Comments on Homework Set #1-4

Homework #1, 3, and 4

1. For details of the Spherical Harmonics (Legendre polynomials) (p.331), spherical bessel functions (p.435), and Coulomb wave functions (p.537), refer to "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, U.S. Department of Commerce, National Bureau of Standards, (1972)". You may find a newer version.

2. Check more carefully the phase shifts of spherical bessel functions at $r \to \infty$. As I mentioned in the class, draw j_0 and j_4 around 50 fm and make sure that two curves coincide.

3. Check more carefully the phase shifts of Coulomb wave functions at $r \to \infty$. Draw rj_0 and F_0^C around 50 fm, measure phase shifts and compare it with the theoretical Coulomb phase shift σ_0.

Homework #2 - Generation of Bound State Wave Functions

1. Obtain the experimentally determined positions of the 3 single particles and 3 single hole states built on the ^{208}Pb core, relative to the ground states of ^{209}Bi, ^{207}Tl, ^{209}Pb and ^{207}Pb.

Answer

$3d_{3/2}$		2.539
$2g_{7/2}$		2.493
$4s_{1/2}$		2.032
$3d_{5/2}$		1.566
$1j_{15/2}$	**	1.422
$1i_{11/2}$		0.778
$2g_{9/2}$		0.000

^{209}Pb

$3p_{1/2}$		0.000
$2f_{9/2}$		0.570
$3p_{3/2}$		0.898
$1i_{13/2}$		1.633
$2f_{7/2}$		2.340
$1h_{9/2}$		3.409

^{207}Pb

$3p_{1/2}$	**	3.604
$3p_{3/2}$	**	3.116
$2f_{5/2}$		2.822
$1i_{13/2}$	**	1.609
$2f_{7/2}$		0.897
$1h_{9/2}$		0.000

^{209}Bi

$3s_{1/2}$	0.000
$2d_{3/2}$	0.351
$1h_{11/2}$	1.341
$2d_{5/2}$	1.674
$1g_{7/2}$	3.474

^{207}Tl

"**" indicates the level which is not a single particle state from the other experimental facts.

2. It is claimed that a Woods-Saxon potential with parameters V_0= 50.9 MeV, V_{SO} = 5.8 MeV, $r_0 = r_{SO}$ = 1.19 fm, and $a_0 = a_{SO}$ = 0.75 fm gives a pretty good description of the states of ^{209}Pb. Verify this using the program NEPTUNE.

Answer

Spec. Not.	Node	$2j$	ℓ	Cal. BE	Ex. Energy	Exp
$3d_{3/2}$	2	3	2	1.496	2.501	2.539
$2g_{7/2}$	1	7	4	1.583	2.414	2.493
$4s_{1/2}$	3	1	0	2.004	1.993	2.032
$3d_{5/2}$	2	5	2	2.485	1.512	1.566
$1j_{15/2}$	0	15	7	-	-	1.422
$1i_{11/2}$	0	11	6	3.478	0.519	0.778
$2g_{9/2}$	1	9	4	3.997	0.000	0.000

The numerically calculated binding energies by the NEPTUNE program agree fairly well with the experimental values of ^{209}Pb.

3. Discuss behavior of wave functions obtained in the tail region, say around $R = 7 - 8$ fm.

Answer

In the region of $R = 7 - 8$ fm, we may not see a general behavior of binding energy effects in the tail region of the wave function, because the centrifugal force still influences in this region. Thus I choose the outer region, say 10-13 fm region, take $4s_{1/2}$, $3s_{1/2}$, and $2s_{1/2}$, and

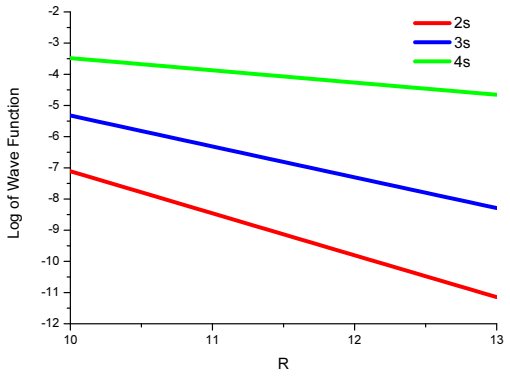

Fig. 1: The tail behavior (10-13 fm) of s-waves

find the binding energies by using NEPTUNE programs, as follows:

Spec. Not.	Node	$2j$	ℓ	Cal. BE
$4s_{1/2}$	3	1	0	2.02
$3s_{1/2}$	2	1	0	17.22
$2s_{1/2}$	1	1	0	33.40

The tail behavior of the s-waves (10-13fm) are plotted in Fig.1. We can see clearly the binding energy effect in the tail region.

4. Use the same potential to see how it works for the hole states of ^{207}Pb.

Answer

Spec. Not.	Node	$2j$	ℓ	Cal. BE	Ex. Energy	Exp
$1h_{9/2}$	0	9	5	11.968	3.407	3.409
$2f_{7/2}$	1	7	3	11.211	2.650	2.240
$1i_{13/2}$	0	13	6	7.845	-0.816	1.633
$3p_{3/2}$	2	3	1	9.321	0.760	0.898
$2f_{5/2}$	1	5	3	9.325	0.764	0.570
$3p_{1/2}$	2	1	1	8.561	0.000	0.000

These calculations show that the potential which gave a good description for the particle state of ^{209}Pb also reproduces reasonably well for the hole states of ^{207}Pb.

5. Use a potential of the same geometry, but input the experimental separation energies and search on the potential depth to get a good description of the single-proton ^{209}Bi states. What is the difference $V_p - V_n$ between this potential and the one of part(2)? If $V_p - V_n = 2V_1(N-Z)/A$, what value do you get for V_1?

Answer

We use a potential of the same geometry as that for Pb nucleus, but input the experimental separation energies, (The binding energy of the ground state of ^{209}Bi was chosen as 3.803 MeV). We then search for the potential depth in the NEPTUNE program and obtain the results as

Spec. Not.	Node	$2j$	ℓ	Exp. Ex. Energy	Input BE	Obtained V_0
$2f_{5/2}$	1	5	3	2.822	0.981	66.04
$2f_{7/2}$	1	7	3	0.897	2.906	66.14
$1h_{9/2}$	0	9	5	0.000	3.803	65.04

We average the obtained 3 values and find

$$\begin{aligned} V_p &= 65.7 \ \text{MeV} \\ V_n &= 50.9 \ \text{MeV}, \quad \text{given in the problem} \end{aligned}$$

The symmetry energy V_1 is usually defined as

$$\begin{aligned} V_p &= V_0 + V_1 \frac{N-Z}{A} \\ V_n &= V_0 - V_1 \frac{N-Z}{A} \\ V_1 &= \frac{1}{2}(V_p - V_n)\frac{A}{N-Z} \end{aligned}$$

We thus obtain

$$V_1 = \frac{1}{2}(65.7 - 50.9)\frac{208}{126-82} = 35.0 \ \text{MeV}$$

This value is higher than the value usually tabulated of 25 MeV. However, if we include the Coulomb correction term, $0.4Z/A^{1/3}$, then we get

$$\begin{aligned} V_1 &= \frac{1}{2}(V_p - V_n - 0.4Z/A^{1/3})\frac{A}{N-Z} \\ &= \frac{1}{2}(65.7 - 50.9 - 0.4 \times 82/(208)^{1/3})\frac{208}{126-82} \\ &= 21.7 \ \text{MeV} \end{aligned}$$

We now have a reasonable value of the symmetry energy.