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Finite nuclei: Just a droplet of nuclear matter?†

H. Müther

Institut für Theoretische Physik der Universität Tübingen D-7400 Tübingen, Germany

Abstract

Differences in the evaluation of the saturation properties of infinite nuclear matter and finite nuclei are discussed. It is demonstrated that ground-state properties of finite nuclei are much more affected by the finite range of realistic nucleon–nucleon interactions than the saturation point of nuclear matter. Therefore very simple local-density approximations in calculating properties of finite nuclei are not reliable. The finite range effects yield a larger incompressibility for finite nuclei as compared to nuclear matter.

1. Introduction

In recent years there have been many controversial discussions on the equation of state (EOS) for nuclear matter at densities well above the saturation density. Most of these controversies have been raised because it seemed that different properties for this equation of state were needed to understand the dynamics of a supernova explosion [1, 2] and some features of central heavy ion collisions at high energies [3, 4].

Of course such observables only yield a rather indirect information on the nuclear equation of state. Early investigations of the supernova required a soft equation of state characterized by a compressibility as low as $K \approx 140\,\mathrm{MeV}$. A stiffer EOS would not allow sufficient energy to be stored during the collapse such that the subsequent explosion would not reach the surface. Taking into account additional features like effects of neutron heating, a different composition or a rotation of the progenitor star may allow for a supernova even with a stiffer EOS. On the other hand it turned out that also the comparison of experimental data of relativistic heavy ion collisions with corresponding results of computer simulations does not provide a unique answer for the properties of the EOS. Early calculations seemed to show a preference for a stiff EOS with a compressibility as

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large as $K \approx 380$ MeV. Attempts to improve such calculations by considering more realistic models for the nucleon–nucleon (NN) interaction, however, indicate that it is rather involved to determine the EOS in this way.

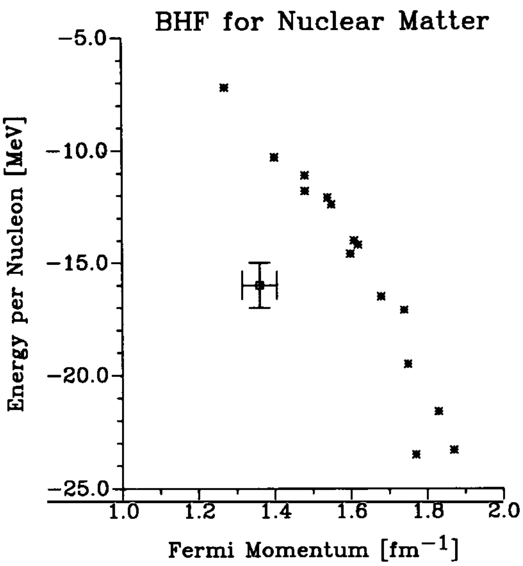
Furthermore one can argue of course that the systems of nuclear material at high densities formed in a supernova and a heavy ion collision are quite different. It has been argued that a different temperature, which in heavy ion reactions is much larger than in a supernova, may be responsible for a different behavior [5]. The main difference between a supernova and a heavy ion collision, however, is the fact that a supernova is infinitely large on the nuclear scale whereas in a heavy ion collision only a very small piece of nuclear matter may be compressed to high densities. This means that one is probing in a supernova the EOS of infinite nuclear matter, while in heavy ion collisions only the compression of small systems is observed.

Therefore in this contribution I would like to discuss differences between the EOS for an infinite system as compared to the EOS for a finite nucleus, both obtained in many-body calculations employing realistic NN interactions.

At first sight the evaluation of ground-state properties of infinite nuclear matter and finite nuclei seem to show very similar features. As an example one may recall that Brueckner–Hartree–Fock (BHF) calculations for the ground-state of nuclear matter exhibit problems almost identical to those of finite nuclei. This is shown in Figs. 1 and 2. If one uses various NN interactions, which are all adjusted to fit NN scattering phase shifts, and calculates the binding energy per nucleon and the saturation density or corresponding Fermi momentum of nuclear matter within the BHF approximation, one obtains saturation points [6] as displayed in Fig. 1. One may find a realistic NN potential, which yields the empirical binding energy per nucleon (-16 MeV), however, at a Fermi momentum which corresponds to a density around twice the experimental value. Other interactions lead to a realistic density or Fermi momentum ($k_F = 1.36\,\mathrm{fm}^{-1}$) but with a result for the energy of only -10 to -11 MeV per nucleon. This phenomenon has been known already for a long time and the collection of calculated results, as in Fig. 1, is frequently called the "Coester band" of infinite nuclear matter [7].

The same phenomenon can also be observed for finite nuclei. If one calculates as an example the ground-state properties of ^{16}O by solving the BHF equation for various realistic NN interactions one obtains results for the energy and the radius of the nucleus as displayed in Fig. 2 [8]. In this figure the result for each NN interaction considered is represented by the value calculated for the binding energy per nucleon and the inverse of the calculated radius for the charge distribution, which yields a representation similar to the one for nuclear matter in Fig. 1. Also in this case the results show the behavior of a "Coester band" as in nuclear matter. For nuclear matter and finite nuclei one finds that NN interactions which exhibit a strong tensor force tend to yield a weak binding energy and small density (Fermi momentum or value for the ratio one over the radius), while NN interactions with a weak tensor force lead to large binding energies and densities.

Many attempts have been made to cure this problem of the "Coester band" by improving the approach used for solving the many-body problem of interacting nucleons or by accounting for sub-nucleonic degrees of freedom [6]. In most of these attempts, however, it was observed that such improvements may modify the calculated saturation point but typically by moving it along the "Coester band". For nuclear matter Tom Kuo and coworkers found some indications that including hole–hole scattering terms the calculated saturation point might be moved off the "Coester band" [9, 10]. This effect, however, has not been confirmed for finite nuclei [11].



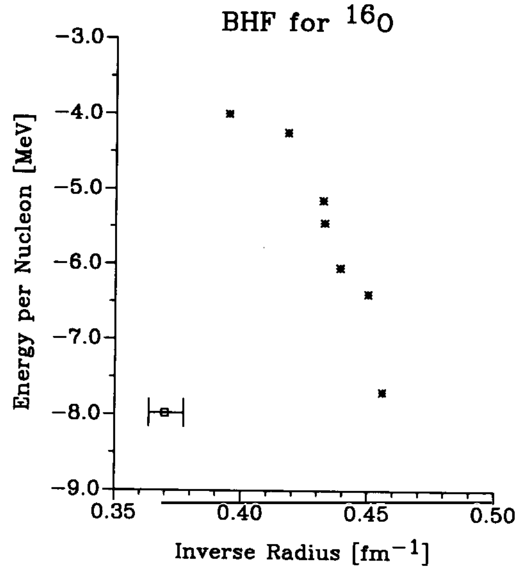


Fig. 1. Binding energy per nucleon and Fermi momentum referring to the calculated saturation density in nuclear matter are displayed as calculated in the BHF approximation for various realistic NN interactions.

Fig. 2. Results on the calculated energy per nucleon and the inverse of the charge radius as obtained from BHF calculations on ^{16}O for various realistic NN interactions.

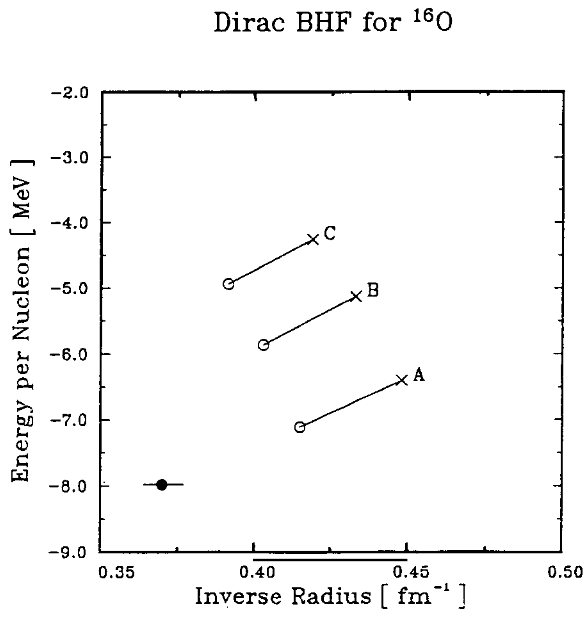


Fig. 3. BHF and DBHF calculations for ^{16}O. Using OBE Potentials A, B and C as defined in Table A.2 of [13] results for the radius of the charge distribution and the binding energy per nucleon obtained in BHF (crosses) and DBHF calculations (circles) are compared to the experimental results.

Attempts to account for relativistic features have been more successful to describe bulk properties of nuclear systems. It has been possible to determine a realistic one-boson-exchange (OBE) potential, which fits NN scattering-phase shifts and yields the empirical saturation properties within the Dirac–Brueckner–Hartree–Fock (DBHF) approach [12]. The relativistic features of the DBHF approach have also been included in investigations of finite nuclei. Employing the same OBE potentials, which were successful in nuclear matter [13], it turned out, that the Dirac effects improved the calculated ground-state properties for ^{16}O as compared to the BHF approach. The relativistic effects, however, were too small to yield a satisfying agreement with the experimental data [14]. This can also be seen from Fig. 3 where results of DBHF calculations (circles) are compared to those of BHF calculations employing the OBE potentials defined in Table A.2 of [13].

The preceding discussion has been made to recall that at first sight the main features of nuclear structure calculations of finite nuclei seem to be rather similar to those of nuclear matter ("Coester band"). Therefore one may ask the question if it is sufficient to consider a finite nucleus just as a droplet of nuclear matter and calculate its properties within such a model. To study this question we will discuss in Section 2 a very simple local density approximation (LDA) and compare its results with a corresponding direct evaluation of finite nuclei. In Section 3 we will explore the sources of the discrepancies between the LDA and the direct calculation. The main conclusion will be summarized in the last section.

2. Compression of nuclei

In order to study the compression modulus of the finite nucleus ^{16}O let us consider as a model wave function the Slater determinant appropriate to describe the ground-state of ^{16}O in terms of harmonic oscillator functions for a given oscillator length b. For each oscillator length one can calculate the radial density distribution $\rho_b(r)$ and the average nucleon density

$$\bar{\rho}_b = \int \rho_b^2(r)\,\mathrm{d}^3r/A\ , \tag{1}$$

with A the number of nucleons (16 in our example). For each model wave function or oscillator parameter b one can furthermore calculate the BHF energy by solving the Bethe–Goldstone equation and the BHF equations directly for the finite nucleus under the constraint that the single-particle energies are oscillator functions for the b parameter under consideration. For the discussion of results presented here we have employed the OBE potential "A" defined in Table A.2 of [13]. The energies per nucleon calculated for various b are displayed by the solid line in Fig. 4 as a function of $\bar{\rho}_b$ defined in Eq. (1). The minimum of this curve is close to the result for the self-consistent BHF calculation obtained without the constraint for the single-particle wave function. In addition the second derivative of this curve gives an idea about the compression modulus for this finite nucleus.

In the next step we now consider the finite nucleus to consist of slices or shells of nuclear matter and calculate the energy in a very simple local density approximation (LDA) as

$$E_{\mathrm{LDA}}(b) = \int \varepsilon_{\mathrm{NM}}(\rho_b(r))\,\mathrm{d}^3r\ , \tag{2}$$

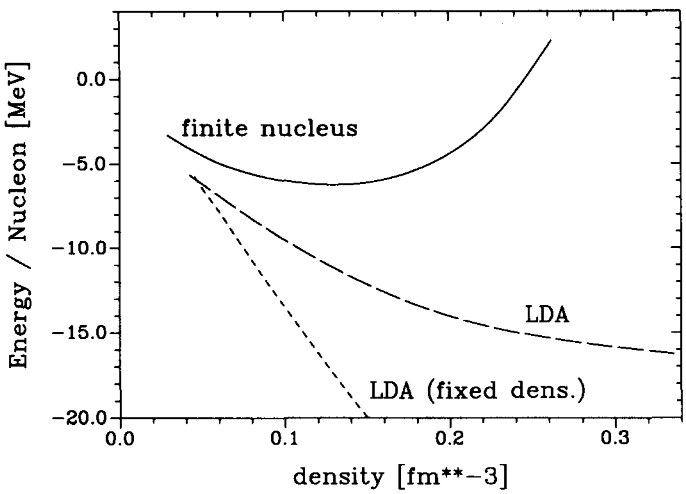


Fig. 4. Binding energy per nucleon for ^{16}O calculated for various mean values of the density $\bar{\rho}$ (see Eq. (1)). The results of the BHF calculation directly for finite nuclei (solid line) and the various local density approximations (LDA, see text) were obtained for the OBE potential A defined in Table A.1 of [13].

where $\varepsilon_{NM}(\rho)$ is the energy density for nuclear matter of density ρ and $\rho_b(r)$ as above stands for the baryonic density calculated for the Slater determinant of oscillator functions with oscillator parameter b. Results for this LDA approximation as a function of density are given by the long-dashed curve in Fig. 4.

One finds that the LDA overestimates the calculated binding energy. At the BHF minimum the difference is about 85 MeV for the total nucleus, which is close to the surface correction contribution in the Bethe–Weizsäcker mass formula for ^{16}O (≈ 118 MeV). This suggests that the LDA approximation misses the major contribution to the surface tension and also the major contribution of the surface effects to the compressibility of a finite nucleus. This is supported by the observation that the LDA result for ^{16}O as a function of density is only slightly above the energy versus density curve of nuclear matter.

Also one sees that the energy versus density curve calculated directly for the finite nucleus is much stiffer than the corresponding curve in the LDA approach or for nuclear matter. This indicates that due to the surface effects it requires more energy per nucleon to compress a small piece of nuclear matter, as is done e.g. in a heavy ion collision, than an infinite system or a stellar object. Therefore this surface contribution to the compressibility may to a large extent explain the difference in the equation of state for nuclear matter derived from heavy ion collisions and studies of astrophysical objects, as discussed above.

Figure 4 also displays a LDA result (short dashes) which is obtained according to Eq. (2) but ignoring the density dependence of the Brueckner G-matrix in $\varepsilon_{NM}(\rho)$, i.e. using the G-matrix as calculated for the density $\rho = 0.5\,\mathrm{fm}^{-3}$. Comparing the two curves for the LDA approximation one can see that the density dependence of the G-matrix is a very important ingredient to obtain the saturation point of finite nuclei as it is for nuclear matter. In addition, however, an additional surface effect is needed for finite nuclei, which is absent in nuclear matter or the LDA approach for finite nuclei.

3. LDA and finite range of interaction

In order to find the origin for the discrepancy between the LDA and the explicit calculation for the finite nucleus one can compare the different contributions to this energy. One finds that the LDA approach for the kinetic energy

$$T_{\mathrm{LDA}}(b) = \int \frac{3}{5}\frac{\hbar^2 k_{\mathrm{F}}^2(r)}{2m}\rho_b(r)\,\mathrm{d}^3 r\,, \tag{3}$$

with the local Fermi momentum $k_{\mathrm{F}}(r)$ determined from the relation

$$\rho_b(r) = (2/3\pi^2)k_{\mathrm{F}}^3(r)\,, \tag{4}$$

is a very good approximation to the kinetic energy of the corresponding harmonic oscillator configuration. Therefore the deviation must originate from the expectation value for the potential energy. To visualize the difference of the two approaches for the calculation of the interaction energy we assume for the moment the two-body interaction, in the BHF approximation the G-matrix, to be local, which means that it depends on the momentum transfer only. In this case the interaction energy can be calculated as

$$\langle V\rangle = \sum_{S,M,T}\int V_{SMT}(q)\phi^{SMT}(q)\,\mathrm{d}q \tag{5}$$

where $V_{SMT}(q)$ is the interaction depending on the momentum transfer q for a given set of spin–isospin quantum numbers S, M, T or a corresponding spin–isospin operator in the interaction V (see e.g. [15]).

The quantity $\phi(q)$ represents a two-body density obtained for a given approximation to calculate $\langle V\rangle$. For this two-body density all variables have been integrated except the momentum transfer. Therefore this $\phi(q)$ is a measure to which extent the local interaction at momentum transfer q contributes to the total interaction energy. In the BHF approximation this $\phi(q)$ can be split into a direct part (related to the Hartree contribution) and an exchange contribution (related to the Fock term)

$$\phi^{SMT}(q) = \phi^{SMT}_{\mathrm{direct}}(q) + \phi^{SMT}_{\mathrm{exch}}(q)\,. \tag{6}$$

For isospin symmetric ($Z = N$, ignoring Coulomb interaction) and spin saturated systems the direct contributions vanish for all spin–isospin channels except the scalar–isoscalar ($S = T = 0$). In nuclear matter the direct term is given by a δ function

$$\phi^{000}_{\mathrm{direct}}(q) = \tfrac{1}{6}(\pi k_{\mathrm{F}}^3/(2\pi)^3)\delta(q) \tag{7}$$

and the exchange contributions are evaluated as

$$\phi^{SMT}_{\mathrm{exch}}(q) = -\frac{1}{(2\pi)^3}\frac{3\pi}{8}\left\{\frac{4}{3}q^2 - \frac{q^3}{k_{\mathrm{F}}} + \frac{q^5}{12k_{\mathrm{F}}^3}\right\} \tag{8}$$

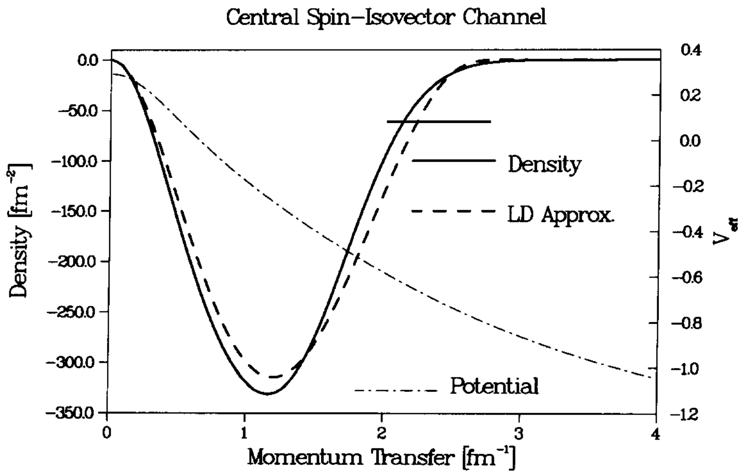


Fig. 5. The two-body density $\phi(q)$ in the central spin isovector channel ($S = T = 1$, M averaged) used to calculate the interaction energy according Eq. (5). Results are presented for the nucleus ^{16}O as obtained in the LDA approach (dashed line) and the exact calculation (solid line). Also given is a local presentation of a realistic G-matrix as a function of the momentum transfer as introduced in [15] (dashed-dotted line). Further details in the text.

for momenta $q \leq 2k_F$ and zero for larger momentum transfers. As the LDA implies an average over nuclear matter energy densities, the LDA result for the two-body density $\phi(q)$ will be a superposition of nuclear matter functions at various densities. This can be seen from Fig. 5, where the dashed line exhibits the LDA approach for $\phi(q)$ in the central spin – isovector channel ($S = T = 1$, averaged over the spin projection M). For this channel one does not obtain a direct contribution for ^{16}O and therefore LDA yields a superposition of functions of the form of Eq. (8) for various densities. The corresponding two-body density approach for the direct calculation of ^{16}O is given by the solid line and we observe that the LDA obviously yields a very good approximation for the two-body density.

The situation is different in the scalar–isoscalar channel ($S = T = 0$) (see Fig. 6). In this channel one also obtains a direct contribution, which in LDA as in nuclear matter (see Eq. (7)) is given by a δ-function and not made visible in the dahsed line of Fig. 6. As the single-particle wave functions of a finite system are not eigenstates of the momentum operator, the direct term in $\phi(q)$ calculated directly for the finite system shows a shape different from a δ function. This can be seen from Fig. 6, in which the two-body density calculated for the oscillator model of ^{16}O (solid line) exhibits quite a different shape from the LDA.

If the two-body interaction in this spin–isospin channel would be of zero range, i.e. the function $V(q)$ would be a constant as a function of momentum transfer, this difference in the $\phi(q)$ would not show up in the calculated energy, since the integral in Eq. (5) would yield the same result for the LDA as for the exact calculation. Realistic interactions and also local representations of the G-matrix [15] exhibit a finite range or a dependence on momentum transfer. A typical function $V_{000}(q)$ is given by the dashed-dotted line (referring to the right scale) in Fig. 6. One can see that such a function is more attractive at $q = 0$, where the LDA yields the largest contribution, than for

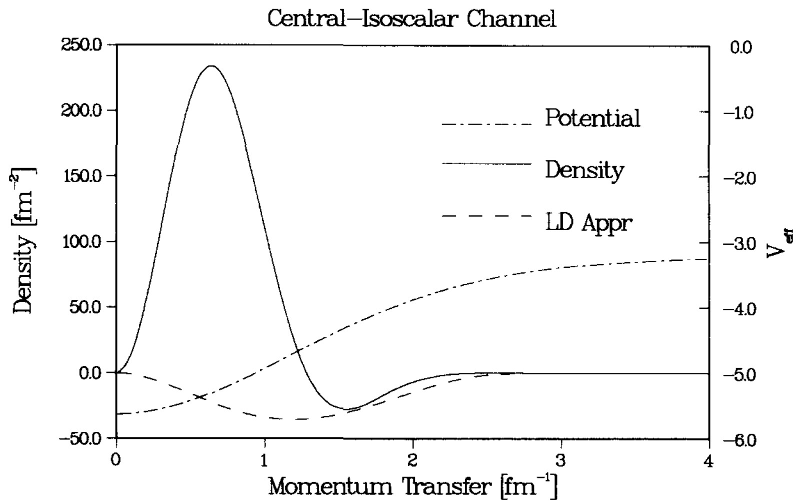


Fig. 6. The two-body density in the scalar–isoscalar channel ($S = T = 0$) used to calculate the interaction energy according Eq. (5). Results are presented for the nucleus ^{16}O as obtained in the LDA approach (dashed line without the direct contribution $\delta(q)$) and the exact calculation (solid line). Further details see Fig. 5.

those momentum transfers, for which the maximal contribution is obtained in the exact calculation. This explains why LDA overestimates the binding energy.

If the nucleus would be compressed, the maximum of $\phi(q)$ for the solid line in Fig. 6 would be shifted to larger momentum transfers, an effect which tends to reduce the calculated energy. On the other hand the LDA still obtains the largest contribution at $q = 0$. Using the language of momentum representation this explains the difference in the calculated compression moduli shown in Fig. 4.

Our investigation demonstrates that the LDA discussed in Section 2 is valid only for an effective interaction of zero range. For realistic interactions of finite range the LDA underestimates the attraction from the direct interaction in the scalar–isoscalar channel.

3. Conclusions

At first sight nuclear structure calculations for infinite nuclear matter and finite nuclei seem to exhibit very similar features (Coester band, effects of relativistic corrections, etc.). A more detailed analysis, however, shows that employing realistic NN interactions substantial differences are obtained in the equation of state for nuclear matter and a finite nucleus. This implies that predictions on the nuclear equation of state obtained from heavy ion experiments cannot directly be compared to those to be used in studies of astrophysical objects.

A simple local density approximation for the calculation of the finite nucleus yields results which overestimate the binding energy and yield a compression modulus too small compared with the exact calculation.

The origin of this discrepancy is traced back to the finite range of a realistic interaction or the local representation of the G-matrix. The LDA seems to be very reliable if effective interactions of zero range (like the Skyrme interactions) are employed. It turns out that binding energies of finite nuclei are much more sensitive to the range of the effective interaction than the saturation point of nuclear matter. Since the DBHF approach is successful in evaluating the saturation point of nuclear matter but fails to predict the ground-state properties of finite nuclei (see Fig. 3), one may argue that a density dependence of the range of the interaction or a change of meson masses in the medium could cure this problem [16].

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