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Few-nucleon correlations in nuclei and nuclear matter

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Summary. — Few-body correlations and clustering have to be taken into account to describe the properties of low-density, low-temperature nuclear systems. A systematic, self-consistent description of clustering is obtained from a quantum statistical approach. The quasiparticle picture is introduced, the role of continuum correlations is discussed, and consequences for the nuclear matter equation of state are shown.

1. – Light elements and clustering in dilute nuclear matter

Nuclear matter at high densities (above the saturation density $n_{\rm sat} = 0.15\,{\rm fm}^{-3}$) and temperatures $T \leq 20\,{\rm MeV}$ may be considered as a degenerate Fermi liquid, with quasiparticle excitations known from the Landau-Migdal theory. Because of degeneracy of the strongly interacting matter, correlations are suppressed (Pauli blocking). At subsaturation baryon densities $n_B \leq n_{\rm sat}$, clusters can be formed, and correlation are relevant to describe the properties of nuclear systems. As a characteristic quantity, the Fermi energy of the non-interacting, ideal gas of neutrons (density n_n) is given by $E_n^{\rm Fermi}(n_n) = (\hbar^2/2m)(3\pi^2 n_n)^{2/3}$ (a similar relation holds for the protons). For given T, the nuclear system behaves classically at low density if the corresponding Fermi energy is small compared to T, and Pauli blocking becomes negligible. In addition, clusters have binding energies (*e.g.*, 28.3 MeV for the free α particle) which compete with the Fermi energy also for degenerate matter at zero temperature. For $T = 0$, α-like clusters are bound up to a critical value $n_B^{\rm critical} = 0.03\,{\rm fm}^{-3}$ of baryon density [1].

We give three examples where clustering in nuclear systems is of relevance: nuclear structure, heavy-ion collisions (HIC), and astrophysics of compact objects, in particular neutron stars and supernova explosions. The cluster structure of light (excited) nuclei, in particular the Hoyle state of ^{12}C, has been extensively investigated, for recent reviews see [2-4]. A challenge is the systematic approach to quartetting (in analogy to pairing) near the surface region of nuclei where the density becomes low [1, 5, 6].

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Cluster formation in HIC has been intensely discussed during the past years. The main problem is that HIC are nonequilibrium processes. Standard transport equations describe adequately the time evolution of single-particle properties such as density, single-nucleon energy distribution, but the quantum description of the formation of clusters is not solved yet. Approximations such as the freeze-out model have been worked out to interpret the yields of clusters in HIC. A central issue is the nuclear matter equation of state (EoS) which takes into account the modification of few-nucleon states in surrounding nuclear matter, which is described by T, n_B and the asymmetry parameter $Y_p = n_p^{\rm total}/n_B$ (the total proton density is given by the sum of free protons and the protons bound in clusters). For the influence of few-body correlations in HIC on the EoS and related properties such as the symmetry energy, in particular for the influence of the formation of light elements, see refs. [7, 8].

An interesting application of cluster formation in low-density nuclear matter is the structure of neutron stars, in particular the inner crust, as well as the time evolution of supernova explosions. Different properties such as neutrino transport and cooling are influenced by the composition of the hot nuclear matter [9]. Correlations and formation of structures in the crust of neutron stars are investigated recently [10].

There are different possibilities to implement the formation of light clusters (mass number $A \leq 4$) in the theory of nuclear matter at subsaturation densities. A simple mass action law considering an ideal mixture of noninteracting nuclei in nuclear statistical equilibrium (NSE) is not applicable in dense systems where the interaction (mean field, Pauli blocking) with the medium modifies the properties of light clusters [11]. A quantum-statistical (QS) approach has been worked out [12] to give a systematic approach to treat few-body correlations in equilibrium. Within a generalized relativistic mean-field theory [13], light elements are included as new degrees of freedom, coupled to the mesonic fields in the Lagrangian. The medium modifications of the light clusters are introduced empirically, for a more recent approach see ref. [14]. An alternative semi-empirical approach to account for medium effects was given in ref. [15] using the concept of excluded volume.

In contrast to the light clusters, the heavy clusters ($A > 4$) are more complex to be described within a first-principle, quantum-statistical approach. Other models such as excluded volume or density functional theory are more appropriate. The heavy clusters can be considered as droplets of a second, high density phase. Heavier clusters become dominant with increasing density in the low-temperature region. The heavy clusters are considered in several papers, see, *e.g.*, ref. [16]. We will not discuss this issue here any further.

In this contribution, we indicate several problems to be discussed in context with the description of few-body correlations and cluster formation. These are:

i) How is the quasiparticle picture correctly introduced?

ii) How can the composition of nuclear matter be defined?

iii) At which critical value of baryon density do the bound cluster states disappear?

iv) Can correlations in the medium be included in the description of few-nucleon clusters?

v) What is the effect of clustering on symmetry energy and β equilibrium (stellar matter)?

2. – Quantum statistical approach and the quasiparticle picture

We discuss here the light clusters with $A \leq 4$, *i.e.*, besides the deuteron ^{2}H also triton ^{3}H, helion ^{3}He, and the α-particle ^{4}He. With the large binding energy 28.3 MeV, the α particle is of particular interest to describe correlations in nuclear systems.

The QS approach considers correlation functions and its Fourier transforms, the spectral function $S_\tau(1,\omega;T,\mu_n,\mu_p)$ with $\tau=\{n,p\}$. The single-nucleon quantum state $|1\rangle$ can be chosen as $1=\{\mathbf{p}_1,\sigma_1,\tau_1\}$ which denotes wave number, spin, and isospin, respectively. A rigorous expression for the nuclear matter equation of state (EoS), which relates the total nucleon numbers $N_\tau^{\rm total}$ (or the particle densities $n_\tau^{\rm total}=N_\tau^{\rm total}/\Omega$) to the chemical potentials μ_τ of neutrons/protons, is found if the spectral function $S_\tau(1,\omega;T,\mu_n,\mu_p)$ is known,

$$n_\tau^{\rm tot}(T,\mu_n,\mu_p)=\frac{1}{\Omega}\sum_{p_1,\sigma_1}\int\frac{{\rm d}\omega}{2\pi}\frac{1}{e^{(\omega-\mu_\tau)/T}+1}S_\tau(1,\omega) \tag{1}$$

(Ω is the system volume, and $k_B=1$). The spectral function is related to the self-energy $\Sigma(1,z)$ for which a systematic approach is possible using diagram techniques [17]:

$$S_\tau(1,\omega)=\frac{2{\rm Im}\Sigma(1,\omega-i0)}{[\omega-E(1)-{\rm Re}\Sigma(1,\omega)]^2+[{\rm Im}\Sigma(1,\omega-i0)]^2}, \tag{2}$$

with $E(1)=\hbar^2p_1^2/2m_1$. The total neutron number density $n_n^{\rm total}$, the total proton number density $n_p^{\rm total}$, and the temperature T are considered as independent thermodynamic variables. The chemical potentials μ_n,μ_p are an alternative to $n_n^{\rm total}$ and $n_p^{\rm total}$ in characterizing thermodynamic equilibrium of warm dense matter. Further thermodynamic variables are consistently derived after a thermodynamic potential (pressure or free energy) is found by integration.

Using the Feynman diagram technique, the self-energy is calculated within perturbation theory. A systematical discussion of different approximations for $\Sigma(1,\omega)$ is found in ref. [18].

The quasiparticle concept follows from the expansion for small Im $\Sigma(1,\omega+i0)$ [19,20],

$$S_\tau(1,\omega)\approx\frac{2\pi\delta[\omega-E^{\rm quasi}(1)]}{1-\frac{\rm d}{{\rm d}z}{\rm Re}\Sigma(1,z)|_{z=E^{\rm quasi}}}-2{\rm Im}\Sigma(1,\omega+i0)\frac{\rm d}{{\rm d}\omega}\frac{\mathcal{P}}{\omega+\mu_1-E^{\rm quasi}(1)} \tag{3}$$

with the quasiparticle energy

$$E^{\rm quasi}(1)=E(1)+{\rm Re}\Sigma(1,\omega)|_{\omega=E^{\rm quasi}}. \tag{4}$$

The single-particle energy $E(1)$ is shifted by the real part of the self-energy. In general, $\Sigma(1,\omega)$ is a function of ω and has to be taken at the quasiparticle energy. The instantaneous part of the interaction with the medium, for instance the Hartree-Fock approximation or the cluster mean-field approximation given below, leads to a self-energy not depending on ω.

In addition, the δ-function is renormalized by the denominator (renormalization factor Z). Consistently with the assumption of small Im $\Sigma(1,\omega+i0)$, we also can consider $\frac{\rm d}{{\rm d}z}{\rm Re}\Sigma(1,z)$ as a small quantity and expand the denominator. As shown in refs. [19,20], this new contribution can be merged with the contribution $\propto \mathrm{Im}\Sigma(1,\omega+i0)$ so that the generalized Beth-Uhlenbeck formula results,

$$n_\tau^{\mathrm{total}}(T,\mu_n,\mu_p) = n_\tau^{\mathrm{quasi}}(T,\mu_n,\mu_p) + n_\tau^{\mathrm{corr}}(T,\mu_n,\mu_p). \tag{5}$$

The quasiparticle contribution $n_\tau^{\mathrm{quasi}} = \frac{1}{\Omega}\sum_{p_1,\sigma_1} f_\tau[E^{\mathrm{quasi}}(1)]$ may be considered as the free nucleon part.

We decompose the correlated part into the contribution of different channels c characterized by spin, isospin, and angular momentum. For instance, we obtain for the deuteron channel

$$\begin{aligned} n_{2,d}^{\mathrm{corr}} &= \frac{2^{3/2}}{\Lambda^3} e^{(\mu_n+\mu_p)/T} \\ &\quad \times 3\left[(e^{-E_d/T}-1) + \int_0^\infty \frac{\mathrm{d}E}{\pi T} e^{-E/T} \left\{ \delta_{2,d}(E) - \frac{1}{2}\sin\left[2\delta_{2,d}(E)\right] \right\} \right] \end{aligned} \tag{6}$$

with $\Lambda^2 = 2\pi\hbar^2/mT$, the deuteron bound state energy $E_d = -2.225\,\mathrm{MeV}$, and $\delta_{2,d}(E)$ denotes the scattering phase shift in the deuteron channel. Because part of the interaction is already taken into account when introducing the quasiparticle energy, one has to subtract the contribution $-\frac{1}{2}\sin[2\delta_c(E)]$ from the ordinary second virial coefficient to avoid double counting, see [19-21].

The quasiparticle picture can also be introduced for the two-particle states occurring in (6). We have to solve an in-medium two-nucleon Schrödinger equation. In general, the A-nucleon in-medium Schrödinger equation

$$\begin{aligned} &[E_{\tau_1}^{\mathrm{quasi}}(p_1;T,\mu_n,\mu_p) + \ldots + E_{\tau_A}^{\mathrm{quasi}}(p_A;T,\mu_n,\mu_p) - E_{A\nu}^{\mathrm{quasi}}(P;T,\mu_n,\mu_p)]\psi_{A\nu P}(1\ldots A) \\ &\quad + \sum_{1'\ldots A'}\sum_{i<j}[1-n(i;T,\mu_n,\mu_p)-n(j;T,\mu_n,\mu_p)]V(ij,i'j')\prod_{k\neq i,j}\delta_{kk'}\psi_{A\nu P}(1'\ldots A') = 0 \end{aligned} \tag{7}$$

is derived from the Green-functions approach [11, 12, 19, 20]. This equation contains the effects of the medium in the single-nucleon quasiparticle shift as well as in the Pauli blocking terms given by the occupation numbers $n(1;T,\mu_n,\mu_p)$ in the phase space of single-nucleon states $|1\rangle$.

In conclusion, the quasiparticle picture can be introduced so that the total density of baryons is expressed in terms of partial contributions from different cluster sizes A, channel number c, and total momentum P [12], *e.g.*, for neutrons ($N_{A,c}$ is the number of neutrons of cluster A, channel c)

$$n_n^{\mathrm{tot}}(T,\mu_n,\mu_p) = \sum_{A,c}\int \frac{\mathrm{d}^3P}{(2\pi)^3} N_{A,c} z_{A,c}^{\mathrm{part.}}(P) \tag{8}$$

With the degeneracy factor in the channel c, $g_{A,c} = 2s_{A,c}+1$, the partial density of the channel c at $\mathbf{P}$,

$$\begin{aligned} &z_{A,c}^{\mathrm{part.}}(P;T,\mu_n,\mu_p) = e^{(N_{A,c}\mu_n + Z_{A,c}\mu_p)/T} \\ &\quad \times \left\{ \sum_\nu^{\mathrm{bound}} g_{A,c,\nu}\left[e^{-E_{A,c,\nu}(P)/T} - e^{-E_{A,c}^{\mathrm{cont}}(P)/T}\right]\Theta\left[E_{A,c}^{\mathrm{cont}}(P) - E_{A,c,\nu}(P)\right] + z_{A,c}^{\mathrm{cont}}(P) \right\}, \end{aligned} \tag{9}$$

contains the intrinsic partition function as sum over all excitations ν. It can be decomposed into the bound-state contribution and the contribution of scattering states $z_{A,c}^{\rm cont}(P;T,\mu_n,\mu_p)$, cf. eq. (6). The region in the parameter space, in particular $\mathbf{P}$, where bound states exist, may be restricted what is expressed by the step function $\Theta(x) = 1, x \geq 0; = 0$ else. The continuum edge of scattering states is denoted by $E_{A,c}^{\rm cont}(P;T,\mu_n,\mu_p)$. Whereas the single nucleons and bound states are treated as quasiparticles with medium-dependent energies, the contribution of scattering states has to be treated with care to avoid double counting.

3. – The composition of nuclear matter

In the low-density limit, in a simplified approach nuclear matter can be considered as a mixture of different components, the free nucleons and the nuclei ("chemical" picture, NSE). The contribution of continuum (scattering states) are usually neglected, but have to be taken into account within a more systematic approach [11, 20]. However, the "chemical" picture is not free of ambiguities with respect to the subdivision into bound state contributions and continuum contributions. This refers to the account of unstable nuclei, but even more to the description of matter at high densities where the bound states may merge with the continuum because of the in-medium shift of the binding energies (mean-field shift and Pauli blocking).

We adapt the prescription according to eq. (8) where the free nucleon contribution is determined by the quasiparticle contribution ($A = 1$). The correlated parts are given by the medium-modified bound-state contributions as far as they exist, but contain also additional contributions $z_{A,c}^{\rm cont}(P;T,\mu_n,\mu_p)$ (9) describing continuum correlations, for instance resonances. The main problem in calculating the contribution of continuum correlations is the reduction to avoid double counting, see eq. (6) for the case of two-body scattering contributions. For $A > 2$, only estimates are given in [12], and more reliable results are expected from a cluster-virial expansion [21].

For instance, the contribution of continuum correlations to the second virial coefficient is reduced if the quasiparticle picture is introduced [20]. A similar behavior is also assumed for higher-order clusters. In contrast to the bound-state contribution, the continuum contributions are difficult to analyze, but they are important to investigate the properties of nuclear systems. A similar problem is also known in plasma physics where the ionization degree of warm dense matter is recently under discussion. The concept of composition becomes fuzzy at high densities if the contribution of correlations in the continuum is not appropriately taken into account.

4. – Depression of binding energy and dissolution of bound states

An important effect of the medium modification of the bound state energies is Pauli blocking which reduces the binding energies. A bound state may merge with the continuum of scattering states if the density increases. The critical value of baryon density where the bound state disappears can be defined by considering the edge of the continuum states $E_{A,c}^{\rm cont}(P;T,\mu_n,\mu_p)$, usually given for zero relative momentum of the nucleons in the cluster, and the medium modified bound state energy $E_{A,c,\nu}(P;T,\mu_n,\mu_p)$ of the cluster. If the binding energy, defined as the difference between both energies, vanishes, the bound state is dissolved. (Note that with the EoS (1) the chemical potentials μ_τ can be replaced by the densities n_τ useful to calculate the energy shifts.)

However, this criterion for the dissolution of bound states has been questioned (also in plasma physics where instead of binding energy the notation ionization potential is used). At zero temperature, where all single-particle states below the Fermi energy are occupied, the bound state cannot disintegrate into free-particle states at the edge of continuum because of the Pauli principle. The bound state is stable until the constituents can disintegrate at the Fermi energy. This criterion has been used also to discuss the α preformation at the surface of heavy nuclei [1].

To solve this contradiction, we have to consider our definition of the correlated contribution to the density (9). In the special case of two-nucleon correlation, the correlated contribution to the density is given by eq. (6). The in-medium scattering phase shifts for a separable potential, adapted to the deuteron channel, are shown in fig. 1, for details see [22]. Instead of the expression $1 - f(1) - f(2) = [1 - f(1)][1 - f(2)] - f(1)f(2)$ for the Pauli blocking (particle-particle RPA), only the particle contribution $[1 - f(1)][1 - f(2)]$ was considered, neglecting the hole-hole contribution $f(1)f(2)$ (Tamm-Dancoff approximation, this is also used in Brueckner Hartree-Fock calculations). At $T = 5\,\mathrm{MeV}$ the bound-state contribution owing to the phase shift π at negative energies is present as long as the bound-state exists, the contribution of scattering states is decreasing with increasing baryon density. In contrast, at $T = 0.1\,\mathrm{MeV}$ a significant contribution appears from the scattering states below the Fermi energy which is similar to a bound state below the Fermi energy.

The particle-particle RPA gives a more complex behavior of the scattering phase shifts, see [22] for details. The jump occurs at the Fermi energy, but the occurrence of negative shifts is discussed as signature of a phase transition to quantum condensates (pairing above a critical density if T is fixed), and new quasiparticle states (Bogoliubov transformation) must be introduced.

In conclusion, the contradiction with respect to the dissolution of bound states is resolved if both, the bound-state contribution as well as the continuum correlation contribution to the intrinsic partition function (9) are considered. The subdivision into a bound state and scattering state contribution is artificially and has no physical relevance. As a model, the concept of composition is of interest but has to be carefully introduced, see eq. (8), where in addition to bound quasiparticle states also scattering states must be considered.

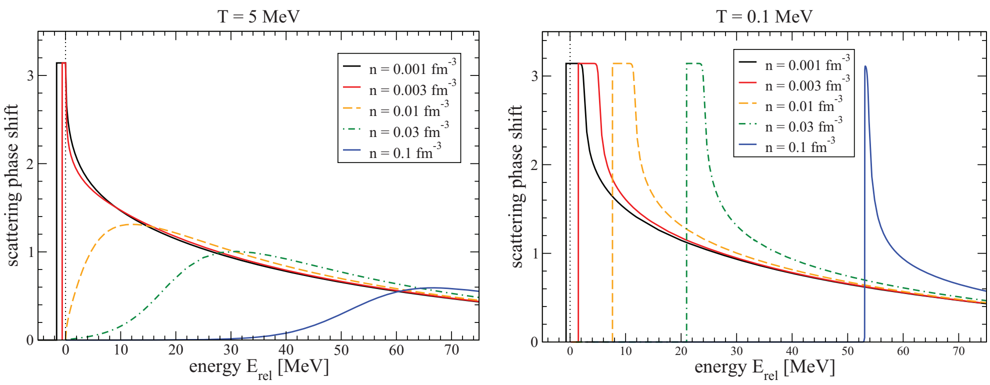

Fig. 1. – Medium-modified two-nucleon scattering phase shifts. A separable potential adapted to the deuteron properties has been used, as well as the Tamm-Dancoff expression for the Pauli blocking.

5. – Correlated medium

In the standard approaches to warm and dense nuclear matter such as RMF, correlations in the surrounding medium are neglected. In particular, the Pauli blocking is given by the Fermi distribution function. In a consistent description of the medium, correlations must be taken into account which modify the occupation of phase space. For instance, it is well-known that owing to the strong interaction, high-energy tails occur for the single-particle distribution function. The occupation $n(i; T, \mu_n, \mu_p)$, see eq. (7), is also modified when clusters are formed. As example, α matter describes α particles in a environment of α particles so that phase space occupation is determined by the α-particle wave function.

There are several attempts to formulate a mean-field theory which takes correlations of the medium into account, including cluster formation. Within a Green-functions approach, a cluster mean-field approximation has been worked out [23,24]. An alternative approach [25] starts from the equation of motion for the Green functions and selects the instantaneous part of the Hamiltonian. The remaining frequency dependent part is considered as perturbation which is responsible for the dynamical evolution. The self-consistent description of few-body clusters moving in a correlated environment, *e.g.*, in cluster mean-field approximation, is of fundamental interest to define an optimal set of basis states. The remaining (frequency-dependent) part of the effective interaction describes transitions between these optimum basis states.

There is only little progress in solving this challenging problem. For nuclei with not too large mass number, a self-consistent description of α-like clustering was given in refs. [3, 4]. In ref. [24], deuteron formation in the medium has been considered, and only minor modifications of the relevant properties are obtained. An approach to model the occupation $n(i; T, \mu_n, \mu_p)$ in phase space using a Fermi distribution function with effective parameter values for T, μ_n, μ_p has been proposed recently [12]. The consistent description of few-body correlations in a correlated environment remains a challenging problem for future investigations.

6. – Application: Influence of light clusters on the β equilibrium of stellar matter

Correlations and, in particular, the formation of light clusters is essential for the symmetry energy at low densities. Because light clusters such as α particles are formed at low temperatures, the binding energy per nucleon in symmetric matter is not going to zero at low densities but takes a finite value corresponding to the composition of nuclear matter. The NSE is valid in the low-density region. It has to be improved taking medium effects into account, as addressed within a QS approach. An experimental verification was given in ref. [8].

A further consequence is the influence of the cluster formation on the β equilibrium of stellar matter. In stellar matter, the process $n \rightleftharpoons p + e + \bar{\nu}_e$ is considered. In thermodynamic equilibrium we have $\mu_n = \mu_p + \mu_e$ (relativistic energies, the neutrinos escape so that the chemical potential is zero). Because of (local) charge neutrality $n_p = n_e$, the thermodynamic parameters of stellar matter are only T, n_B, and Y_p follows from β equilibrium. Compared to the ideal, noninteracting Fermi gas model for nuclear matter, the proton fraction Y_p is increased if a mean-field approximation (Skyrme, Relativistic

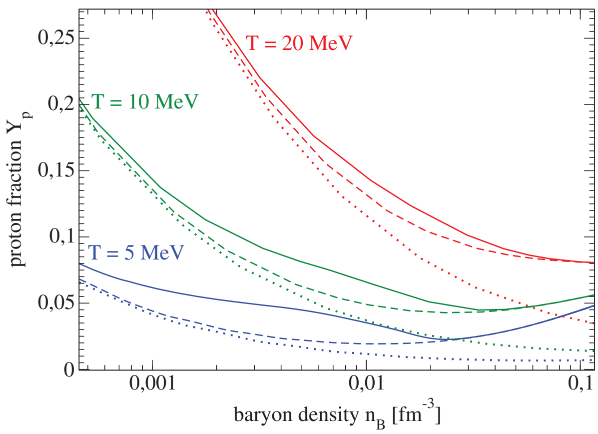

Fig. 2. – Proton fraction Y_p for stellar matter in β equilibrium, for different $T = 5, 10, 20\,\mathrm{MeV}$ as function of the baryon density n_B. Ideal Fermi gas (dotted) is compared to the relativistic mean-field (RMF, dashed) and the account of light cluster ($A \leq 4$) formation (QS, full) according ref. [12].

mean-field (RMF), etc.) is considered, because nuclear matter is stronger bound for increasing Y_p. Calculations based on the QS approach [12] are presented in fig. 2. This tendency is enhanced if formation of light clusters is taken into account, because the binding of protons is further increased if they are bound in clusters such as α particles, see fig. 2. This effect was considered already some time ago [26] but is also of interest in recent investigations of properties of the crust of neutron stars. The inclusion of heavier clusters has been discussed, for references see [16]. The account of light cluster formation is also of interest to explore the inner crust of neutron stars where nuclear pasta phases can be expected, see [10, 14, 16] and references given there.

In conclusion, the inclusion of light clusters to describe the properties of nuclear systems remains an interesting area of research. The limiting cases of low density, where the NSE is applicable, and high density, where a Fermi liquid approach to strongly degenerate systems can be applied, are known. Interpolations between both limiting cases have been performed in a semi-empirical way. A systematic quantum statistical approach has been worked out, and simple approximations have to be improved. The account of continuum correlations and the self-consistent treatment of correlations in the environment of a few-body cluster need further investigations. Alternatively, perturbation theory to evaluate Green functions can be avoided performing numerical solutions (Fermion molecular dynamics, Antisymmetrized molecular dynamics, etc.) for correlation functions in the strongly interacting, degenerate nuclear system. A challenge is the description of clustering in non-equilibrium processes like HIC.

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