattering **Fresnel** **Fraunhöfer**

Elastic scattering: S-matrix elements

Elastic (nuclear) S-matrix (`fort.7`): $f_{el}^L(r) = I_L(r) - S_{el}^L O_L(r)$

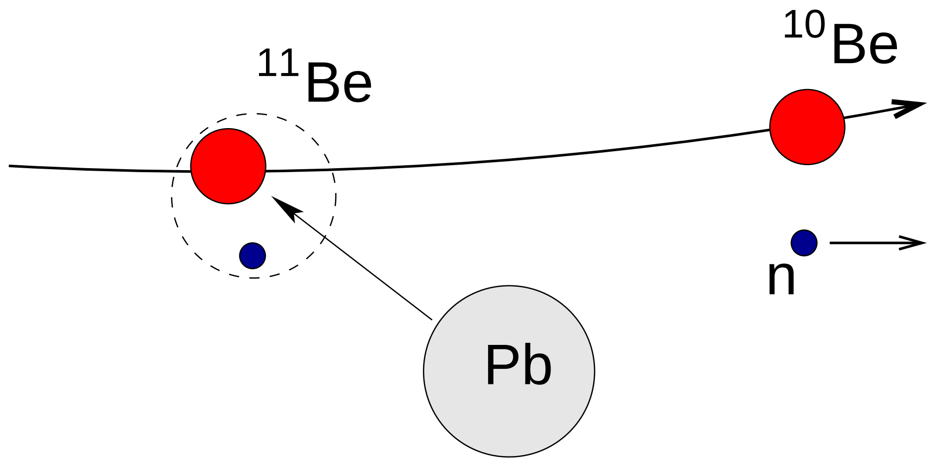




$$kR_g\,(1 - 2\eta/kR_g) \approx L_g + 1/2$$

$\Rightarrow$ the number of partial waves required for convergence grows approximately as $\sqrt{E}$

Inelastic scattering to bound states



Coupled-channels method

- The Hamiltonian: $H = T_R + h(\xi) + \Delta(\mathbf{R}, \xi)$
- Internal states: $h(\xi)\phi_n(\xi) = \epsilon_n \phi_n(\xi)$
- Model wavefunction: $\Psi(\mathbf{R}, \xi) = \phi_0(\xi)\chi_0(\mathbf{R}) + \sum_{n>0} \phi_n(\xi)\chi_n(\mathbf{R}) + \ldots$
- Coupled equations: $[H - E]\Psi(\mathbf{R}, \xi)$

$$\boxed{[E - \epsilon_n - T_R - V_{n,n}(\mathbf{R})]\,\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- Coupling potentials:

$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\xi \phi_{n'}(\xi)^* \Delta(\mathbf{R}, \xi)\phi_n(\xi)}$$

☞ $\phi_n(\xi)$ *will depend on the structure model (collective, single-particle,etc).*

Boundary conditions and scattering amplitude

- Boundary conditions:

$$\begin{array}{lcll} \chi_0^{(+)}(\mathbf{R}) & \to & e^{i\mathbf{K}_0 \cdot \mathbf{R}} + A_{0,0}(\theta)\dfrac{e^{iK_0 R}}{R} & \text{(elastic)} \\ \chi_n^{(+)}(\mathbf{R}) & \to & A_{n,0}(\theta)\dfrac{e^{iK_n R}}{R}, \quad n \neq 0 & \text{(non} - \text{elastic)} \end{array}$$

☞ If Coulomb is present, then

$$\frac{e^{iKR}}{R} \to \frac{1}{R} e^{i(KR - \eta 2KR)}$$

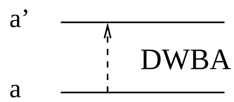
- Cross sections:

$$\frac{d\sigma_n(\theta)}{d\Omega} = \frac{K_n}{K_0} |A_{n,0}(\theta)|^2$$

DWBA approximation

- DWBA approximation:

$$
\begin{aligned}
\left[E - \epsilon_n - T_n - V_{n:n}(\mathbf{R})\right] \widetilde{\chi}_n(\mathbf{K},\mathbf{R}) &= 0 \\
\left[E - \epsilon_{n'} - T_{n'} - V_{n':n'}(\mathbf{R})\right] \widetilde{\chi}_{n'}(\mathbf{K}',\mathbf{R}) &= 0
\end{aligned}
$$



- Scattering amplitude:

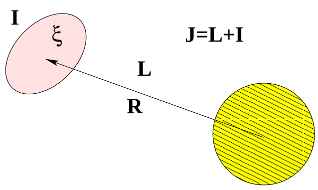
$$
\boxed{A(\mathbf{K}',\mathbf{K})_{n',n} = -\frac{2\mu}{4\pi\hbar^2}\int d\mathbf{R}\,\widetilde{\chi}^{(-)}_{n'}(\mathbf{K}',\mathbf{R})V_{n':n}(\mathbf{R})\widetilde{\chi}^{(+)}_{n}(\mathbf{K},\mathbf{R})}
$$

☞ *The DWBA approximation amounts at solving the CC equations to first order (Born approximation)*

☞ *In practice, phenomenological optical potentials that fit the elastic cross section in the respective channels are used instead of* $V_{n,n}$ *and* $V_{n',n'}$

$$
V_{n,n}(\mathbf{R}) \equiv (n|\Delta(\mathbf{R},\xi)|n) \rightarrow U_n(\mathbf{R})
$$

Partial wave decomposition: the channel basis



- The channel basis:

$$
\Phi^{JM_J}_{nLI}(\hat{R},\xi) = \sum_{M_I M_L} i^L Y_{LM_L}(\hat{R})|nIM_I\rangle\langle LM_L IM_I|JM_J\rangle
$$

- Partial wave expansion of the total WF:

$$
\Psi(\mathbf{R},\xi) = \sum_{nLIJM} C^{JM_J}\frac{f^J_{nLI}(R)}{R}\Phi^{JM}_{nLI}(\hat{R},\xi)
$$

Coupled equations

- The coupled equations: (Comp. Phys. Comm. 40 (1986) 233-262)

$$\boxed{\left(-\frac{\hbar^2}{2\mu}\frac{d^2}{dR^2}+\frac{\hbar^2 L(L+1)}{2\mu R^2}+\epsilon_n-E\right)f^J_\beta(R)+\sum_{\beta'}V^J_{\beta,\beta'}(R)f^J_{\beta'}(R)=0}$$

$\beta \equiv \{n, L, I\}$

- Coupling potentials:

$$\boxed{V^J_{\beta,\beta'}(R)=\int d\hat{R}\, d\xi \Phi^{JM_J}_\beta(\hat{R},\xi)^*\Delta(\mathbf{R},\xi)\Phi^{JM_J}_{\beta'}(\hat{R},\xi)}$$

Boundary conditions

Solution of the coupled equations:

1. Integrate the differential equation for $R \in [0, R_m]$ with the condition:

$$\lim_{R\to 0} f^J_{\beta;\beta_i}(R)=0$$

2. Match the solution at R_m with the asymptotic form $\Rightarrow$ S-matrix:

$$f^J_{\beta;\beta_i}(R)\to \delta_{\beta,\beta_i}I_\beta(R)-S^J_{\beta,\beta_i}O_\beta(R)$$

$$\begin{aligned} I_\beta(R) &= (K_nR)h^*_L(K_nR)/\sqrt{v_n} \\ O_\beta(R) &= (K_nR)h_L(K_nR)/\sqrt{v_n} \end{aligned}$$

Scattering wavefunction

- Wavefunction that corresponds to the experimental condition:

$$\Psi_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi) = e^{i\mathbf{K}_i\cdot\mathbf{R}}|n_iI_iM_i\rangle + \chi^{(+)}_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi)$$

- The outgoing wave:

$$\begin{aligned}\chi^{(+)}_{\mathbf{K}_i,n_iI_iM_i}(\mathbf{R},\xi) &= \sum_{JM_JL_iN_i} \frac{2\pi i\sqrt{v_i}}{k_iR}\langle L_iN_iI_iM_i|JM_J\rangle Y^*_{L_iN_i}(\hat{K}_i) \\ &\times \sum_{nIL}\left(\delta_{n,n_i}\delta_{I,I_i}\delta_{n,n_i} - S^J_{nIL:n_iI_iL_i}\right)\Phi^{JM_J}_{nLI}(\hat{R},\xi)O_{nIL}(R)\end{aligned}$$

- The scattering amplitude:

$$\begin{aligned}A(\mathbf{K}_i,\mathbf{K})_{n_iI_iM_i;nIM} &= \frac{2\pi i}{\sqrt{KK_i}}\sum_{JM_JL_iN_i}\langle L_iN_iI_iM_i|JM_J\rangle Y^*_{L_iN_i}(\hat{K}_i) \\ &\times \sum_{nILN}\langle LNIM|JM_J\rangle Y_{LN}(\hat{K})\left(\delta_{n,n_i}\delta_{I,I_i}\delta_{L,L_i} - S^J_{nIL;n_iI_iL_i}\right)\end{aligned}$$

Scattering amplitude and cross sections

- Elastic and inelastic cross sections:

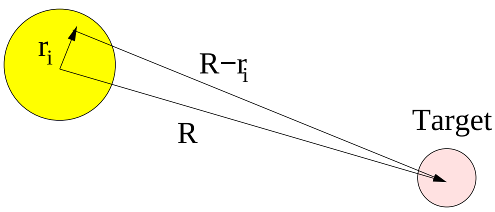
$$\left.\frac{d\sigma}{d\Omega}\right._{i\to n} = \frac{1}{2I_i+1}\sum_{MM_i}|A(\mathbf{K}_i,\mathbf{K})_{n_iI_iM_i;nIM}|^2$$

☞ *Coupled channels calculations give elastic and inelastic cross sections, if the states are properly described, if the interactions are known, and if all "relevant" channels are included*

Inelastic scattering: collective models

- Projectile-target Coulomb interaction:

$$V(\mathbf{R},\xi) = \frac{Ze^2}{4\pi\epsilon_0}\sum_i \frac{1}{|\mathbf{R}-\mathbf{r}_i|}; \qquad \xi \equiv \{\mathbf{r}_i\}$$



- Multipolar expansion:

$$\frac{1}{|\mathbf{R}-\mathbf{r}_i|} = \sum_{\lambda\mu} \frac{r_i^\lambda}{R^{\lambda+1}} \frac{4\pi}{2\lambda+1} Y_{\lambda\mu}(\hat{r}_i) Y^*_{\lambda\mu}(\hat{R}) \qquad (R > r_i)$$

Inelastic scattering: collective models

- Electric multipole operator: $M(E\lambda,\mu) = e\sum_i r_i^\lambda Y_{\lambda\mu}(\hat{r}_i)$
- Monopole and transition operator:

$$V(\mathbf{R},\xi) = V_0(R) + \Delta(\mathbf{R},\xi) = \frac{Zze^2}{4\pi\epsilon_0 R} + \frac{Ze}{\epsilon_0}\sum_{\lambda\neq 0,\mu} \frac{M(E\lambda,\mu)}{2\lambda+1}\frac{Y^*_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

- Transition potentials:

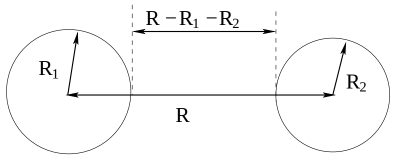
$$\boxed{\Delta_{nm}(\mathbf{R}) = \frac{Ze}{\epsilon_0}\sum_{\lambda\neq 0,\mu} \frac{\langle nI_nM_n|M(E\lambda,\mu)|mI_mM_m\rangle}{2\lambda+1}\frac{Y^*_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}}$$

Inelastic scattering: collective models

- Central potential: Typically $U_{\rm nuc}(\mathbf{R}) = V(R-R_0)$, $R_0 = R_1 + R_2$.

☞ Eg: WS parametrization

$$U(R) = -\frac{V_0}{1+\exp\left(\frac{R-R_0}{a_r}\right)} - i\frac{W_0}{1+\exp\left(\frac{R-R_i}{a_i}\right)}$$



Inelastic scattering: collective models

- Non-spherical nucleus → deformed surface $r(\theta,\phi) = R_0 + \sum_{\lambda\mu} \hat{\delta}_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)$

  ($\hat{\delta}_{\lambda\mu}$=deformation length operators)

- Deformed potential: $V(\mathbf{R},\xi) = V(R - r(\theta,\phi))$
- Multipole expansion of the potential:

$$V(\mathbf{R},\xi) = U_0(R-R_0) - \frac{dU_0(R-R_0)}{dR}\sum_{\lambda\mu}\hat{\delta}_{\lambda\mu}Y^*_{\lambda\mu}(\theta,\phi)$$

- Transition potential:

$$\boxed{\Delta_{nm}(\mathbf{R}) \equiv \langle n|V|m\rangle = -\frac{dU_0(R-R_0)}{dR}\sum_{\lambda}\langle nI_nM_n|\hat{\delta}_{\lambda\mu}|mI_mM_m\rangle Y^*_{\lambda\mu}(\hat{R})}$$

☞*The nuclear transition potentials are proportional to the matrix element of the deformation length operator*.

Physical ingredients for collective excitations

- Coulomb excitation → electric reduced matrix elements

$$\Delta_{nm}(\mathbf{R}) = \frac{Ze}{\epsilon_0} \sum_{\lambda \neq 0, \mu} \frac{\langle n I_n M_n | M(E\lambda, \mu) | m I_m M_m \rangle}{2\lambda + 1} \frac{Y^*_{\lambda\mu}(\hat{R})}{R^{\lambda+1}}$$

$$\boxed{\langle n I_n || M(E\lambda) || m I_m \rangle = \sqrt{(2I_n + 1) B(E\lambda; I_n \to I_m)}}$$

- Nuclear excitation (collective model) → deformation lengths

$$\Delta_{nm}(\mathbf{R}) = -\frac{dV_0(R - R_0)}{dR} \sum_{\lambda} \langle n I_n M_n | \widehat{\delta}_{\lambda\mu} | m I_m M_m \rangle Y^*_{\lambda\mu}(\hat{R})$$

within the rotational model:

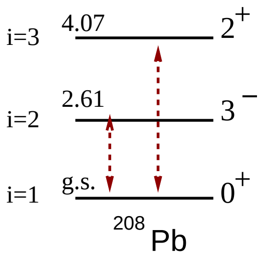
$$\boxed{\langle n I_n || \widehat{\delta}_{\lambda} || m I_m \rangle = \delta_\lambda \sqrt{2I_n + 1} \langle I_n K \lambda 0 | I_m K \rangle} \qquad \delta_\lambda = \beta_\lambda R$$

What do we learn by measuring inelastic scattering?

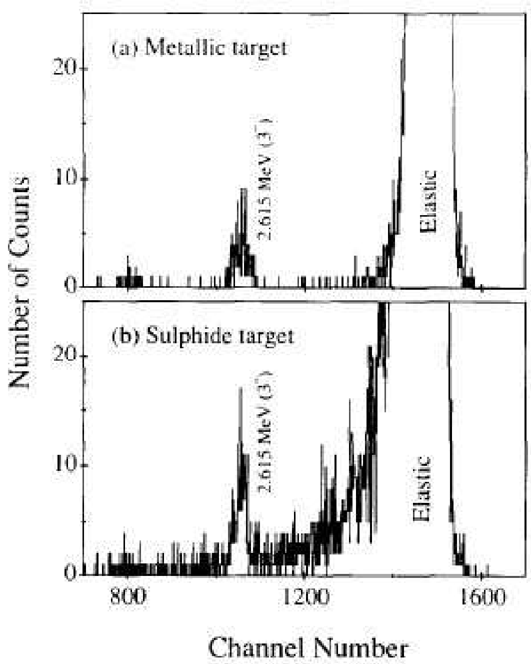
- Structure: Properties of $h(\xi)$ and its eigenstates: Spin and parity of excited states, $M(E\lambda)$ matrix elements, deformation parameters.
- Interaction: Properties of $V(\vec{r}, \xi)$: Response of the nucleus to the nuclear and coulomb field (transition potentials). Coulomb- nuclear interference effects.
- Reaction mechanism: Dynamics of the collision: States that should be included in the calculation, Dynamic polarization effects due to the coupling (coupled channels effects).

^{16}O + ^{208}Pb inelastic scattering

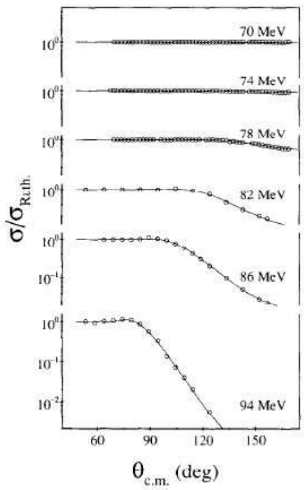
Physical example: $^{16}O + ^{208}Pb \rightarrow ^{16}O + ^{208}Pb(3^-,2^+)$

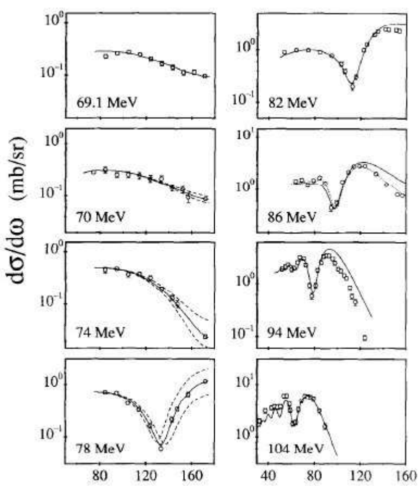


Nucl. Phys. A517 (1990) 193



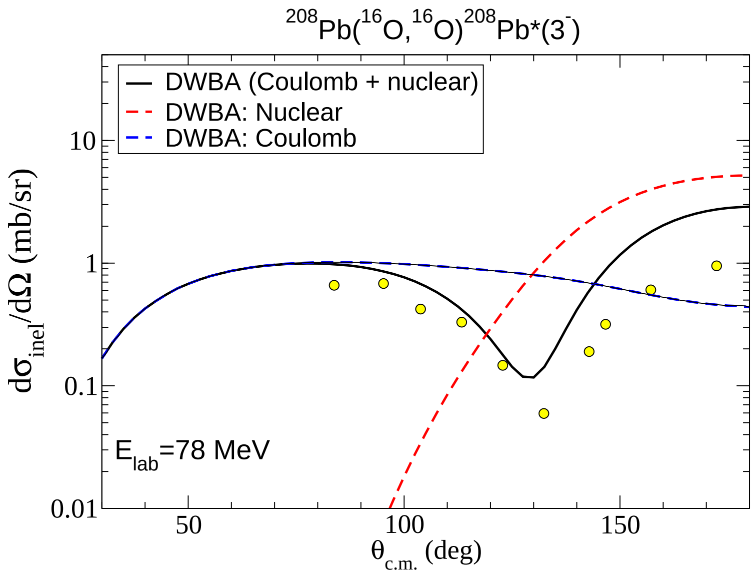
Collective excitations: example





^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

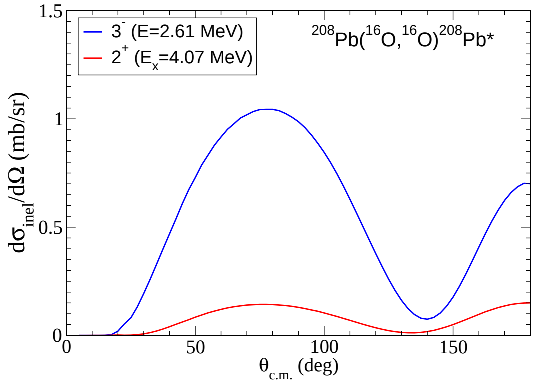
Coulomb and Nuclear excitations can produce <u>constructive</u> or <u>destructive</u> interference:



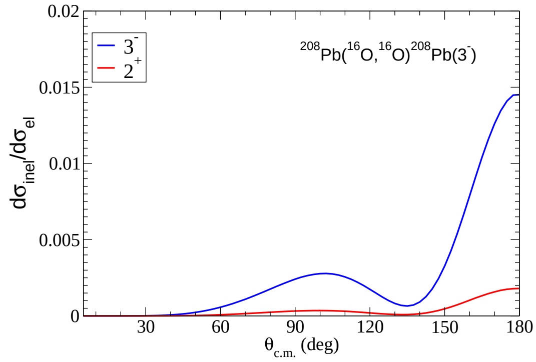
^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Angular distribution of the ejectile in c.m. frame

Absolute cross section



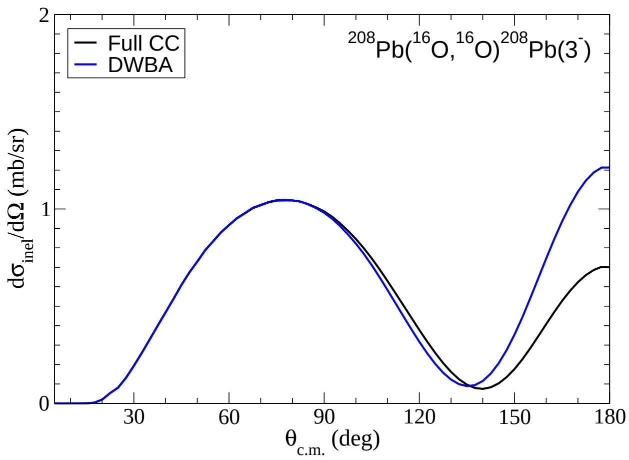
Ratio to elastic cross section



^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

CC versus DWBA:

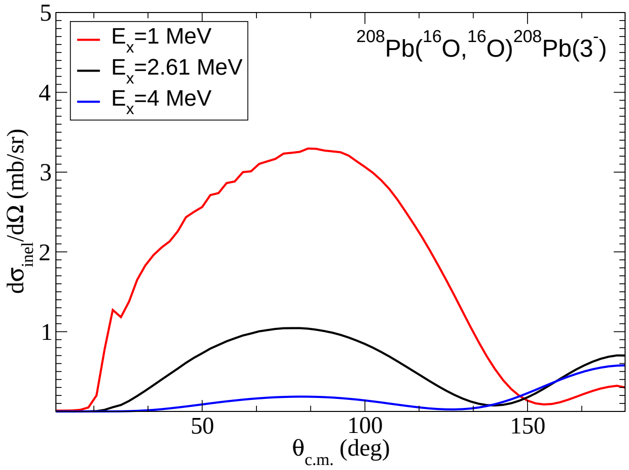
- Full coupled-channels: `iblock = 3`, `iter = 0`
- DWBA: `iblock = 0`, `iter = 1`



Inelastic scattering

Effect of the excitation energy:

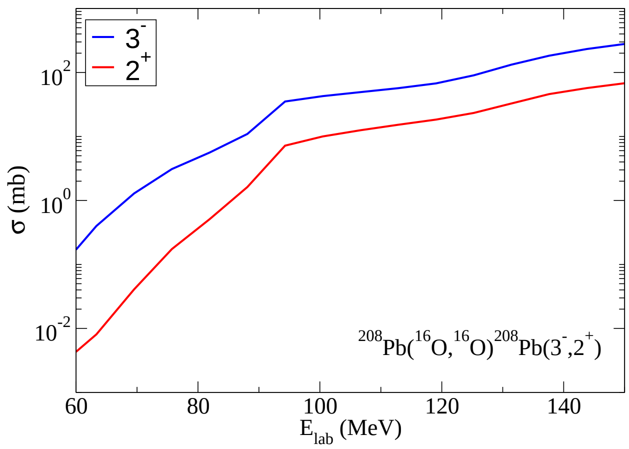
```
&STATES jp=0.0 copyp=1 ep=0.0000 cpot=1 jt=3.0 bandt=-1 et=2.6100 /
```



^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Effect of the incident energy: In FRESCO main output:

```
0CUMULATIVE REACTION cross section = 11.22270 <L> = 47.07 <L**2> = 3441.3
0CUMULATIVE outgoing cross sections in partition 1 : 0.00000 7.67943 0.99138
```



Inelastic scattering: cluster model

- Some nuclei permit a description in terms of two or more clusters: d=p+n, ^{6}Li=α+d, ^{7}Li=α+^{3}H.
- Projectile-target interaction:

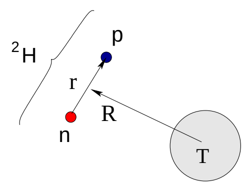
$$V(\mathbf{R},\mathbf{r}) = U_1(\mathbf{R}_1) + U_2(\mathbf{R}_2)$$

- Ej:Deuteron case:

$$\mathbf{R}_1 = \mathbf{R} + \frac{1}{2}\mathbf{r}; \qquad \mathbf{R}_2 = \mathbf{R} - \frac{1}{2}\mathbf{r}$$

- Internal states: $[T_{\mathbf{r}} + V_{pn}(\mathbf{r}) - \epsilon_n]\phi_n(\mathbf{r}) = 0$
- Transition potentials:

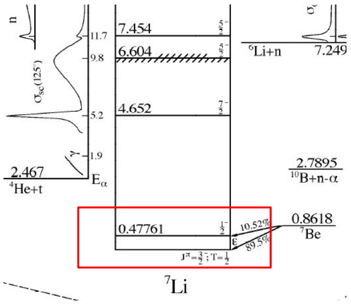
$$\boxed{V_{n,n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n^*(\mathbf{r})V(\mathbf{r},\mathbf{R})\phi_{n'}(\mathbf{r})}$$

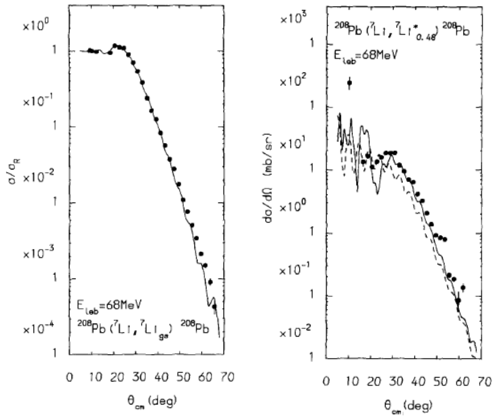


Inelastic scattering: cluster model

Example 1: ^{7}Li(α+t) +^{208}Pb at 80 MeV (Phys. Lett. 139B (1984) 150):

$$V_{n,n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n^*(\mathbf{r})\,[V_\alpha(\mathbf{r}_\alpha) + V_t(\mathbf{r}_t)]\,\phi_{n'}(\mathbf{r}); \quad n = 0, 1$$

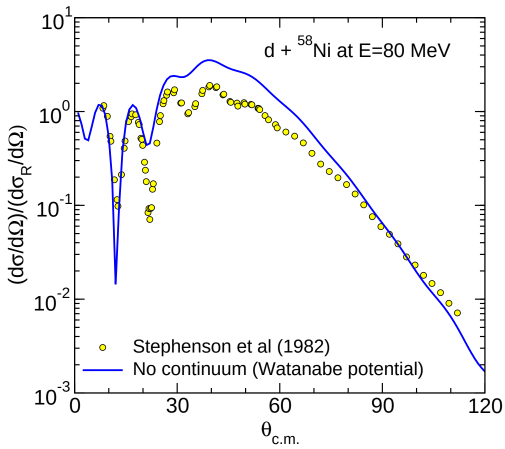




Failure of the calculations without continuum

Example: Three-body calculation (p+n+^{58}Ni) with Watanabe potential:

$$V_{dt}(\mathbf{R}) = \int d\mathbf{r}\phi_{gs}(\mathbf{r})\,(V_{pt} + V_{nt})\,\phi_{gs}(\mathbf{r})$$



☞*Three-body calculations omitting breakup channels fail to describe the experimental data.*

Example for inelastic scattering

Proposed exercise: For the reaction p+^{12}C at $E_{\rm lab}$=185 MeV calculate:

1. The elastic scattering angular distribution, using Kooning-Delaroche optical potential (you can get the parameters from RIPL-2 database: *http://www-nds.iaea.org/RIPL-2/*
2. The angular distribution for the inelastic scattering populating the first excited state located at $E_x = 4.44$ MeV and with $J^\pi = 2^+$. Assume that this is a collective quadrupole excitation, and that the DWBA approximation is valid. Compare with the experimental data from Nucl. Phys. 69 (1965) 81-102.

Data: Deformation parameter: $\beta_2 = 0.6$. For the radius of the ^{12}C assume $R = 1.2 \times 12^{1/3}$.

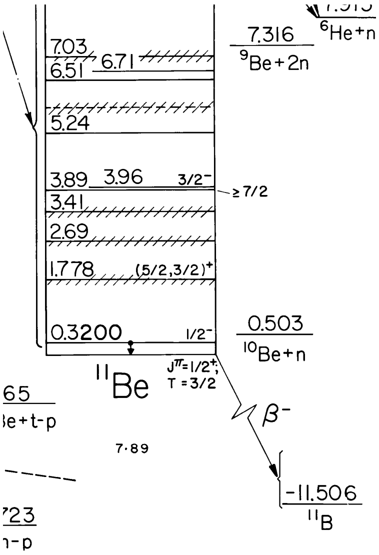
Input example for inelastic scattering with single-particle form factors

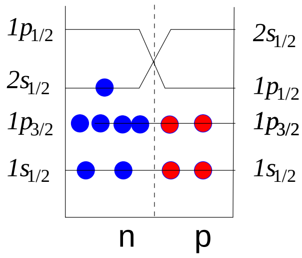
^{11}Be+ ^{12}C $\rightarrow$ ^{11}Be(1/2$^+$,1/2$^-$) + ^{12}C at 49.3 MeV/A

Inelastic scattering example: ^{11}Be+^{12}C

Example: ^{11}Be+ ^{12}C $\rightarrow$ ^{11}Be(1/2$^+$,1/2$^-$) + ^{12}C at 49.3 MeV/A
Phys. Rev. C 67, 037601 (2003)

Input file: `be11c12_inel.in`





^{11}Be+^{12}C inelastic scattering

General variables:

```
&FRESCO hcm=0.05 rmatch=60.0 jtmin=0.0
      jtmax=150.0 thmin=0.00 thmax=45.00 thinc=0.50
      iblock=2 nnu=24 chans=1 smats=2 xstabl=1
      elab=542.3 /
```

- `iblock=2`: number of channels coupled *exacly*.

Partitions & states:

```
&PARTITION namep='11Be' massp=11.0 zp=4 namet='12C' masst=12.0000 zt=6 nex=2 /
&STATES jp=0.5 bandp=1 cpot=1 jt=0.0 bandt=1 /
&STATES jp=0.5 bandp=-1 ep=0.3200 cpot=1 jt=0.0 copyt=1 /
```

- `nex=2`: This partition will contain two pairs of states.
- `copy=1`: The target of the second pair of states is just the same (a copy) of the first target stat.

```
&PARTITION namep='10Be' massp=10.0000 zp=4 namet='12C+n' masst=13.0000 zt=6 nex=1 /
&STATES jp=0.0 bandp=1 cpot=2 jt=0.0 bandt=1 /
```

^{11}Be+^{12}C inelastic scattering

Projectile-target Coulomb potential (monopole):

```
&POT kp=1 ap=11.000 at=12.000 rc=1.111 /
```

Neutron-target & core-target potentials:

```
&POT kp=3 ap=0.000 at=12.000 rc=1.111 /
&POT kp=3 type=1 p1=37.400 p2=1.200
      p3=0.750 p4=10.000 p5=1.300 p6=0.600 /
```

```
&POT kp=2 ap=10.000 at=12.000 rc=1.111 /
&POT kp=2 type=1 p1=123.000 p2=0.750
      p3=0.800 p4=65.000 p5=0.780 p6=0.800 /
```

Neutron binding potential:

```
&POT kp=4 ap=0 at=10.000 rc=1.0 /
&POT kp=4 type=1 p1=87.0 p2=1.0 p3=0.53 /
```


^{11}Be+^{12}C inelastic scattering

Bound state wave functions:

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=2 sn=0.5 l=0 j=0.5
      kbpot=4 be=0.500 isc=1 /
&OVERLAP kn1=2 ic1=1 ic2=2 in=1 nn=1 l=1 sn=0.5
      j=0.5 kbpot=4 be=0.180 isc=1 ipc=2 /
```

- `kn1=1,2`: Index for this WF
- `ic1/ic2`: Index of partition containing core (^{10}Be) / composite (^{11}Be)
- `in=1/2`: WF for projectile/target
- `nn,sn,l,j`: Quantum numbers for bound state
- `be`: separation energy.
- `kbpot=3`: Index KP of binding potential.

^{11}Be+^{12}C inelastic scattering

Couplings:

```
&COUPLING icto=1 icfrom=2 kind=3 ip1=4 ip2=1 p1=3.0 p2=2.0 /
```

- kind=3: Single-particle excitations of projectile
- icto=1: Partition containing nucleus being excited (^{11}Be)
- icfrom=2: Partition containing core (^{10}Be)
- ip1=4: Maximum multipole for coupling potentials
- p1/p2: KP index for fragment-target / core-target potentials

Spectroscopic amplitudes:

```
&CFP in=1 ib=1 ia=1 kn=1 a=1.000 /
&CFP in=1 ib=2 ia=1 kn=2 a=1.000 /
```

- in=1/2: Projectile/target
- ib/ia: Index for composite/core state
- a=1.0: Spectroscopic amplitude

Input example for inelastic scattering with collective form factors

Example: ^{208}Pb(^{16}O,^{16}O)^{208}Pb(3^-,2^+)

Inelastic scattering

Input example 2: ^{208}Pb(^{16}O,^{16}O)^{208}Pb(3^-,2^+) (o16pb_cc1a.in)

```
o16pb_cc1a.in: 16O+208Pb 80 MeV
NAMELIST
&FRESCO hcm=0.05 rmatch=100.0
      jtmin=0.0 jtmax=300.0
      thmin=5.00 thmax=-180.00 thinc=2.50
      iblock=3
      smats=2 xstabl=1
      elab= 80.0 /

&PARTITION namep='16-O' massp=15.9949 zp=8
        namet='PB-208' masst=207.9770 zt=82
        nex=3 /
&STATES jp=0.0 bandp=1 ep=0.0000 cpot=1
      jt=0.0 bandt=1 et=0.0000 /
&STATES jp=0.0 copyp=1 ep=0.0000 cpot=1
      jt=3.0 bandt=-1 et=2.6100 fexch=F /
&STATES jp=0.0 copyp=1 bandp=1 ep=0.0000 cpot=1
      jt=2.0 bandt=1 et=4.0700 /
&partition /
```

```
&POT kp=1 itt=F ap=208.000 at=16.000 rc=1.200 /
&POT kp=1 type=13 shape=10 itt=F p2=54.45 p3=815.0 /
&STEP ib=1 ia=2 k=3 str=815.0 /
&STEP ib=2 ia=1 k=3 str=815.0 /
&STEP ib=1 ia=3 k=2 str=54.45 /
&STEP ib=3 ia=1 k=2 str=54.45 /
&step /
&POT kp=1 type=1 shape=1 p4=10.000 p5=1.000 p6=0.400 /
&POT kp=1 type=-1 p1=60.500 p2=1.179 p3=0.658 /
&POT kp=1 type=13 shape=11 p2=0.400 p3=0.8 /
&STEP ib=1 ia=2 k=3 str=0.8 /
&STEP ib=2 ia=1 k=3 str=0.8 /
&STEP ib=1 ia=3 k=2 str=0.4 /
&STEP ib=3 ia=1 k=2 str=0.4 /
&step /

&pot /

&overlap /
&coupling /
```


^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

General variables:

```
&FRESCO hcm=0.05 rmatch=100.0
        jtmin=0.0 jtmax=300.0
        thmin=5.00 thmax=-180.00 thinc=2.50
        iblock=3
        smats=2 xstabl=1
        elab= 80.0 /
```

`iblock`: Number of states (including gs) that will be coupled to all orders.

- `iblock=1`: only elastic scattering
- `iblock=2`: elastic scattering + 1st inelastic channel (^{208}Pb(3^-))
- `iblock=3`: elastic scattering + ^{208}Pb(3^-) + ^{208}Pb(2^+)

^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Partitions and states:

```
&PARTITION namep='16-O' massp=15.9949 zp=8 namet='PB-208' masst=207.9770 zt=82
         nex=3 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=+1 et=0.00 /
&STATES    copyp=1       cpot=1 jt=3.0 bandt=-1 et=2.61 /
&STATES    copyp=1       cpot=1 jt=2.0 bandt=+1 et=4.07 /
&partition /
```

- `nex`: number of states within the partition
- `ep, et`: excitation energy for projectile / target
- `copyp=1` tells `FRESCO` that the 2nd and 3rd projectile states are just a copy of the ground state.

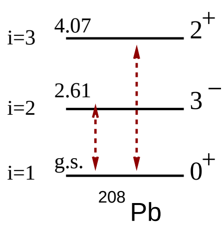
^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Coulomb excitation:

```
&POT kp=1 ap=208.000 at=16.000 rc=1.2 /
&POT kp=1 type=13 shape=10 p2=54.45 p3=815.0 p4=0 p5=0 p6=0 /
```

- `type=13`: couple target states by deforming previous potential
- `p1,...,p6`: consider couplings for multipolarities k with `pk`$\neq 0$
- `shape=10`: usual deformed charge sphere: $\Delta_{nm}(R) \propto M(Ek)/R^{k+1}$

```
&STEP ib=1 ia=2 k=3 str=815.0 /
&STEP ib=2 ia=1 k=3 str=815.0 /
&STEP ib=1 ia=3 k=2 str=54.45 /
&STEP ib=3 ia=1 k=2 str=54.45 /
&step /
```



- `ia, ib`: couple from state number `ia` to state `ib`
- `k`: multipolarity
- `str`=$\langle \mathrm{ib}||\mathrm{M(Ek)}||\mathrm{ia}\rangle = \sqrt{(2\mathrm{I_a}+1)\mathrm{B(E\lambda; ia \to ib)}}$

^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

Nuclear excitation:

```
&POT kp=1 type=1 shape=1 p4=10.000 p5=1.000 p6=0.400 /
&POT kp=1 type=-1 shape=0 p1=60.500 p2=1.179 p3=0.658 /
&POT kp=1 type=13 shape=10 itt=F p2=0.400 p3=0.8 /
```

- `type=13`: couple target states by deforming preceding potential
- `shape=10`: usual deformed nuclear potential: $\Delta_{nm}(R) \propto \delta_k dU(R)/dR$

```
&STEP ib=1 ia=2 k=3 str=0.8 /
&STEP ib=2 ia=1 k=3 str=0.8 /
&STEP ib=1 ia=3 k=2 str=0.4 /
&STEP ib=3 ia=1 k=2 str=0.4 /
```

- `str`=$\langle ib||\delta_k||ia\rangle$ (reduced deformation length)

^{208}Pb(^{16}O,^{16}O)^{208}Pb inelastic scattering

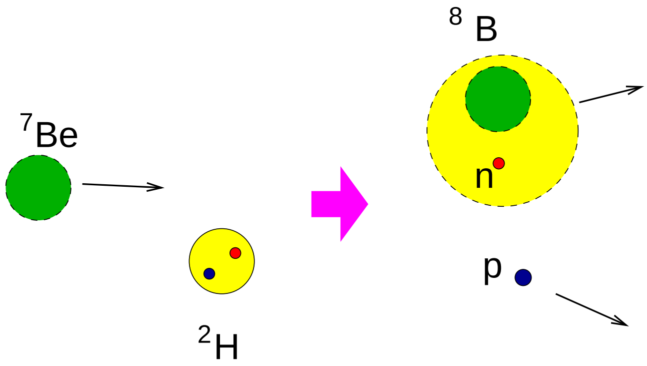
Useful output files:

- Main output file:

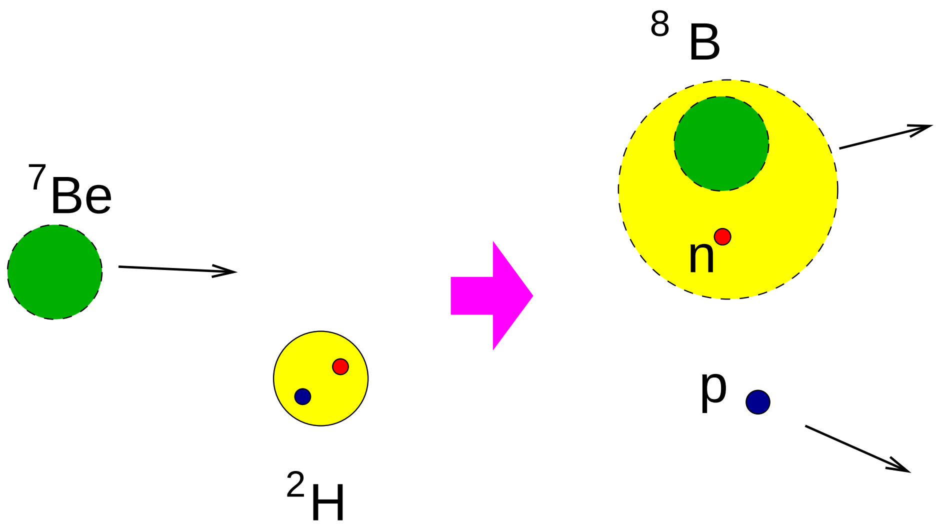
```
0CUMULATIVE REACTION cross section                 =   11.22270  <L> =    47.07
0CUMULATIVE outgoing cross sections in partition 1 :    0.00000    7.67943    0.99138
0Cumulative ABSORBTION by Imaginary Potentials     =    2.55189  <L> =     6.99
```

- Angular distributions:
  - `fort.201` : Elastic scattering angular distribution
  - `fort.202` : 1st state angular distribution
  - `fort.203` : 2nd excited state angular distribution
- `fort.56`: 3 columns: Fusion (absorption), reaction and inelastic cross section for each total angular momentum J.

$$\sigma_{\rm reac} = \sigma_{\rm inel} + \sigma_{\rm abs}$$

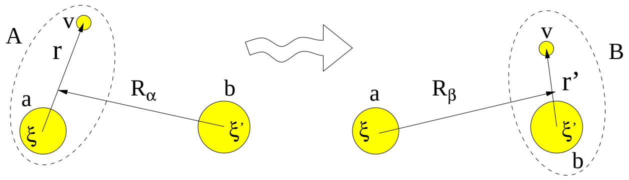


Transfer reactions



Formalism for transfer reactions

- Transfer process:. $\underbrace{(a+v)}_{A}+b \rightarrow a+\underbrace{(b+v)}_{B}$



- Projectile–target interaction:
  - Prior form: $\mathbf{V}_{\rm prior} = V_{vb} + U_{ab} = U_\alpha + \underbrace{(V_{vb} + U_{ab} - U_\alpha)}_{\boldsymbol{\Delta}_{\rm prior}}$
  - Post form: $\mathbf{V}_{\rm post} = V_{av} + U_{ab} = U_\beta + \underbrace{(V_{av} + U_{ab} - U_\beta)}_{\boldsymbol{\Delta}_{\rm post}}$

☞ U_α, U_β: average proyectile-target interaction in entrance/exit channel

Coupled Reaction Channels

- Model wavefunction:

$$\Psi = \phi_A(\xi, \mathbf{r})\phi_b(\xi')\chi_\alpha(\mathbf{R}_\alpha) + \phi_a(\xi)\phi_B(\xi', \mathbf{r}')\chi_\beta(\mathbf{R}_\beta)$$

- Coupled-reaction channels (CRC) equations: $[H - E]\,\Psi = 0$

$$\begin{aligned}
[E - \epsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\,\chi_\alpha(\mathbf{R}_\alpha) &= \int d\mathbf{R}_\beta K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\beta(\mathbf{R}_\beta) \\
[E - \epsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)]\,\chi_\beta(\mathbf{R}_\beta) &= \int d\mathbf{R}_\alpha K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta)\chi_\alpha(\mathbf{R}_\alpha)
\end{aligned}$$

- Non-local kernels:

$$K_{\alpha,\beta}(\mathbf{R}_\beta, \mathbf{R}_\alpha) = \int d\xi d\xi' d\mathbf{r}\ \phi_a(\xi)\phi_B(\xi', \mathbf{r}')(H - E)\phi_A(\xi, \mathbf{r})\phi_b(\xi')$$

☞ *CRC equations have to be solved iteratively due to NL kernels.*

DWBA approximation

- Distorted wave Born approximation:

$$\begin{aligned}
[E - \epsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)]\,\widetilde{\chi}_\alpha(\mathbf{R}_\alpha) &= 0 \\
[E - \epsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)]\,\widetilde{\chi}_\beta(\mathbf{R}_\beta) &= 0
\end{aligned}$$

- DWBA amplitude (prior):

$$\boxed{T_{\text{prior}} = \int\int \widetilde{\chi}_\beta^{(-)}(\mathbf{R}_\beta)(\phi_a\phi_B|\Delta_{\text{prior}}|\phi_A\phi_b)\widetilde{\chi}_\alpha^{(+)}(\mathbf{R}_\alpha)d\mathbf{R}_\alpha d\mathbf{r}}$$

- Structure form-factor:

$$(\phi_a\phi_B|\Delta_{\text{prior}}|\phi_A\phi_b) \equiv \int d\xi d\xi'\ \phi_a(\xi)\phi_B(\xi', \mathbf{r}')\Delta_{\text{prior}}\phi_A(\xi, \mathbf{r})\phi_b(\xi')$$

Spectroscopic factors

- Parentage amplitudes:
  - Projectile: $\phi_A^{JM}(\xi,\mathbf{r}) = \frac{1}{\sqrt{n_A}}\sum_{I\ell j} A_{IJ;\ell sj}\left[\phi_a^I(\xi)\otimes\varphi_{\ell sj}(\mathbf{r})\right]_{JM}$
  - Target: $\phi_B^{J'M'}(\xi',\mathbf{r}') = \frac{1}{\sqrt{n_B}}\sum_{I\ell j} A_{IJ';\ell sj}\left[\phi_b^I(\xi')\otimes\varphi_{\ell sj}(\mathbf{r}')\right]_{J'M'}$

☞ $A_{IJ;\ell sj}$ = spectroscopic amplitudes

☞ $S_{IJ;\ell sj} = |A_{IJ;\ell sj}|^2$ = spectroscopic factors

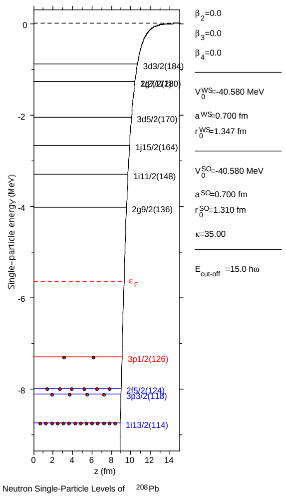
- DWBA amplitude (prior)

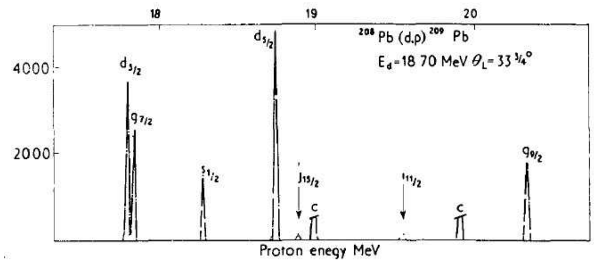
$$\boxed{T_{\rm prior} = A_{IJ;\ell sj}A_{I'J';\ell' sj'}\int\int \widetilde{\chi}_\beta^{(-)}(\mathbf{R}_\beta)\varphi_{\ell' sj'}(\mathbf{r}')\Delta_{\rm prior}\varphi_{\ell sj}(\mathbf{r})\widetilde{\chi}_\alpha^{(+)}(\mathbf{R}_\alpha)d\mathbf{R}_\alpha d\mathbf{r}}$$

☞ *In DWBA, the transfer cross section is proportional to the spectroscopic factors* $S_{IJ;\ell sj}S_{I'J';\ell' sj'}$

Extracting structure information from transfer reactions

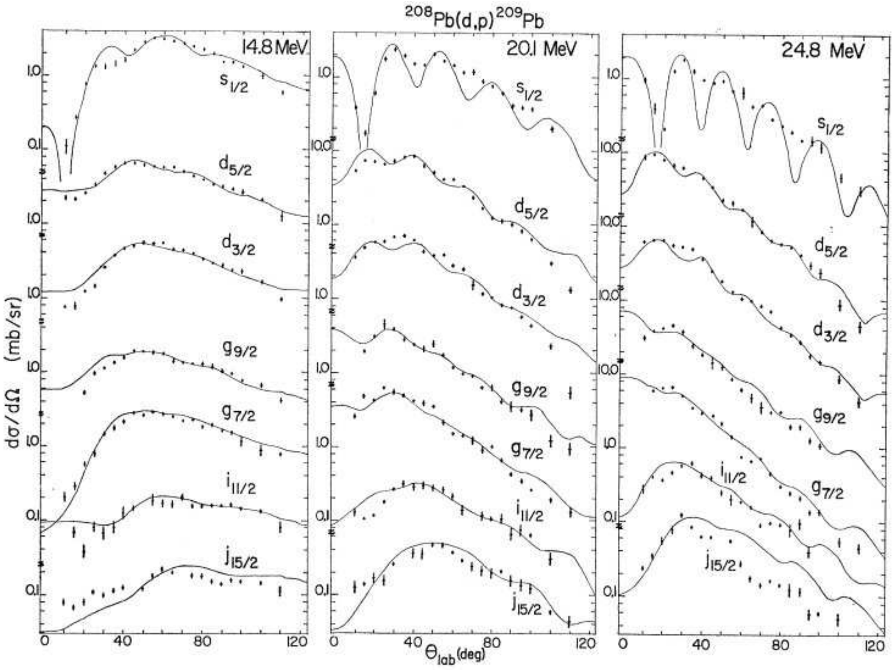
Example: d+^{208}Pb $\longrightarrow$ p + ^{209}Pb





Jeans et al, NPA 128 (1969) 224

Extracting structure information from transfer reactions



☞*Angular distributions of transfer cross sections are very sensitive to the single-particle configuration of the transferred nucleon/s.*

Beyond DWBA: CCBA formalism

When there are strongly coupled excited states in the initial or final partition, the CC and DWBA formalisms can be combined → CCBA

Ej: ^{172}Yb(p,d) Ascuitto et al, Nucl Phys. A226 (1974) 454

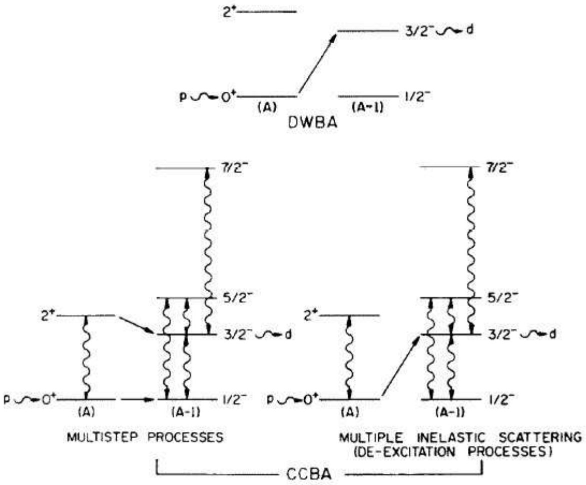
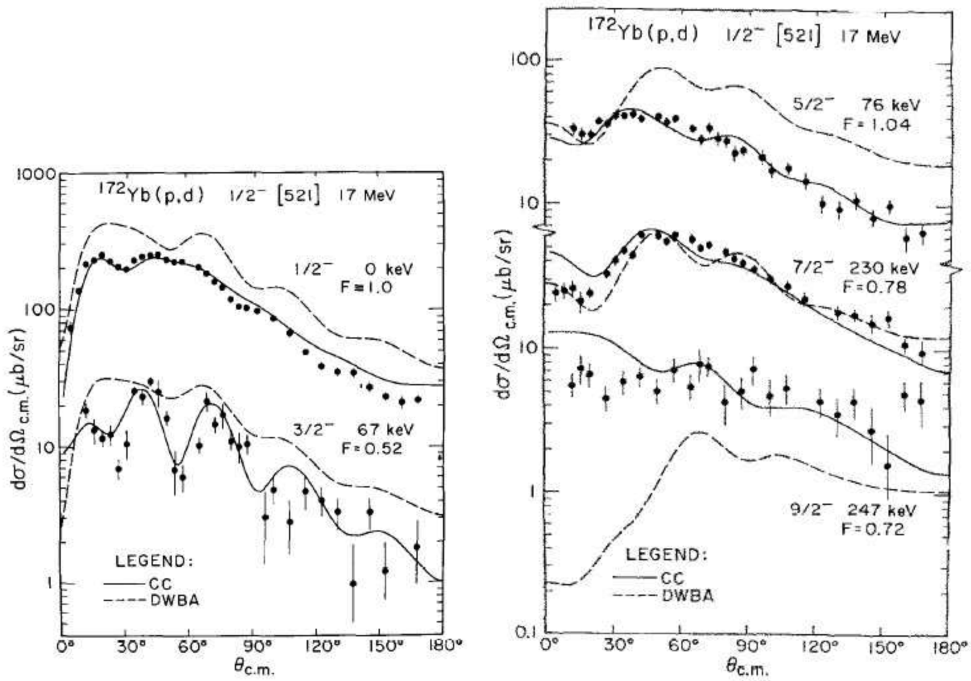


Fig. 1. Schematic representation of processes *explicitly* included in a DWBA analysis and those in a CCBA analysis. The multistep processes (for simplicity we show a single route) are predominantly determined by parentage conditions, while the de-excitation processes depend on the strength of the inelastic coupling.



Brief summary on transfer reactions

- Inclusion of transfer couplings in the Schrodinger equation gives rise to a set of coupled equations with non-local kernels (Coupled Reactions Channels)

- If transfer couplings are weak, the CRC equations can be solved in Born approximation $\Rightarrow$ DWBA approximation

- The DWBA amplitude is proportional to the product of the projectile and target spectroscopic factors.

- The analysis of transfer reactions provide information on:
  - Spectroscopic factors.
  - Quantum number for single-particle configurations (n, ℓ, j).
  - Binding interactions.
  - Reactions mechanisms.

Transfer example

Physical example: ^{56}Fe(d,p)^{57}Fe at $E_d = 12$ MeV

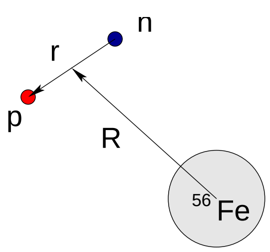
Transfer example: ^{56}Fe(d,p)^{57}Fe

DWBA transfer amplitude:

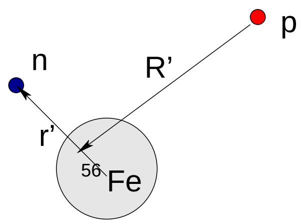
$$\boxed{T^{\mathrm{DWBA}} = A_i A_f \langle \chi^{(-)}_{\mathrm{p}-{}^{57}\mathrm{Fe}} \phi_{{}^{57}\mathrm{Fe}} | V_{\mathrm{prior/post}} | \chi^{(+)}_{\mathrm{d}-{}^{56}\mathrm{Fe}} \phi_d \rangle}$$

- $\chi_{\mathrm{d}-{}^{56}\mathrm{Fe}}, \chi_{\mathrm{p}-{}^{57}\mathrm{Fe}}$: initial and final distorted waves
- ϕ_d: projectile bound wavefunction($p-n$)
- $\phi_{{}^{57}\mathrm{Fe}}$: final (residual) wavefunction (n+^{56}Fe)
- A_i, A_f: initial / final spectroscopic amplitudes.
- $V_{\mathrm{prior/post}}$: transition potential in PRIOR or POST form

PRIOR



POST



$$V_{\mathrm{prior}} = V_{\mathrm{n-56Fe}} + \underbrace{U_{\mathrm{p-56Fe}} - U_{\mathrm{d-56Fe}}}_{\texttt{remnant}}$$

$$V_{\mathrm{post}} = V_{\mathrm{p-n}} + \underbrace{U_{\mathrm{p-56Fe}} - U_{\mathrm{p-57Fe}}}_{\texttt{remnant}}$$

Transfer example: ^{56}Fe(d,p)^{57}Fe

Essential physical ingredients in a DWBA calculation:

- **Potentials (5):**
  - Distorted potential for <u>entrance</u> channel (complex): d+^{56}Fe
  - Distorted potential for <u>exit</u> channel (complex): p+^{57}Fe
  - Core-core interaction (complex): p+^{56}Fe
  - Binding potential for projectile (real): p+n
  - Binding potential for target (real): n+^{56}Fe
- **Spectroscopic amplitudes:** A_i, A_f

Transfer example: ^{56}Fe(d,p)^{57}Fe

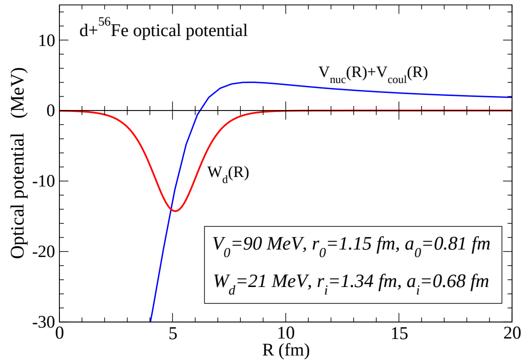
Physical ingredients: Optical and binding potentials (NPA(1971) 529)

System	$\mathbf{V}_0$ (MeV)	$\mathbf{r}_0$ (fm)	$\mathbf{a}_0$ (fm)	$\mathbf{W}_d$ (MeV)	$\mathbf{r}_i$ (fm)	$\mathbf{a}_i$ (fm)	$\mathbf{r}_C$ (fm)
d+^{56}Fe	90	1.15	0.81	21.0	1.34	0.68	1.15
p+^{56,57}Fe	47.9	1.25	0.65	11.5	1.25	0.47	1.15
$p+n$ [a]	72.15	0.00	1.484	-	-	-	
n+^{56}Fe	B.E.	1.25	0.65	-	-	-	

$\Rightarrow$ $$U(R) = -V_0 f_{WS}(R) + 4\,i\,a\,W_d \frac{df_{WS}(R)}{dR}$$

$$f_{WS}(R) = \frac{1}{1+\exp\left(\frac{R-R_0}{a}\right)}$$

[a] Gaussian geometry: $V(r) = -V_0 \exp[-(r/a_0)^2]$.



Transfer example: ^{56}Fe(d,p)^{57}Fe

Spectroscopic factors:

$$\phi_B^{JM}(\xi,\mathbf{r}) = \sum_{I\ell j} A^{IJ}_{\ell s j}\left[\phi_b^I(\xi)\otimes\varphi_{\ell s j}(\mathbf{r})\right]_{JM}$$

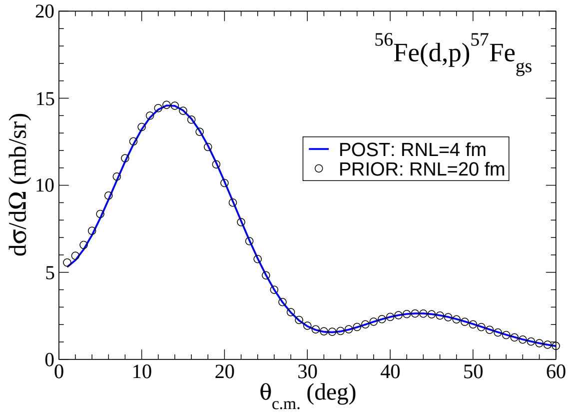
So, for example:

$$|^{57}\mathrm{Fe};1/2^-\rangle = \alpha\left[|^{56}\mathrm{Fe};\mathrm{gs}\rangle\otimes|\nu 2p_{1/2}\rangle\right]_{1/2^-} + \beta\left[|^{56}\mathrm{Fe};2^+\rangle\otimes|\nu 2p_{3/2}\rangle\right]_{1/2^-} + \ldots$$

- α, β, $\ldots$: spectroscopic amplitudes
- $|\alpha|^2$, $|\beta|^2$, $\ldots$: spectroscopic factors

Transfer example: ^{56}Fe(d,p)^{57}Fe

Post and prior equivalence:

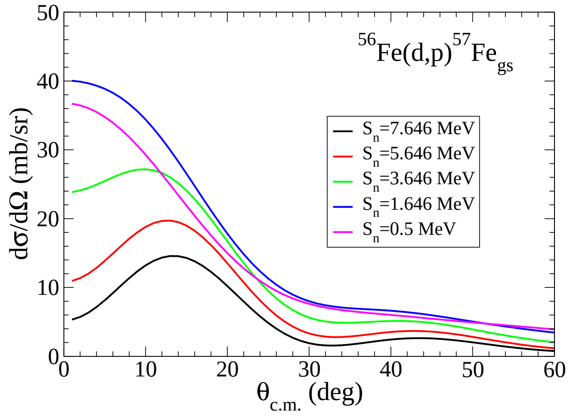


☞ *Post and prior give identical results, provide that the parameters are adequate for convergence.*

Transfer example: ^{56}Fe(d,p)^{57}Fe

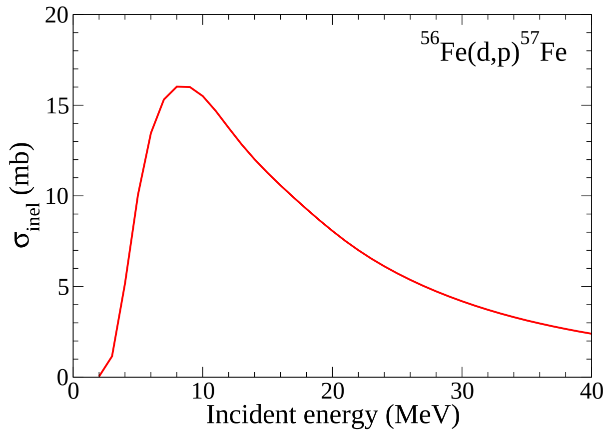
Dependence with binding energy:

```
&OVERLAP kn1=2 (...) be=7.646 /
&OVERLAP kn1=2 (...) be=5.646 /
&OVERLAP kn1=2 (...) be=5.646 /
&OVERLAP kn1=2 (...) be=3.646 /
&OVERLAP kn1=2 (...) be=1.646 /
&OVERLAP kn1=2 (...) be=0.100 /
```



Transfer example: ^{56}Fe(d,p)^{57}Fe

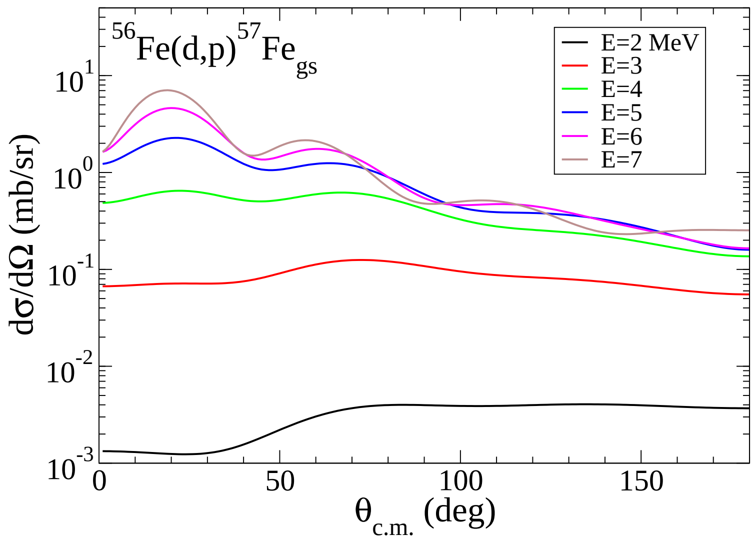
Dependence with beam energy:



Transfer example: ^{56}Fe(d,p)^{57}Fe

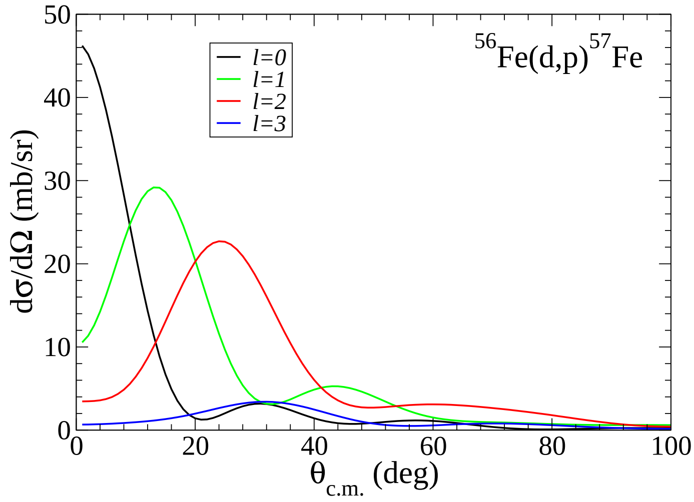
Dependence with beam energy

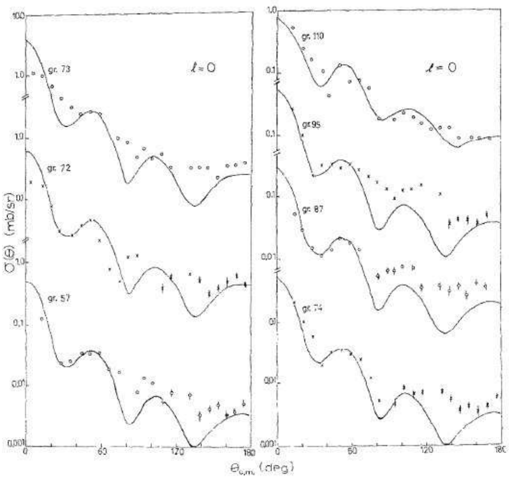
- $E \gg V_b$: diffractive structure, forward peaked.
- $E \ll V_b$: smooth dependence with θ, backward peaked.

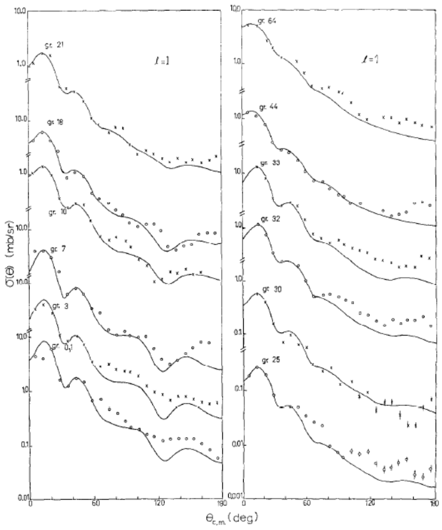


Transfer example: ^{56}Fe(d,p)^{57}Fe

Selectivity of ℓ:



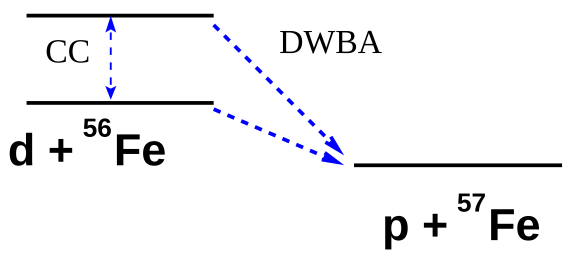




H.M. Sen Gupta et al, Nucl. Phys. A160, 529 (1971)

Beyond DWBA: CCBA formalism

When there are strongly coupled excited states in the initial or final partition, the CC and DWBA formalisms can be combined → CCBA



Transfer example: ^{56}Fe(d,p)^{57}Fe

Appendix: ^{56}Fe(d,p)^{57}Fe with FRESCO: `fe56dp_dwba.in`

```
6Fe(d,p)57Fe @ Ed=12 MeV;
AMELIST
FRESCO hcm=0.1 rmatch=20.000
      rintp=0.20 hnl=0.100 rnl=4 centre=-0.45
      jtmax=15
      thmin=1.00 thmax=180.00 thinc=1.00
      it0=1 iter=1
      chans=1 smats=2 xstabl=1
      elab= 12 /

PARTITION namep='d' massp=2.014 zp=1 namet='56Fe'
      masst=55.934 zt=26 nex=1 /
STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0
      bandt=1 et=0.0 /

PARTITION namep='p' massp=1.0078 zp=1 namet='57Fe'
      masst=56.935 zt=26 qval=5.421 pwf=F nex=1 /
STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5
      bandt=-1 et=0.0 /
partition /

POT kp=1 itt=F at=56 rc=1.15 /
POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /
POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /

POT kp=2 itt=F at=57 rc=1.15 /
POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

```
POT kp=3 itt=F at=56 rc=1.00 /
POT kp=3 type=1 itt=F p1=65.0 p2=1.25 p3=0.65 /
POT kp=4 itt=F ap=1.0000 at=0.0000 rc=1.0000 /
POT kp=4 type=1 shape=2 itt=F p1=72.1500
     p2=0.0000 p3=1.4840 /
POT kp=5 itt=F at=56 rc=1.15 /
POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
pot /

OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 sn=0.5
     j=0.5 kbpot=4 be=2.2250 isc=1 /
OVERLAP kn1=2 ic1=1 ic2=2 in=2 nn=2 l=1 sn=0.5
     j=0.5 kbpot=3 be=7.646 isc=1 /
overlap /

COUPLING icto=2 icfrom=1 kind=7 ip2=-1 ip3=5 /
CFP in=1 ib=1 ia=1 kn=1 a=1.0000 /
CFP in=2 ib=1 ia=1 kn=2 a=1.0000 /
cfp /

coupling /
```


Transfer example: ^{56}Fe(d,p)^{57}Fe

General variables:

```
56Fe(d,p)57Fe @ Ed=12 MeV;
NAMELIST
&FRESCO hcm=0.1 rmatch=20.000
      rintp=0.20 hnl=0.100 rnl=4 centre=-0.45
      jtmax=15
      thmin=1.00 thmax=180.00 thinc=1.00
      it0=1 iter=1
      chans=1 smats=2 xstabl=1
      elab= 12 /
```

- `rnl`: range of non-locality
- `centre,rintp,hnl`: parameters for numerical integration (see `fresco` manual)
- `iter`: Number of iterations so, for DWBA, `iter=1`

Transfer example: ^{56}Fe(d,p)^{57}Fe

Partitions and states:

- Incoming (initial) partition: d+^{56}Fe

```
&PARTITION namep='d' massp=2.014 zp=1
        namet='56Fe' masst=55.934 zt=26 nex=1 /
&STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- Outgoing (final) partition: p+^{57}Fe

```
&PARTITION namep='p' massp=1.0078 zp=1
        namet='57Fe'masst=56.935 zt=26
        qval=5.421 nex=1 /
&STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5 bandt=-1 et=0.0 /
```

• `qval`: Q-value for gs-gs transfer

Transfer example: ^{56}Fe(d,p)^{57}Fe

Interactions:

- Entrance channel distorted potential: d+^{56}Fe

```
&POT kp=1 itt=F at=56 rc=1.15 /
&POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /
&POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /
```

- Exit channel distorted potential: p+^{57}Fe

```
&POT kp=2 itt=F at=57 rc=1.15 /
&POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
&POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

- Core-core potential: p+^{56}Fe

```
&POT kp=5 itt=F at=56 rc=1.15 /
&POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
&POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

Transfer example: ^{56}Fe(d,p)^{57}Fe

Interactions (continued ...)

Binding potentials:

- n+ ^{56}Fe: Woods-Saxon

```
&POT kp=3 at=56 rc=1.0 /
&POT kp=3 type=1 p1=65.0 p2=1.25 p3=0.65 /
```

- n+p: Gaussian

```
&POT kp=4 ap=1.0 at=0.0 /
&POT kp=4 type=1 shape=2 p1=72.15 p2=0.00 p3=1.484 /
```


Transfer example: ^{56}Fe(d,p)^{57}Fe

Bound wavefunctions (overlaps):

- d=p+n: simple 1S model

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 l=0 sn=0.5 j=0.5
      kbpot=4 be=2.2250 isc=1 /
```

- ^{57}Fe=^{56}Fe+n: assume $2p_{1/2}$ configuration

```
&OVERLAP kn1=2 ic1=1 ic2=2 in=2 nn=2 l=1 sn=0.5 j=0.5
      kbpot=3 be=7.646 isc=1 /
```

  - `in=1`: projectile
    `in=2`: target
  - `nn,l,sn,j`: quantum numbers: $\vec{l} + \vec{sn} = \vec{j}$
  - `be`: binding (separation) energy
  - `kbpot`: potential index

Transfer example: ^{56}Fe(d,p)^{57}Fe

Transfer coupling between the two partitions:

```
&COUPLING icfrom=1 icto=2 kind=7 ip1=0 ip2=-1 ip3=5 /
```

- `icfrom`: index for partition of initial state
- `icto`: index for partition of final state
- `kind`: kind of coupling. kind=7 means finite-range transfer.
- `ip1=0`: post representation
  `ip1=1`: prior
- `ip2=-1`: include full remnant
- `ip3`: index for core-core potential (p+^{56}Fe)

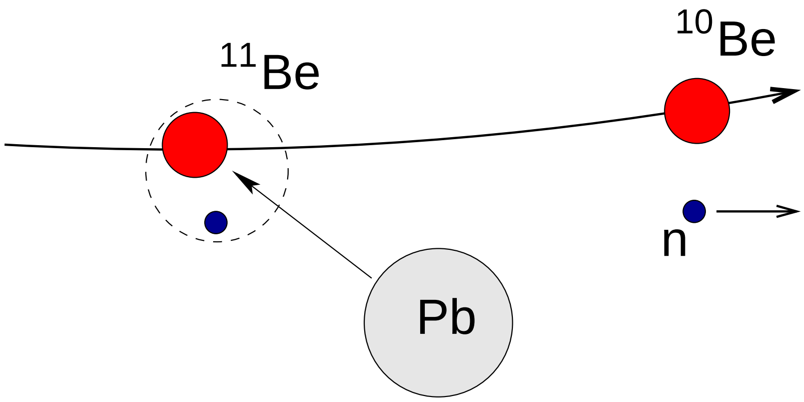
Transfer example: ^{56}Fe(d,p)^{57}Fe

Spectroscopic amplitudes:

```
&CFP in=1 ib=1 ia=1 kn=1 a=1.0 /
&CFP in=2 ib=1 ia=1 kn=2 a=1.0 /
```

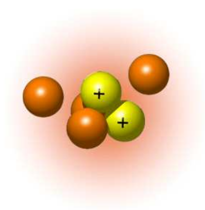
- `in=1`: projectile state
  `in=2`: target state
- `ib`: index for state of composite
  `ia`: index for state of core
- `a`: spectroscopic amplitude

Inelastic scattering to the continuum

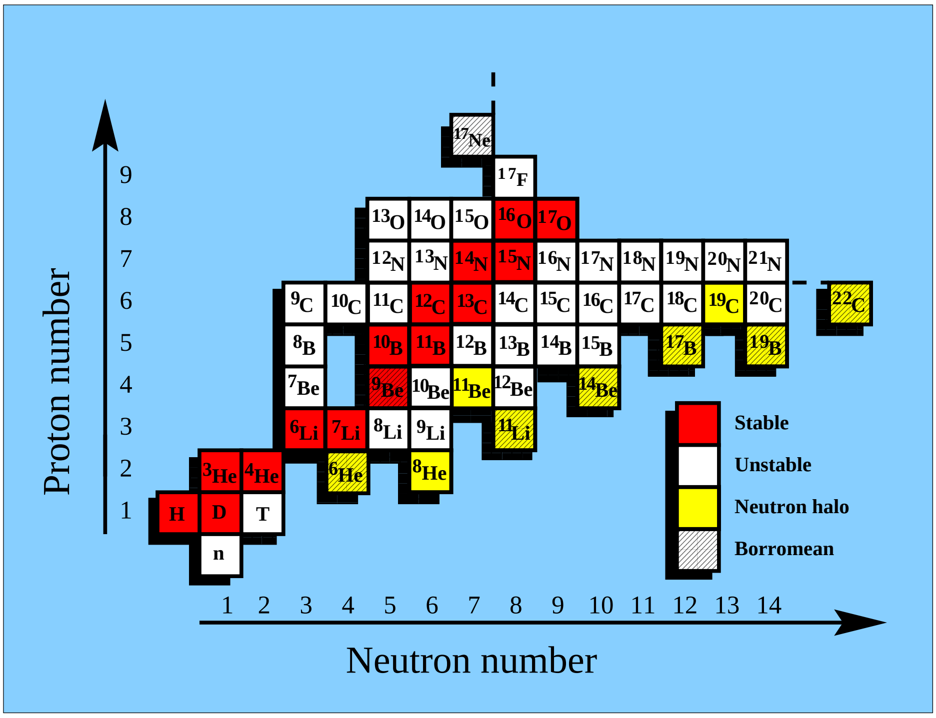


Exotic nuclei, halo nuclei and Borromean systems

- Radioactive nuclei: they typically decay by β emission.
  E.g.: ^{6}He $\xrightarrow{\beta^-}$ ^{6}Li ($\tau_{1/2} \simeq$807 ms)
- Weakly bound: typical separation energies are around 1 MeV or less.
- Spatially extended
- Halo structure: one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.
- Borromean nuclei: Three-body systems with no bound binary sub-systems.

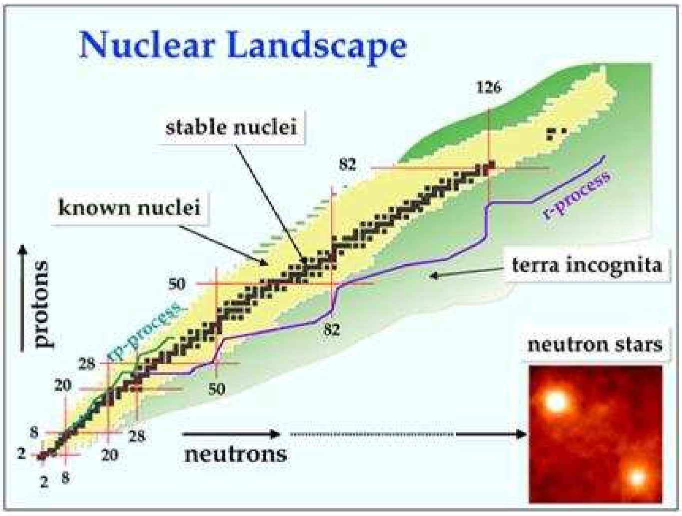


Exotic nuclei, halo nuclei, and Borromean systems



Why study reactions with exotic nuclei?

- Many properties of nuclear structure (level spacing, magic numbers,etc) could be different from normal nuclei.
- Many reactions of astrophysical interest are known to involve nuclei far from the stability valley.



Some difficulties inherent to the study of reactions with exotic beams

Experimentally:

- Exotic nuclei are short-lived and difficult to produce. Beam intensities are typically small.

Theoretically:

- Exotic nuclei are easily broken up in nuclear collisions $\Rightarrow$ coupling to the continuum plays an important role.
- Effective NN interactions, level schemes, etc are different from stable nuclei.
- Many exotic nuclei exhibit complicated cluster (few-body) structure.

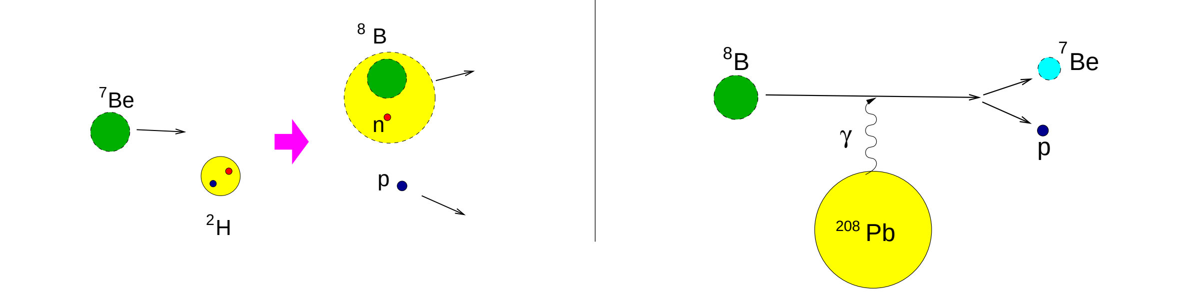
Indirect measurements for nuclear astrophysics

- Many reactions of astrophysical interest at energies too low to be measured e at present experimental facilities.
- Coulomb breakup and transfer reactions provide indirect information for these processes.

Example: The rate for the ^{7}Be+ p $\rightarrow$ ^{8}B + γ reaction depends mainly on the overlap (^{8}B|^{7}Be), which can be investigated by means of other direct reactions:

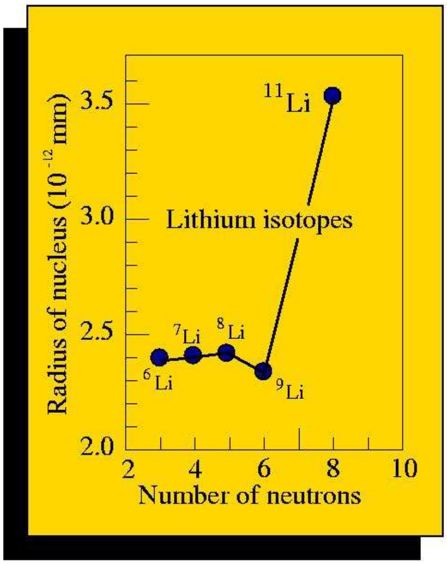
Transfer reactions:
E.g.: d(^{7}Be, ^{8}B)n

Coulomb breakup reactions:
E.g.: ^{8}B+^{208}Pb $\rightarrow$ ^{7}Be+ p + ^{208}Pb



Some difficulties inherent to the study of reactions with exotic beams

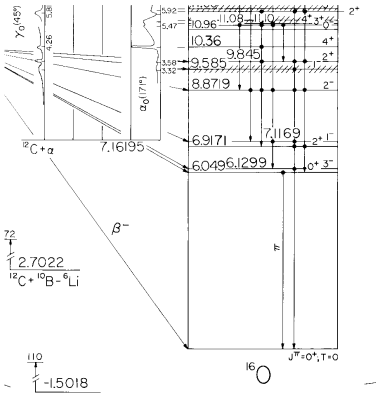
☞ First evidences of the existence of halo nuclei came from reaction cross sections measurements.

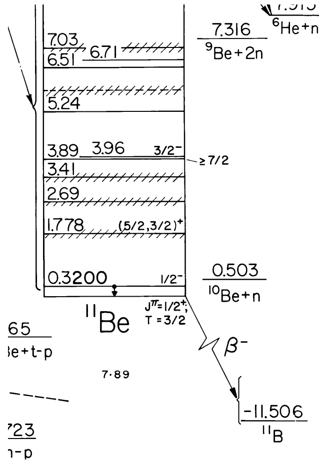


Tanihata *et al*, PRL55, 2676 (1985)

Inelastic scattering of weakly bound nuclei

- Single-particle (or cluster) excitations become dominant.
- Excitation to continuum states important.



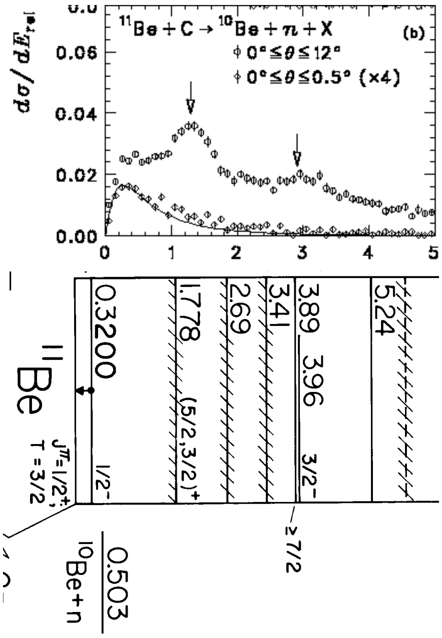


☞*Exotic nuclei are weakly bound ⇒ coupling to continuum states becomes an important reaction channel*

Inelastic scattering example: ^{11}Be+^{12}C

Fukuda et al, Phys. Rev. C70 (2004) 054606)

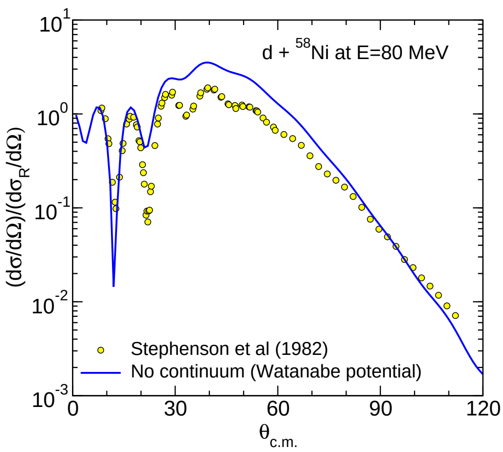
^{11}Be+^{12}C $\to$ (^{10}Be+n) + ^{12}C



Failure of the calculations without continuum

Three-body calculation (p+n+^{58}Ni) with Watanabe potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r}\phi_{gs}(\mathbf{r})\,(V_{pt} + V_{nt})\,\phi_{gs}(\mathbf{r})$$



☞*Three-body calculations omitting breakup channels fail to describe the experimental data.*

The role of the continuum in the scattering of weakly bound nuclei

The origins of the CDCC method:

- Pioneering work of Johnson & Soper for deuteron scattering: PRC1,976(1970) ⇒ p-n continuum represented by a single s-wave state.

PHYSICAL REVIEW C VOLUME 1, NUMBER 3 MARCH 1970

Contribution of Deuteron Breakup Channels to Deuteron Stripping and Elastic Scattering

R. C. Johnson
Department of Physics, University of Surrey, Guildford, Surrey, England
AND
P. J. R. Soper*
International Centre for Theoretical Physics, Trieste, Italy

(Received 10 November 1969)

We present a model of deuteron stripping and elastic scattering which treats explicitly the contributions from channels in which the deuteron is broken up into a relative S state and the target is in its ground state. An adiabatic treatment of these channels leads to a description of deuteron stripping which resembles

The role of the continuum in the scattering of weakly bound nuclei

- More realistic formulation by G.H. Rawitscher [PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)].

PHYSICAL REVIEW C VOLUME 9, NUMBER 6 JUNE 1974

Effect of deuteron breakup on elastic deuteron-nucleus scattering

George H. Rawitscher*
Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139,[†]
and Department of Physics, University of Surrey, Guildford, Surrey, England

(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements $V_{ab}(R)$ of the breakup potential V_N taken between states $\phi_a(\vec{r})$ and $\phi_b(r)$ are examined. Here $\phi_a(\vec{r})$ are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy ϵ_a are positive (continuum states) or negative (bound deuteron); $V_N(\vec{r}, \vec{R})$ is the sum of the phenomenological proton nucleus $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$ and neutron nucleus $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$ optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transi-

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

Inclusion of the continuum in CC calculations: continuum discretization

Quantum Hamiltonian

Bound states	Unbound states
- Discrete	- Continuous
- Finite	- Infinite
- Normalizable	- Non-normalizable

Continuum discretization: represent the continuum by a finite set of square-integrable states

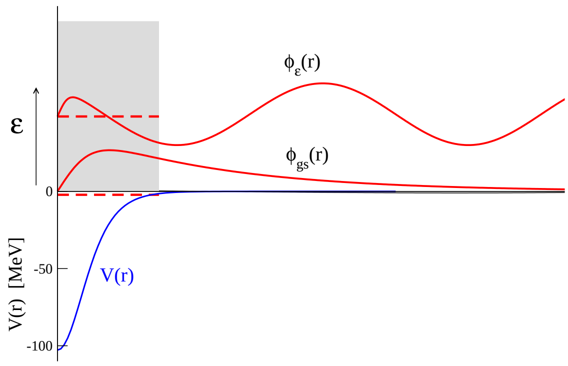
***True* continuum**	$\longrightarrow$	**Discretized continuum**
Non normalizable	$\longrightarrow$	Normalizable
Continuous	$\longrightarrow$	Discrete

Continuum discretization

SOME POPULAR METHODS OF CONTINUUM DISCRETIZATION:

- Box method:
  Eigenstates of the H in a large box.
- Sturmian basis
- Gamow states: complex-energy eigenstates of the Schrödinger equation.
- Pseudostate method:
  Expand continuum states in a complete basis of square-integrable states (eg. HO)
- Bin method:
  Square-integrable states constructed from scattering states.

Bound versus scattering states

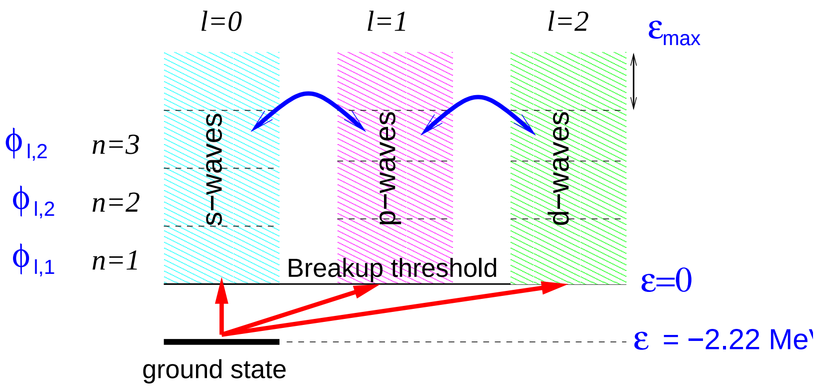


Unbound states are not suitable for CC calculations:

- Continuous (infinite) distribution in energy.
- Non-normalizable: $\langle \phi_k(r)^* | \phi_{k'}(r) \rangle \propto \delta(k-k')$

CDCC formalism

Example: discretization of the deuteron continuum in terms of energy bins.

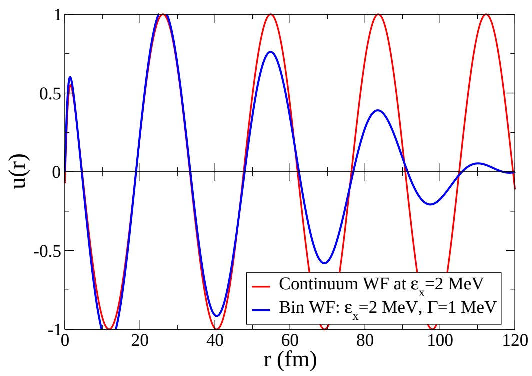


CDCC formalism

Bin wavefunction:

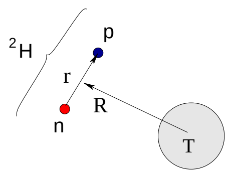
$$u_{\ell sj,n}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{\ell sj,k}(r) dk$$

- k: linear momentum
- $u_{\ell sj,k}$: scattering states (radial part)
- $w(k)$: weight function



CDCC formalism for deuteron scattering

- Hamiltonian: $H = T_R + h_r + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- Model wavefunction:



$$\Psi(\mathbf{R}, \mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

- Coupled equations: $[H - E]\Psi(\mathbf{R}, \mathbf{r})$

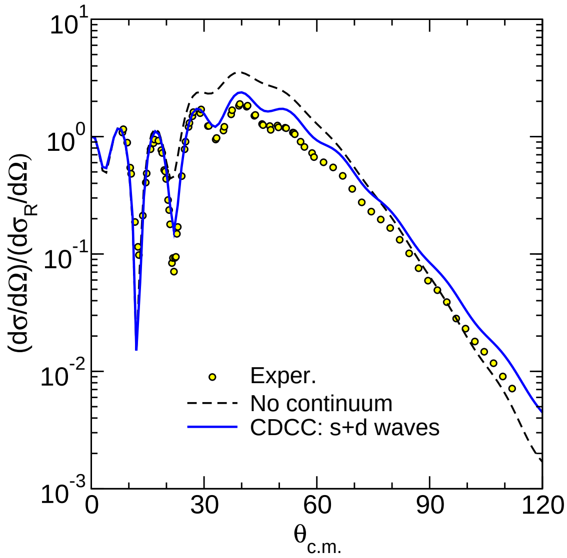
$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2}\right) + \epsilon_n - E\right] f_{\alpha J}(R) + \sum_{\alpha'} i^{L'-L}\, V^J_{\alpha:\alpha'}(R) f_{\alpha' J}(R) = 0$$

α= $\{L, \ell, s, j, n\}$

- Transition potentials:

$$V^J_{n;n'}(\mathbf{R}) = \int d\mathbf{r} \phi_n(\mathbf{r})^* \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2})\right] \phi_{n'}(\mathbf{r})$$

Application of the CDCC formalism: d+ ^{58}Ni

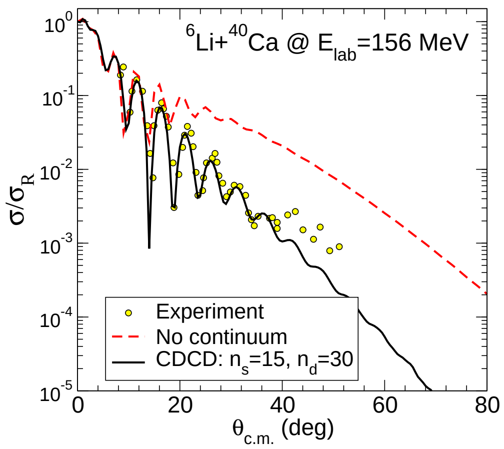


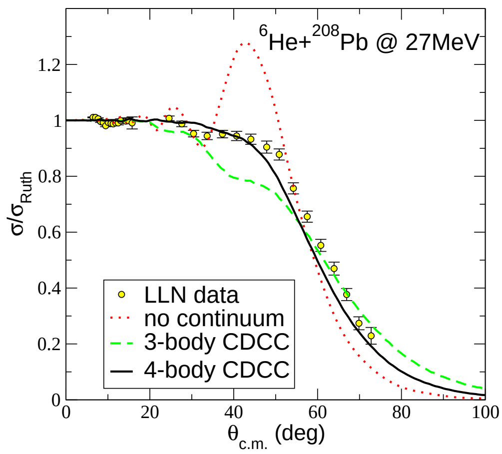
☞Inclusion of the continuum is important to describe the data

Application of the CDCC method: ^{6}Li and ^{6}He scattering

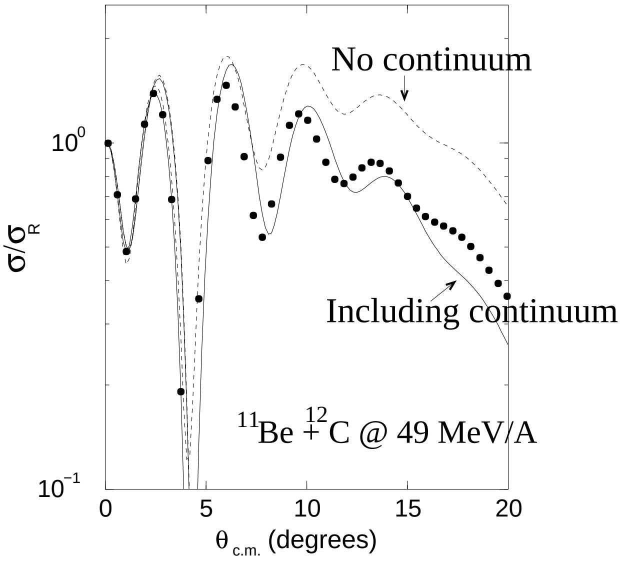
☞ The CDCC has been also applied to nuclei with a cluster structure:

- ^{6}Li=α + d
- ^{6}He=α + n + n





Normal versus halo nuclei: Fraunhofer

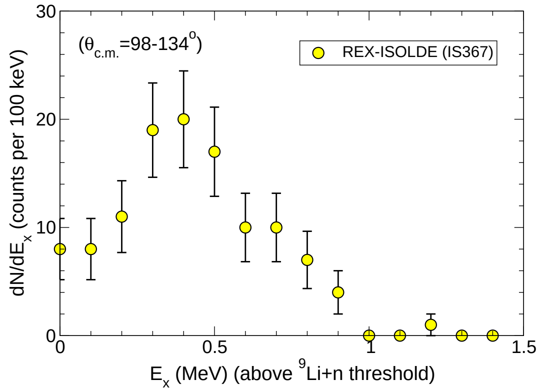


☞ In Fraunhofer scattering the presence of the continuum produces a reduction of the elastic cross section

Breakup observables: resonant and non-resonant continuum

What is a resonance?

Definition 1: (Experimentalist) It is a maximum in the cross section as a function of the energy



Ej: $^9\text{Li} + d \to {}^{10}\text{Li} + p$ (H. Jeppesen et al). Searching for resonances in ^{10}Li.

...but

- Not all the maxima in the cross section can be associated to resonances. Coupling to a non-resonant continuum produces a maximum at some energy, which is related to the size of the system, and the properties of the interaction.
- For weakly bound systems, resonances can be very broad (1 MeV), and occur al relatively low excitation energies. It is not clear, a priori, whether a bump in the cross section is a signature of a genuine resonance, or not.

What is a resonance?

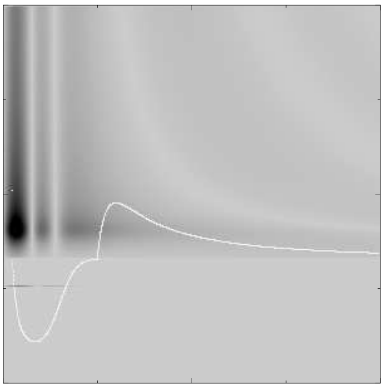
Definition 2: (Theoretician) It is a pole on the S-matrix (of the n+^{9}Li system)

...but

- The fact that there is a pole in the S-matrix in the complex E plane does not allow, by itself, to calculate the cross section to the resonance.
- The wave function corresponding to a pole in the complex E-plane is not square-normalizable.
- Finding poles in a complex energy-plane for multi-channel or three-body systems is difficult.

What is a resonance?

Definition 3: It is a structure on the continuum
which may, or may not, produce a maximum in the cross section, depending on the reaction mechanism and the phase space available.

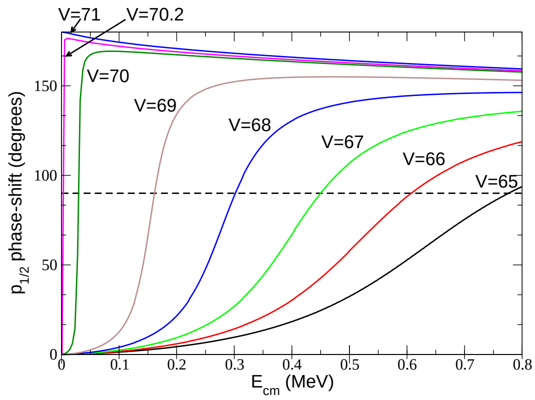


Cuts and areas ordered by size

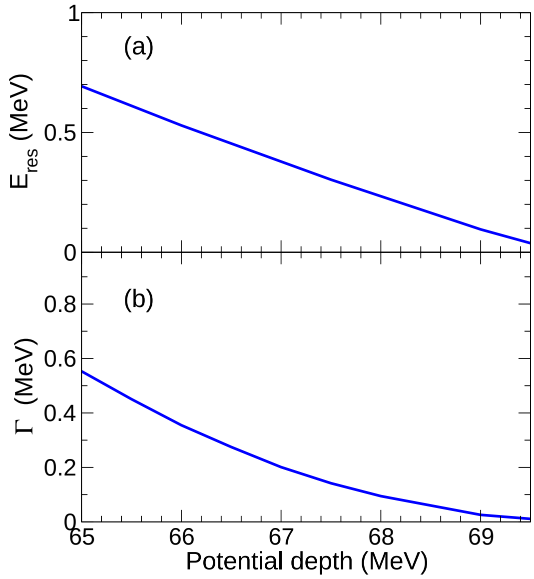
- The resonance occurs in the range of energies for which the phase shift is close to $\pi/2$.
- In this range of energies, the continuum wavefunctions have a relatively large probability of being in the radial range of the potential.
- The continuum wavefunctions are not square normalizable. However, a normalized "bin" of wavefunctions can be constructed to represent the resonance.

^{10}Li p-wave phase-shifts

$\boxed{p_{1/2} \text{ resonance}}$



- $E_{\rm res} \longrightarrow \delta(E_{\rm res}) = \frac{\pi}{2}$
- $\frac{2}{\Gamma} = \frac{d\delta}{dE}$



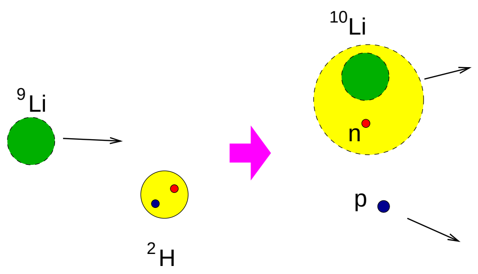
hola

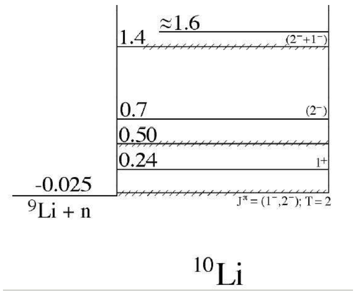
Why measuring transfer reactions with final unbound states?

- Structures in the continuum (resonances, virtual states).
- Some exotic systems are unbound, but contain resonances and other structures that can be studied by means of transfer reactions

  Examples: ^{5}He, ^{8}Be, ^{10}Li, etc

Example: ^{9}Li+p $\rightarrow$ ^{10}Li+p





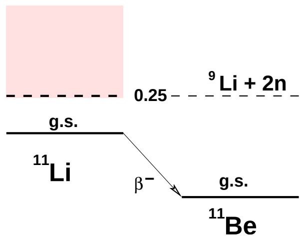
Spectroscopy to unbound states

The ^{10}Li and ^{11}Li systems

- ^{11}Li radioactive:
  $^{11}\text{Li} \xrightarrow{\beta^-} {}^{11}\text{Be}$ $\quad (t_{1/2} \simeq 8.5$ ms)

- ^{11}Li key example of Borromean nucleus:
  - n+n and n+^{9}Li unbound but,
  - n+n+^{9}Li has a (weakly) bound state.

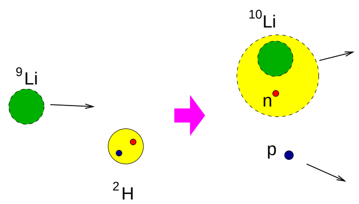
- The structure of ^{11}Li depends critically on:
  - n+n $\longrightarrow$ well understood.
  - n+^{9}Li $\longrightarrow$ dominated by: $\left\{ \begin{array}{l} \text{p-wave resonance} \\ \text{s-wave virtual state} \end{array} \right\} \longrightarrow$ not well understood

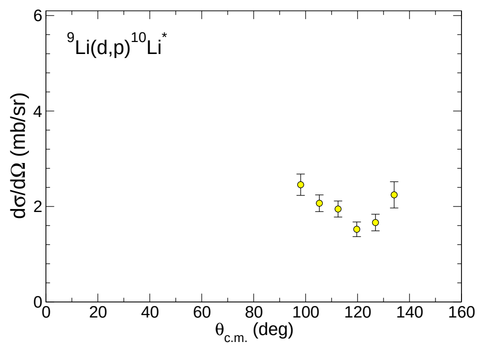


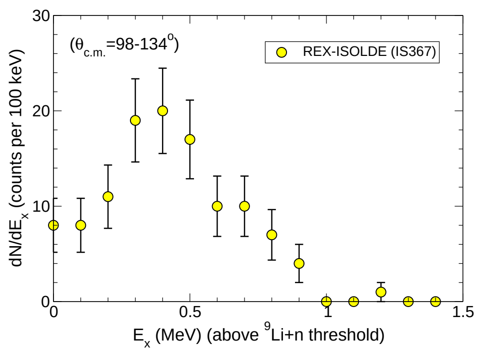
Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

The Experiment:

- REX-ISOLDE (2002)
- ^{9}Li beam on D target at $E = 2.75$ MeV/u
- The experiment provided angular and energy distributions for protons $\Rightarrow$ ^{10}Li.



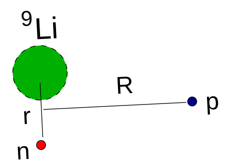




The transfer to the continuum (TC) amplitude

- Exact scattering amplitude (prior form):

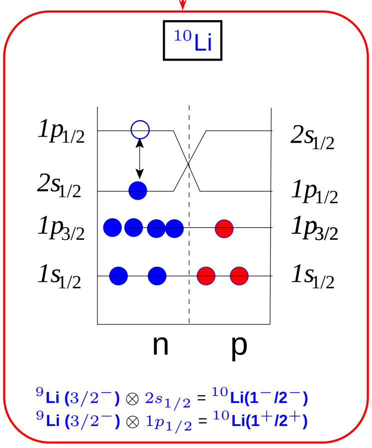
$$\boxed{T_{if} = \langle \Psi_f^{(-)} | V_{\mathrm{n}+{}^9\mathrm{Li}} + U_{\mathrm{p}+{}^9\mathrm{Li}} - U_\alpha | \chi_d^{(+)} \phi_d \rangle}$$

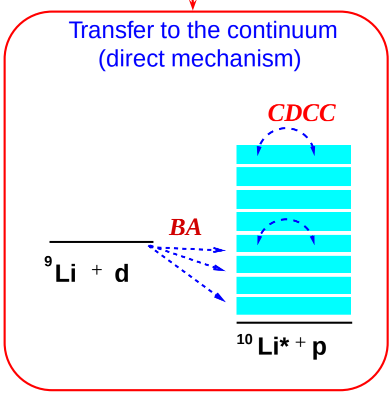


- Numerical evaluation of T:
  - $\Psi_f^{(-)}$ calculated in 3-body model: p+n+^{9}Li
  - $\Psi_f^{(-)} \approx \Psi_f^{\mathrm{CDCC}}(\mathbf{r},\mathbf{R}) = \sum_i \chi^i_{\mathrm{p}-{}^{10}\mathrm{Li}}(\mathbf{R}) \phi^i_{{}^{10}\mathrm{Li}}(\mathbf{r})$
    [$\phi^i_{{}^{10}\mathrm{Li}}(\mathbf{r})$: continuum bins for n-^{9}Li unbound states]
  - $U_\alpha(\mathbf{R})$ taken to reproduce the elastic data *

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

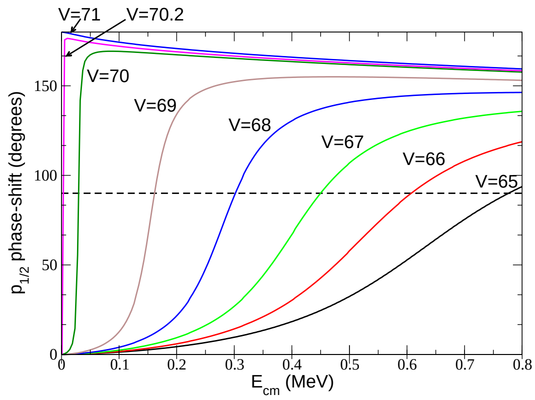
STRUCTURE + REACTION

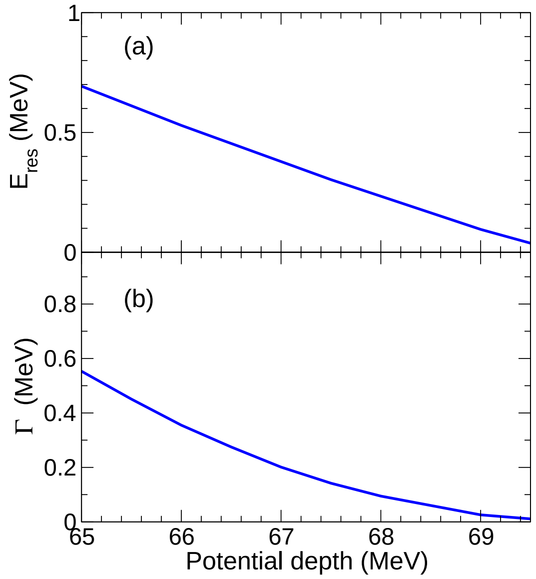




Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

$p_{1/2}$ resonance

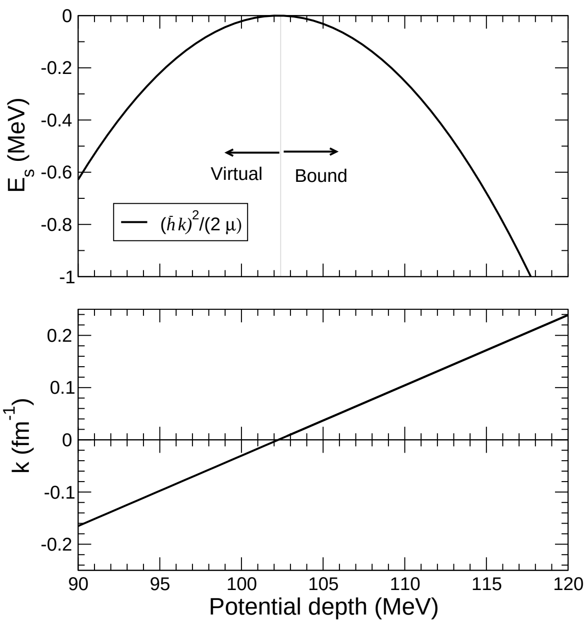


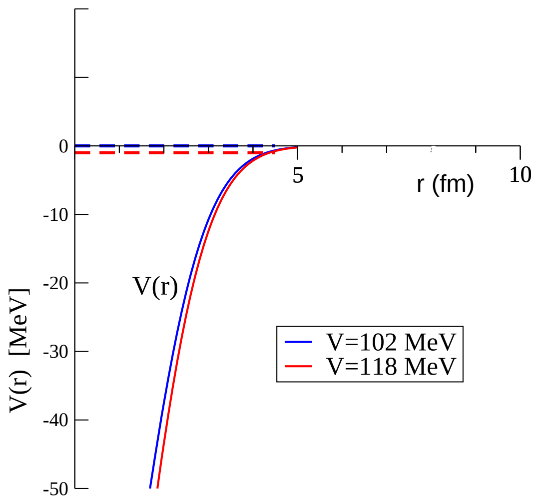


- $E_{\rm res} \to \delta(E_{\rm res}) = \frac{\pi}{2}$
- $\frac{2}{\Gamma} = \frac{d\delta}{dE}$

Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

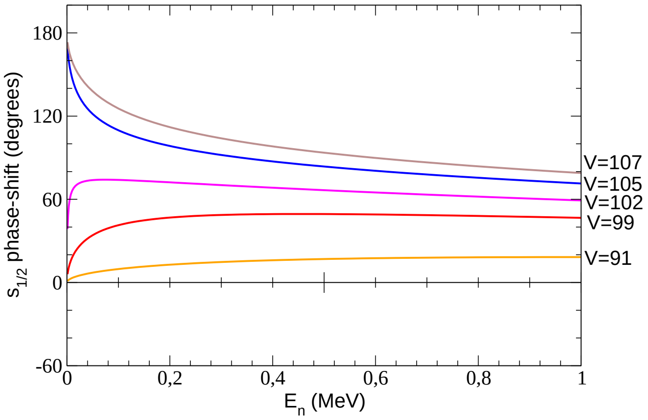
Appearance of a virtual state in ^{10}Li=^{9}Li+n:

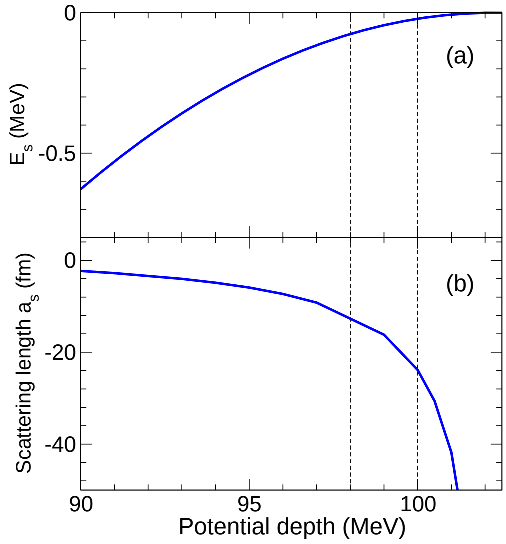




Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

V_s (virtual state)





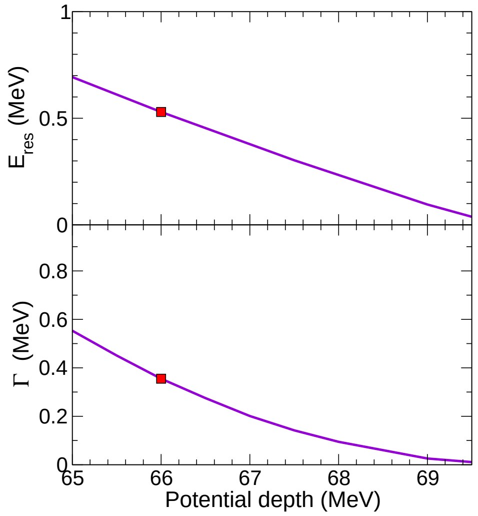
Scattering length:

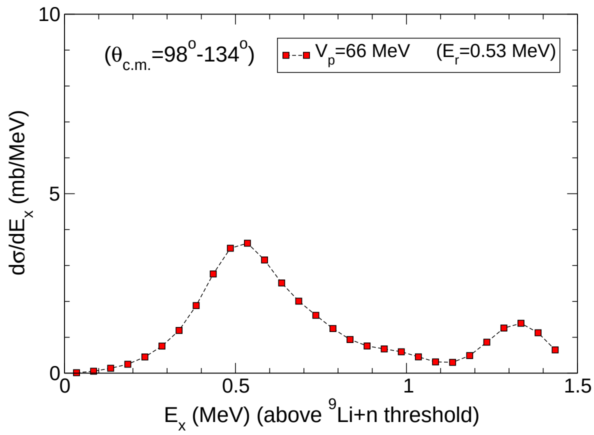
$$a_s = -\lim_{k \to 0} \tan \frac{\delta(k)}{k}$$

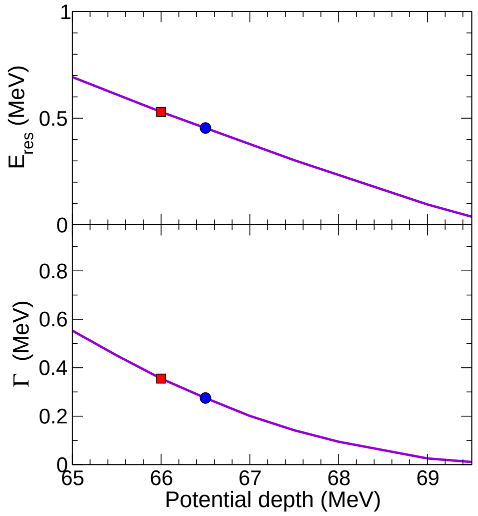
.

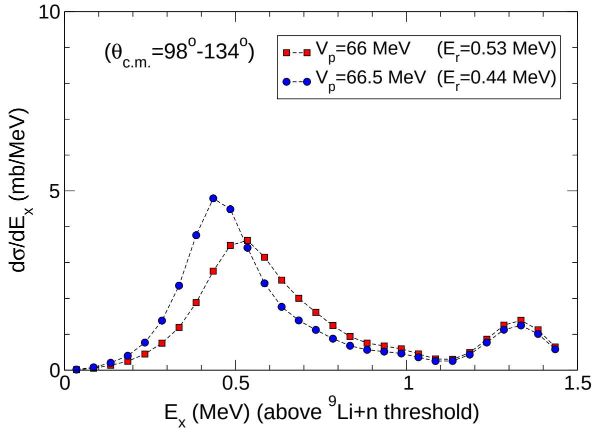
Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

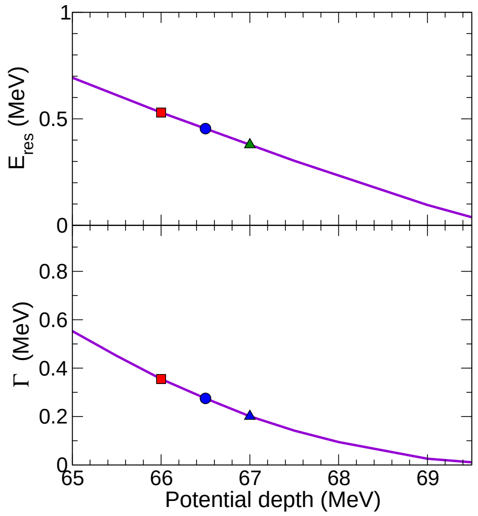
Structure: $p_{1/2}$ resonance → Reaction

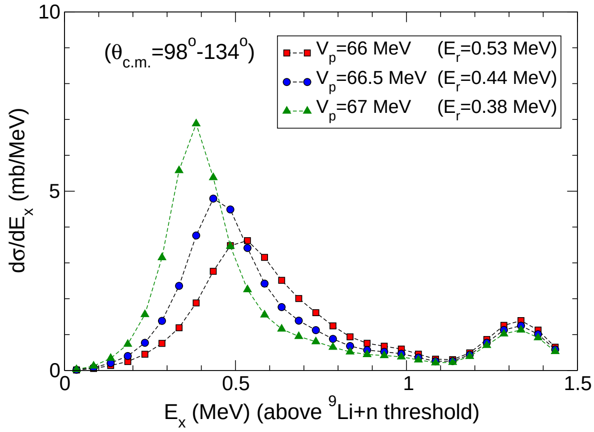






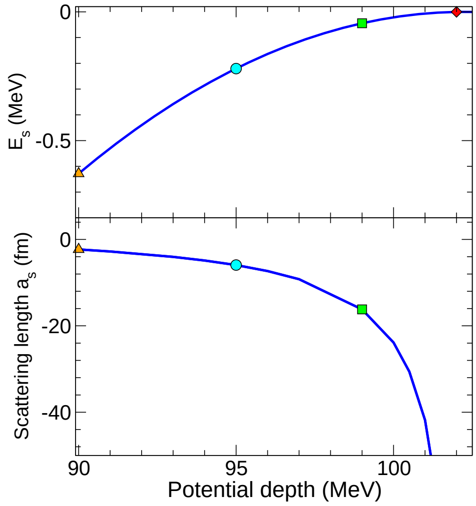


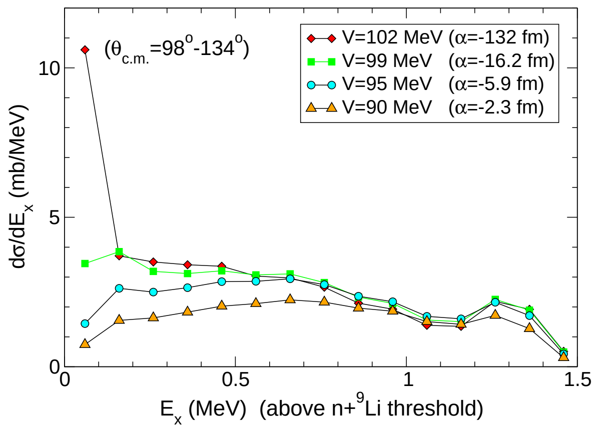




Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li case

Structure: $s_{1/2}$ v.s. → Reaction





Spectroscopy to unbound states: ^{9}Li(d,p)^{10}Li

BEST FIT RESULTS: HP.Jeppesen et al, PLB642 (2006) 449

- $p_{1/2}$ resonance ($1^+/2^+$ doublet): $E_r \simeq 0.38$ MeV, $\Gamma = 0.2$ MeV
- $s_{1/2}$ virtual state ($1^-/2^-$ doublet): $a_s \simeq -24$ fm