Unitary limit Fermions under trap And Dripline

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NLEFT collaboration

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Nuclear Lattice Effective Field Theory Collaboration

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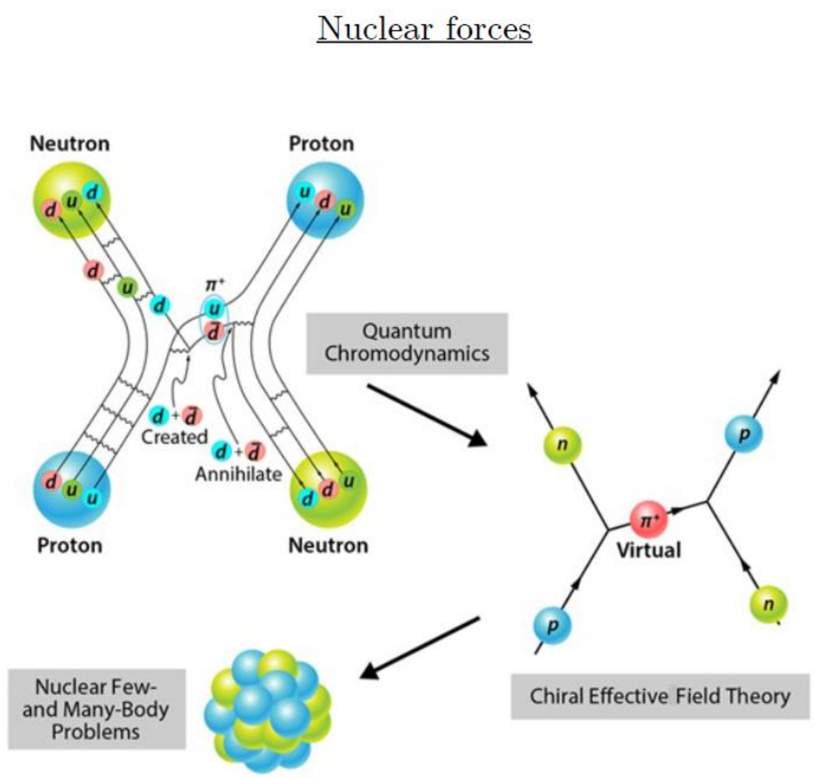
- Shihang Shen (Juelich)
- Gianluca Stellin(CEA Paris-Saclay)
- And More…

Ab-initio method

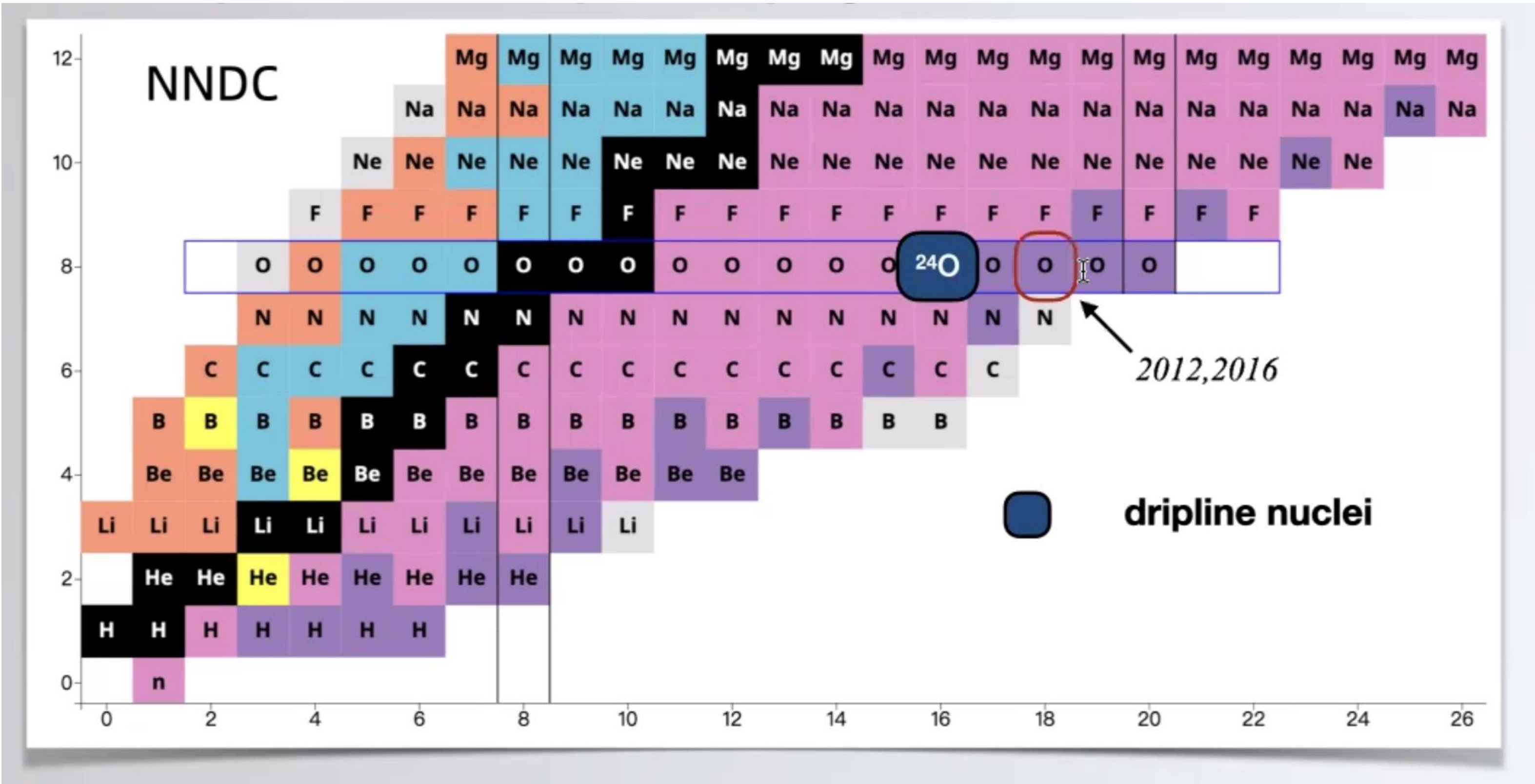


- *ab-initio* Nuclear Physics
  - (1) nucleon degrees of freedom
  - (2) nucleon-nucleon interaction

Goal: predict wide range(structure, reaction, nuclear matter) of nuclear phenomena (without parameter fitting, model assumption) from nuclear interaction (for 2-body,3-body, many-body, based on QCD)

Direct connection between
Nuclear Force ↔ Nuclear Phenomena

Dripline

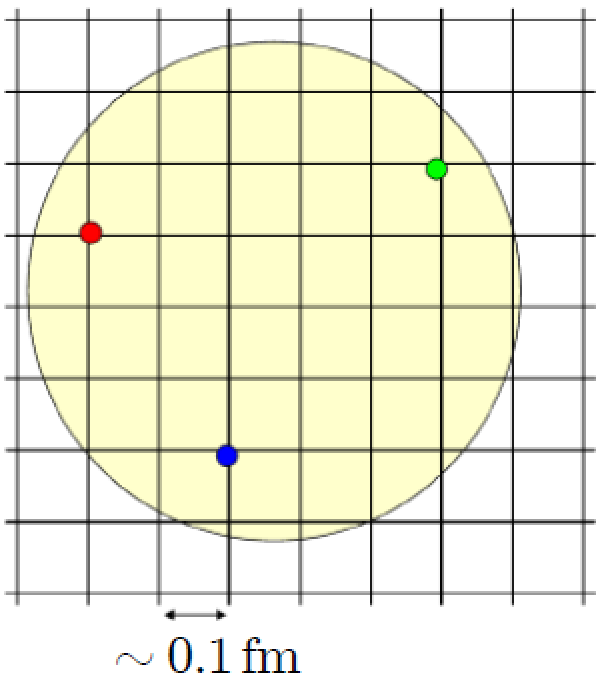


Can we explain the dripline in NLEFT?
Can we find a effective mean-field from NLEFT?

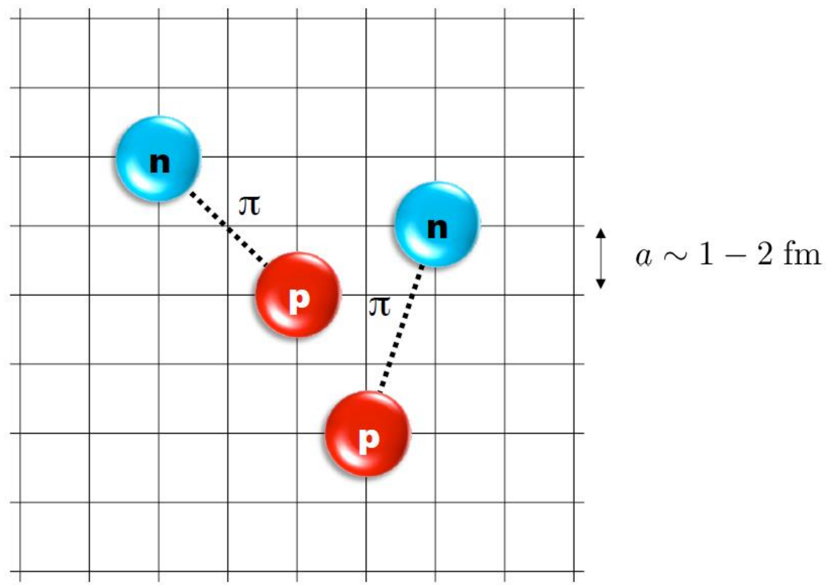
Nuclear Lattice Effective Field Theory

- One of ab initio method for many fermion system

Nucleon in LQCD



Nucleons as point particles on the lattice



Path integral

Correlator function for A Nucleons $Z_A(t) = \langle \Psi_A | \exp(-tH) | \Psi_A \rangle$

Slater Determinants for A free Nucleons

Ground state energy by time derivative of the correlator

$$E(t) = -\frac{d}{dt} \ln Z_A(t)$$

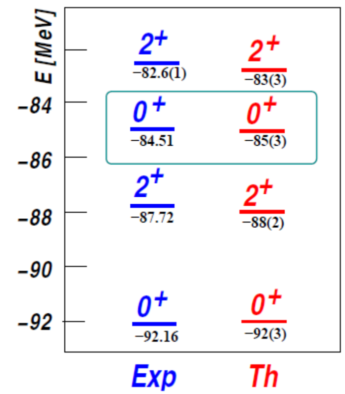
At large time only ground states survive $\Rightarrow$ $E_A^0 = \lim_{t\to\infty} E_A(t)$

$$\begin{aligned} |\Psi_A\rangle &= \sum_n c_n |n\rangle_A , \\ Z_A(t) &= \langle \Psi_A | e^{-tH} | \Psi_A \rangle = \sum_n c_n e^{-tE_n} \end{aligned}$$

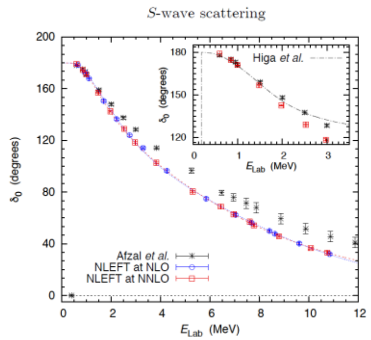
Applications of NLEFT

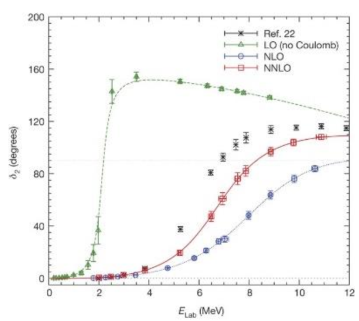
- Has been successfully applied to
  - Nuclear matter, Cold atom, dilute fermion system
  - Finite nuclei (A<=60)
  - First ab-initio calculation of Hoyle state
  - Cluster structure of ^{12}C and ^{16}O
  - NN scattering, N-D scattering
  - Alpha-alpha scattering
  - radiative capture, fusion
  - Etc.

The first ab-initio calculation of Hoyle state



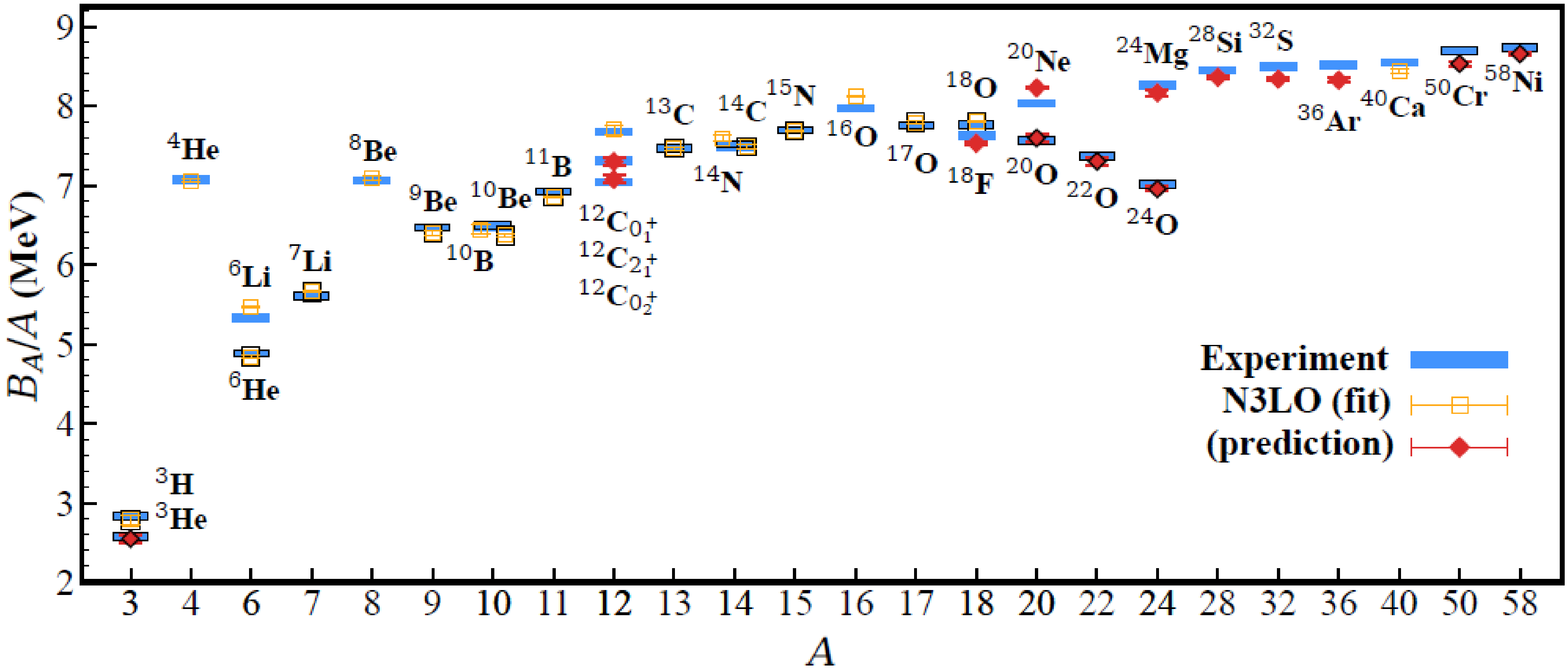
Epelbaum, Krebs, Lähde, Lee, Meißner:
Phys. Rev. Lett. 109, 252501 (2012)





Ab initio alpha-alpha scattering
(Nature 528, 111-114(2015))

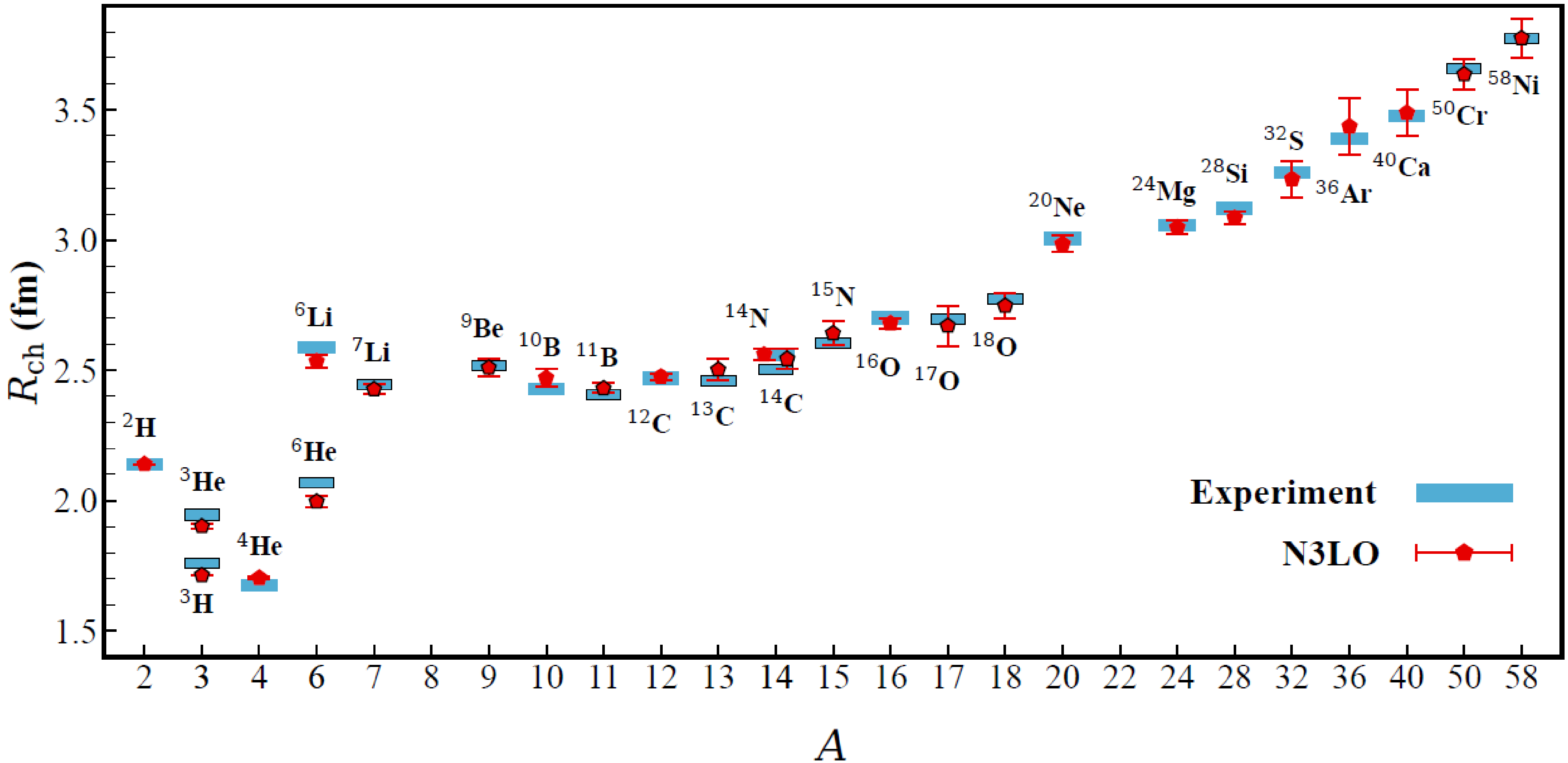
BE/A from WFM



RMSD ~ 0.1 MeV per nucleon

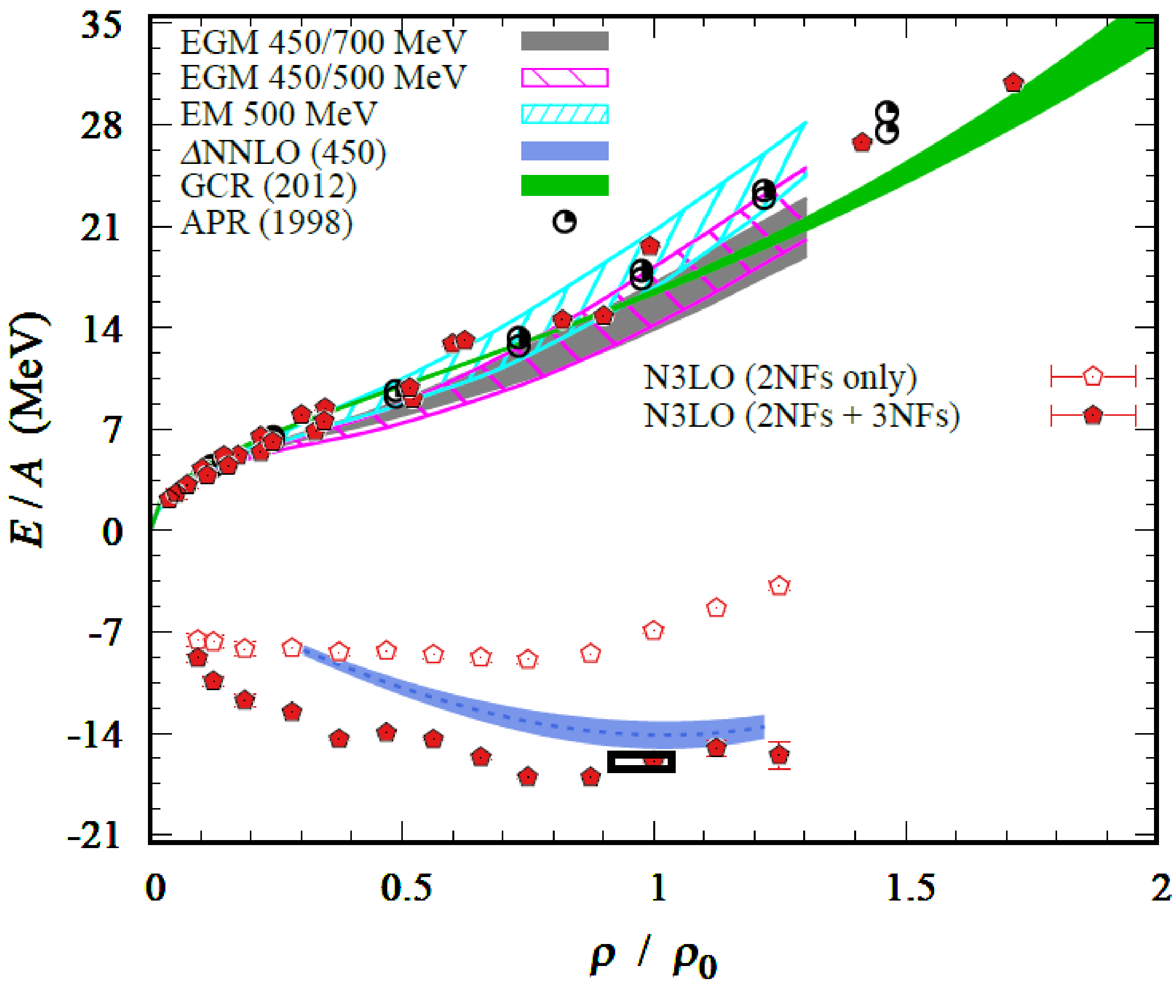
Accepted by Nature recently

Charge Radius



No fitting! RMSD~ 0.03 fm

Nuclear/Neutron Matter

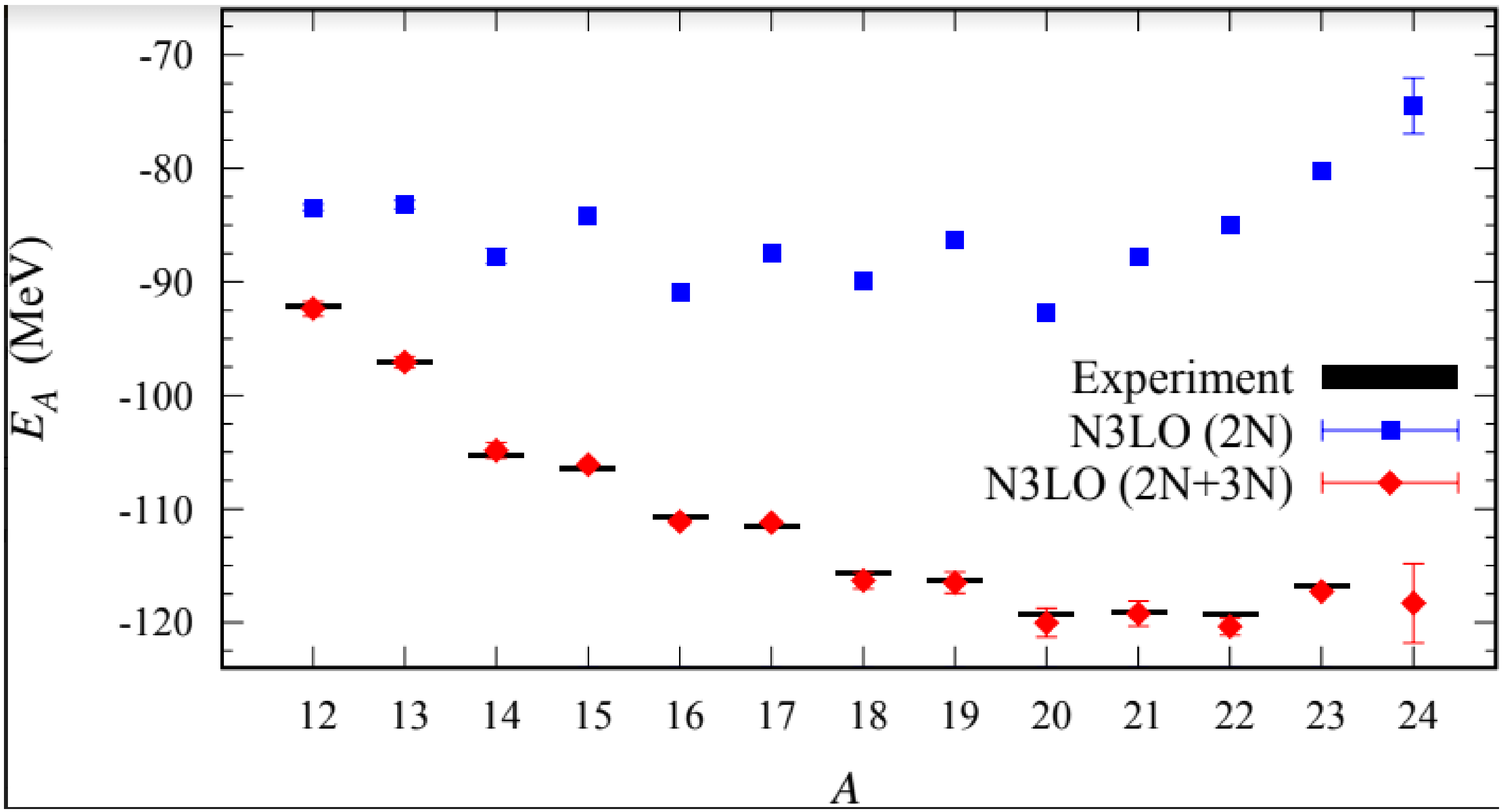


Neutron matter:
A=4~80
box size 6.6 ~ 13.2 fm.

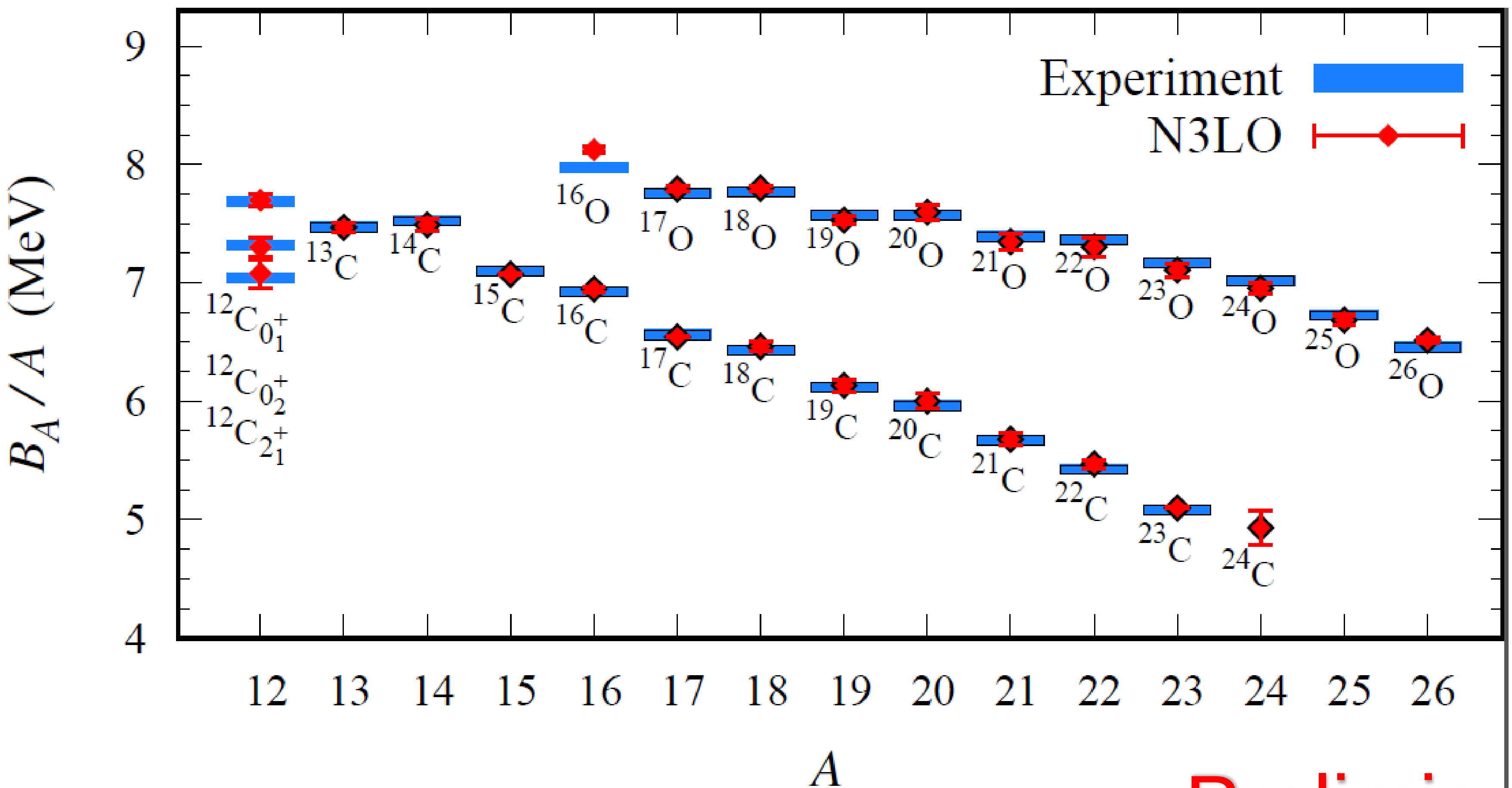
Nuclear matter:
A=4 ~ 160
Box size 7.92~9.24 fm.

- Uncertainties from finite system size correction

Carbon isotopes



Carbon and Oxygen



Preliminary

How can we understand the dripline?

- Although fully ab initio calculation of dripline was possible, Mean field picture is very useful to understand the general beh avior.
  - For example, core potential in a shell model with core.

- How can we find more simple picture which describe the dri pline? What is the most important property of the potential t o get the dripline?
  - Fully ab-initio approach
  - Half ab-initio/phenomenological approach

- Warning: This is on-going work and no final results are avail able.

Dripline for unitary Fermions

- Unitary Fermion Gas :
  - A strongly coupled conformal system
  - Physically interesting multi-fermion system with simple interaction

$$\mathcal{A} = \frac{4\pi}{M} \frac{1}{p\cot\delta - ip}$$

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2 + O(p^4)$$

- p $\cot\delta = 0$: Unitary limit

→largest possible scattering amplitude. Infinite scattering length.
(Realistic nuclear force is close to unitary limit.)

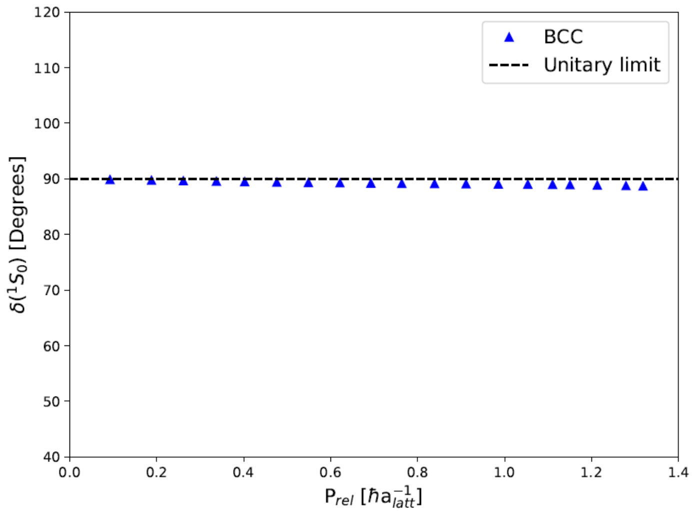
Universality of unitary limit of many fermion system
→ Bertsch Parameter

$r_e \ll n^{-\frac{1}{3}} \ll a_s,$

$E_0 = \xi E_{FG},$

Bertsch parameter

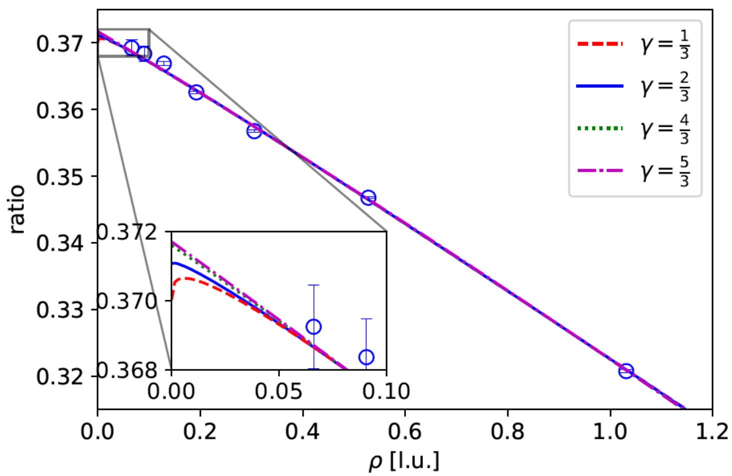
Unitary limit Hamiltonian



1S0 NN scattering phase shift is fitted for unitary limit .

→ Very large scattering length.

In cubic and BCC lattice



N=66 neutrons (33 spin up, 33 spin down)

L = 4,…, 10

$$\xi = E_0 / E_{\rm FG}$$

$\xi_{33,33}^{\rm thermo} = 0.369(2)$ and $\xi_{33,33}^{\rm finite} = 0.372(2)$. Cubic lattice

$\xi^{\rm therm} = 0.369(2)$. $\xi^{\rm few} = 0.371(2)$. BCC lattice

Unitary Fermions under trap

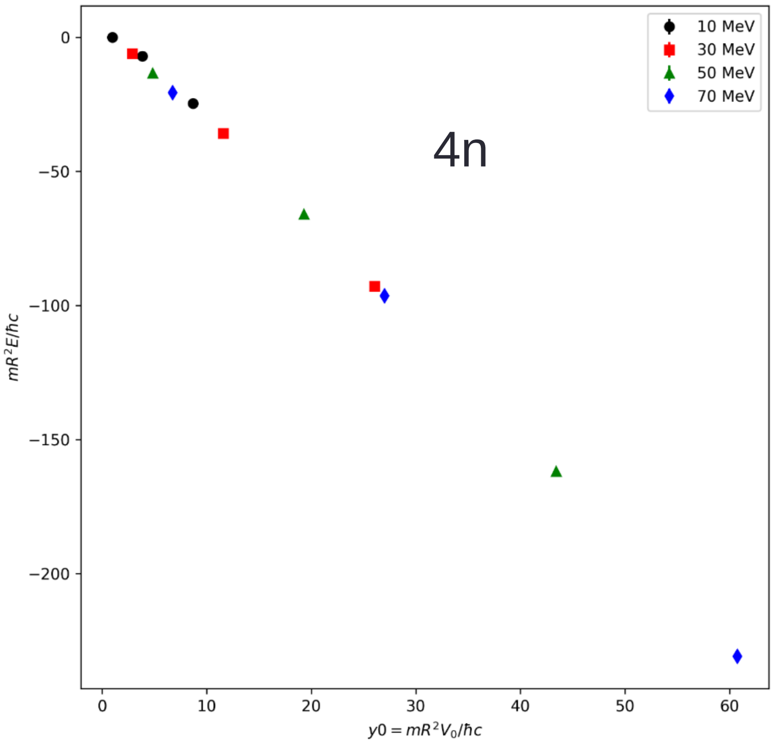
- Unitary Fermion itself does not form a bound nuclei
  → Introduce an external spherical well potential
  Can we find a relation between trap shape and dripline?

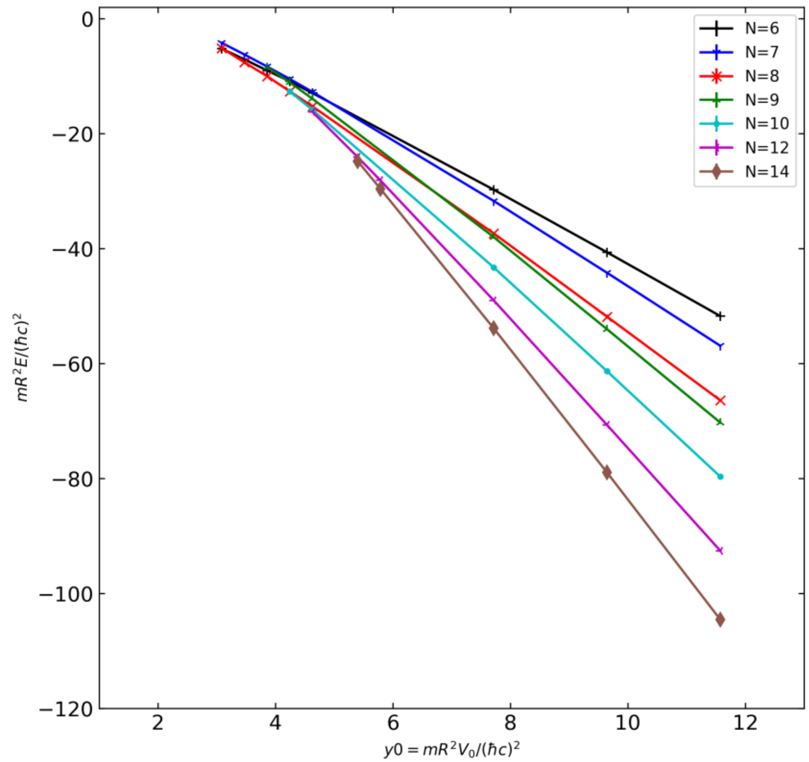
- Separation energy have to be calculated accurately
  → Use an re-weighting method to reduce error in separation energy.
  (correlated error estimation in energy difference.)

- Unfortunately, realistic mean-field potential is rather expensive.
  (reaching dripline in Unitary limits is difficult.)
  → limited to shallow external potential.
  Try to see the dependence on parameter : $y0 = m R^2 V$

(Work in progress. Only preliminary results)

Neutrons in trap



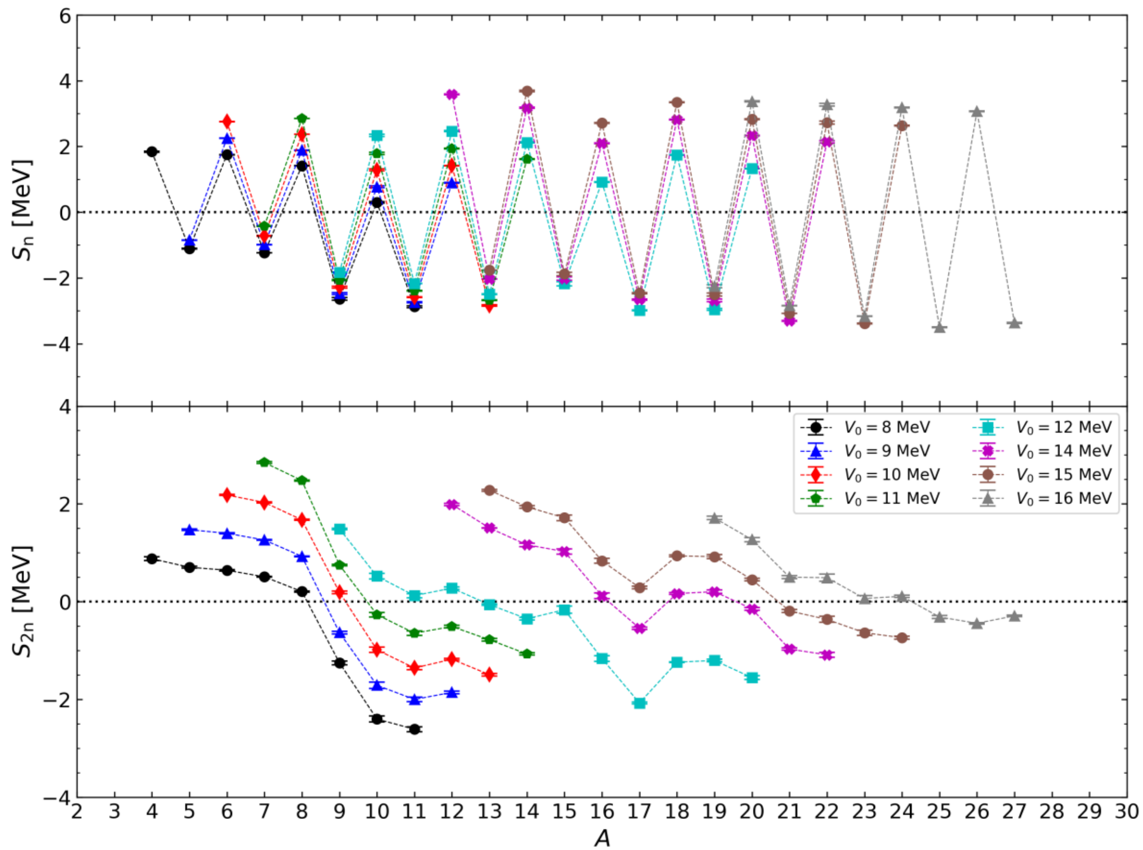


Separation energies

Lattice spacing
100 MeV

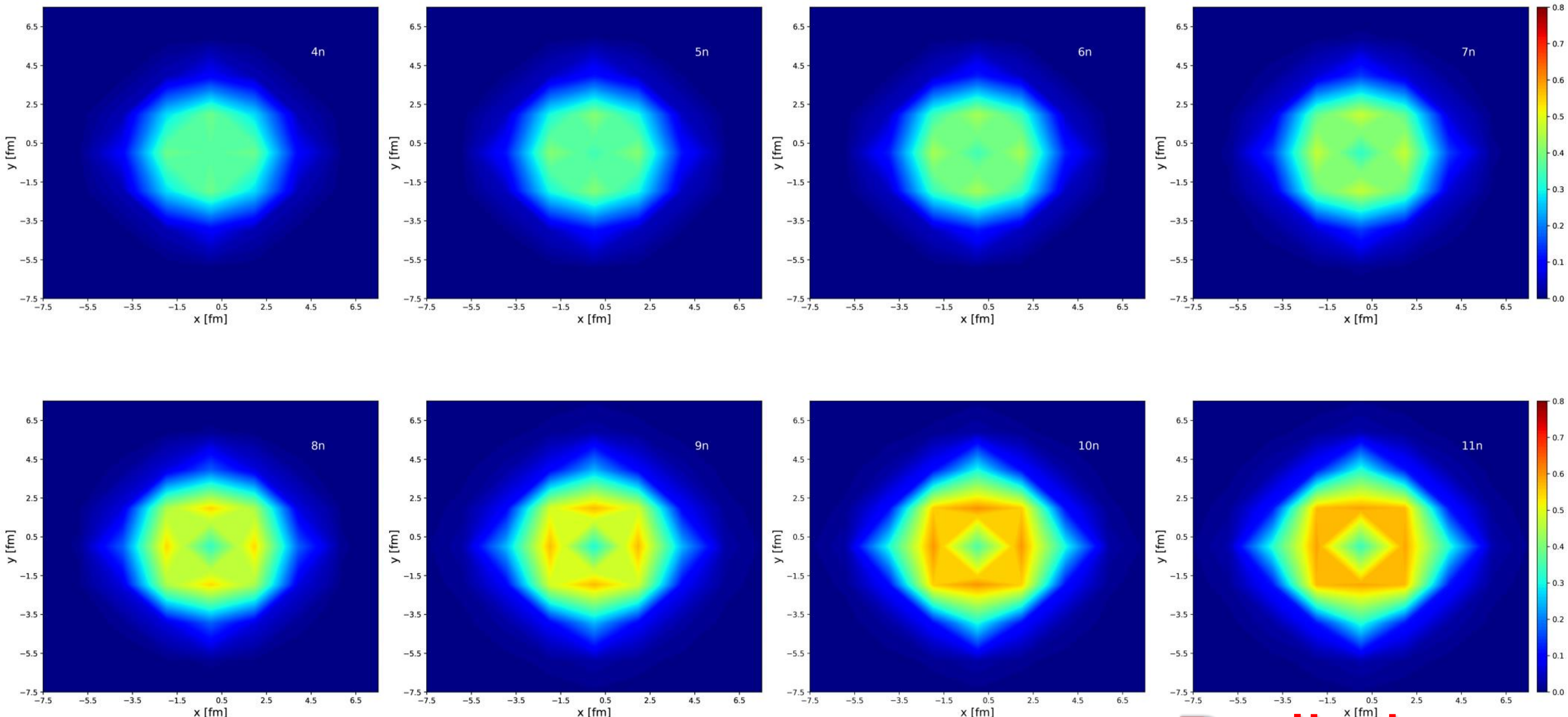
Spherical well
radius~ 4fm

- Unitary limit interaction has too strong pairing!



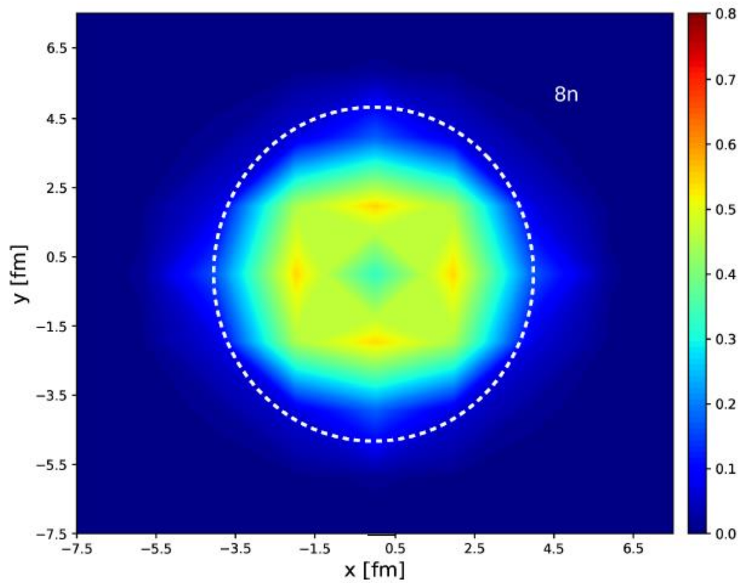
Preliminary

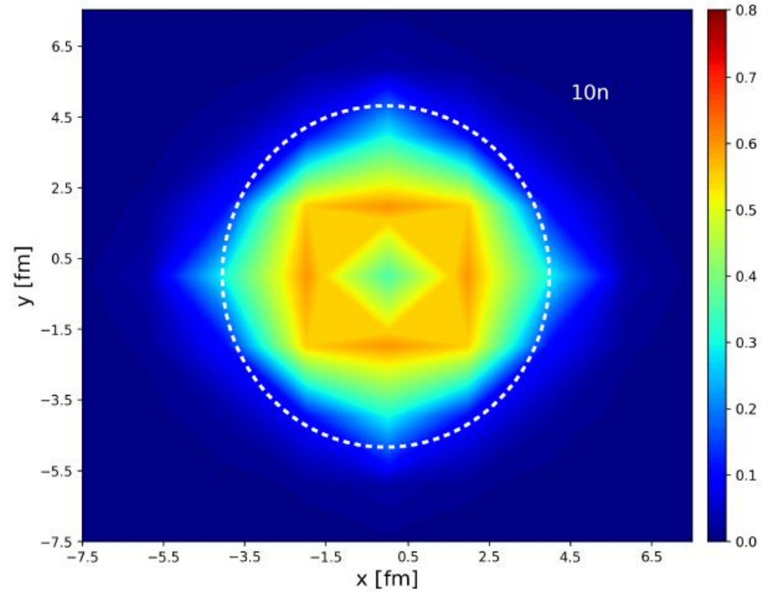
V0=8MeV density contour



Preliminary

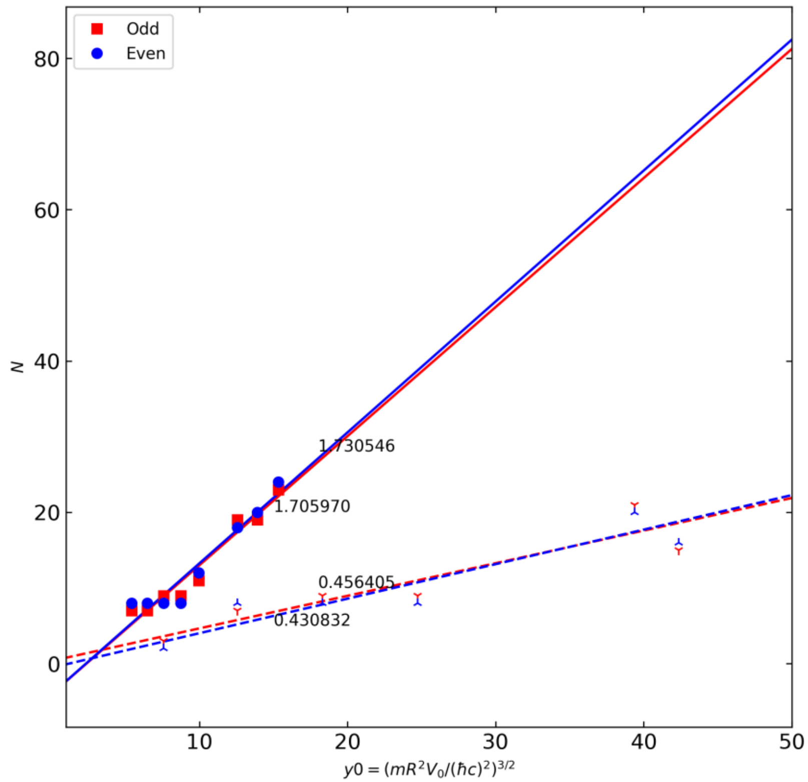
V0=8MeV density contour





Preliminary

Extrapolation?



- Linear Fit
- Needs More data
- Unitary limit interaction has too strong pairing.

- getting mean field potential for the last neutron by using inver se method→ not accurate.

Preliminary

Mean field potential from protons?

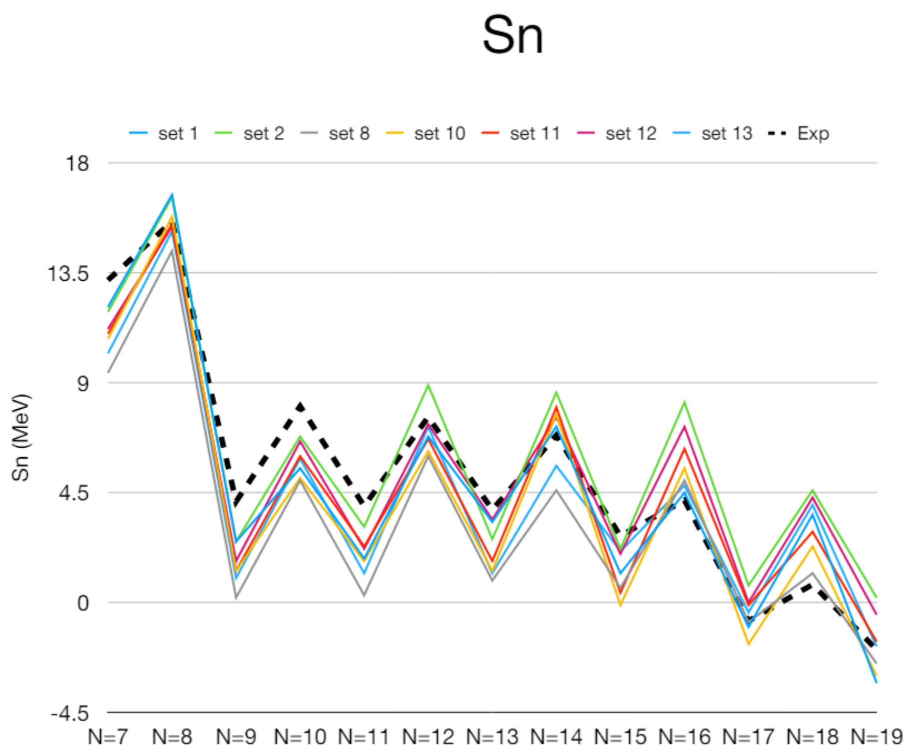
- Instead of inverse method to get mean field potential,
  let us try to use half-phenomenological approach.

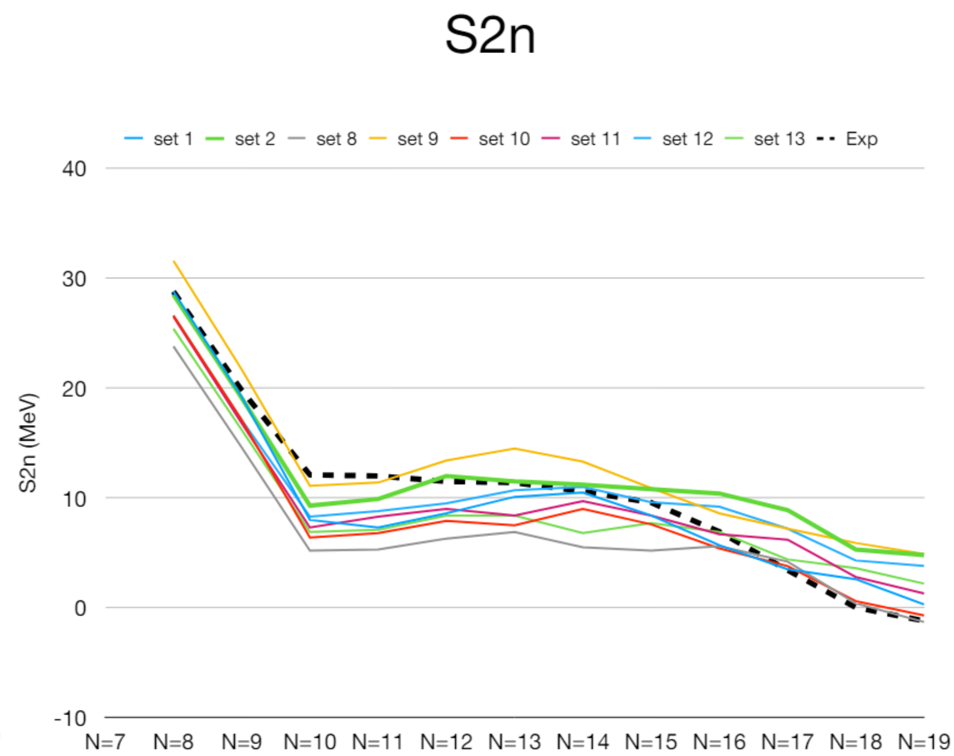
- Oxygen isotopes as
  - Fully interacting neutrons (WFM interaction up to N3LO)
  - Replace proton contributions into WS potential
  - → N neutrons under effective proton mean field.

- What kind of effective potential will give experimental separati on energies?
  - Searching for the best Woods-Saxon potential form

Preliminary

Mean field potential from protons?





Variation of WS potential parameters
depth : -45 ~ -50 MeV,
Diffuseness : 0.5 ~ 0.7 fm
Radius : 2.52 fm

Preliminary

Summary

- How one can get a simple mean field picture of dripline from ab initio ?
- Unitary limit Fermion system: simple but too strong pairing for dripline.
- Half-Phenomenological approach:
  - Searching for the basic property of mean field potential.
  - Still preliminary stage.