THE IMAGINARY PART OF THE NUCLEON OPTICAL POTENTIAL IN NUCLEAR MATTER ☆

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We show that the energy dependence of the real part of the optical potential, or equivalently the effective mass of nucleons in nuclear matter, gives significant corrections to the imaginary part calculated with either impulse approximation or Brueckner's theory. These corrections greatly reduce the difference between theoretical and empirical strengths of the imaginary potential.

Calculations of the imaginary part $W_0(E_p)$ of the optical potential for nucleons in nuclear matter generally start with Fermi's golden rule for the rate of transitions from a one-particle state $|p\rangle$ to two-particle–one-hole states $|p'p''h\rangle$. The total transition rate for transitions from the state $|p\rangle$ is given by:

$$\frac{2\pi}{\hbar}\int \frac{d^3h}{(2\pi)^3}\int \sin\theta\, d\theta\, d\phi\, |\langle p|H|p'p''h\rangle|^2 \rho(E_p + E_h)$$
$$= 1/\tau_p \equiv 2W(E_p)/\hbar\,. \tag{1}$$

The momenta p, p', p'' and h are illustrated in fig. 1, and the integrals in (1) are to be carried out over regions where $|h| < k_F$, and $|p'|$ and $|p''|$ are $> k_F$. For the sake of brevity we assume normalization in unit volume. E_p and E_h are the energies of the particles in states p and h, respectively, and $\rho(E_p + E_h)$ is the density of final states at $E_{p'} + E_{p''} = E_p + E_h$.

The total transition rate equals the inverse of the lifetime τ_p of the state $|p\rangle$. Eq. (1) also defines a potential $W(E_p)$ which will generate the lifetime τ_p. In the past $W(E_p)$ has been identified with $W_0(E_p)$. The purpose of this letter is to show that this identification is incorrect, and that

$$W_0(E_p) = [m^*(E_p)/m]\, W(E_p)\,. \tag{2}$$

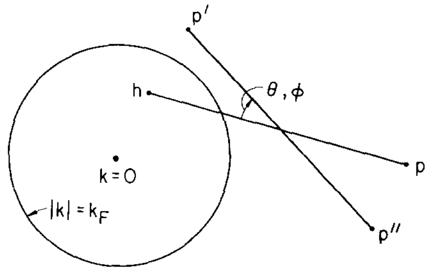

Fig. 1. Typical scattering event that contributes to W.

Here $m^*(E_p)$ is the effective mass of nucleons of energy E_p in nuclear matter. We also show that a significantly improved agreement between the theoretical $W_0(E_p)$ and the empirical $W_0(E_p)$ is obtained after taking this effect into account.

Low-energy ($E_p \lesssim 150$ MeV) nucleons have an effective mass $m^* \approx 0.7\, m$ in nuclear matter [1,2] so that E_p is given by:

$$E_p = (\hbar^2/2m^*)p^2 - V\,, \tag{3}$$

or

$$(\hbar^2/2m)p^2 = E_p + (m^*/m)V - (1 - m^*/m)E_p$$
$$\equiv E_p + V_0(E_p)\,. \tag{4}$$

It is known [1] that the real part of the optical potential $V_0(E_p)$ is well reproduced by $52 - 0.3\, E_p$, or $m^* \approx 0.7\, m$. This approximation may not be valid in a

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narrow energy region at $E_p \approx E_F \approx -16$ MeV [3], but that has little consequence on this discussion. The density of states $\rho(E_p + E_h)$ is then given by:

$$\rho(E_p + E_h) = (m^*/4\hbar^2)|\boldsymbol{p} - \boldsymbol{h}|/(2\pi)^3 . \quad (5)$$

Within the impulse approximation [4] we obtain:

$$W_I(E_p) = \pi \int \frac{d^3h}{(2\pi)^3} \int \sin\theta \, d\theta \, d\phi \, \frac{m}{4\hbar^2} \frac{|\boldsymbol{p} - \boldsymbol{h}|}{(2\pi)^3} \times |\langle ph|t_f|p'p''\rangle|^2 , \quad (6)$$

where t_f is the free t-matrix, and in lowest-order Brueckner theory [5]

$$W_B(E_p) = \pi \int \frac{d^3h}{(2\pi)^3} \int \sin\theta \, d\theta \, d\phi \, \frac{m^*}{4\hbar^2} \frac{|\boldsymbol{p} - \boldsymbol{h}|}{(2\pi)^3} \times |\langle ph|G|p'p''\rangle|^2 , \quad (7)$$

where G is the reaction matrix. At $E_p \gtrsim 50$ MeV the difference between t_f and G does not appear to be large, and indeed $W_B(E_p) \approx m^* W_I(E_p)/m$ as can be seen from fig. 2. In the formalism of ref. [5] $W_B(E_p)$ is given by $m \, \mathrm{Im}(M(p, E_p))/\bar{m}$, where $M(p, E)$ is the mass operator, and m^* is given by $\tilde{m}(\bar{m}/m)$. However, in the energy range of interest in this work $\bar{m} \approx m$, and $m^* \approx \tilde{m}$.

The velocity of nucleons of energy E_p in nuclear matter is given by the group velocity $v(E_p)$. From eq. (3) we find:

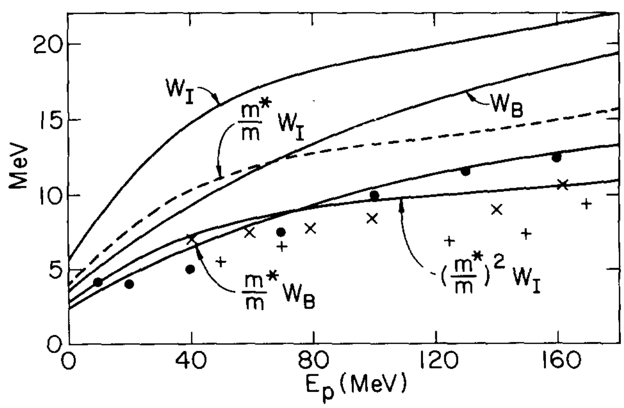

Fig. 2. The curves labeled $(m^*/m)\,W_B$ and $(m^*/m)^2\,W_I$ give theoretical approximations to the W_0 in nuclear matter, while the •, x and + give the strengths of the Woods–Saxon potential used to fit the observed scattering data.

$$v(E_p) = \hbar^{-1}\, dE_p/dp = \hbar p/m^* , \quad (8)$$

and this gives the mean free path λ

$$\lambda(E_p) = v(E_p)\tau_p = \hbar^2 p/2m^* W(E_p) . \quad (9)$$

The flux of nucleons decreases by a factor e over a path-length λ. We may approximate $W(E_p)$ in eq. (9) with either $W_B(E_p)$ or $m^* W_I(E_p)/m$.

In the optical model elastic scattering is analyzed with the Schrödinger equation:

$$E_p \psi(r) = [-(\hbar^2/2m)\nabla^2 - V_0(E_p) - iW_0(E_p)]\psi(r) . \quad (10)$$

The plane-wave solutions of this equation are:

$$\psi(r) = \exp[i(k + i/2\lambda)z] , \quad (11)$$

$$k^2(E_p) = 1/4\lambda^2 + (2m/\hbar^2)[E_p + V_0(E_p)] \approx p^2 , \quad (12)$$

(we assume $p \gg 1/\lambda$), and

$$\lambda(E_p) = \hbar^2 p/2mW_0(E_p) . \quad (13)$$

Comparing (13) with (9) we obtain the relation (2) between $W(E_p)$ and $W_0(E_p)$.

Reasonable approximations to $W_0(E_p)$ are $m^* \times W_B(E_p)/m$ or $W_I(E_p)(m^*/m)^2$, and we see in fig. 2 that these are in much better agreement with experiment. The •, x and + signs in fig. 2 show the magnitude of the Woods–Saxon terms $W_0(r, E_p)$ in the nucleon–nucleus potential. The • are from the compilation by Bohr and Mottelson [1], while the x and + signs are from proton scattering on ^{208}Pb and ^{40}Ca [6]. The proton energy is approximately corrected for Coulomb deceleration (≈20 MeV in ^{208}Pb and 10 MeV in ^{40}Ca). The theoretical $W_0(E_p)$ is a bit on the higher side, particularly with respect to the Indiana data [6], but the difference is not as large as was thought previously by comparing $W_I(E_p)$ or $W_B(E_p)$ with $W_0(E_p)$. One obviously has to improve upon the calculation of $W(E_p)$ if $W_0(E_p)$ is indeed as small as suggested by the Indiana analysis.

We conclude with a more general discussion of the factor m^*/m in eq. (2). If we have a particle moving in a momentum dependent potential $\Sigma(p)$, and its hamiltonian is

$$H = (-\hbar^2/2m)\nabla^2 + \Sigma(p) + \delta H , \quad (14)$$

where δH is a perturbation, then the complex momenta q associated with the energy E are given by:

$$q^2 = (2m/\hbar^2)[E - \Sigma(q) - \delta H] . \tag{15}$$

We assume for simplicity that

$$\Sigma(p) = \Sigma_0 + (\hbar^2/2m)p^2 a , \tag{16}$$

so that

$$m^*/m = (1 + a)^{-1} , \tag{17}$$

and the solution of eq. (15) is:

$$q^2(E) = (2m/\hbar^2)(1 + a)^{-1}(E - \Sigma_0 - \delta H) . \tag{18}$$

The motion of the particle may be described by an equivalent hamiltonian H' using an energy dependent potential $\Sigma(E)$:

$$\Sigma(E) = (m^*/m)\Sigma_0 + (1 - m^*/m)E , \tag{19}$$

$$H' = (-\hbar^2/2m)\nabla^2 + \Sigma(E) + (m^*/m)\delta H . \tag{20}$$

In order to obtain the correct $q^2(E)$, δH must be multiplied by m^*/m in H'. Following this argument the Coulomb [5], and the isovector [7] terms in the optical-model Schrödinger equation have been multiplied by m^*/m. The factor m^*/m in the imaginary term was probably left out by a mere oversight. After this manuscript was written we learned that this effect has also been recently noticed by Yazaki and Negele [8].

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