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Uncertainty Quantification of the Experimental Spectroscopic Factor from Transfer Reaction Models

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We study the uncertainty stemming from different theoretical models in the spectroscopic factors extracted from experiments. We use three theoretical approaches, the distorted wave Born approximation (DWBA), the adiabatic distorted wave approximation (ADWA) and the continuum discretized coupled-channels method (CDCC), and analyze the $^{12}C(d,p)^{13}C$, $^{14}C(d,p)^{15}C$ reactions. We find that the uncertainty associated with the adopted theoretical models is less than 20%. We also investigate the contribution from the remnant term and observe that it gives less than 10% uncertainty. We finally make an attempt to explain the discrepancy in the spectroscopic factors of $^{17}C(\frac{3}{2}^+)$ between the ones extracted from experiments and from shell model calculations by analyzing the $^{16}C(d,p)^{17}C$ reaction.

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I. INTRODUCTION

Several new or improved rare isotope (RI) facilities will be completed worldwide in coming years. One of the new facilities, called RAON (Rare isotope Accelerator complex for ON-line experiments), is under construction by the Rare Isotope Science Project (RISP) in Korea. RAON will provide many exotic neutron- or proton-rich nuclei for deeper understanding of nuclear structures and reactions and related fields such as astrophysics.

One of the main goals of RAON is to investigate the r-process, which is responsible for the existence of about half of the stable nuclei heavier than iron. Most studies of the r-process have concentrated on the reaction flow of heavy unstable nuclei. In Refs. [1,2], however, it was shown that light neutron-rich nuclei can also play an important role in r-process nucleosynthesis in models with a short dynamic time scale. For example, a sensitivity study [1,2] showed that the chain of neutron capture reactions on carbon isotopes, $^{\mathrm{A}}\mathrm{C}(n,\gamma)^{\mathrm{A+1}}\mathrm{C}(\mathrm{A}=12-19)$, can affect the final r-process abundances. Because the direct measurement of these reactions at the energies of astrophysical interest is difficult, alternative indirect techniques, such as the asymptotic normalization coefficient(ANC) method for (d,p) reactions, could be valuable tools [3–5].

Nucleon transfer reactions such as (d,p) or (d,n) have been important tools for providing spectroscopic information on nuclei [6,7] because the angular distribution of the transfer reaction is sensitive to the transferred nucleon's orbital angular momentum. Traditionally, the spectroscopic factor is extracted by comparing results from experiments with those from the theoretical distorted wave born approximation (DWBA). However, the validity of the DWBA is not fully confirmed for exotic nuclei. Thus, testing the validity of the DWBA and developing a new method suitable for exotic nuclei, if necessary, would be important. Because exotic nuclei near the neutron drip-line have weak binding energies, the couplings to the continuum breakup states is expected to be more important than it is for stable nuclei. The adiabatic distorted wave born approximation (ADWA) and the continuum discretized coupled channel (CDCC) are suitable formalisms to include such breakup channel effects.

Because of the role of the (d,p) reaction in astrophysics and nuclear structure study, identifying the validity or uncertainty of available methods in the analysis of nuclear reactions involving exotic nuclei is important. In this work, we study the $^{12}C(d,p)^{13}C$ and $^{14}C(d,p)^{15}C$ reactions by using three theoretical methods, the DWBA, ADWA and CDCC methods, and try to quantify a theoretical uncertainty. A similar comparative study between the DWBA and ADWA was made in Refs. [8,9]. We include the CDCC method for our comparative study in addition to those used in Refs. [8, 9]. We also study the contributions from remnant corrections that are not considered in Refs. [8,9]. We then compare our results with the experimental ones to es-

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timate the uncertainty associated with the theoretical methods used in this work. In addition, as a related topic, we study the $^{16}C(d,p)^{17}C$ reaction by using the 2nd order DWBA for the possibility of resolving the discrepancy of the spectroscopic factor between theory and experiment shown in the knockout reaction [10].

In Sec. II, we briefly introduce the three methods. We present our results for the $^{12}C(d,p)^{13}C$, $^{14}C(d,p)^{15}C$ and $^{16}C(d,p)^{17}C$ reactions in subsequent sections. A summary of this work is given in Sec. VI.

II. FORMALISM

The nuclear transfer reaction is, in principle, a complex quantum many-body process. However, a three-body model approximation is customarily used for the transfer reaction in practical applications. Let us consider a transfer process $A(d,p)B$, where a neutron in $d = n + p$ is transferred to the target A to form a bound state $B = A + n$. Then, the transfer matrix elements in post form can be written as

$$T_{(\mathrm{post})} = \langle \Phi_B(\boldsymbol{r}_{nA}) \chi^{(-)}_{k_f}(R_f) | V_{np} + V_{pA} - U_f | \Psi^{(+)} \rangle, \quad (1)$$

where $\Phi_B(\boldsymbol{r}_{nA})$ is an overlap function of B from A, $\chi^{(-)}_{k_f}(R_f)$ is a distorted wave function of the exit channel with an optical potential U_f, V_{np} is the binding potential of the proton and the neutron in deuteron, and V_{pA} is the potential between the proton and the target. A dominant contribution of the transfer matrix elements comes from the deuteron binding potential V_{np}. The remnant correction $V_{pA} - U_f$ is usually neglected by using an approximation $V_{pA} \simeq U_f$. This expression of $T_{(\mathrm{post})}$ is exact as long as the three-body wave function $\Psi^{(+)}$ is exact.

In the DWBA, the three-body wave function $\Psi^{(+)}$ is approximated as a product of a deuteron bound state wave function $\phi_d(\boldsymbol{r}_{np})$ and a distorted wave $\chi^{(+)}_{DW}(R_i)$ that is computed from the deuteron-target optical potential U_i for the entrance channel. This method has been successfully applied to reactions with stable nuclei. However, for weakly bound nuclei, the couplings with the continuum breakup channels is expected to be important for transfer reaction studies.

The ADWA, developed by Johnson and Soper [11] for zero-range and by Johnson and Tandy [12] for finite range, accounts for such breakup channel effects on the transfer reaction by replacing $\Psi^{(+)}$ with the adiabatic wave function $\chi^{(+)}_{AD}(\boldsymbol{R})$ that is obtained from an adiabatic potential. The adiabatic potential is formally defined as

$$U_i^{\mathrm{AD}}(\boldsymbol{R}) = \frac{\langle \phi_d | V_{np}(U_n + U_p) | \phi_d \rangle}{\langle \phi_d | V_{np} | \phi_d \rangle}, \quad (2)$$

where U_n (U_p) is the optical potential between neutron (proton) and the target nucleus at half the incident deuteron energy. Note that this adiabatic potential cannot describe the elastic scattering data by construction.

While the ADWA includes the effects of the breakup channel implicitly, the CDCC formalism treats the continuum coupling effects explicitly by solving coupled channel equations including break up channels as averaged scattering wave functions over a range of momenta. Unlike the ADWA, the CDCC can treat both elastic and transfer channels consistently. The full wave function $\Psi^{(+)}$ is approximated as the CDCC wave function

$$\Psi^{(+)}_{\mathrm{CDCC}}(\boldsymbol{r}, \boldsymbol{R}) = \sum_{i=0}^{N} \phi_i(\boldsymbol{r}) \chi_i(\boldsymbol{R}), \quad (3)$$

where $\phi_i(\boldsymbol{r})$ are np states either bound ($i = 0$) or discretized continuum states ($i \neq 0$) and $\chi_i(\boldsymbol{R})$ is the projectile-target wave function for each channel i. $\chi_i(\boldsymbol{R})$ is obtained by solving coupled channel equations including all bound-continuum couplings and continuum-continuum couplings. Though doing fully coupled channel calculations including both breakup and transfer channels is possible, in this work, we solve the coupled channel equations for the elastic channel while we apply the DWBA method for transfer reactions. Thus, this approach is named 'CDCC-BA' or simply 'CDCC' in this work. As in the ADWA, the CDCC equation requires U_n, U_p potentials as inputs instead of the deuteron-target optical potential U_i in the entrance channel.

Once the theoretical transfer reaction cross section is calculated, one may extract the spectroscopic factor for a particular state by comparing experimental cross sections. Experimentally, the spectroscopic factor S_{exp} is defined as the ratio between the experimental data and the theoretical single-particle cross section :

$$\frac{d\sigma_{\mathrm{exp}}}{d\Omega} = S_{\mathrm{exp}} \frac{d\sigma_{\mathrm{s.p.}}}{d\Omega}. \quad (4)$$

One can also extract the asymptotic normalization constant (ANC), which is defined as the ratio of the overlap function and the Whittaker function. If we assume that the transfer reaction is sensitive only to the long range tail of the wave function, the spectroscopic factor S can be related as

$$S = \frac{C^2}{b^2}, \quad (5)$$

where C is the ANC and b is a single particle ANC which is defined as the ratio of the single-particle wave function and the Whittaker function. In principle, the overlap function and spectroscopic factor can be obtained from ab-initio nuclear structure studies or shell model calculations. However, in this work, we study the uncertainty in the extraction of the spectroscopic factor from experiments by using the reaction models. Thus, we obtained the single-particle wave functions by solving Schrödinger equation with a simple Woods-Saxon potential with radius $R_0 = r_0 A^{1/3}$ and diffuseness parameter

Table 1. Experimental spectroscopic factors from each reaction model and references for the experimental data on the $^{12}C(d,p)^{13}C$ reaction.

E (MeV)	S.F. (DWBA)	S.F. (ADWA)	S.F. (CDCC-no rem)	S.F. (CDCC-rem)	Ref.
4	0.86	0.77	0.96	0.89	[20]
4.5	0.55	0.50	0.61	0.57	[21]
7.15	0.98	0.93	1.09	1.02	[22]
8.9	0.95	0.94	1.06	0.98	[23]
10.2	0.87	0.85	0.94	0.87	[24]
11.8	0.63	0.61	0.67	0.62	[25]
12	0.90	0.89	0.95	0.88	[26]
12.4	0.83	0.81	0.87	0.81	[24]
14.7	0.76	0.74	0.78	0.72	[24]
14.8	0.81	0.79	0.83	0.77	[27]
15	0.76	0.70	0.77	0.72	[28]
16.6	0.66	0.61	0.65	0.61	[29]
19.6	0.71	0.63	0.69	0.66	[29]
25.9	0.78	0.73	0.80	0.74	[30]
30	0.75	0.65	0.77	0.75	[31]
51	1.20	0.91	1.53	1.78	[32]
56	1.38	1.11	1.53	1.71	[33]

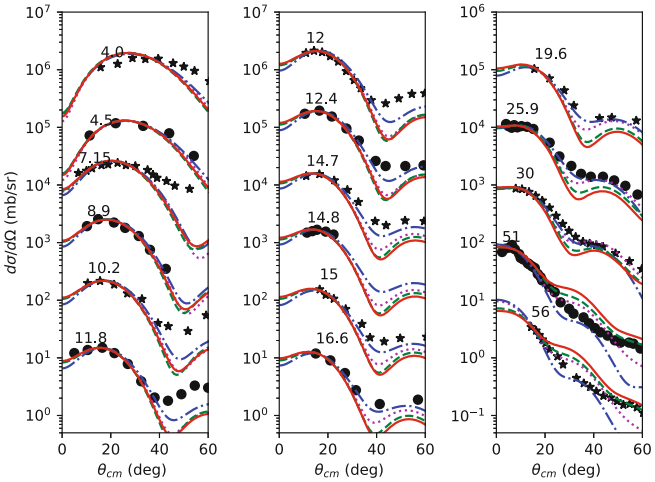

Fig. 1. (Color online) Angular distributions for the $^{12}C(d,p)^{13}C$ reaction for each reaction model and beam energy. The spectroscopic factor is included in this figure. The lines correspond to the DWBA (magenta dotted), ADWA (blue dash dotted), CDCC without remnant correction(green dashed) and CDCC with remnant correction (red solid) results, respectively. The number for each line represents the beam energy. Each graph is shifted by a factor of 10 from nearby lines. The result at the bottom corresponds to an overall normalization factor 1. References and the values of the spectroscopic factor are listed in Table 1.

a_0 and adjust the depth of the potential to reproduce the experimental neutron separation energy, while the spin-orbit potential depth is fixed as 6 MeV. We will discuss the uncertainties associated with parameters (r_0, a_0) in Sec. IV.

III. CALCULATIONS FOR $^{12}C(d,p)^{13}C$

Because many experimental data are available for the $^{12}C(d,p)^{13}C$ reaction at various energies, it may serve as a good testing ground for different theoretical methods. However, the ^{12}C is a deformed nucleus and so the couplings with the excited states of ^{12}C can be large. Because our focus in this work is to see the dependence on the nuclear reaction theories not a precise reproduction of the experimental data, the core excitation effects are not considered for the moment. Also, we treat the $^{13}C(\frac{1}{2}^-)$ ground state as a bound state of the $^{12}C(0^+)$ core and a single neutron in the $1p_{1/2}$ state. We take $r_0 = 1.25$ fm and $a_0 = 0.65$ fm for the Woods-Saxon potential and use the same parameters for the spin-orbit interaction with a potential depth of 6 MeV.

The DWBA and the ADWA calculations are done by using a modified version of the TWOFNR code [13] in the local energy approximation (LEA). The values of the zero-range strength and range parameters are adopted from the Reid soft-core neutron-proton interaction. On the other hand, the CDCC calculations are done by using the FRESCO code [14] using a finite range transfer option with and without remnant corrections. In the case of the CDCC, the deuteron bound state and continuum states are described by using the Gaussian-form potential $V_0 \exp(-r^2/R^2)$, with $V_0 = 72.15$ MeV and $R = 1.484\ A^{\frac{1}{3}}$ fm, which reproduces the deuteron binding energy and s-wave phase shifts [15]. It was shown that the usage of the more realistic Reid soft core potential does not give any significant difference in the elastic and the transfer reactions [16]. Continuum bins are chosen only for $l = 0, 2$ partial waves because the other partial wave contributions are found to be small [16,17]. We adopt the continuum bins truncated up to a relative energy of 15 MeV, which is divided into 6-8 bins with a equal momentum difference.

Liu *et al.* [8] studied the experimental spectroscopic factors of $^{12}C(d,p)^{13}C$ in the DWBA and the ADWA at various energies. They could obtain rather consistent spectroscopic factors within 15% by using a global optical potential consistently, though the reported spectroscopic factors for each experiment varied from 0.3 to 1.4, where the adopted optical potential were fitted to elastic scattering data for each measurement. They argued that one can avoid the spurious energy dependence stemming from detailed coupled-channel or compound resonance effects by using the global optical potential. Following this example and to study the dependence on different theoretical methods, we adopt global optical potentials in this work. In the case of the DWBA, the deuteron global optical potential of Daehnick *et al.* [18] is used for the entrance channel and the nucleon global optical

potential of Koning and Delaroche (KD02) [19] for the exit channel. In the case of the ADWA and the CDCC, KD02 nucleon global optical potentials are adopted for both entrance and exit channels.

The experimental data used in this work are listed in Table 1. The transfer reaction cross section is calculated for each experiment, and the experimental spectroscopic factors are extracted by using the most forward angle data or the first peak because the angular distributions at larger angles are more sensitive to the details of the optical potential, the structure of core nuclei and coupled channel effects. The angular distribution of the experimental data and the fitted reaction cross section are shown in Fig. 1. It is noticeable that the angular distributions of the transfer reaction at low energy below $E_d \simeq 10$ MeV and high energy above $E_d \simeq 50$ MeV do not show good agreement between theory and experiment. This might be attributed to the significant effects of compound nuclear reactions or resonances at low energy. On the other hand, the reaction at high energy would have more non-peripheral contributions and would not be suited for extracting spectroscopic factor. Thus, as in Ref. [9], we use the data between 10 MeV and 30 MeV to obtain the average spectroscopic factors. Table 1 and Fig. 2 show the spectroscopic factors extracted from the analysis and their average value.

The average spectroscopic factors are 0.77 ± 0.08 for the DWBA, 0.73 ± 0.09 for the ADWA, 0.79 ± 0.10 for the CDCC without the remnant correction, and 0.74 ± 0.09 for the CDCC with the remnant correction. Overall, the DWBA, ADWA and CDCC agree with each other within $\sim 8\%$, and the remnant correction contribution is about $\sim 6\%$. Thus, we may conclude that the uncertainties in the spectroscopic factors due to different theoretical methods are less than 20%. However, a comparison to the shell model values of the spectroscopic factor (*i.e.*, 0.62 from Cohen and Kurath [34]), suggests that the contribution from higher order reactions is non-negligible for the transfer reaction. Most likely, an important contribution is expected from the inelastic channel of the core excitation of ^{12}C.

IV. CALCULATIONS FOR $^{14}C(d,p)^{15}C$

The radiative neutron capture reaction $^{14}C(n,\gamma)^{15}C$ involving a loosely bound halo nucleus ^{15}C plays important roles in many astrophysical applications. Some examples are the production of heavier nuclei in the inhomogeneous big-bang nucleosynthesis model [35] and production of seed nuclei for a possible r-process in core-collapse supernovae [1]. The $^{14}C(d,p)^{15}C$ reaction can be used as an indirect method to obtain the ANC for the $^{14}C(n,\gamma)^{15}C$ reaction, and also can be used for a spectroscopy of one neutron halo ^{15}C. In this section, we extract spectroscopic factors from existing experimental data [36–39] by using the reaction models and try

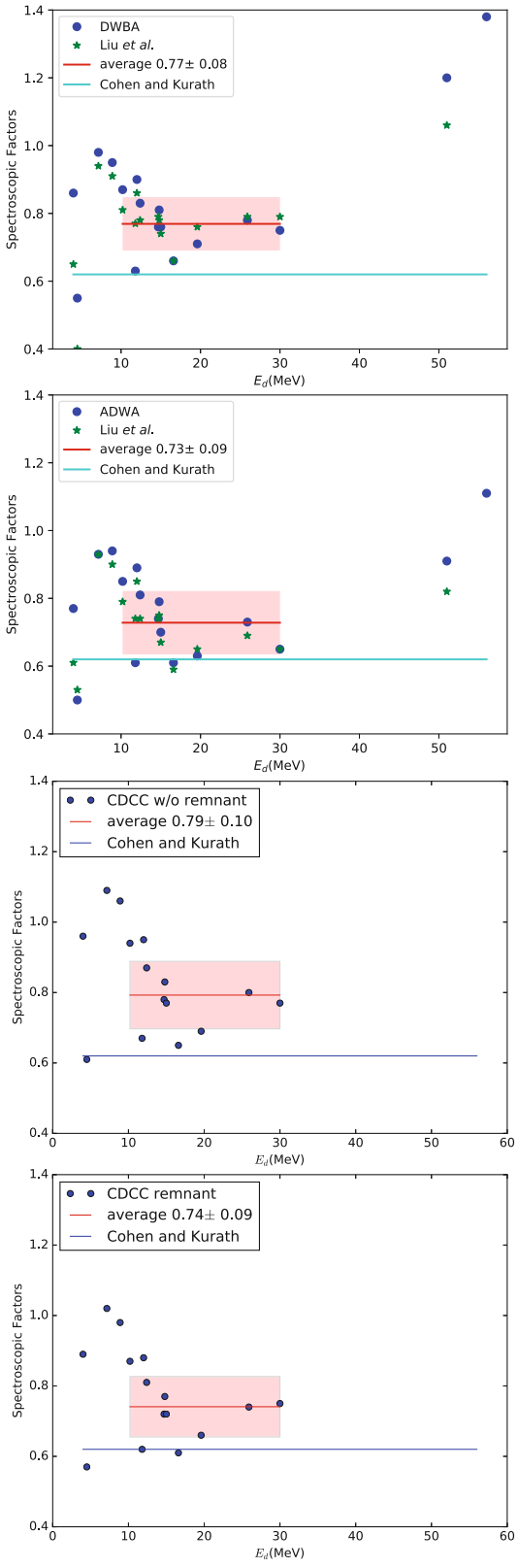

Fig. 2. (Color online) Spectroscopic factors from experimental data of $^{12}C(d,p)^{13}C$ and averaged values for each reaction model compared with the shell model results of Cohen and Kurath [34]. Liu *et al.* represents the results from Ref. [8].

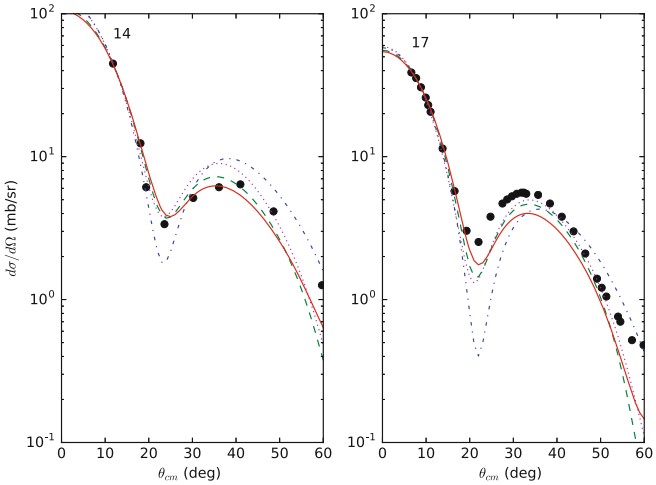

Fig. 3. (Color online) Experimental differential cross sections of the $^{14}C(d,p)^{15}C$ reaction at 14 MeV and 17.06 MeV and corresponding theoretical results from various reaction models in the same notational convention as in Fig. 1. The theoretical results are adjusted using the spectroscopic factors in Table 2.

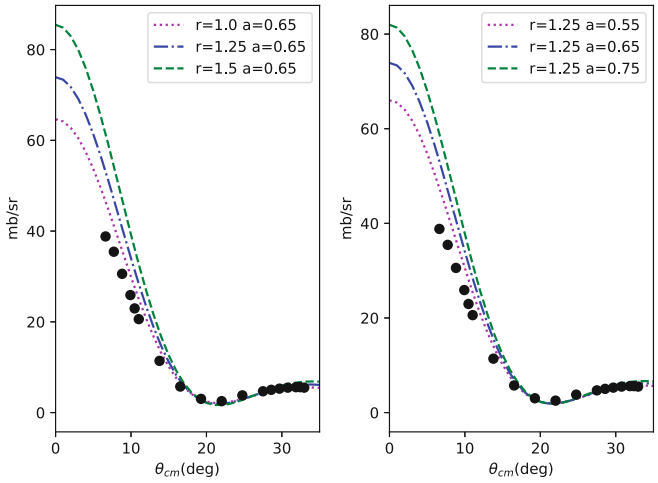

Fig. 4. (Color online) Angular distribution of the $^{14}C(d,p)^{15}C$ reaction in CDCC without remnant correction for various radius and diffuseness parameters of the ^{15}C binding potential. Left panel shows the dependence on r_0 with fixed $a = 0.65$ fm and right panel shows the dependence on a with fixed $r_0 = 1.25$ fm. Spectroscopic factor is not multiplied and the experimental data is also shown for a guide.

Table 2. Experimental spectroscopic factors from each reaction model and references for the experimental data for $^{14}C(d,p)^{15}C$ reaction.

E (MeV)	S.F. (DWBA)	S.F. (ADWA)	S.F. (CDCC-no rem.)	S.F. (CDCC-rem)	Ref.
14	1.22	1.11	0.98	0.91	[37]
17.06	0.87	0.71	0.75	0.71	[39]

to determine the uncertainties coming from the reaction models.

Much of the theoretical treatment in this section is similar to that for the $^{12}C(d,p)^{13}C$ reaction. The Daehnick and the KD02 global optical potentials are adopted for the DWBA, and KD02 optical potential is used for the ADWA and the CDCC calculation. The deuteron binding potential V_{np} has a Gaussian form. $^{15}C(\frac{1}{2}^+)$ is modeled as a composite of a $^{14}C(0^+)$ core and a valence neutron in the $2S_{\frac{1}{2}}$ state. As in ^{13}C, the depth of the Woods-Saxon potential is adjusted to reproduce the neutron separation energy ($S_n = 1.22$ MeV) of ^{15}C. Here, we also investigate other shape parameters of Woods-Saxon potential in addition to the standard values $r_0 = 1.25$ fm and $a = 0.65$ fm due to the shallow binding of ^{15}C.

As in the $^{12}C(d,p)^{13}C$ case, we find that the theoretical angular distribution of the cross section at low energy ($E_d \leq 10$ MeV) does not show agreement with the experimental data [36], even at the forward angle. Also, the data in Ref. [38] do not contain forward angle (≤ 15 degree) results and are not suitable for spectroscopic analysis. Thus, we only extract spectroscopic factors from 14 MeV and 17.06 MeV data. The resulting spectroscopic factors are summarized in Table 2, and the corresponding differential cross sections are shown in Fig. 3. Though all model results reasonably agree with experiments, the ADWA is noted to show the worst agreement in angular distribution, which is consistent with the results in Refs. [39,40] . For the spectroscopic factor, as summarized in Table 2, the CDCC shows less variation in its value depending on the energy than the DWBA and the ADWA. However, because only data for two energies are available, taking average would be less meaningful. We note that the uncertainty, which depends on the reaction model, is likely to be less than ∼20%. The contribution from the remnant correction seems to be less than ∼10%.

However, because the ^{15}C is a shallow bound nucleus, testing the dependence from the binding potential geometry may be useful. Figure 4 shows the dependence of the angular distribution on the radius and the diffuseness parameters of the binding potential between a neutron and the ^{14}C core in the case of CDCC without remnant correction. The result shows that changes in the radius parameter and diffuseness parameters can affect the extracted spectroscopic factor. Within the range of parameters tested, the uncertainty from the geometric parameter is ∼ 20%. Interestingly, the closest differential cross section to the experimental data can be obtained in the case of $r_0 = 1.0$ fm and $a = 0.55$ fm. This tendency of small geometric parameters is also observed in Ref. [41]. Also, $r_0 = 1.223$ fm and $a_0 = 0.50$ fm is adopted in the analysis of Coulomb breakup of ^{15}C [42] and neutron knockout of ^{15}C [43]. These values are inferred from the mean field Hartree-Fock calculations in Ref. [43]. Thus, having a stringent constraint on the wave function of ^{15}C

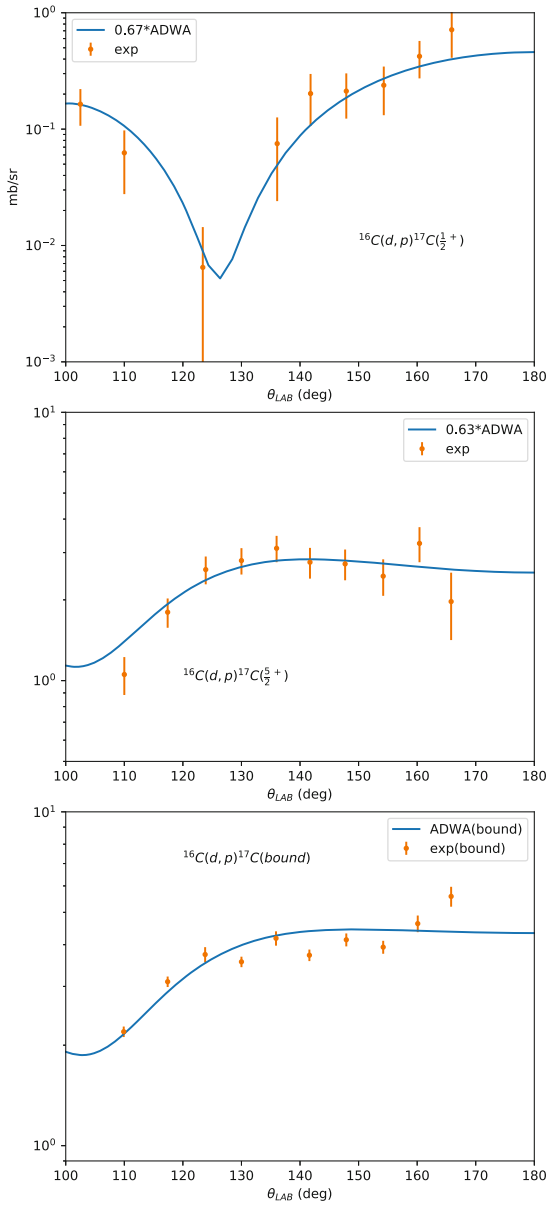

Fig. 5. (Color online) Differential cross section of the $^{16}C(d,p)^{17}C$ reaction in lab angle. The first and the second are for the transfer reaction to the first and the second excited states of ^{17}C, respectively. The third is a combined cross section for the bound state of ^{17}C. The ADWA cross sections are multiplied by the spectroscopic factors described in the text.

other than the separation energy to reduce uncertainty in the spectroscopic factor will be important.

V. CALCULATIONS FOR $^{16}C(d,p)^{17}C$

As already mentioned in the introduction, the $^{16}C(d,p)^{17}C$ reaction is important for production of seed nuclei for the r-process. In addition, because the existence of shell gaps in the single-particle spectrum affects the excitation energy of a nuclei, the excitations of ^{17}C may be used to study the shell evolution to test the existence of $N = 14$ shell gaps.

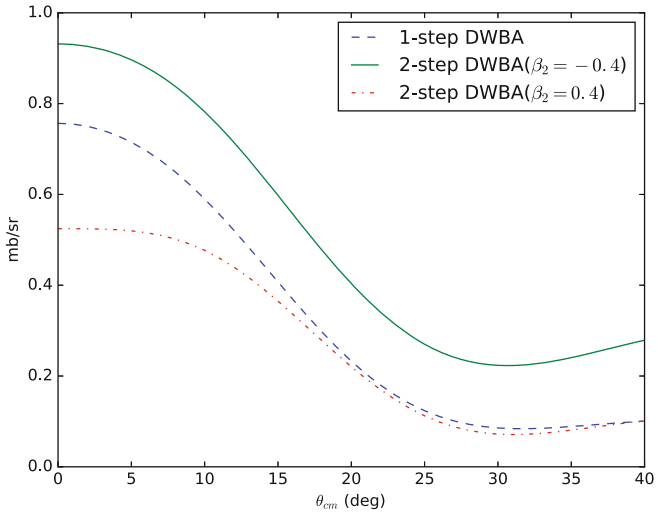

Fig. 6. (Color online) One-step and two-step DWBA calculations of the $^{16}C(d,p)^{17}C$ reaction in center of mass angle. The two-step DWBA calculation includes the contribution from core excitation of $^{16}C(2^+)$.

Another interesting result for ^{17}C is that a large discrepancy exists between experimental and the theoretical spectroscopic factors observed in knockout experiments [10]. According to a shell-model calculation, the spectroscopic factor of $^{17}C(\frac{3}{2}^+; g.s.) = {}^{16}C(0^+) + n(1d_{\frac{3}{2}})$ is very small (~ 0.03). However, the observed knockout cross section of $^{17}C(\frac{3}{2}^+; g.s.)$ is roughly 10 times larger than the theoretical prediction [10]. The origin of this discrepancy is not yet clearly understood. Thus, testing whether the same discrepancy can be seen in the neutron transfer reaction would be interesting. Unfortunately, not many experimental data have been published for $^{16}C(d,p)^{17}C$ reaction other than Ref. [44], which use ^{16}C beam at 17.2 MeV in inverse kinematics. Also, the authors of Ref. [44] show only the combined cross section of the ground state $^{17}C(\frac{3}{2}^+; g.s.)$ and excited states $^{17}C(\frac{1}{2}^+; Ex = 0.217)$ and $^{17}C(\frac{5}{2}^+; Ex = 0.335)$. Thus, for the $^{16}C(d,p)^{17}C$ reaction, we do not try to test uncertainty of spectroscopic factor of ^{17}C for reaction models. Instead, we compare the one-step DWBA with the two-step DWBA including core excitation of $^{16}C(2^+)$ in the $^{16}C(d,p)^{17}C$ reaction to see whether the higher order corrections give meaningful corrections.

The ADWA analysis of $^{16}C(d,p)^{17}C$ is already done in Ref. [44]. We repeat the ADWA calculation and get the results consistent with those in Ref. [44]. Similar to the previous $^{12}C(d,p)^{13}C$ and $^{14}C(d,p)^{15}C$ reactions, Daehnick and KD02 global optical potentials are used for distorted waves, a Gaussian potential for deuteron and the Woods-Saxon potential with $r_0 = 1.25$ fm and

$a = 0.65$ is used for ^{17}C. Figure 5 shows the results from the ADWA analysis of neutron transfer reaction to the first excited state and second excited states of ^{17}C. The values of the spectroscopic factors in Ref. [44], 0.67 for $^{17}C(\frac{1}{2}^+; Ex = 0.217)$ and 0.63 for $^{17}C(\frac{1}{2}^+; Ex = 0.217)$, seem to be reasonable, as shown in the first two graphs in Fig. 5. The combined cross section, which is the sum of the cross sections of all bound states, assuming the spectroscopic factor 0.53 for $^{17}C(\frac{3}{2}^+; g.s.)$, also gives reasonable agreement with experimental data. Thus, we can conclude that compared to the result from the shell model, the transfer reaction shows a spectroscopic factor for $^{17}C(\frac{3}{2}^+; g.s.)$ that is more than 10 times larger. A possible source of this large discrepancy is coupled channel effects on the transfer reaction through the core excitation of $^{16}C(2^+)$. To estimate the contribution, we use the shell model code NUSHELL [45] with the WBT interaction in 'spsdpf' model space to compute the spectroscopic amplitudes of $^{17}C(\frac{3}{2}^+)$. We obtain

$$\left|^{17}C(\frac{3}{2}^+)\right\rangle \simeq -0.16\left|^{16}C(0^+)\otimes n(1d_{\frac{3}{2}})\right\rangle +1.19\left|^{16}C(2^+)\otimes n(1d_{\frac{5}{2}})\right\rangle +0.39\left|^{16}C(2^+)\otimes n(2s_{\frac{1}{2}})\right\rangle . \quad (6)$$

The deformation parameter β_2 of ^{16}C is not well determined. A theoretical prediction from relativistic Hartree-Fock-Bogoliubov calculations [46] showed that it varied from 0 to ~ 0.35. Because our motivation is to test whether the coupling with the core excitation resolves the above-mentioned discrepancy, not to do an exact calculation of the reaction cross section, considering the uncertainty in β_2, we choose $\beta_2 = -0.4$ and $\beta_2 = +0.4$ for a collective rotor model of the ^{16}C core and use the FRESCO code to compute the second-order DWBA contribution with shell-model spectroscopic amplitudes. Figure 6 shows the one-step DWBA and two-step DWBA differential cross sections via core excitation $^{16}C(2^+)$ of $^{16}C(d,p)^{17}C$. Though the inclusion of the core-excitation coupling with $\beta_2 = -0.4$ increases the cross section and reduces the discrepancy, the correction is not large enough to explain the discrepancy in the spectroscopic factor. For $\beta_2 = +0.4$, the discrepancy becomes even worse. However, considering the naive model used in this estimate, we can only conclude that more precise approach would be necessary to understand the discrepancy, which will be a future study.

VI. SUMMARY

In this work, using different theoretical models, we investigated the uncertainty in the spectroscopic factors extracted from experiments. To study the model dependence, we compared the results from the DWBA, ADWA and CDCC for the $^{12}C(d,p)^{13}C$, $^{14}C(d,p)^{15}C$ reactions and observed that the uncertainty coming from the model dependence is less than 20%. We also found that the contribution from the remnant correction is less than 10%. Thus, inclusion of the break up channel could be important for precise extraction of spectroscopic information of weakly bound nuclei from experiments. However, as seen in the case of $^{14}C(d,p)^{15}C$, the uncertainty in the potential parameters (r_0, a) of bound state wave functions can also give an uncertainty of about 20%. We also tried to explain the recent discrepancy of the spectroscopic factors of $^{17}C(\frac{3}{2}^+)$ between the experiment and the shell-model calculation. However, we found that the naive model estimate of core excitation effects of ^{16}C cannot resolve the discrepancy. Thus, future study with a more precise approach for the (d,p) reaction for the reliable extraction of spectroscopic information of rare isotopes is demanded.

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