

# **TRIUMF Summer Institute**

## **Nuclear Reactions: applications and examples**

**(<http://departamento.us.es/famn/tsi08/>)**

**Antonio M. Moro**

***Universidad de Sevilla***

## *My plan for the talks...*

- Lecture 1: Elastic scattering
  - ✦ Phenomenology: Rutherford, Fresnel and Fraunhofer scattering
  - ✦ The optical model
- Lecture 2: Inelastic scattering: coupled-channels (CC) method
  - ✦ Collective model
  - ✦ Cluster model
- Lecture 3: Transfer reactions: DWBA method
- Lecture 4: Reaction involving continuum
  - ✦ Breakup: CDCC method
  - ✦ Spectroscopy to unbound states: the *transfer to the continuum* method

## Lecture 1: Elastic scattering: optical model calculations

- ❖ Direct versus compound reactions
- ❖ Elastic scattering
- ❖ Effective interaction
- ❖ Rutherford scattering
- ❖ Fraunhofer scattering
- ❖ Fresnel scattering
- ❖ The optical model
- ❖ The scattering amplitude
- ❖ Halo versus normal nuclei
- ❖ Fresco code
- ❖ OM with Fresco
- ❖ Optical potential
- ❖ Input example for  $^4\text{Ni} + ^{58}\text{Ni}$
- ❖ Xfresco interface
- ❖ Dynamical effects
- ❖ angular distributions
- ❖ S-matrix
- ❖ Proposed homework:  $^8\text{Li} + ^{208}\text{Pb}$
- ❖  $^8\text{Li} + ^{208}\text{Pb}$  exercise

# Lecture 1: Elastic scattering: optical model calculations

## *Direct versus compound reactions*

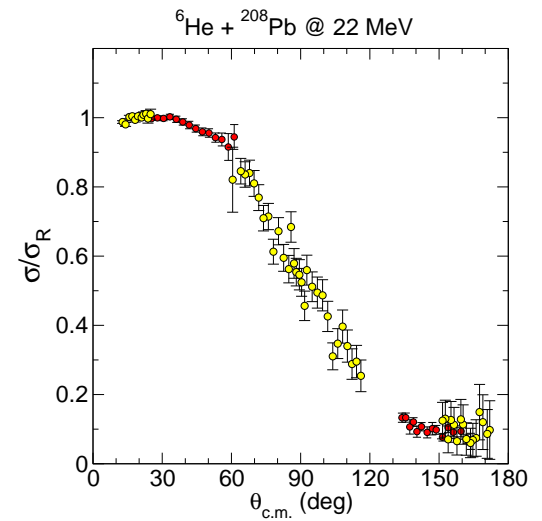
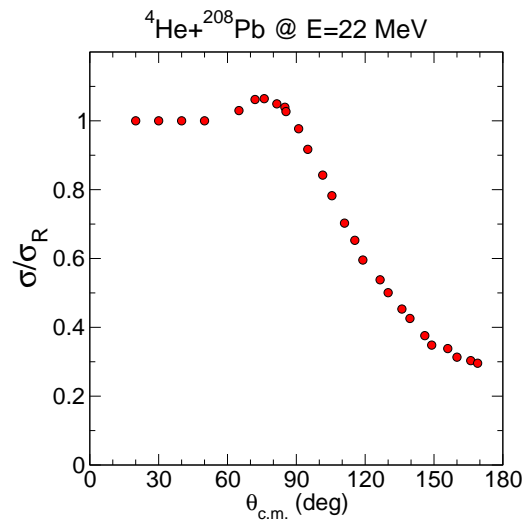
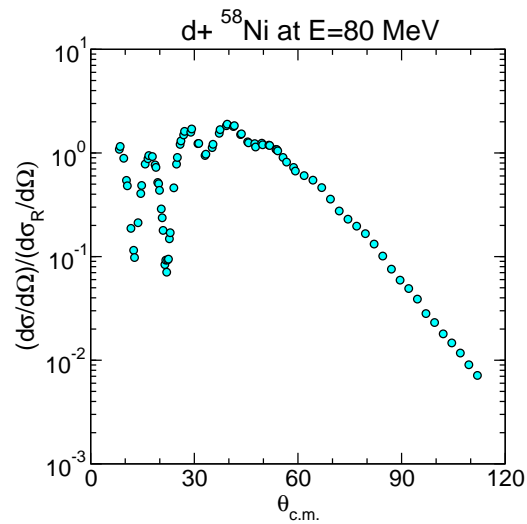
**DIRECT:** elastic, inelastic, transfer, . . .

- only a few modes (degrees of freedom) involved
- small momentum transfer
- angular distribution asymmetric about  $\pi/2$  (peaked forward)

**COMPOUND:** complete, incomplete fusion.

- many degrees of freedom involved
- large amount of momentum transfer
- "lose of memory"  $\Rightarrow$  almost symmetric distributions forward/backward

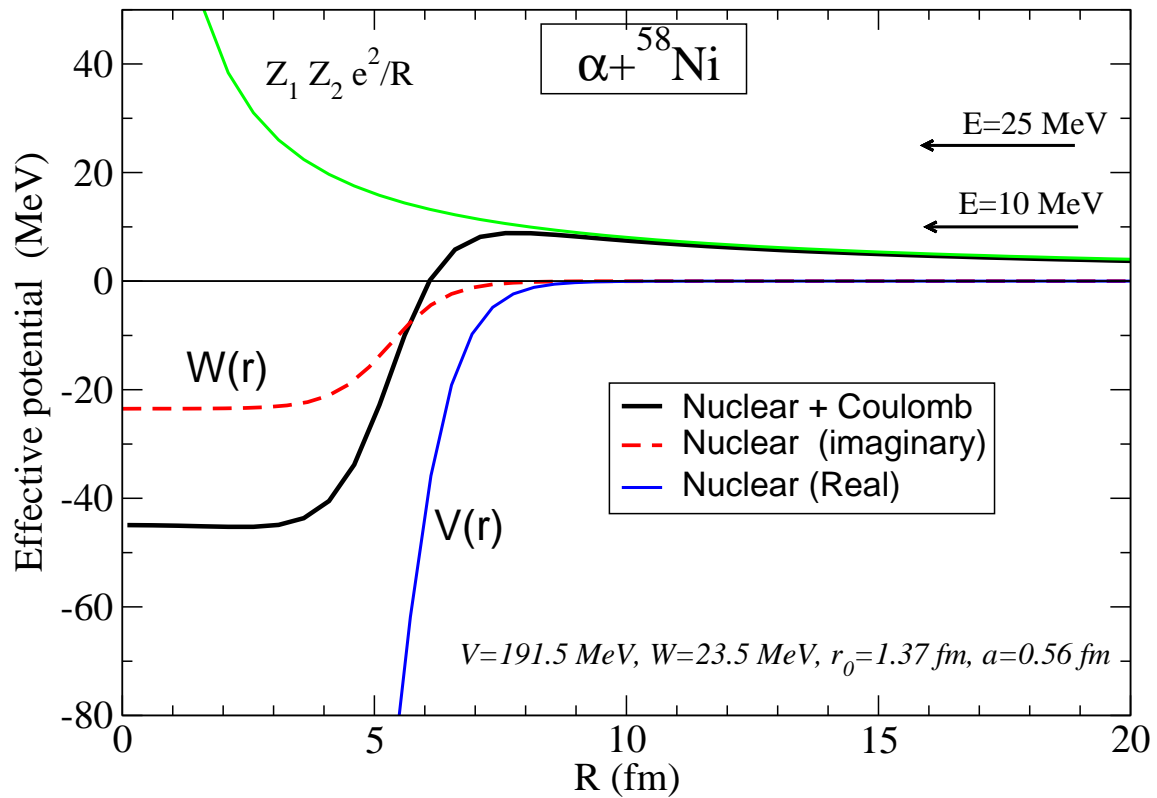
# Elastic scattering



What can we learn from an optical model analysis of the elastic cross section?

# Elastic scattering: phenomenology

## EFFECTIVE PROJECTILE-TARGET INTERACTION:



## Elastic scattering: phenomenology

➡ Depending on the bombarding energy  $E$  and the charges of the interacting nuclei, we observe different types of elastic scattering.

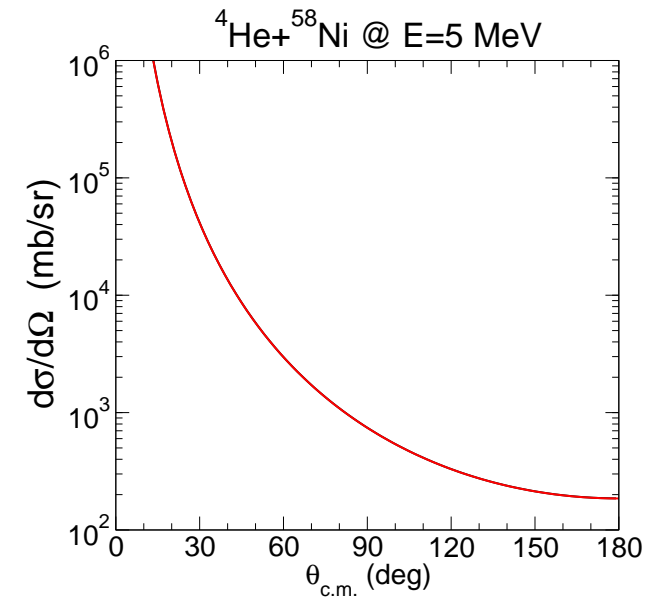
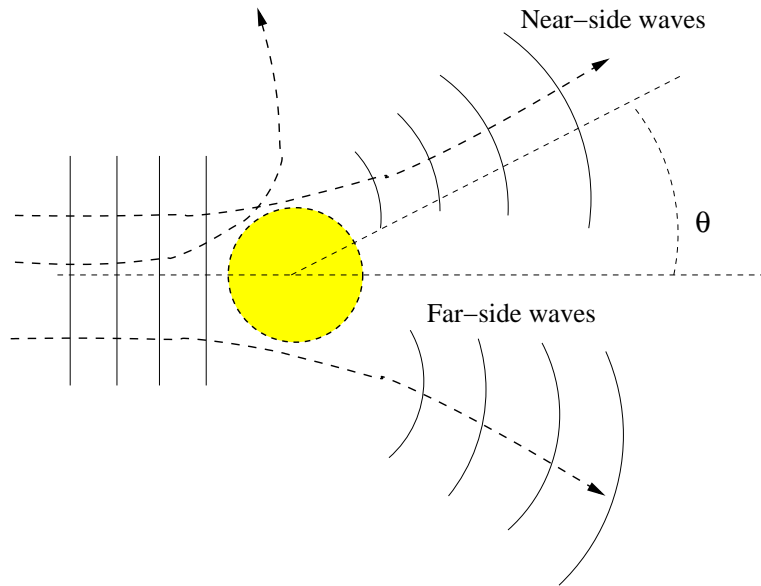
➡ This can be characterized in terms of the Coulomb (or Sommerfeld) parameter:

$$\eta = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 \hbar v}$$

- $E$  well above the Coulomb barrier ( $\eta \lesssim 1$ )  $\Rightarrow$  Fraunhofer scattering
- $E$  around the Coulomb barrier ( $\eta \gg 1$ )  $\Rightarrow$  Fresnel scattering
- $E$  well below the Coulomb barrier ( $\eta \gg \gg 1$ )  $\Rightarrow$  Rutherford scattering

# Elastic scattering: phenomenology

## RUTHERFORD SCATTERING



- Purely Coulomb potential ( $\eta \gg 1$ )
- Bombarding energy well below the Coulomb barrier
- Obeys Rutherford law:

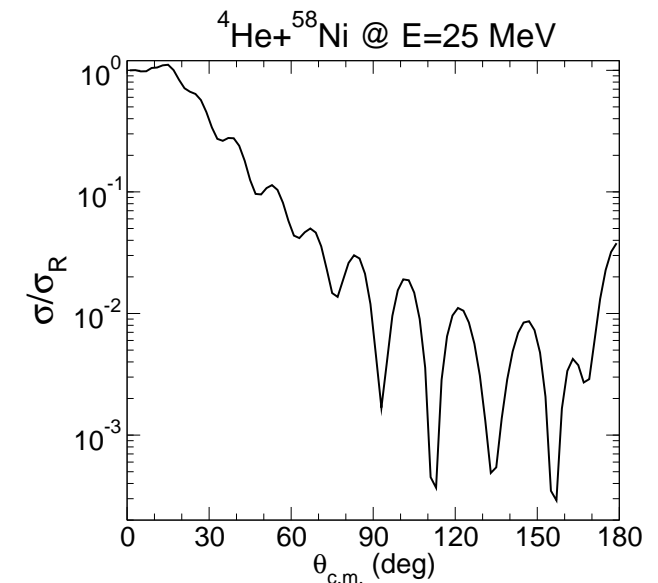
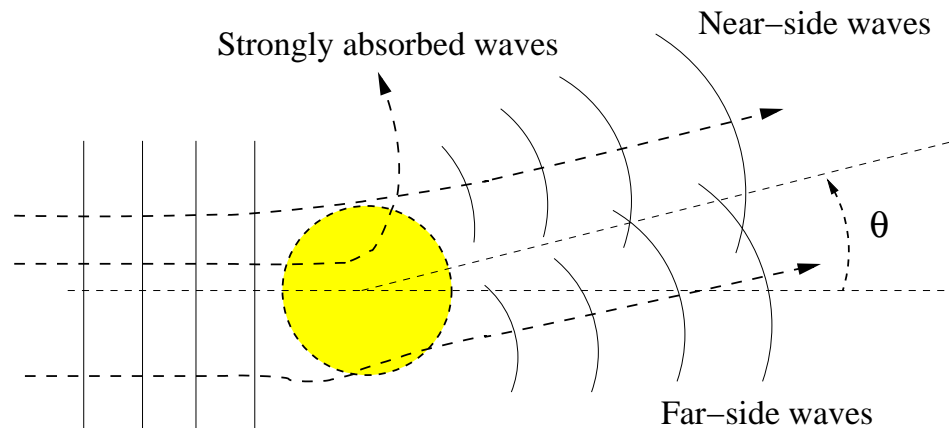
$$\frac{d\sigma}{d\Omega} = \frac{zZe^2}{4E} \frac{1}{\sin^4(\theta/2)}$$

.



# Elastic scattering: phenomenology

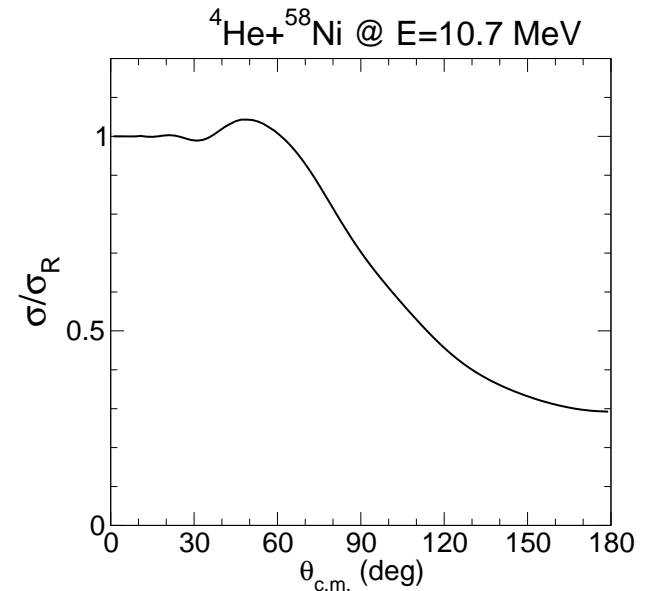
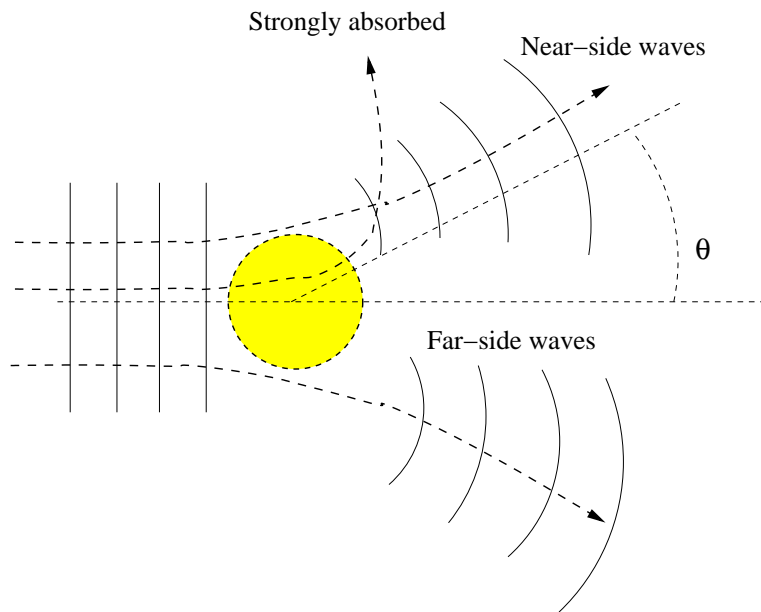
## FRAUNHOFER SCATTERING:



- Bombarding energy well above Coulomb barrier
- Coulomb weak ( $\eta \lesssim 1$ )
- Nearside/farside interference pattern (diffractive)

# Elastic scattering: phenomenology

## FRESNEL SCATTERING:



- Bombarding energy around or near the Coulomb barrier
- Coulomb strong ( $\eta \gg 1$ )
- 'Illuminated' region  $\Rightarrow$  interference pattern (near-side/far-side)
- 'Shadow' region  $\Rightarrow$  strong absorption

## Elastic scattering: optical model

How does one describe the motion of a particle in quantum mechanics?

- **Hamiltonian:**  $H = T_R + U(R)$

$U(R)$ : optical model  $\Rightarrow$  effective projectile-target interaction

- **Schrodinger equation:**  $[H - E]\Psi(\mathbf{R}) = 0$
- **Partial wave expansion of the model wavefunction:**

$$\Psi(\mathbf{R}) = \sum_{LM} C^{LM} \frac{f^L(R)}{R} Y_{LM}(\hat{R})$$

- $f^L(R)$  obtained as solution of:

$$\left[ -\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} + \frac{\hbar^2 L(L+1)}{2\mu R^2} + U(R) - E \right] f^L(R) = 0$$

## Elastic scattering: optical model

Numerical procedure:

1. Fix a *matching radius*,  $R_m$ , such that  $V_{\text{nuc}}(R_m) \ll$
2. Integrate  $f(R)$  from  $R = 0$  up to  $R_m$ , starting with the condition:

$$\lim_{R \rightarrow 0} f^L(R) = 0$$

3. At  $R = R_m$  impose the boundary condition:

$$f^L(R) \rightarrow I_L(R) - S_L O_L(R)$$

☞  $S_L$  = scattering matrix

☞  $I_L$  and  $O_L$  are the so called incoming and outgoing waves:

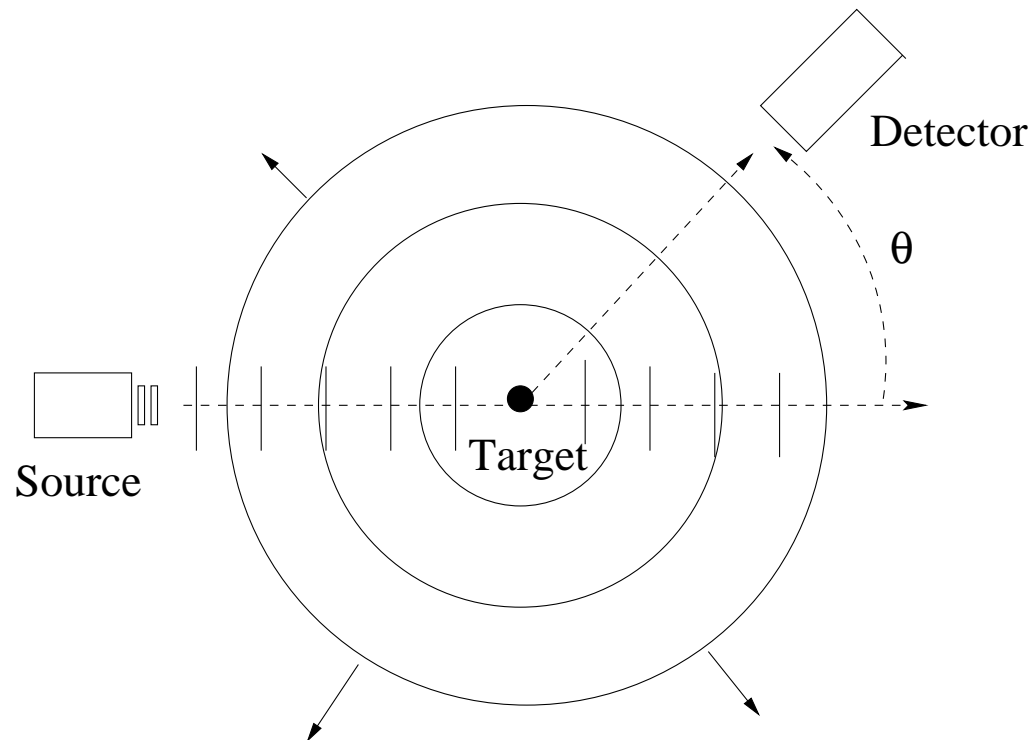
$$\begin{aligned} I_L(R) &= \frac{1}{\sqrt{v}}(KR) h_L^*(KR) \propto e^{-i(KR - \eta \log 2KR)} \\ O_L(R) &= \frac{1}{\sqrt{v}}(KR) h_L(KR) \propto e^{i(KR - \eta \log 2KR)} \end{aligned}$$

## The S-matrix

- $S_L$  = coefficient of the outgoing wave for partial wave  $L$ .
- Phase-shifts:  $S_L = e^{2i\delta_L}$
- $U(R) = 0 \Rightarrow$  No scattering  $\Rightarrow S_L = 1 \Rightarrow \delta_L = 0$
- $U$  real  $\Rightarrow |S_L| = 1 \Rightarrow \delta_L$  real
- $U$  complex  $\Rightarrow |S_L| < 1 \Rightarrow \delta_L$  complex
- For  $L \gg \Rightarrow S_L \rightarrow 1$

## Elastic scattering: the scattering amplitude

Which one of the many solutions of Schrödinger equation is the one that correspond to a scattering experiment?



$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = e^{i\mathbf{K}_i \cdot \mathbf{R}} + \chi_{\mathbf{K}_i}^{(+)}(\mathbf{R})$$

## Elastic scattering: the scattering amplitude

- Scattering amplitude:  $A(\theta)$

$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = e^{i\mathbf{K}_i \cdot \mathbf{R}} + \chi_{\mathbf{K}_i}^{(+)}(\mathbf{R}) \rightarrow e^{i\mathbf{K}_i \cdot \mathbf{R}} + A(\theta) \frac{e^{iK_i R}}{R}$$

- Partial wave decomposition:

$$\Psi_{\mathbf{K}_i}(\mathbf{R}) = \frac{1}{R} \sum_{LM} C^{LM} f^L(R) Y_{LM}(\hat{R}) \rightarrow \frac{1}{R} \sum_{LM} C^{LM} [I_L(R) - S_L O_L(R)] Y_{LM}(\hat{R})$$

- Incident plane wave:

$$\begin{aligned} e^{i\mathbf{K}_i \cdot \mathbf{R}} &= \sum_{LM} 4\pi Y_{LM}^*(\hat{K}_i) i^L Y_{LM}(\hat{R}) j_L(KR) \\ &= \sum_{LM} \frac{2\pi i \sqrt{v}}{KR} Y_{LM}^*(\hat{K}_i) i^L Y_{LM}(\hat{R}) [I_L(R) - O_L(R)] \end{aligned}$$

- Outgoing spherical waves:

$$\chi_{\mathbf{K}_i}^{(+)}(\mathbf{R}) \rightarrow \sum_{LM} \frac{2\pi i \sqrt{v}}{KR} Y_{LM}^*(\hat{K}_i) (1 - S^L) Y_{LM}(\hat{R}) O_L(R)$$

# Scattering amplitude and cross sections

- Scattering amplitude:

- ❖ Nuclear potential alone:

$$A(\theta) = \frac{i}{2K} \sum_L (2L + 1) P_L(\cos \theta) (1 - S^L)$$

- ❖ Nuclear+Coulomb:  $A(\theta) = A_C(\theta) + A'(\theta)$

$$A_C(\theta) = \frac{i}{2K} \sum_L (2L + 1) (1 - e^{2i\sigma_L}) P_L(\cos \theta)$$

$$A'(\theta) = \frac{i}{2K} \sum_L (2L + 1) e^{2i\sigma_L} (1 - S^L) P_L(\cos \theta)$$

- Differential cross section:

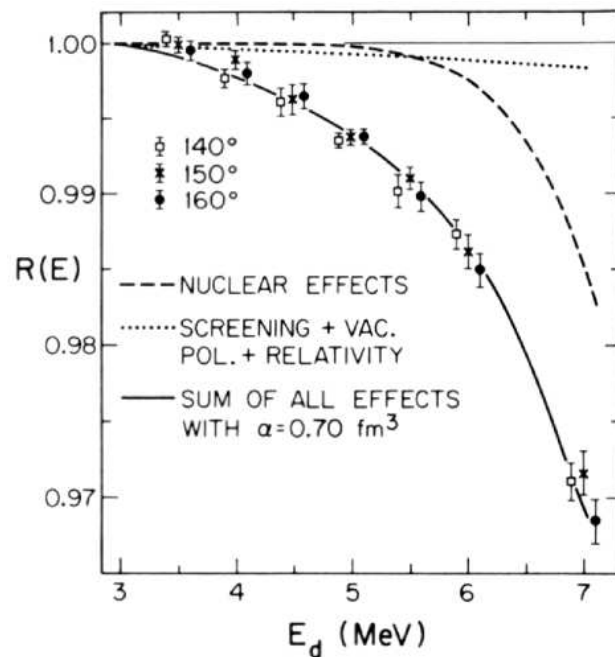
$$\frac{d\sigma}{d\Omega} = |A(\theta)|^2$$



# Extracting structure information from elastic scattering measurements

Eg: deuteron polarizability from  $d+^{208}\text{Pb}$ :

- Deuteron polarizability:  $\mathbf{P} = \alpha \mathbf{E}$
- For  $E < V_b$ , the main deviation from Rutherford scattering comes from dipole polarizability.
- In the adiabatic limit ( $E_x \gg$ ):  $V_{\text{dip}} = -\alpha \frac{Z_1 Z_2 e^2}{2R^4}$



Rodning, Knutson, Lynch and Tsang,  
PRL49, 909 (1982)  
 $\alpha = 0.70 \pm 0.05 \text{ fm}^3$

## Halo and Borromean nuclei: the ${}^6\text{He}$ case

- Radioactive:



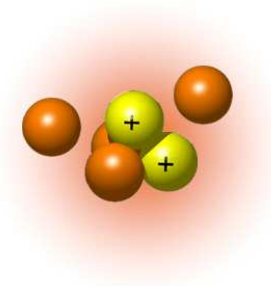
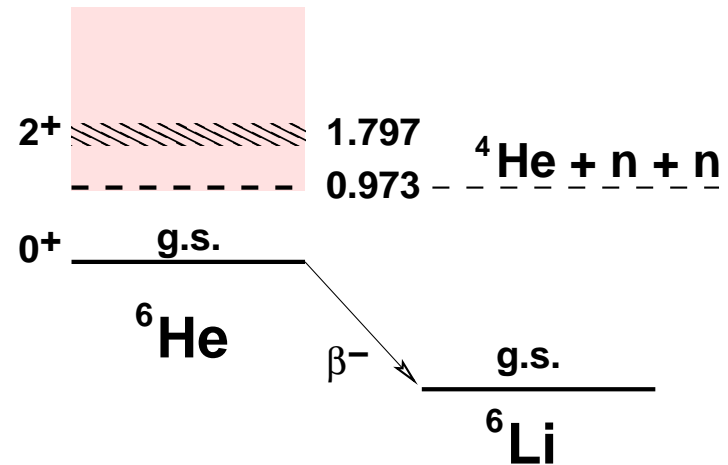
- Weakly bound:

$$\epsilon_b = -0.973 \text{ MeV}$$

- Neutron halo

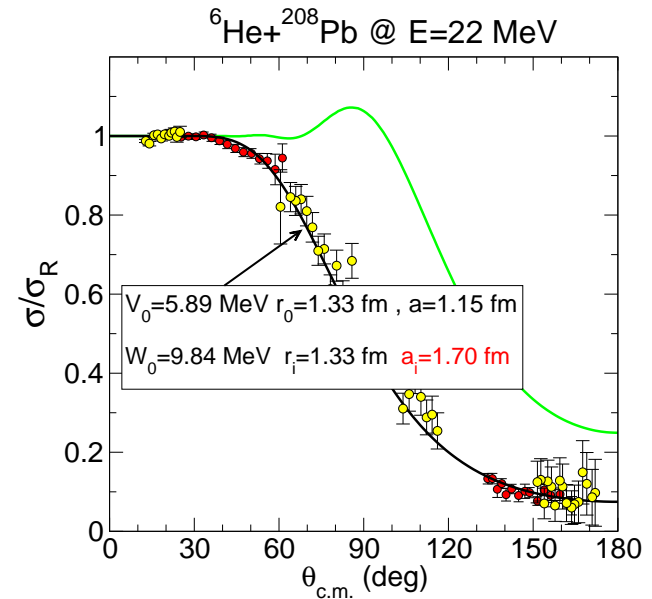
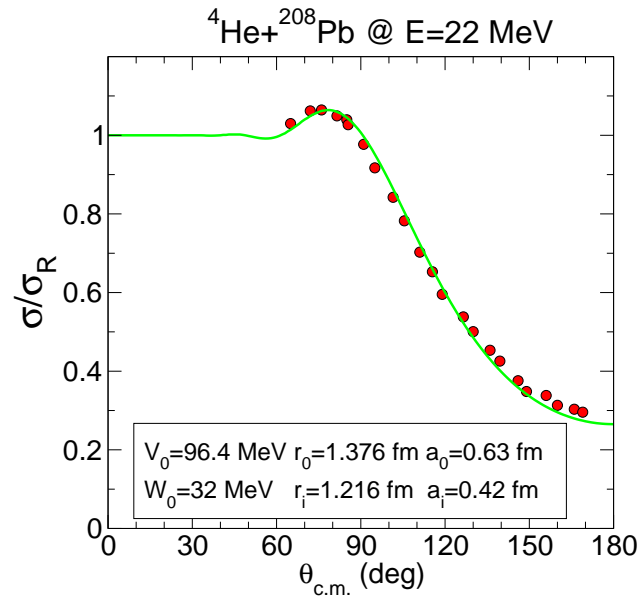
- Borromean system:  
n-n and  $\alpha$ -n unbound

- $\sim 3$  body system:  
 $\alpha$  almost inert



## Normal versus halo nuclei

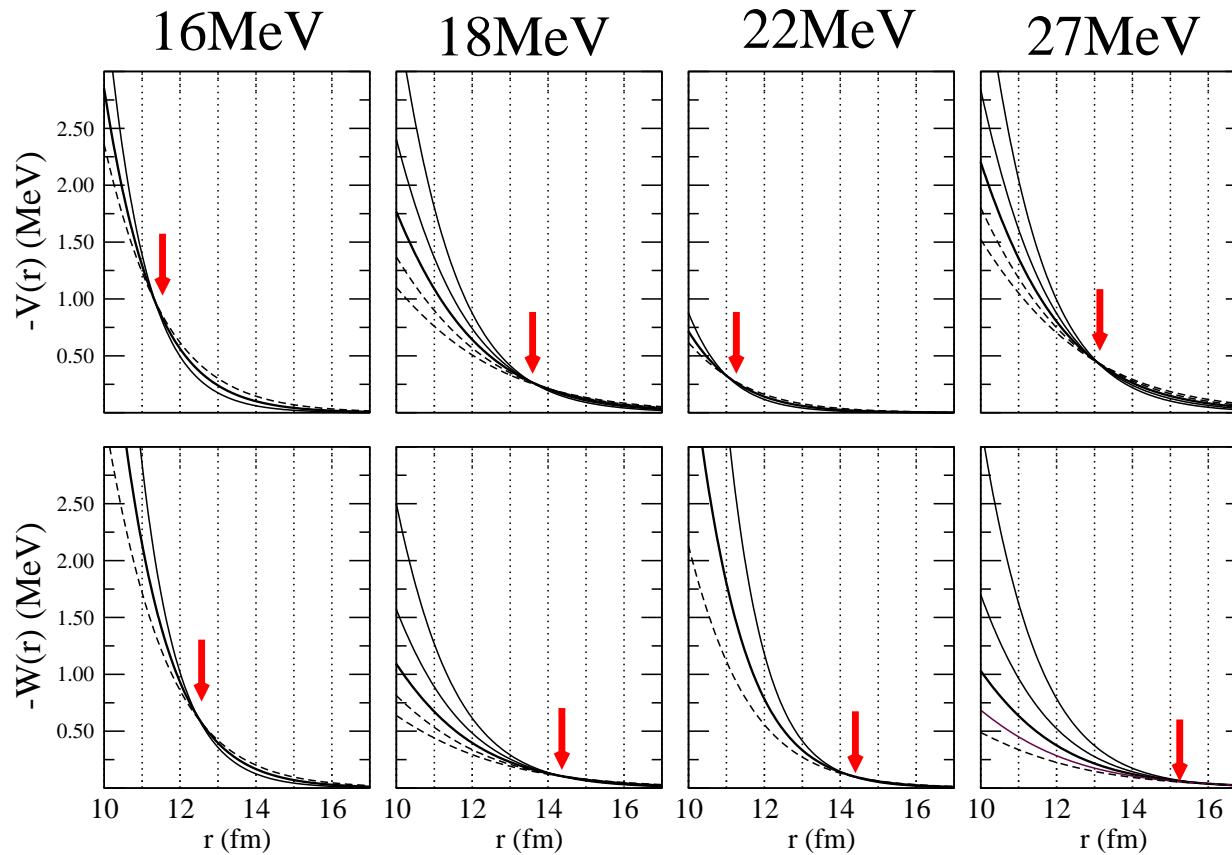
How does the halo structure affect the elastic scattering?



- $^4\text{He} + ^{208}\text{Pb}$  shows typical Fresnel pattern → *strong absorption*
- $^6\text{He} + ^{208}\text{Pb}$  shows a prominent reduction in the elastic cross section due to the flux going to other channels (mainly break-up)
- $^6\text{He} + ^{208}\text{Pb}$  requires a large imaginary diffuseness → *long-range absorption*

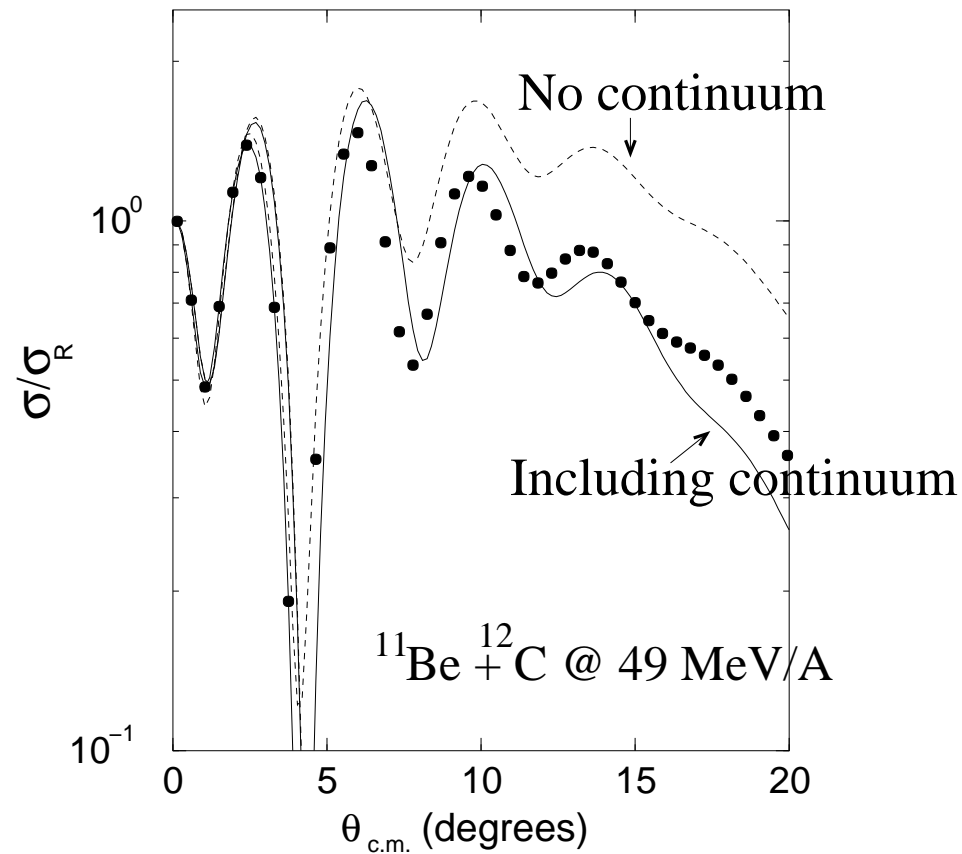
# Optical model calculations for ${}^6\text{He} + {}^{208}\text{Pb}$

## RADIUS OF SENSITIVITY OF $V(r)$ AND $W(r)$



*Imaginary part  $\Rightarrow$  long range compared to strong absorption radius*

## Normal versus halo nuclei: Fraunhofer



☞ In Fraunhofer scattering the presence of the continuum produces a reduction of the elastic cross section

- What is FRESKO?

Program developed by Ian Thompson since 1983, to perform coupled-reaction channels calculations in nuclear physics.

- Some general features:

- ❖ Multi-platform (Windows, Linux, Unix, VAX)
- ❖ Treats many direct reaction models: elastic scattering (optical model), transfer, inelastic excitation to bound and unbound states, etc
- ❖ Can be run in text mode and graphical mode (XFRESKO interface)
- ❖ FRESKO and XFRESKO can be freely downloaded at <http://www.fresco.org.uk/>
- ❖ SFRESKO: Extension of Fresco, to provide  $\chi^2$  searches of potential and coupling parameters.

## Essential ingredients of an OM calculation:

- **Physical:**

- Identify projectile and target (mass, spin, etc)
- Incident energy
- Parametrization of the optical potential

- **Numerical:**

- Radial step for numerical integration (HCM in fresco)
- Maximum radius  $R$  for integration (RMATCH)
- Maximum angular momentum  $L$ . (JTMAX)

RMATCH and JTMAX are linked by:  $kR_g (1 - 2\eta/kR_g) \approx L_g + 1/2$   
( $L_g$ =grazing angular momentum)

## Elastic scattering: optical model

Effective potential:  $U(R) = U_{\text{nuc}}(R) + U_{\text{coul}}(R)$

- Coulomb potential: charge sphere distribution

$$U_c(R) = \begin{cases} \frac{Z_1 Z_2 e^2}{2R_c} \left( 3 - \frac{R^2}{R_c^2} \right) & \text{if } R \leq R_c \\ \frac{Z_1 Z_2 e^2}{R} & \text{if } R \geq R_c \end{cases}$$

- Nuclear potential (complex): Woods-Saxon parametrization

$$U_{\text{nuc}}(R) = V(r) + iW(r) = -\frac{V_0}{1 + \exp\left(\frac{R-R_0}{a_0}\right)} - i \frac{W_0}{1 + \exp\left(\frac{R-R_i}{a_i}\right)}$$

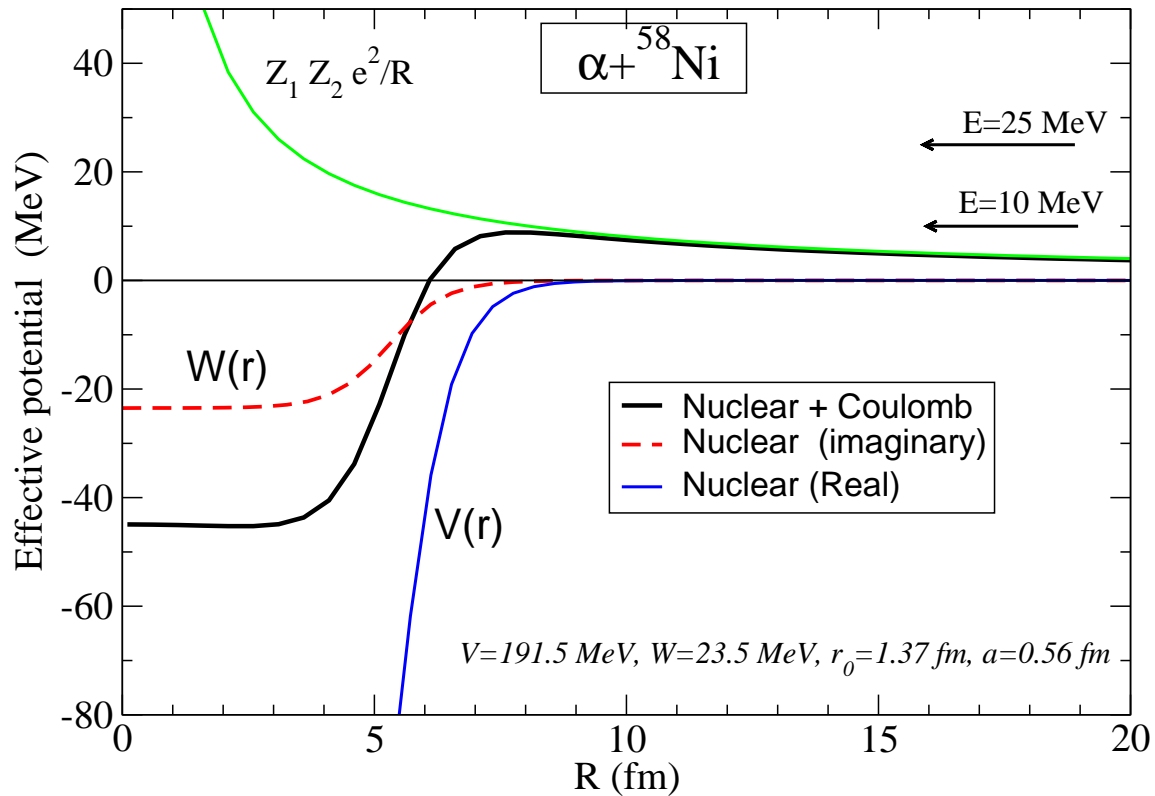
Typically:  $R_0 = r_0(A_p^{1/3} + A_t^{1/3})$

- ❖  $r_0$ =reduced radius ( $r_0 \sim 1.1 - 1.4$  fm)
- ❖  $A_p, A_t$ : projectile, target masses (amu)



## Elastic scattering: effective potential

Effective potential:  $U(R) = U_{nuc}(R) + U_{coul}(R)$



## OM example: ${}^4\text{He} + {}^{58}\text{Ni}$

### Input example: 4he58ni\_e10.in

```
4he58ni_e10.in: 4He + 58Ni elastic scattering Ecm=10.0 MeV
NAMELIST
&FRESCO hcm=0.1 rmatch=25.0 jtmax=30
        thmin=1.0 thmax=180.0 thinc=2.0
        smats=2 xstabl=1
        elab=10.7 /

&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28 nex=1 /
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
&partition /

&POT kp=1 at=58 rc=1.4 /
&POT kp=1 type=1
        p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /
&pot /

&overlap /
&coupling /
```

# Elastic scattering example

## General variables

```
&FRESKO hcm=0.1 rmatch=25.0 jtmax=30  
        thmin=1.00 thmax=180.00 thinc=2.00  
        smats=2 xstabl=1  
        elab=10.7 /
```

## Mass partitions & states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28  
          nex=1 /  
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /  
&partition /
```

## Potentials

```
&POT kp=1 itt=F at=58 rc=1.4 /  
&POT kp=1 type=1  
      p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /  
&pot /
```

## Elastic scattering example

### Essential input variables: FRESKO namelist

```
&FRESKO hcm=0.1 rmatch=25.0 jtmax=30  
      thmin=1.00 thmax=180.00 thinc=2.00  
      smats=2 xstabl=1  
      elab=10.7 /
```

- `hcm`: step for integration of radial equations.
- `rmatch`: matching radius (for  $R > R_{MATCH}$  asymptotic behaviour is assumed)
- `elab`: laboratory energy
- `jtmax`: maximum total angular momentum (projectile+target+relative)
- `smats`: trace variable  
`smats=2` → print elastic S-matrix
- `xstabl`: trace variable  
`xstabl=1` → print cross sections

# Elastic scattering with Fresco

## Essential input variables: partitions and states

```
&PARTITION namep='ALPHA' massp=4 zp=2 namet='58Ni' masst=58 zt=28  
          nex=1 /
```

- `namep` / `namet`: projectile / target name
- `massp` / `masst`: projectile / target mass (amu)
- `zp` / `zt`: projectile / target charge
- `nex`: number of (pairs) of states in this partition

```
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- `jp` / `jt`: projectile / target spins
- `bandp` / `bandt`: projectile / target parities ( $\pm 1$ )
- `cpot`: index of potential for this pair of states.

## Elastic scattering with fresco

```
&POT kp=1 type=0 ap=0 at=58 rc=1.4 /  
&POT kp=1 type=1 shape=0  
      p1=191.5 p2=1.37 p3=0.56 p4=23.5 p5=1.37 p6=0.56 /  
&pot /
```

- **kp**: index to identify this potential
- **ap**, **at**: projectile and target mass, for conversion from reduced to physical radii:  
$$R = r(ap^{1/3} + at^{1/3})$$
- **type**, **shape**: potential category and shape:  $\Rightarrow$ 
  - ❖ **type=0**: Coulomb potential  
**shape=0**: uniform charge sphere
  - ❖ **type=1**: volume nuclear potential  
**shape=0**: Woods-Saxon shape
- **rc**: reduced radius for charge distribution
- **p1**, **p2**, **p3**:  $V_0, r_0, a_0$  (real part)
- **p4**, **p5**, **p6**:  $W_0, r_i, a_i$  (imaginary part)

## General variables:

File Edit Run Options About

Integration Trace CC, iterations... Partitions Potentials Overlaps Couplings

**Integration**

Radial step: HCM 0.1

Matching radius: RMATCH 25.0

Intervals for N-L kernels (RINTP): 0.5

Step size for NL range: HNL 0

Center for NL range: CENTRE 0

NL range: RNL 0

Step size for 2N distance: HNN 0

Min. radius for 2N distance: RMIN 0

Max. radius for 2N distance: RNN 0

State radius for s.p. states: RSP 0

☐ Use Coupled Coulomb w.f. CCWF parameters ...

**J interval**

JMIN (=J1): 0

JMAX (=J5): 30

☐ Use absend 0

☐ Include only incoming channel for J<JMIN

J intervals ...

**Near-side / Far-side analysis**

Elastic channel Usual cross sections

**Angular range**

THMIN 1.00

THMAX 180.00

THINC 2.00

**Incoming channel**

Energy intervals: ELAB 10.7 0 0 0 NLAB: 0 0 0

Incoming plane waves are present in partition (PEL) 1 with excitation pair (EXL) 1

Specified energies refer to (LIN) projectile for partition (LAB) 1 in excitation pair (LEX) 1

OK

# Optical model with XFRESKO

## Partitions & states:

File Edit Run Options About

Integration Trace CC, iterations... **Partitions** Potentials Overlaps Couplings

Projectile  
Nucleus A Z  
ALPHA 4 2

Target  
Nucleus A Z  
58Ni 58 28

Q-value: 0.0000 ☐ PWF  
Readstates:  Do not print xsec  
☐ for this partition [NEX<0]

Add  
Replace  
Insert  
Delete

Projectile	Mass	Z	Target	Mass	Z	Q value	PWF	xsec?	Readstates
ALPHA	4	2	58Ni	58	28	0.0000	F	T	

Excited states for selected partition

Index: 1 J Copy Band E K T

Projectile 0.0  + 0.0

Target 0.0  + 0.0

☐ FEXCH ☐ IGNORE

INFAM=0  0

OUTFAM=0

Optical potential [CPOT] : 1

Replace  
Insert after  
Add  
Delete

J proj.	Copy P.	Band P.	E proj.	K proj.	T proj.	cpot	J targ.	Copy T	Band T.	E targ.	K targ.	T targ.	EX
0.0	1	0.0			1	0.0		1	0.0				F

OK



# Optical model with XFRESKO

## Potentials:

File Edit Run Options About

Integration Trace CC, iterations... Partitions Potentials Overlaps Couplings

Pot. Index (kP): 1

☐ Add to previous

Potential Type: 1.-Central potential, volume

Shape: 0.-Woods-Saxon

Parameters:

p1 (Vo)	p2 (ro)	p3 (ao)
191.5	1.37	0.56
p4 (W)	p5 (ri)	p6 (ai)
23.5	1.37	0.56

p7: 0

☐ ITT

Add Insert Replace Delete

KP	Type	Shape	itt	p1-Vo	p2-r0	p3-a0	p4-W	p5-ri	p6-ai	p7	Add prev?
1	0	0	F	0	58	1.4	0				
1	1	0	F	191.5	1.37	0.56	23.5	1.37	0.56	0	F

Table of couplings:

Couple state: IB = 2

with state IA = 1

Multipolarity (k): 1

Strength (STR):

Add Insert Replace Delete

IB	IB-Desc	IA	IA-Desc	k	STR
----	---------	----	---------	---	-----

OK

## *Useful output information in OM calculations*

### Useful output files:

- Main output file (stdout)
- `fort.201` : Elastic scattering angular distribution
  - ❖ `thmax > 0`: relative to Rutherford.
  - ❖ `thmax < 0`: absolute units (mb/sr).
- `fort.7`: Elastic S-matrix (real part, imaginary part, angular momentum)
- `fort.56`: Fusion (absorption), reaction and inelastic cross section for each angular momentum

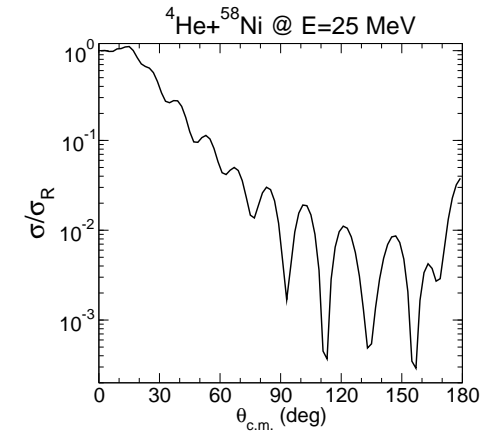
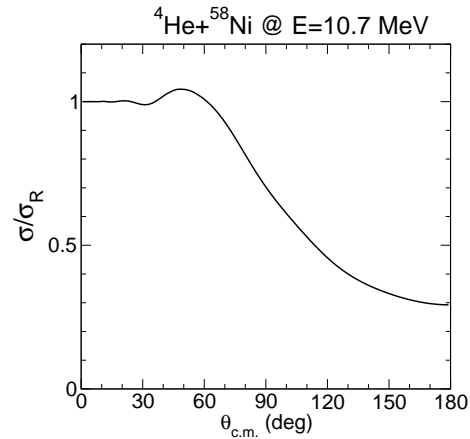
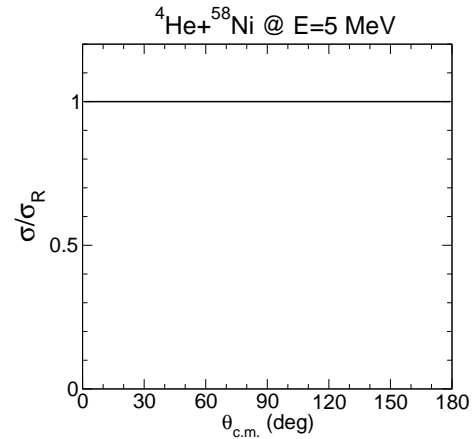
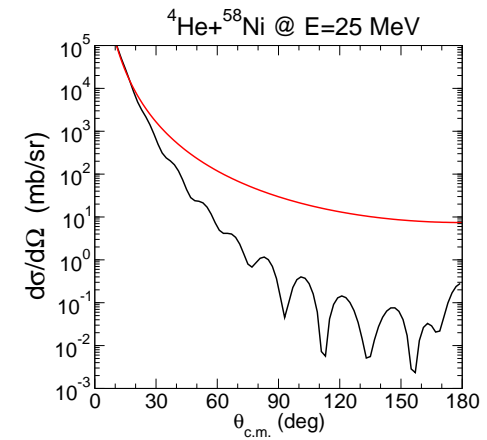
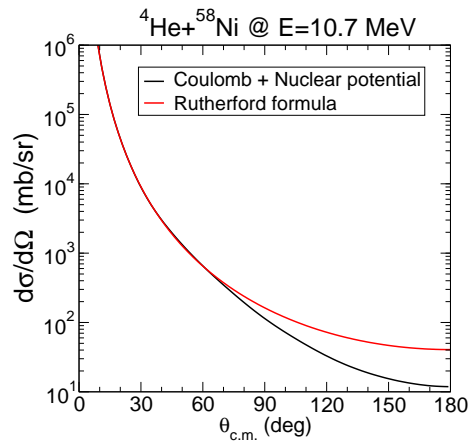
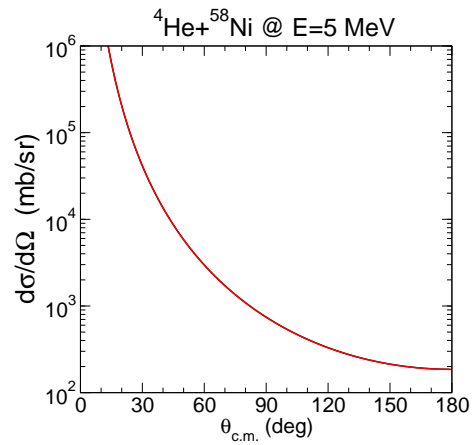
## Elastic scattering: optical model

**Dynamical effects:**  $^4\text{He} + ^{58}\text{Ni}$  at  $E=5, 10.7, 25$  and  $50$  MeV

$E_{\text{lab}}$ (MeV)	$\eta$	$k$ (fm <sup>-1</sup> )	$\bar{\lambda} = 1/k$ (fm)	$2a_0$ (fm)
5	7.95	0.920	1.087	17.2
10.7	5.62	1.34	0.746	8.06
25	3.55	2.06	0.485	3.44
50	2.51	2.91	0.343	1.69

- $\eta \gg 1$ : Rutherford scattering:  $\sigma(\theta) \propto 1/\sin^4(\theta/2)$
- $\eta \gg 1$ : Fresnel scattering (rainbow)
- $\eta \leq 1$ : Fraunhofer scattering (oscillatory behaviour):

# Elastic scattering: energy dependence



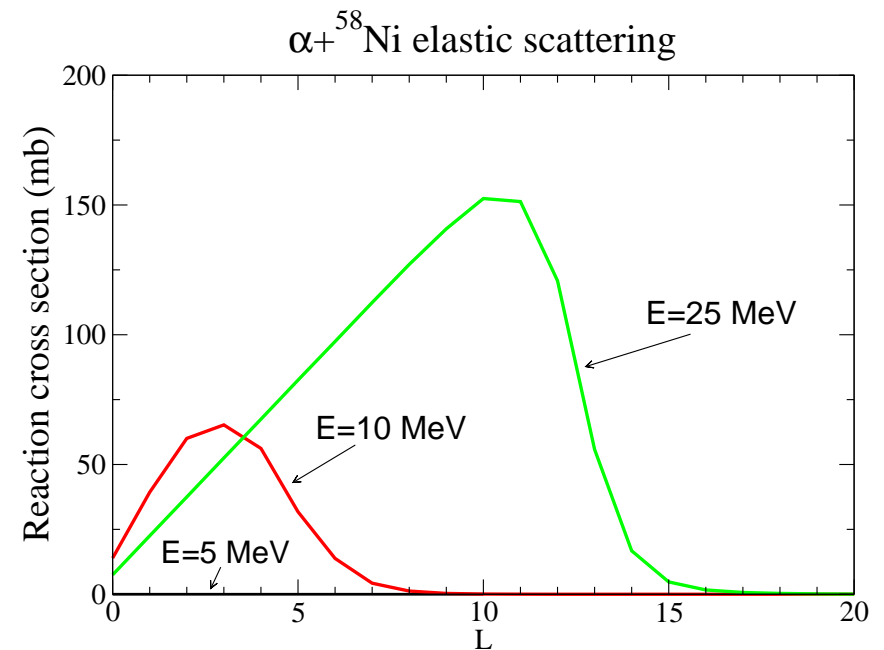
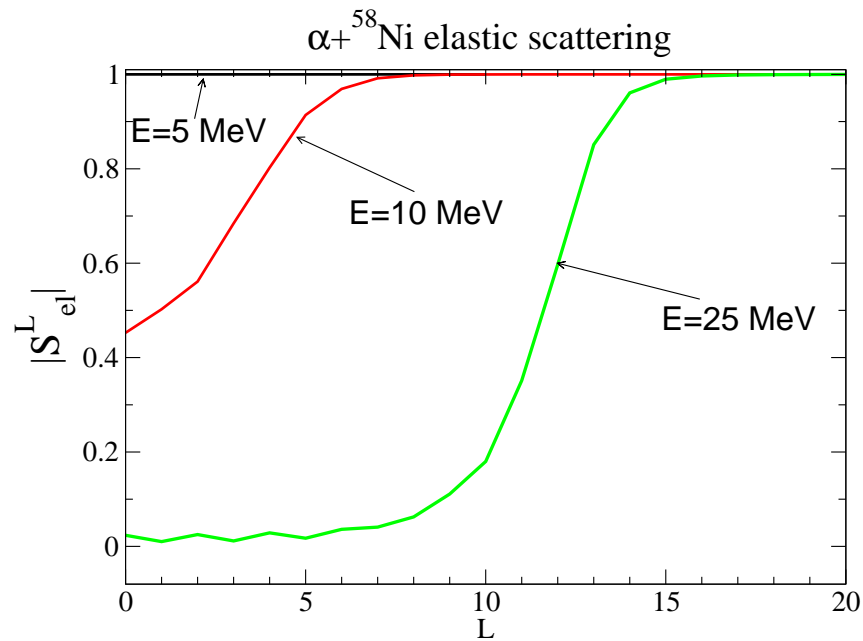
Rutherford scattering

Fresnel

Fraunhofer

## Elastic scattering: S-matrix elements

Elastic (nuclear) S-matrix (fort. 7):  $f_{el}^L(r) = I_L(r) - S_{el}^L O_L(r)$



$$kR_g (1 - 2\eta/kR_g) \approx L_g + 1/2$$

$\Rightarrow$  the number of partial waves required for convergence grows approximately as  $\sqrt{E}$

## Elastic scattering: exercise

### Proposed exercise:

- Calculate elastic angular distribution ( $d\sigma/d\Omega$ ) for  ${}^7\text{Li}+{}^{208}\text{Pb}$  OMP at 33 MeV.
- Elastic distribution and reaction cross section for  ${}^8\text{Li}+{}^{208}\text{Pb}$  at 33 MeV using the  ${}^7\text{Li}$  potential.
- Elastic distribution and reaction cross section for  ${}^8\text{Li}+{}^{208}\text{Pb}$  at 33 MeV using the  ${}^8\text{Li}$  potential.

System	$V_0$	$r_x$	$a_0$	$W_0$	$a_i$
${}^7\text{Li}+{}^{208}\text{Pb}$	15.4	1.3	0.65	13.2	0.65
${}^8\text{Li}+{}^{208}\text{Pb}$	15.4	1.3	0.65	58.4	0.70

$$r_C = 1.25 \text{ fm}, R_x = r_x(A_p^{1/3} + A_t^{1/3})$$

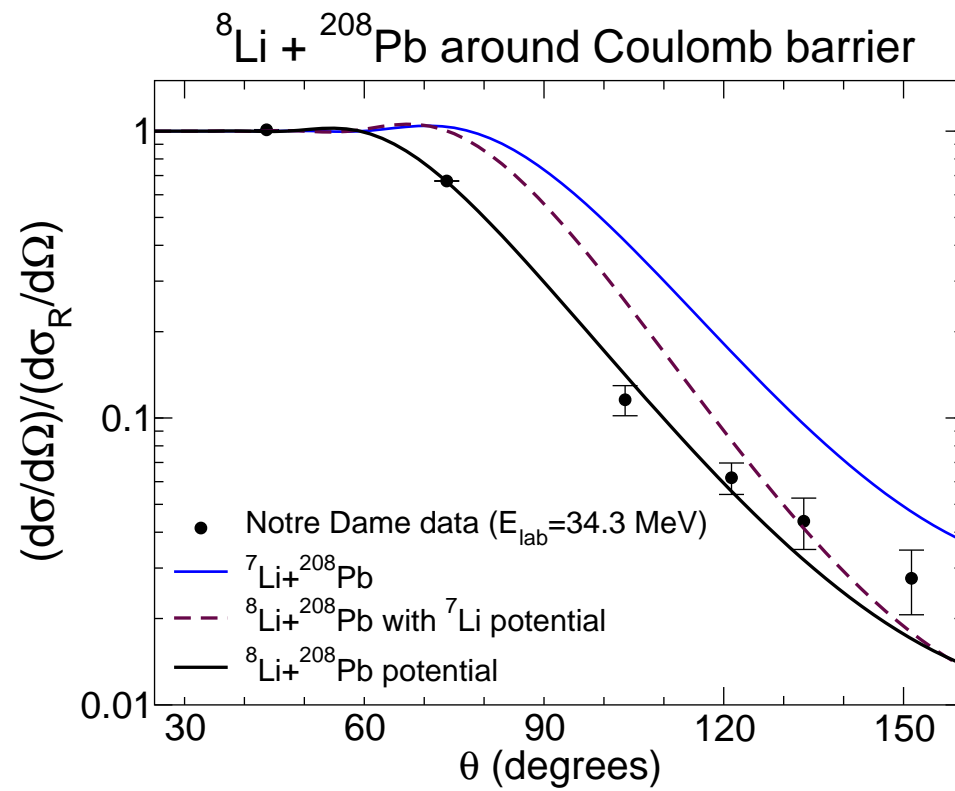
## FRESCO input file: li8pb\_e34.in

```
NAMELIST
&FRESCO hcm=0.05 rmatch=40.0 jtmax=60
      thmin=5.00
      thmax=160.00 thinc=2.00
      smats=2 xstabl=1
      elab= 34.404 /

&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
      masst=208 zt=82 qval=0.0000 pwf=T nex=1 /
&STATES jp=2.0 bandp=1 ep=0.000 cpot=1 jt=0.0
      bandt=1 et=0.000 fexch=F /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=58.3 p5=1.3 p6=0.7 /
&pot /
```





**SFRESKO:** Can be used together with FRESKO to do determine automatically optical model parameters by means of a  $\chi^2$  analysis of experimental angular distribution.

We need 3 input files:

1. FRESKO input file: `li8pb_e34.in`
2. MINUIT input file: `sfresco.in`
3. SEARCH input file: `search.in`

`sfresco.in`  $\Rightarrow$  `search.in`  $\Rightarrow$  `li8pb_e34.in`

## Performing fits with SFRESCO:

1.- FRESCO input file: `li8pb_e34.in` (previous slide)

2.- MUNUIT input file: `sfresco.in`

```
search.in  
min  
fix  
migrad  
end  
q  
show  
plot
```

## Performing fits with SFRESCO (continued):

### 3.- SEARCH input file: search.in

```
'li8pb_e34.in' 'li8pb_e34.out' 2 1

&variable kind=1 name='V' kp=1 pline=2 col=1 valmin=5.0 valmax=150.0 step=0.2/
&variable kind=1 name='W' kp=1 pline=2 col=4 valmin=5.0 valmax=100.0 step=0.2 /

&data type=0 iscale=2 idir=1 lab=F abserr=T/
43.7      1.01026 0.014
73.76     0.67003 0.014
103.537   0.11577 0.01394
121.296   0.06194 0.00778
133.351   0.04369 0.00888
151.332   0.02763 0.00701
&
```

```
sfresco < sfresco.in > sfresco.out
```

Lecture 2: Inelastic  
scattering: DWBA and  
Coupled-Channels  
method

---

- ❖ Coupled-channels  
method
- ❖ Boundary conditions
- ❖ DWBA approximation
- ❖ Partial wave  
decomposition
- ❖ Scattering  
wavefunction
- ❖ Scattering amplitude
- ❖ Cluster models
- ❖ Collective excitations
- ❖  $^{11}\text{Be} + ^{12}\text{C}$  inelastic  
scattering
- ❖  $^{16}\text{O} + ^{208}\text{Pb}$   
inelastic scattering
- ❖ Coulomb vs Nuclear
- ❖ Effect of excitation  
energy
- ❖ Effect of incident  
energy

## Lecture 2: Inelastic scattering: DWBA and Coupled-Channels method

## Coupled-channels method

- The Hamiltonian:  $H = T_R + h(\xi) + \Delta(\mathbf{R}, \xi)$
- Internal states:  $h(\xi)\phi_\alpha(\xi) = \epsilon_\alpha\phi_\alpha(\xi)$
- Model wavefunction:  $\Psi(\mathbf{R}, \xi) = \phi_\alpha(\xi)\chi_\alpha(\mathbf{R}) + \phi_{\alpha'}(\xi)\chi_{\alpha'}(\mathbf{R}) + \dots$
- Coupled equations:  $[H - E]\Psi(\mathbf{R}, \xi)$

$$[E - \epsilon_\alpha - T_R - V_{\alpha,\alpha}(\mathbf{R})]\chi_\alpha(\mathbf{R}) = \sum_{\alpha' \neq \alpha} V_{\alpha,\alpha'}(\mathbf{R})\chi_{\alpha'}(\mathbf{R})$$

- Coupling potentials:

$$V_{\alpha,\alpha'}(\mathbf{R}) = \int d\xi \phi_{\alpha'}(\xi)^* \Delta(\mathbf{R}, \xi) \phi_\alpha(\xi)$$

☞  $\phi_\alpha(\xi)$  will depend on the structure model (collective, single-particle, etc).

## Boundary conditions and scattering amplitude

- Boundary conditions:

$$\chi_0^{(+)}(\mathbf{R}) \rightarrow e^{i\mathbf{K}_0 \cdot \mathbf{R}} + A_{0,0}(\theta) \frac{e^{iK_0 R}}{R} \quad (\text{elastic})$$

$$\chi_n^{(+)}(\mathbf{R}) \rightarrow A_{n,0}(\theta) \frac{e^{iK_n R}}{R}, \quad n \neq 0 \quad (\text{non - elastic})$$

☞ If Coulomb is present, then

$$\frac{e^{iKR}}{R} \rightarrow \frac{1}{R} e^{i(KR - \eta 2KR)}$$

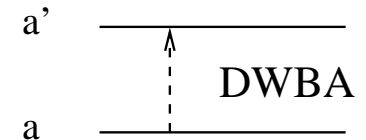
- Cross sections:

$$\frac{d\sigma_n(\theta)}{d\Omega} = \frac{K_n}{K_0} |A_{n,0}(\theta)|^2$$

## DWBA approximation

- DWBA approximation:

$$\begin{aligned} [E - \epsilon_\alpha - T_\alpha - V_{\alpha:\alpha}(\mathbf{R})] \tilde{\chi}_\alpha(\mathbf{K}, \mathbf{R}) &= 0 \\ [E - \epsilon_{\alpha'} - T_{\alpha'} - V_{\alpha':\alpha'}(\mathbf{R})] \tilde{\chi}_{\alpha'}(\mathbf{K}', \mathbf{R}) &= 0 \end{aligned}$$



- Scattering amplitude:

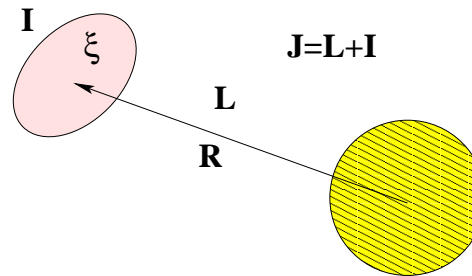
$$A(\mathbf{K}', \mathbf{K}) = -\frac{2\mu}{4\pi\hbar^2} \int d\mathbf{R} \tilde{\chi}_{\alpha'}^{(-)}(\mathbf{K}', \mathbf{R}) V_{\alpha':\alpha}(\mathbf{R}) \tilde{\chi}_\alpha^{(+)}(\mathbf{K}, \mathbf{R})$$

☞ The DWBA approximation amounts at solving the CC equations to first order (Born approximation)

☞ In practice, phenomenological optical potentials that fit the elastic cross section in the respective channels are used instead of  $V_{\alpha,\alpha}$  and  $V_{\alpha',\alpha'}$

$$V_{\alpha,\alpha}(\mathbf{R}) \equiv (\alpha|\Delta(\mathbf{R}, \xi)|\alpha) \rightarrow U_\alpha(\mathbf{R})$$

## Partial wave decomposition: the channel basis



- The channel basis:

$$\Phi_{nLI}^{JM_J}(\hat{R}, \xi) = \sum_{M_I M_L} i^L Y_{LM_L}(\hat{R}) |nIM_I\rangle \langle LM_L IM_I | JM_J\rangle$$

- Partial wave expansion of the total WF:

$$\Psi(\mathbf{R}, \xi) = \sum_{nLIJM} C^{JM_J} \frac{f_{nLI}^J(R)}{R} \Phi_{nLI}^{JM}(\hat{R}, \xi)$$



## Coupled equations

- The coupled equations:

$$\left( -\frac{\hbar^2}{2\mu} \frac{d^2}{dR^2} + \frac{\hbar^2 L(L+1)}{2\mu R^2} + \epsilon_n - E \right) f_\beta^J(R) + \sum_{\beta'} V_{\beta,\beta'}^J(R) f_{\beta'}^J(R) = 0$$

$$\beta \equiv \{n, L, I\}$$

- Coupling potentials:

$$V_{\beta,\beta'}^J(R) = \int d\hat{R} d\xi \Phi_\beta^{JM_J}(\hat{R}, \xi)^* \Delta(\vec{R}, \xi) \Phi_{\beta'}^{JM_J}(\hat{R}, \xi)$$

## Boundary conditions

Solution of the coupled equations:

1. Integrate the differential equation for  $R \in [0, R_m]$  with the condition:

$$\lim_{R \rightarrow 0} f_{\beta; \beta_i}^J(R) = 0$$

2. Match the solution at  $R_m$  with the asymptotic form  $\Rightarrow$  S-matrix:

$$f_{\beta; \beta_i}^J(R) \rightarrow \delta_{\beta, \beta_i} I_{\beta}(R) - S_{\beta, \beta_i}^J O_{\beta}(R)$$

$$\begin{aligned} I_{\beta}(R) &= (K_n R) h_L^*(K_n R) / \sqrt{v_n} \\ O_{\beta}(R) &= (K_n R) h_L(K_n R) / \sqrt{v_n} \end{aligned}$$

# Scattering wavefunction

- Wavefunction that corresponds to the experimental condition:

$$\Psi_{\mathbf{K}_i, n_i I_i M_i}(\mathbf{R}, \xi) = e^{i\mathbf{K}_i \cdot \mathbf{R}} |n_i I_i M_i\rangle + \chi_{\mathbf{K}_i, n_i I_i M_i}^{(+)}(\mathbf{R}, \xi)$$

- The outgoing wave:

$$\begin{aligned} \chi_{\mathbf{K}_i, n_i I_i M_i}^{(+)}(\mathbf{R}, \xi) &= \sum_{JM_J L_i N_i} \frac{2\pi i \sqrt{v_i}}{k_i R} \langle L_i N_i I_i M_i | JM_J \rangle Y_{L_i N_i}^*(\hat{K}_i) \\ &\times \sum_{nIL} \left( \delta_{n, n_i} \delta_{I, I_i} \delta_{n, n_i} - S_{nIL; n_i I_i L_i}^J \right) \Phi_{nLI}^{JM_J}(\hat{R}, \xi) O_{nIL}(R) \end{aligned}$$

- The scattering amplitude:

$$\begin{aligned} A(\mathbf{K}_i, \mathbf{K})_{n_i I_i M_i; n I M} &= \frac{2\pi i}{\sqrt{K K_i}} \sum_{JM_J L_i N_i} \langle L_i N_i I_i M_i | JM_J \rangle Y_{L_i N_i}^*(\hat{K}_i) \\ &\times \sum_{nILN} \langle LNIM | JM_J \rangle Y_{LN}(\hat{K}) \left( \delta_{n, n_i} \delta_{I, I_i} \delta_{L, L_i} - S_{nIL; n_i I_i L_i}^J \right) \end{aligned}$$

# Scattering amplitude and cross sections

- Elastic and inelastic cross sections:

$$\frac{d\sigma}{d\Omega_{i \rightarrow n}} = \frac{1}{2I_i + 1} \sum_{M M_i} |A(\mathbf{K}_i, \mathbf{K})_{n_i I_i M_i; n I M}|^2$$

☞ *Coupled channels calculations give elastic and inelastic cross sections, if the states are properly described, if the interactions are known, and if all “relevant” channels are included*

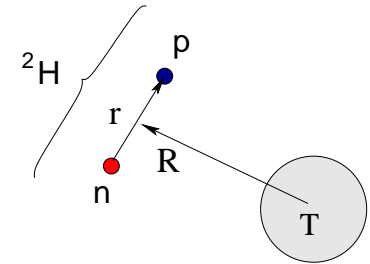
## Inelastic scattering: cluster model

- Some nuclei permit a description in terms of two or more clusters:  
 $d=p+n$ ,  ${}^6\text{Li}=\alpha+d$ ,  ${}^7\text{Li}=\alpha+{}^3\text{H}$ .

- Projectile-target interaction:

$$V(\mathbf{R}, \mathbf{r}) = U_1(\mathbf{R}_1) + U_2(\mathbf{R}_2)$$

- Internal states:  $[\hbar - \epsilon_\alpha]\phi_\alpha(\mathbf{r}) = 0$



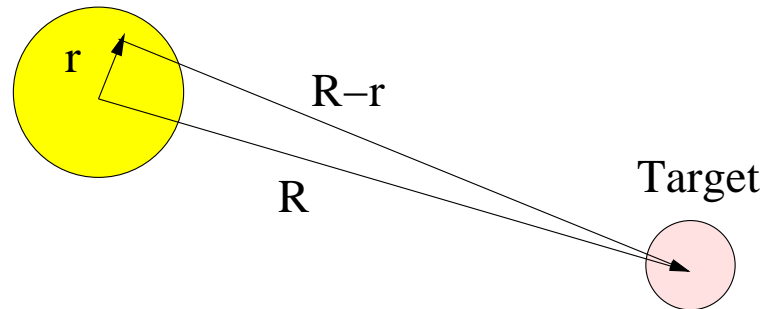
- Transition potentials:

$$V_{\alpha,\alpha'}(\mathbf{R}) = \int d\mathbf{r} \phi_\alpha^*(\mathbf{r}) V(\mathbf{r}, \mathbf{R}) \phi_{\alpha'}(\mathbf{r})$$

## Inelastic scattering: collective models

- Projectile-target Coulomb interaction:

$$V(\mathbf{R}, \xi) = \frac{Ze^2}{4\pi\epsilon_0} \sum_i \frac{1}{|\mathbf{R} - \mathbf{r}_i|}$$



- Multipolar expansion:

$$\frac{1}{|\mathbf{R} - \vec{r}_i|} = \sum_{\lambda\mu} \frac{r_i^\lambda}{R^{\lambda+1}} \frac{4\pi}{2\lambda+1} Y_{\lambda\mu}(\hat{r}_i) Y_{\lambda\mu}^*(\hat{R}) \quad (R > r_i)$$

## Inelastic scattering: collective models

- Electric multipole operator:  $M(E\lambda, \mu) = e \sum_i r_i^\lambda Y_{\lambda\mu}(\hat{r}_i)$
- Monopole and transition operator:

$$V(\mathbf{R}, \xi) = V_0(R) + \Delta(\mathbf{R}, \xi) = \frac{Zze^2}{4\pi\epsilon_0 R} + \frac{Ze}{\epsilon_0} \sum_{\lambda \neq 0, \mu} \frac{M(E\lambda, \mu)}{2\lambda + 1} \frac{Y_{\lambda\mu}^*(\hat{R})}{R^{\lambda+1}}$$

- Transition potentials:

$$\Delta_{nm}(\mathbf{R}) = \frac{Ze}{\epsilon_0} \sum_{\lambda \neq 0, \mu} \frac{\langle nI_n M_n | M(E\lambda, \mu) | mI_m M_m \rangle}{2\lambda + 1} \frac{Y_{\lambda\mu}^*(\hat{R})}{R^{\lambda+1}}$$

## Inelastic scattering: collective models

- Multipole expansion:  $r(\theta, \phi) = R_0 + \sum_{\lambda\mu} \hat{\delta}_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi)$

( $\hat{\delta}_{\lambda\mu}$ =deformation length operators)

- Transition operator:

$$V(\mathbf{R}, \xi) = V_0(R - R_0) - \frac{dV_0(R - R_0)}{dR} \sum_{\lambda\mu} \hat{\delta}_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi)$$

- Central and transition potential:

$$\Delta_{nm}(\mathbf{R}) = -\frac{dV_0(R - R_0)}{dR} \sum_{\lambda} \langle nI_n M_n | \hat{\delta}_{\lambda\mu} | mI_m M_m \rangle Y_{\lambda\mu}^*(\hat{R})$$

☞ *The nuclear transition potentials are proportional to the matrix element of the deformation length operator.*



## Physical ingredients for collective excitations

- Coulomb excitation → electric reduced matrix elements

$$\Delta_{nm}(\mathbf{R}) = \frac{Ze}{\epsilon_0} \sum_{\lambda \neq 0, \mu} \frac{\langle nI_n M_n | M(E\lambda, \mu) | mI_m M_m \rangle}{2\lambda + 1} \frac{Y_{\lambda\mu}^*(\hat{R})}{R^{\lambda+1}}$$

$$\langle nI_n || M(E\lambda) || mI_m \rangle = \sqrt{(2I_n + 1)B(E\lambda; I_n \rightarrow I_m)}$$

- Nuclear excitation (collective model) → deformation lengths

$$\Delta_{nm}(\mathbf{R}) = -\frac{dV_0(R - R_0)}{dR} \sum_{\lambda} \langle nI_n M_n | \hat{\delta}_{\lambda\mu} | mI_m M_m \rangle Y_{\lambda\mu}^*(\hat{R})$$

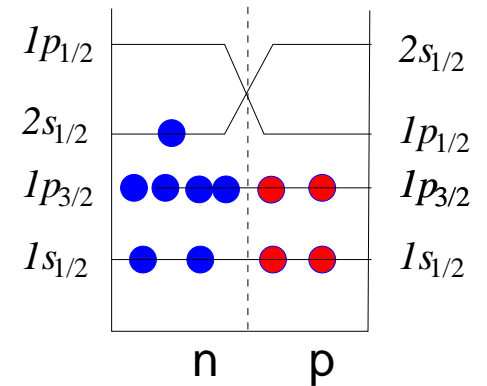
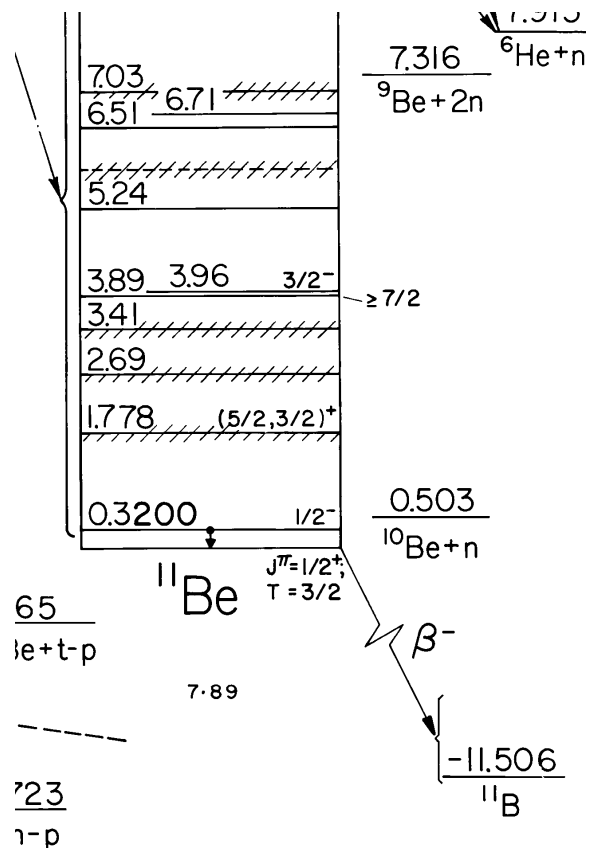
within the rotational model:

$$\langle nI_n || \hat{\delta}_{\lambda} || mI_m \rangle = \delta_{\lambda} \sqrt{2I_n + 1} \langle I_n K \lambda 0 | I_m K \rangle \quad \delta_{\lambda} = \beta_{\lambda} R$$

# Inelastic scattering example: $^{11}\text{Be} + ^{12}\text{C}$

**Example 1:**  $^{11}\text{Be} + ^{12}\text{C} \rightarrow ^{11}\text{Be}(1/2^+, 1/2^-) + ^{12}\text{C}$  at 49.3 MeV/A  
 Phys. Rev. C 67, 037601 (2003)

**Input file:** `be11c12_inel.in`



## <sup>11</sup>Be+<sup>12</sup>C inelastic scattering

### General variables:

```
&FRESKO hcm=0.05 rmatch=60.0 jtmin=0.0  
      jtmax=150.0 thmin=0.00 thmax=45.00 thinc=0.50  
      iblock=2 nnu=24 chans=1 smats=2 xstabl=1  
      elab=542.3 /
```

- `iblock=2`: number of channels coupled *exactly*.

### Partitions & states:

```
&PARTITION namep='11Be' massp=11.0 zp=4 namet='12C' masst=12.0000 zt=6 nex=2 /  
&STATES jp=0.5 bandp=1 cpot=1 jt=0.0 bandt=1 /  
&STATES jp=0.5 bandp=-1 ep=0.3200 cpot=1 jt=0.0 copyt=1 /
```

- `nex=2`: This partition will contain two pairs of states.
- `copy=1`: The target of the second pair of states is just the same (a copy) of the first target stat.

```
&PARTITION namep='10Be' massp=10.0000 zp=4 namet='12C+n' masst=13.0000 zt=6 nex=1 /  
&STATES jp=0.0 bandp=1 cpot=2 jt=0.0 bandt=1 /
```

## <sup>11</sup>Be+<sup>12</sup>C inelastic scattering

### Projectile-target Coulomb potential (monopole):

```
&POT kp=1 ap=11.000 at=12.000 rc=1.111 /
```

### Neutron-target & core-target potentials:

```
&POT kp=3 ap=0.000 at=12.000 rc=1.111 /  
&POT kp=3 type=1 p1=37.400 p2=1.200  
      p3=0.750 p4=10.000 p5=1.300 p6=0.600 /
```

```
&POT kp=2 ap=10.000 at=12.000 rc=1.111 /  
&POT kp=2 type=1 p1=123.000 p2=0.750  
      p3=0.800 p4=65.000 p5=0.780 p6=0.800 /
```

### Neutron binding potential:

```
&POT kp=4 ap=0 at=10.000 rc=1.0 /  
&POT kp=4 type=1 p1=87.0 p2=1.0 p3=0.53 /
```

## <sup>11</sup>Be+<sup>12</sup>C inelastic scattering

### Bound state wave functions:

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=2 sn=0.5 l=0 j=0.5  
          kbpot=4 be=0.500 isc=1 /  
&OVERLAP kn1=2 ic1=1 ic2=2 in=1 nn=1 l=1 sn=0.5  
          j=0.5 kbpot=4 be=0.180 isc=1 ipc=2 /
```

- $kn1=1, 2$ : Index for this WF
- $ic1/ic2$ : Index of partition containing core (<sup>10</sup>Be) / composite (<sup>11</sup>Be)
- $in=1/2$ : WF for projectile/target
- $nn, sn, l, j$ : Quantum numbers for bound state
- $be$ : separation energy.
- $kbpot=3$ : Index KP of binding potential.

# <sup>11</sup>Be+<sup>12</sup>C inelastic scattering

## Couplings:

```
&COUPLING icto=1 icfrom=2 kind=3 ip1=4 ip2=1 p1=3.0 p2=2.0 /
```

- `kind=3`: Single-particle excitations of projectile
- `icto=1`: Partition containing nucleus being excited (<sup>11</sup>Be)
- `icfrom=2`: Partition containing core (<sup>10</sup>Be)
- `ip1=4`: Maximum multipole for coupling potentials
- `p1/p2`: KP index for fragment-target / core-target potentials

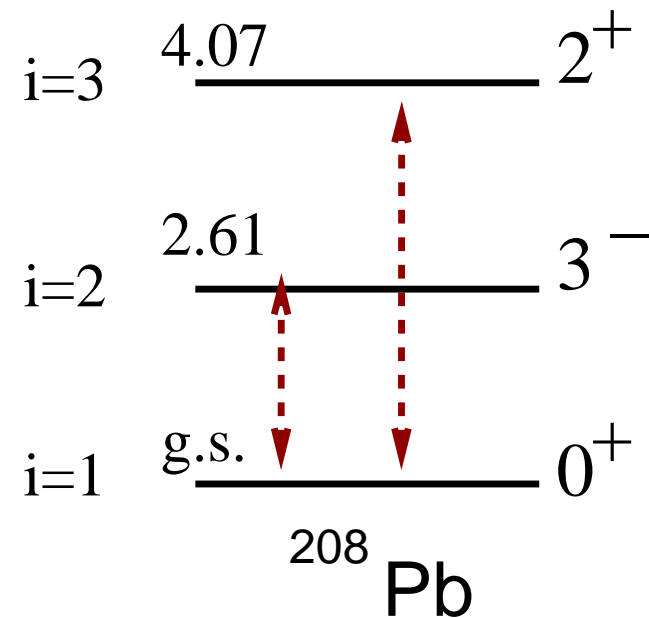
## Spectroscopic amplitudes:

```
&CFP in=1 ib=1 ia=1 kn=1 a=1.000 /  
&CFP in=1 ib=2 ia=1 kn=2 a=1.000 /
```

- `in=1/2`: Projectile/target
- `ib/ia`: Index for composite/core state
- `a=1.0`: Spectroscopic amplitude

## $^{16}\text{O} + ^{208}\text{Pb}$ inelastic scattering

**Physical example:**  $^{16}\text{O} + ^{208}\text{Pb} \rightarrow ^{16}\text{O} + ^{208}\text{Pb}(3^-, 2^+)$



# Inelastic scattering

Input example 2:  $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}(3^-, 2^+)$  (o16pb\_cc1a.in )

```
o16pb_cc1a.in: 16O+208Pb 80 MeV
NAMELIST
&FRESKO hcm=0.05 rmatch=100.0
      jtmin=0.0 jtmax=300.0
      thmin=5.00 thmax=-180.00 thinc=2.50
      iblock=3
      smats=2 xstabl=1
      elab= 80.0 /

&PARTITION namep='16-O' massp=15.9949 zp=8
      namet='PB-208' masst=207.9770 zt=82
      nex=3 /
&STATES jp=0.0 bandp=1 ep=0.0000 cpot=1
      jt=0.0 bandt=1 et=0.0000 /
&STATES jp=0.0 copyp=1 ep=0.0000 cpot=1
      jt=3.0 bandt=-1 et=2.6100 fexch=F /
&STATES jp=0.0 copyp=1 bandp=1 ep=0.0000 cpot=1
      jt=2.0 bandt=1 et=4.0700 /
&partition /
```

```
&POT kp=1 itt=F ap=208.000 at=16.000 rc=1.200 /
&POT kp=1 type=13 shape=10 itt=F p2=54.45 p3=815.0 /
&STEP ib=1 ia=2 k=3 str=815.0 /
&STEP ib=2 ia=1 k=3 str=815.0 /
&STEP ib=1 ia=3 k=2 str=54.45 /
&STEP ib=3 ia=1 k=2 str=54.45 /
&step /
&POT kp=1 type=1 shape=1 p4=10.000 p5=1.000 p6=0.400 /
&POT kp=1 type=-1 pl=60.500 p2=1.179 p3=0.658 /
&POT kp=1 type=13 shape=11 p2=0.400 p3=0.8 /
&STEP ib=1 ia=2 k=3 str=0.8 /
&STEP ib=2 ia=1 k=3 str=0.8 /
&STEP ib=1 ia=3 k=2 str=0.4 /
&STEP ib=3 ia=1 k=2 str=0.4 /
&step /

&pot /

&overlap /
&coupling /
```



## $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ inelastic scattering

### General variables:

```
&FRESCO hcm=0.05 rmatch=100.0  
  jtmin=0.0 jtmax=300.0  
  thmin=5.00 thmax=-180.00 thinc=2.50  
  iblock=3  
  smats=2 xstabl=1  
  elab= 80.0 /
```

**iblock**: Number of states (including gs) that will be coupled to all orders.

- **iblock=1**: only elastic scattering
- **iblock=2**: elastic scattering + 1st inelastic channel ( $^{208}\text{Pb}(3^-)$ )
- **iblock=3**: elastic scattering +  $^{208}\text{Pb}(3^-)$  +  $^{208}\text{Pb}(2^+)$

## Partitions and states:

```
&PARTITION namep='16-O' massp=15.9949 zp=8 namet='PB-208' masst=207.9770 zt=82  
      nex=3 /  
&STATES jp=0.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=+1 et=0.00 /  
&STATES      copyp=1      cpot=1 jt=3.0 bandt=-1 et=2.61 /  
&STATES      copyp=1      cpot=1 jt=2.0 bandt=+1 et=4.07 /  
&partition /
```

- **nex**: number of states within the partition
- **ep**, **et**: excitation energy for projectile / target
- **copyp=1** tells FRESKO that the 2nd and 3rd projectile states are just a copy of the ground state.

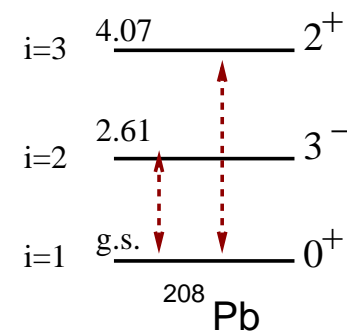
## $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ inelastic scattering

### Coulomb excitation:

```
&POT kp=1 ap=208.000 at=16.000 rc=1.2 /  
&POT kp=1 type=13 shape=10 p2=54.45 p3=815.0 p4=0 p5=0 p6=0 /
```

- **type=13**: couple target states by deforming previous potential
- **p1, ..., p6**: consider couplings for multipolarities  $k$  with  $p_k \neq 0$
- **shape=10**: usual deformed charge sphere:  $\Delta_{nm}(R) \propto M(Ek)/R^{k+1}$

```
&STEP ib=1 ia=2 k=3 str=815.0 /  
&STEP ib=2 ia=1 k=3 str=815.0 /  
&STEP ib=1 ia=3 k=2 str=54.45 /  
&STEP ib=3 ia=1 k=2 str=54.45 /  
&step /
```



- **ia, ib**: couple from state number **ia** to state **ib**
- **k**: multipolarity
- **str** =  $\langle ib || M(Ek) || ia \rangle = \sqrt{(2I_a + 1)B(E\lambda; ia \rightarrow ib)}$

## $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ inelastic scattering

### Nuclear excitation:

```
&POT kp=1 type=1 shape=1 p4=10.000 p5=1.000 p6=0.400 /  
&POT kp=1 type=-1 shape=0 p1=60.500 p2=1.179 p3=0.658 /  
&POT kp=1 type=13 shape=10 itt=F p2=0.400 p3=0.8 /
```

- **type=13**: couple target states by deforming preceding potential
- **shape=10**: usual deformed nuclear potential:  $\Delta_{nm}(R) \propto \delta_k dU(R)/dR$

```
&STEP ib=1 ia=2 k=3 str=0.8 /  
&STEP ib=2 ia=1 k=3 str=0.8 /  
&STEP ib=1 ia=3 k=2 str=0.4 /  
&STEP ib=3 ia=1 k=2 str=0.4 /
```

- **str** =  $\langle ib || \delta_k || ia \rangle$  (reduced deformation length)

## Useful output files:

- Main output file:

```
OCUMULATIVE REACTION cross section          = 11.22270  <L> = 47.07
OCUMULATIVE outgoing cross sections in partition 1 : 0.00000  7.67943  0.99138
OCumulative ABSORBTION by Imaginary Potentials = 2.55189  <L> = 6.99
```

- Angular distributions:

- `fort.201` : Elastic scattering angular distribution
- `fort.202` : 1st state angular distribution
- `fort.203` : 2nd excited state angular distribution

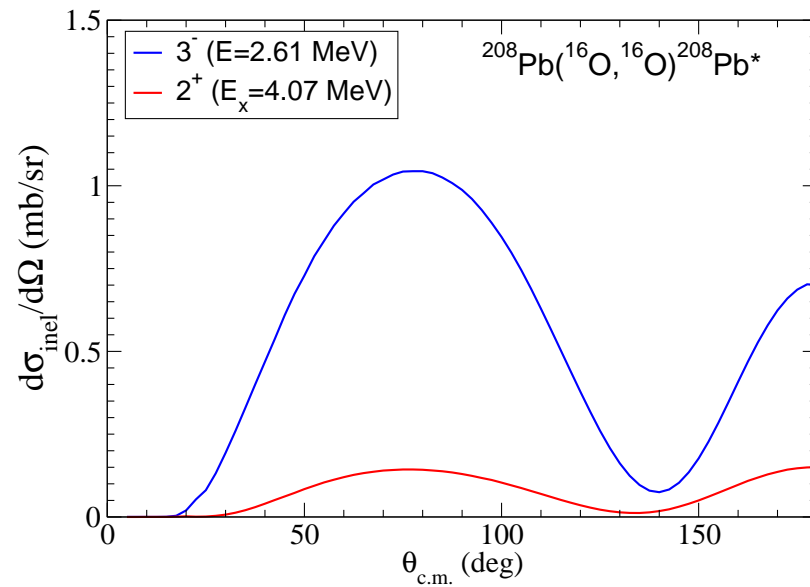
- `fort.56`: 3 columns: Fusion (absorption), reaction and inelastic cross section for each total angular momentum J.

$$\sigma_{\text{reac}} = \sigma_{\text{inel}} + \sigma_{\text{abs}}$$

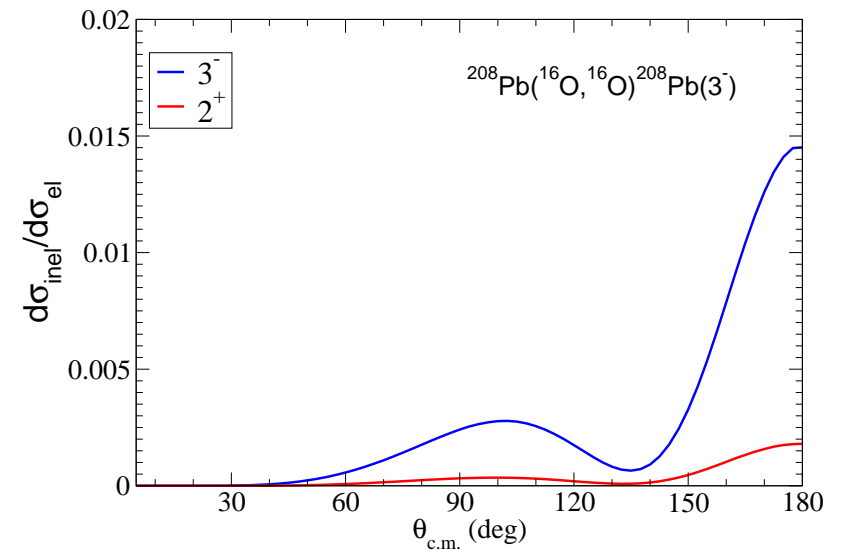
# $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ inelastic scattering

Angular distribution of the ejectile in c.m. frame

Absolute cross section

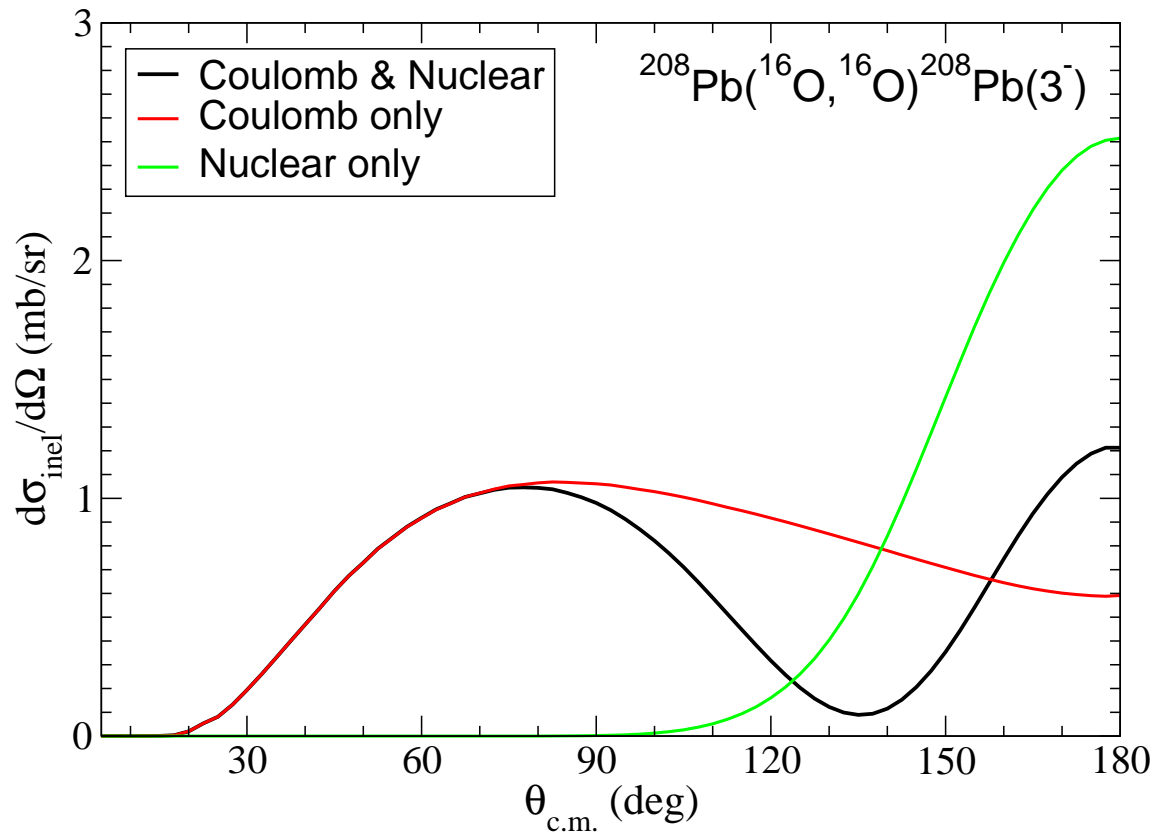


Ratio to elastic cross section



## $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ inelastic scattering

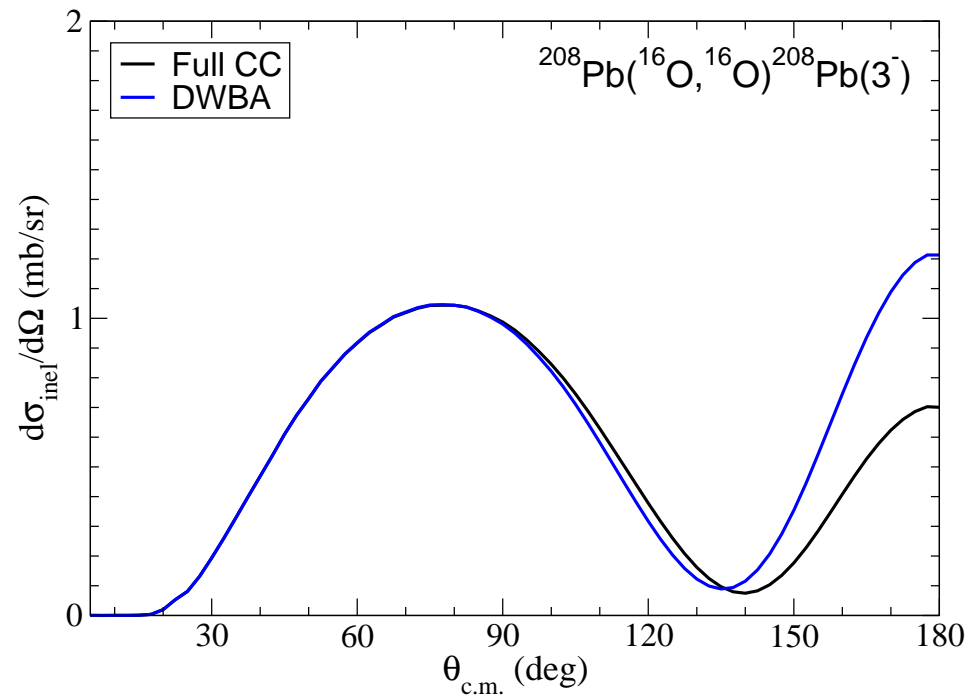
Coulomb and Nuclear excitations can produce constructive or destructive interference:



## $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ inelastic scattering

CC versus DWBA:

- Full coupled-channels: `iblock = 3, iter = 0`
- DWBA: `iblock = 0, iter = 1`

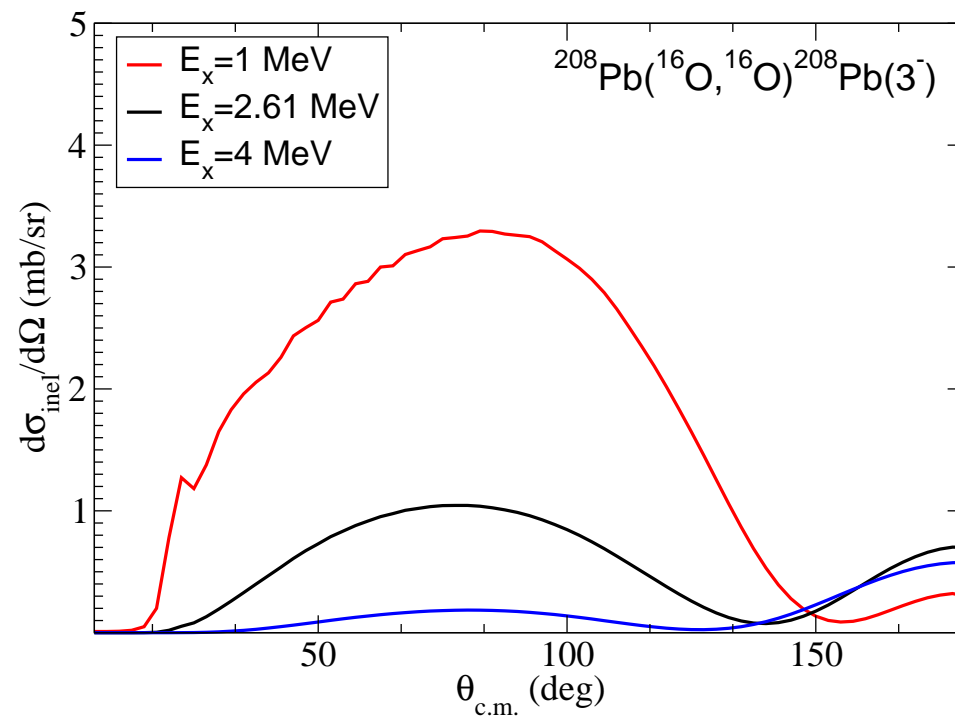




# Inelastic scattering

Effect of the excitation energy:

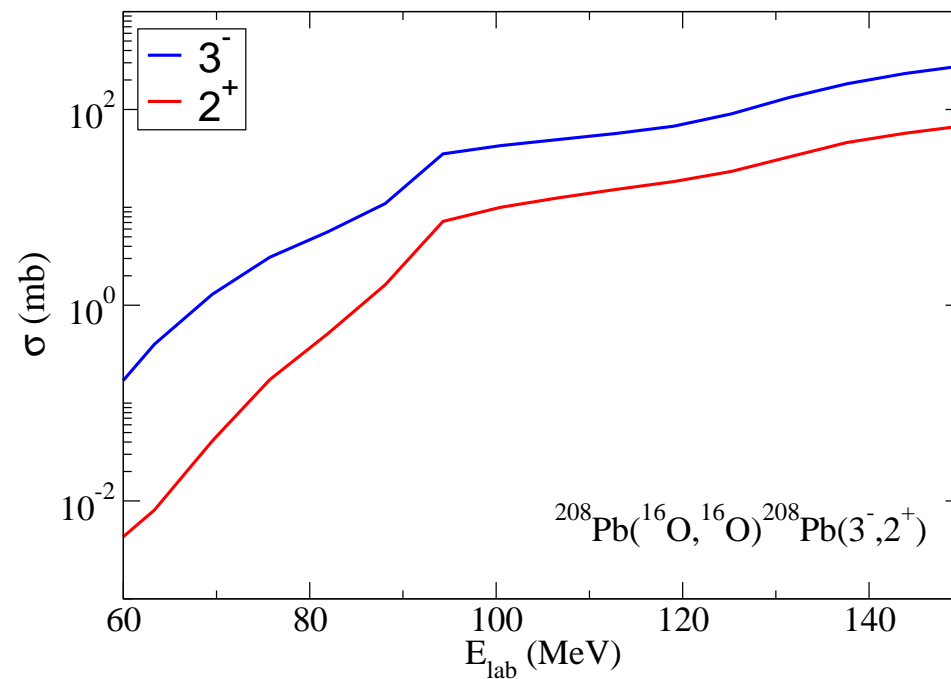
```
&STATES jp=0.0 copy=1 ep=0.0000 cpot=1 jt=3.0 bandt=-1 et=2.6100 /
```



## $^{208}\text{Pb}(^{16}\text{O}, ^{16}\text{O})^{208}\text{Pb}$ inelastic scattering

Effect of the incident energy: In FRESKO main output:

```
OCUMULATIVE REACTION cross section = 11.22270 <L> = 47.07 <L**2> = 3441.3  
OCUMULATIVE outgoing cross sections in partition 1 : 0.00000 7.67943 0.99138
```



## Example for inelastic scattering

### Proposed exercise:

For the reaction  ${}^8\text{Li} + {}^{208}\text{Pb}$  at  $E_{\text{c.m.}} = 24 \text{ MeV}$  and  $33 \text{ MeV}$  evaluate the inelastic cross section to the first excited state of  ${}^8\text{Li}$  ( $E_x = 0.98 \text{ MeV}$ ,  $1^+$ ) in the following situations:

- Coulomb excitation alone
- Only nuclear couplings
- Coulomb + nuclear excitation

Data:  $B(E2; 2^+ \rightarrow 1^+) = 30 \text{ e}^2 \text{ fm}^4$  and  $\delta_2 = 1.75 \text{ fm}$ .

*(further details from Phys.Rev. C 68, 034614 (2003))*

## $^8\text{Li} + ^{208}\text{Pb}$ inelastic scattering

### Input example: li8pb\_inel.in

```
8Li+208Pb quasielastic
NAMELIST
&FRESCO hcm=0.1 rmatch=100 jtmax=150
        thmin=2.00 thmax=-180.00 thinc=1.00 iblock=2
        smats=2 xstabl=1 elab=34.404 /

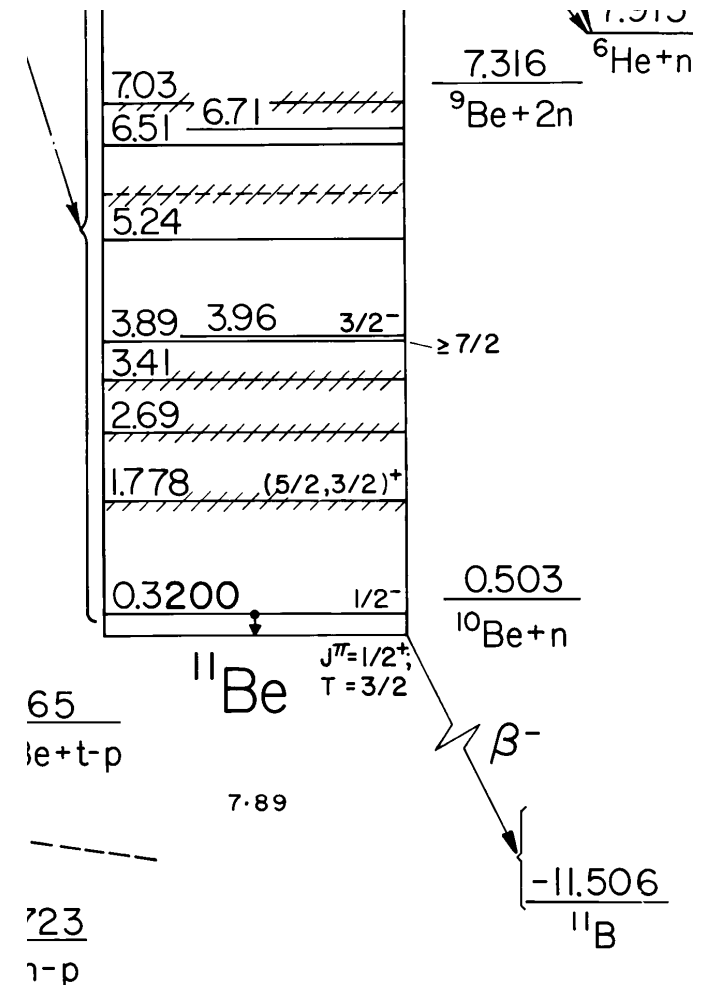
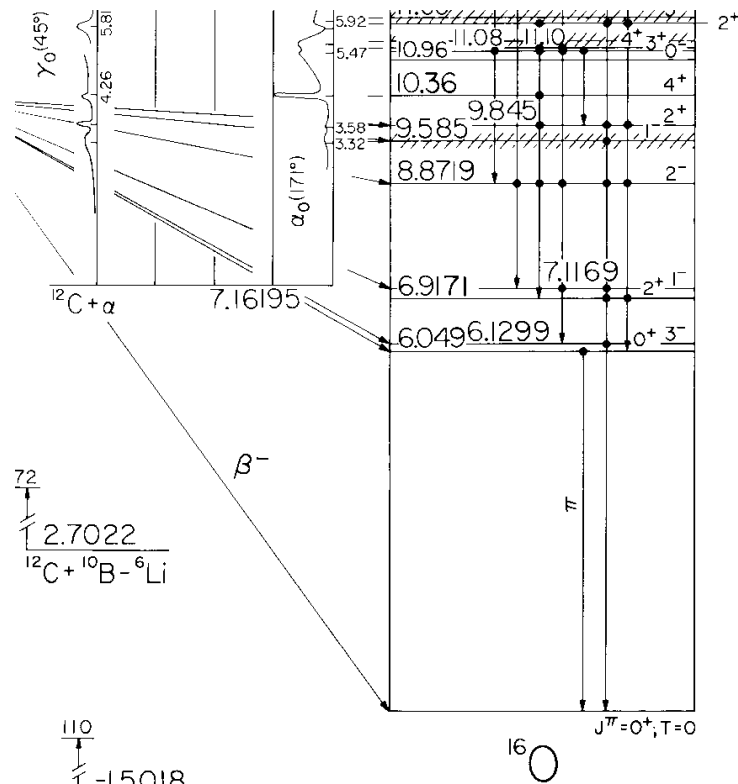
&PARTITION namep='Li-8' massp=8 zp=3 namet='Pb-208'
        masst=207.977 zt=82 qval=0.0000 pwf=F nex=2 /
&STATES jp=2.0 bandp=1 ep=0.000 kkp=1 cpot=1 jt=0.0 bandt=1 et=0.000 /
&STATES jp=1.0 bandp=1 ep=0.981 kkp=1 cpot=1 copyt=1 /

&partition /

&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=10 shape=10 itt=F p2=10.00 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=80 p5=1.3 p6=0.70 /
&POT kp=1 type=10 shape=11 p2=1.75 /
&pot /

&overlap /
&coupling /
```

# Inelastic scattering in exotic nuclei



➡ Exotic nuclei are weakly bound  $\Rightarrow$  coupling to continuum states becomes an important reaction channel

### Lecture 3: Transfer reactions: the DWBA method

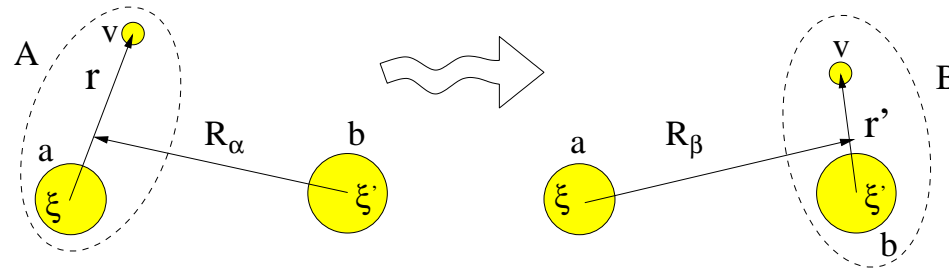
---

- ❖ Post/prior representation
- ❖ CRC equations
- ❖ DWBA approximation
- ❖ Spectroscopic factors
- ❖ Extracting structure information
- ❖ Example:  $d + {}^{56}\text{Fe} \rightarrow p + {}^{57}\text{Fe}$
- ❖ Post-prior equivalence
- ❖ Dependence with binding energy
- ❖ Dependence with beam energy
- ❖ Sensitivity with  $l$
- ❖ CCBA method
- ❖ Summary

## Lecture 3: Transfer reactions: the DWBA method

## Transfer reactions: prior/post representation

- Transfer process:  $\underbrace{(a + v)}_A + b \rightarrow a + \underbrace{(b + v)}_B$



- Projectile–target interaction:

❖ Prior form:  $V_{\text{prior}} = V_{vb} + U_{ab} = U_{\alpha} + \underbrace{(V_{vb} + U_{ab} - U_{\alpha})}_{\Delta_{\text{prior}}}$

❖ Post form:  $V_{\text{post}} = V_{av} + U_{ab} = U_{\beta} + \underbrace{(V_{av} + U_{ab} - U_{\beta})}_{\Delta_{\text{post}}}$

☞  $U_{\alpha}, U_{\beta}$ : average projectile-target interaction in entrance/exit channel

# Coupled Reaction Channels

- Model wavefunction:

$$\Psi = \phi_A(\xi, \mathbf{r})\phi_b(\xi')\chi_\alpha(\mathbf{R}_\alpha) + \phi_a(\xi)\phi_B(\xi', \mathbf{r}')\chi_\beta(\mathbf{R}_\beta)$$

- Coupled-reaction channels (CRC) equations:  $[H - E] \Psi = 0$

$$\begin{aligned} [E - \epsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)] \chi_\alpha(\mathbf{R}_\alpha) &= \int d\mathbf{R}_\beta K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta) \chi_\beta(\mathbf{R}_\beta) \\ [E - \epsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)] \chi_\beta(\mathbf{R}_\beta) &= \int d\mathbf{R}_\alpha K_{\alpha,\beta}(\mathbf{R}_\alpha, \mathbf{R}_\beta) \chi_\alpha(\mathbf{R}_\alpha) \end{aligned}$$

- Non-local kernels:

$$K_{\alpha,\beta}(\mathbf{R}_\beta, \mathbf{R}_\alpha) = \int d\xi d\xi' d\mathbf{r} \phi_a(\xi)\phi_B(\xi', \mathbf{r}')(H - E)\phi_A(\xi, \mathbf{r})\phi_b(\xi')$$

☞ CRC equations have to be solved iteratively due to NL kernels.



## DWBA approximation

- Distorted wave Born approximation:

$$\begin{aligned}[E - \epsilon_\alpha - T_R - U_\alpha(\mathbf{R}_\alpha)] \tilde{\chi}_\alpha(\mathbf{R}_\alpha) &= 0 \\ [E - \epsilon_\beta - T_R - U_\beta(\mathbf{R}_\beta)] \tilde{\chi}_\beta(\mathbf{R}_\beta) &= 0\end{aligned}$$

- DWBA amplitude (prior):

$$T_{\text{prior}} = \int \int \tilde{\chi}_\beta^{(-)}(\mathbf{R}_\beta) (\phi_a \phi_B | \Delta_{\text{prior}} | \phi_A \phi_b) \tilde{\chi}_\alpha^{(+)}(\mathbf{R}_\alpha) d\mathbf{R}_\alpha d\mathbf{r}$$

- Structure form-factor:

$$(\phi_a \phi_B | \Delta_{\text{prior}} | \phi_A \phi_b) \equiv \int d\xi d\xi' \phi_a(\xi) \phi_B(\xi', \mathbf{r}') \Delta_{\text{prior}} \phi_A(\xi, \mathbf{r}) \phi_b(\xi')$$

# Spectroscopic factors

- Parentage amplitudes:

- ◆ **Projectile:**  $\phi_A^{JM}(\xi, \mathbf{r}) = \frac{1}{\sqrt{n_A}} \sum_{I\ell j} A_{IJ;\ell sj} [\phi_a^I(\xi) \otimes \varphi_{\ell sj}(\mathbf{r})]_{JM}$

- ◆ **Target:**  $\phi_B^{J'M'}(\xi', \mathbf{r}') = \frac{1}{\sqrt{n_B}} \sum_{I\ell j} A_{IJ';\ell sj} [\phi_b^I(\xi') \otimes \varphi_{\ell sj}(\mathbf{r}')]_{J'M'}$

☞  $A_{IJ;\ell sj}$  = spectroscopic amplitudes

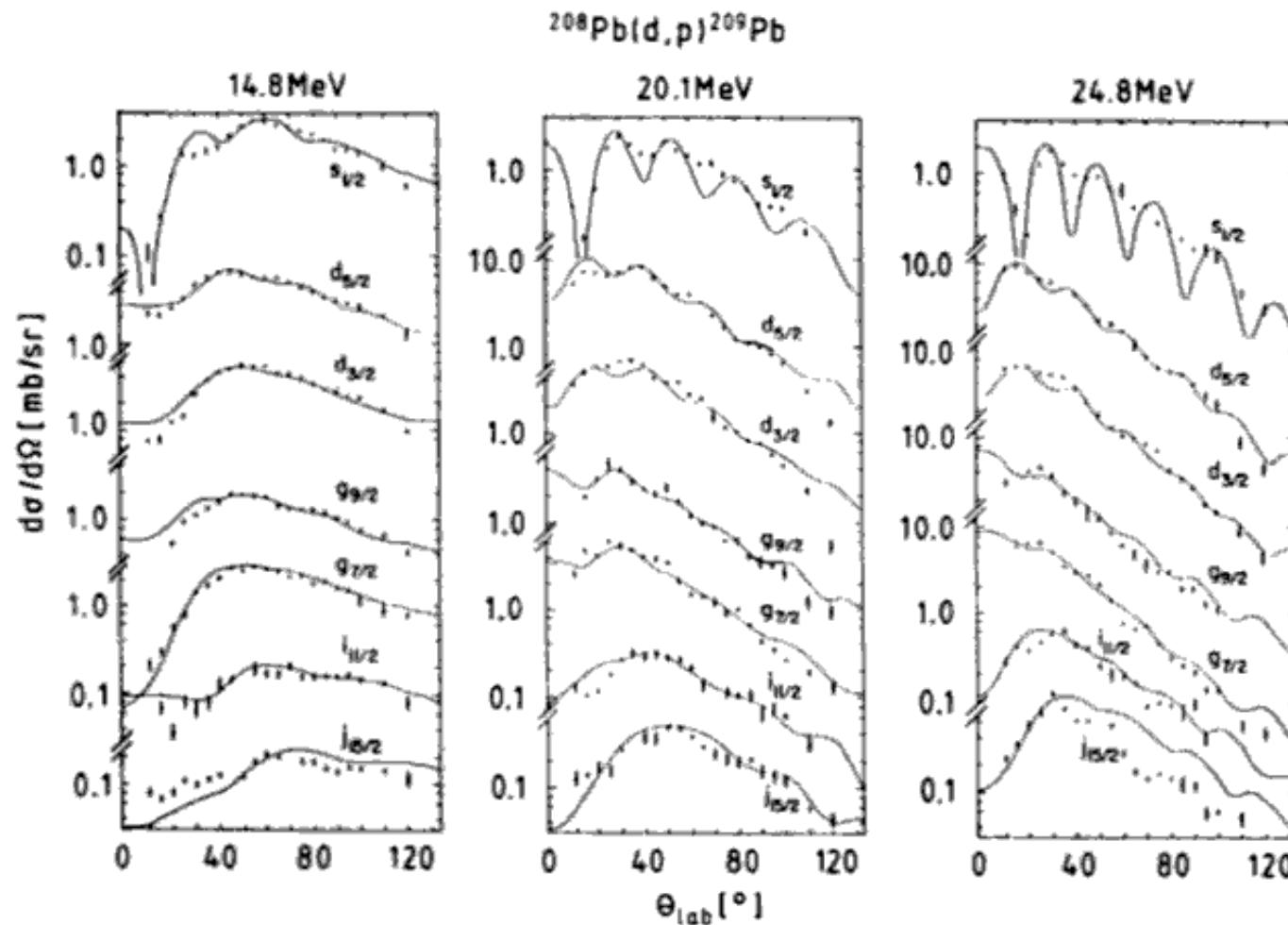
☞  $S_{IJ;\ell sj} = |A_{IJ;\ell sj}|^2$  = spectroscopic factors

- DWBA amplitude (prior)

$$T_{\text{prior}} = A_{IJ;\ell sj} A_{I'J';\ell' sj'} \int \int \tilde{\chi}_{\beta}^{(-)}(\mathbf{R}_{\beta}) \varphi_{\ell' sj'}(\mathbf{r}') \Delta_{\text{prior}} \varphi_{\ell sj}(\mathbf{r}) \tilde{\chi}_{\alpha}^{(+)}(\mathbf{R}_{\alpha}) d\mathbf{R}_{\alpha} d\mathbf{r}$$

☞ *In DWBA, the transfer cross section is proportional to the spectroscopic factors  $S_{IJ;\ell sj} S_{I'J';\ell' sj'}$*

## Extracting structure information from transfer reactions



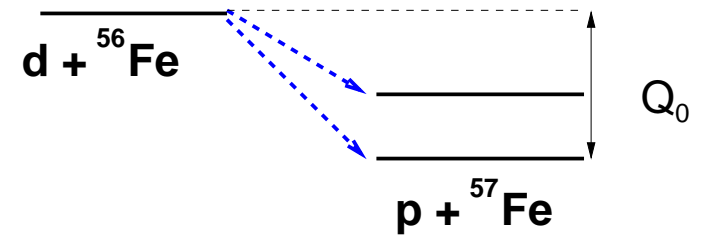
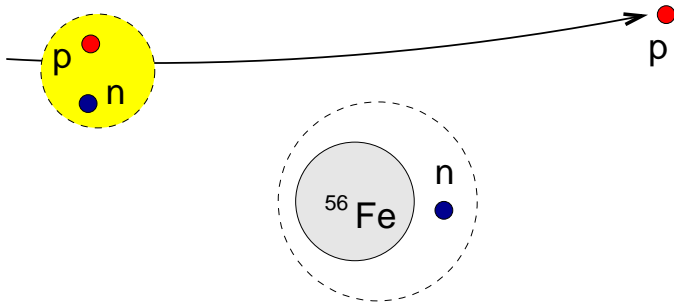
➡ Angular distributions of transfer cross sections is very sensitive to the single-particle configuration of the transferred nucleon/s.

## *Brief summary on transfer reactions*

- Inclusion of transfer couplings in the Schrodinger equation gives rise to a set of coupled equations with non-local kernels (Coupled Reactions Channels)
- If transfer couplings are weak, the CRC equations can be solved in Born approximation  $\Rightarrow$  DWBA approximation
- The DWBA amplitude is proportional to the product of the projectile and target spectroscopic factors.
- The analysis of transfer reactions provide information on:
  - ❖ Spectroscopic factors
  - ❖ Quantum number for single-particle configurations  $(n, \ell, j)$ .

## Transfer example

Physical example:  $^{56}\text{Fe}(d,p)^{57}\text{Fe}$  at  $E_d = 12\text{ MeV}$



## ***Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$***

DWBA transfer amplitude:

$$T^{\text{DWBA}} = A_i A_f \langle \chi_{p-^{57}\text{Fe}}^{(-)} \phi_{^{57}\text{Fe}} | V_{\text{prior/post}} | \chi_{d-^{56}\text{Fe}}^{(+)} \phi_d \rangle$$

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

DWBA transfer amplitude:

$$T^{\text{DWBA}} = A_i A_f \langle \chi_{p-^{57}\text{Fe}}^{(-)} \phi_{^{57}\text{Fe}} | V_{\text{prior/post}} | \chi_{d-^{56}\text{Fe}}^{(+)} \phi_d \rangle$$

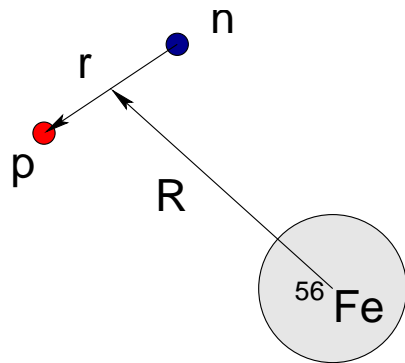
- $\chi_{d-^{56}\text{Fe}}, \chi_{p-^{57}\text{Fe}}$ : initial and final distorted waves
- $\phi_d$ : projectile bound wavefunction(  $p - n$ )
- $\phi_{^{57}\text{Fe}}$ : final (residual) wavefunction ( $n+^{56}\text{Fe}$ )
- $A_i, A_f$ : initial / final spectroscopic amplitudes.
- $V_{\text{prior/post}}$ : transition potential in PRIOR or POST form

# Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

DWBA transfer amplitude:

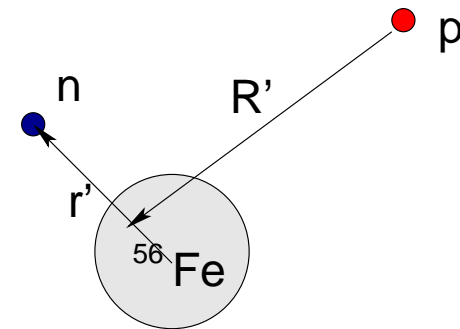
$$T^{\text{DWBA}} = A_i A_f \langle \chi_{p-^{57}\text{Fe}}^{(-)} \phi_{^{57}\text{Fe}} | V_{\text{prior/post}} | \chi_{d-^{56}\text{Fe}}^{(+)} \phi_d \rangle$$

PRIOR



$$V_{\text{prior}} = V_{n-^{56}\text{Fe}} + \underbrace{U_{p-^{56}\text{Fe}} - U_{d-^{56}\text{Fe}}}_{\text{remnant}}$$

POST



$$V_{\text{post}} = V_{p-n} + \underbrace{U_{p-^{56}\text{Fe}} - U_{p-^{57}\text{Fe}}}_{\text{remnant}}$$



## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### Essential physical ingredients in a DWBA calculation:

- Potentials (5):

- Distorted potential for entrance channel (complex):  $d+^{56}\text{Fe}$
- Distorted potential for exit channel (complex):  $p+^{57}\text{Fe}$
- Core-core interaction (complex):  $p+^{56}\text{Fe}$
- Binding potential for projectile (real):  $p+n$
- Binding potential for target (real):  $n+^{56}\text{Fe}$

- Spectroscopic amplitudes:  $A_i, A_f$

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

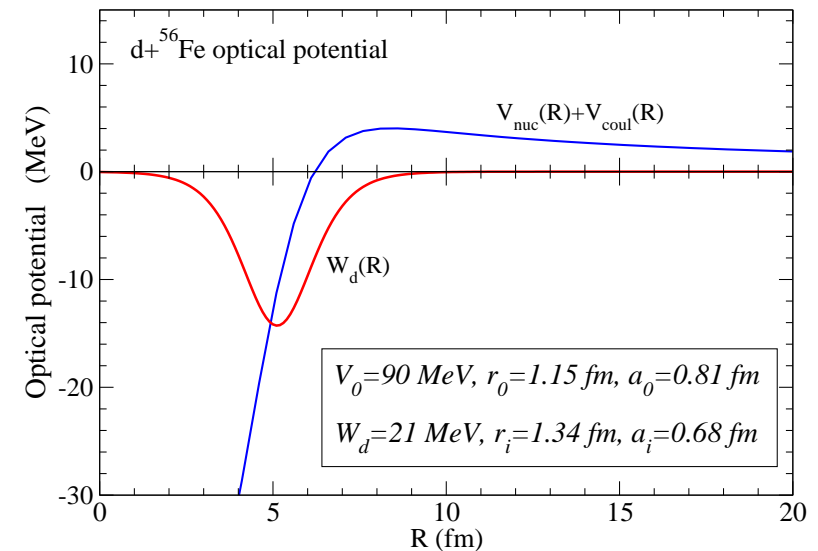
Physical ingredients: Optical and binding potentials (NPA(1971) 529)

System	$V_0$ (MeV)	$r_0$ (fm)	$a_0$ (fm)	$W_d$ (MeV)	$r_i$ (fm)	$a_i$ (fm)	$r_C$ (fm)
$d+^{56}\text{Fe}$	90	1.15	0.81	21.0	1.34	0.68	1.15
$p+^{56,57}\text{Fe}$	47.9	1.25	0.65	11.5	1.25	0.47	1.15
$p+n^1$	72.15	0.00	1.484	-	-	-	-
$n+^{56}\text{Fe}$	B.E.	1.25	0.65	-	-	-	-

$$\Rightarrow U(R) = -V_0 f_{WS}(R) + 4i a W_d \frac{df_{WS}(R)}{dR}$$

$$f_{WS}(R) = \frac{1}{1 + \exp\left(\frac{R-R_0}{a}\right)}$$

<sup>1</sup>Gaussian geometry:  $V(r) = -V_0 \exp[-(r/a_0)^2]$ .



## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Input example for  $^{56}\text{Fe}(d,p)^{57}\text{Fe}$ : fe56dp\_dwba.in

```
6Fe(d,p)57Fe @ Ed=12 MeV;
AMELIST
FRESCO hcm=0.1 rmatch=20.000
      rintp=0.20 hnl=0.100 rnl=4 centre=-0.45
      jtmax=15
      thmin=1.00 thmax=180.00 thinc=1.00
      it0=1 iter=1
      chans=1 smats=2 xstabl=1
      elab= 12 /

PARTITION namep='d' massp=2.014 zp=1 namet='56Fe'
      masst=55.934 zt=26 nex=1 /
STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0
      bandt=1 et=0.0 /

PARTITION namep='p' massp=1.0078 zp=1 namet='57Fe'
      masst=56.935 zt=26 qval=5.421 pwf=F nex=1 /
STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5
      bandt=-1 et=0.0 /
partition /

POT kp=1 itt=F at=56 rc=1.15 /
POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /
POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /

POT kp=2 itt=F at=57 rc=1.15 /
POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

```
POT kp=3 itt=F at=56 rc=1.00 /
POT kp=3 type=1 itt=F p1=65.0 p2=1.25 p3=0.65 /
POT kp=4 itt=F ap=1.0000 at=0.0000 rc=1.0000 /
POT kp=4 type=1 shape=2 itt=F p1=72.1500
      p2=0.0000 p3=1.4840 /
POT kp=5 itt=F at=56 rc=1.15 /
POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /
POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
pot /

OVERLAP knl=1 icl=1 ic2=2 in=1 nn=1 sn=0.5
      j=0.5 kbpot=4 be=2.2250 isc=1 /
OVERLAP knl=2 icl=1 ic2=2 in=2 nn=2 l=1 sn=0.5
      j=0.5 kbpot=3 be=7.646 isc=1 /
overlap /

COUPLING icto=2 icfrom=1 kind=7 ip2=-1 ip3=5 /
CFP in=1 ib=1 ia=1 kn=1 a=1.0000 /
CFP in=2 ib=1 ia=1 kn=2 a=1.0000 /
cfp /

coupling /
```

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### General variables:

```
56Fe(d,p)57Fe @ Ed=12 MeV;  
NAMELIST  
&FRESCO hcm=0.1 rmatch=20.000  
        rintp=0.20 hnl=0.100 rnl=4 centre=-0.45  
        jtmax=15  
        thmin=1.00 thmax=180.00 thinc=1.00  
        it0=1 iter=1  
        chans=1 smats=2 xstabl=1  
        elab= 12 /
```

- **rnl**: range of non-locality
- **centre, rintp, hnl**: parameters for numerical integration (see fresco manual)
- **iter**: Number of iterations so, for DWBA, **iter**=1

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### Partitions and states:

- Incoming (initial) partition:  $d+^{56}\text{Fe}$

```
&PARTITION namep='d' massp=2.014 zp=1  
          namet='56Fe' masst=55.934 zt=26 nex=1 /  
&STATES jp=1.0 bandp=1 ep=0.0 cpot=1 jt=0.0 bandt=1 et=0.0 /
```

- Outgoing (final) partition:  $p+^{57}\text{Fe}$

```
&PARTITION namep='p' massp=1.0078 zp=1  
          namet='57Fe' masst=56.935 zt=26  
          qval=5.421 nex=1 /  
&STATES jp=0.5 bandp=1 ep=0.0 cpot=2 jt=0.5 bandt=-1 et=0.0 /
```

- **qval**: Q-value for gs-gs transfer

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### Interactions:

- Entrance channel distorted potential:  $d+^{56}\text{Fe}$

```
&POT kp=1 itt=F at=56 rc=1.15 /  
&POT kp=1 type=1 itt=F p1=90 p2=1.15 p3=0.81 /  
&POT kp=1 type=2 itt=F p4=21 p5=1.34 p6=0.68 /
```

- Exit channel distorted potential:  $p+^{57}\text{Fe}$

```
&POT kp=2 itt=F at=57 rc=1.15 /  
&POT kp=2 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /  
&POT kp=2 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

- Core-core potential:  $p+^{56}\text{Fe}$

```
&POT kp=5 itt=F at=56 rc=1.15 /  
&POT kp=5 type=1 itt=F p1=47.9 p2=1.25 p3=0.65 /  
&POT kp=5 type=2 itt=F p4=11.5 p5=1.25 p6=0.47 /
```

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### Interactions (continued ...)

#### Binding potentials:

- $n + ^{56}\text{Fe}$ : Woods-Saxon

```
&POT kp=3 at=56 rc=1.0 /  
&POT kp=3 type=1 p1=65.0 p2=1.25 p3=0.65 /
```

- $n+p$ : Gaussian

```
&POT kp=4 ap=1.0 at=0.0 /  
&POT kp=4 type=1 shape=2 p1=72.15 p2=0.00 p3=1.484 /
```

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### Bound wavefunctions (overlaps):

- $d=p+n$ : simple 1S model

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 l=0 sn=0.5 j=0.5  
          kbpot=4 be=2.2250 isc=1 /
```

- $^{57}\text{Fe}=^{56}\text{Fe}+n$ : assume  $2p_{1/2}$  configuration

```
&OVERLAP kn1=2 ic1=1 ic2=2 in=2 nn=2 l=1 sn=0.5 j=0.5  
          kbpot=3 be=7.646 isc=1 /
```

- ❖  $in=1$ : projectile  
 $in=2$ : target
- ❖  $nn, l, sn, j$ : quantum numbers:  $\vec{l} + s\vec{n} = \vec{j}$
- ❖  $be$ : binding (separation) energy
- ❖  $kbpot$ : potential index



## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Transfer coupling between the two partitions:

```
&COUPLING icfrom=1 icto=2 kind=7 ip1=0 ip2=-1 ip3=5 /
```

- `icfrom`: index for partition of initial state
- `icto`: index for partition of final state
- `kind`: kind of coupling. `kind=7` means finite-range transfer.
- `ip1=0`: post representation  
    `ip1=1`: prior
- `ip2=-1`: include full remnant
- `ip3`: index for core-core potential ( $p+^{56}\text{Fe}$ )

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Spectroscopic factors:

$$\phi_B^{JM}(\xi, \mathbf{r}) = \sum_{I\ell j} A_{\ell sj}^{IJ} \left[ \phi_b^I(\xi) \otimes \varphi_{\ell sj}(\mathbf{r}) \right]_{JM}$$

So, for example:

$$|^{57}\text{Fe}; 1/2^- \rangle = \alpha [ |^{56}\text{Fe}; \text{gs} \rangle \otimes | \nu 2p_{1/2} \rangle ]_{1/2^-} + \beta [ |^{56}\text{Fe}; 2^+ \rangle \otimes | \nu 2p_{3/2} \rangle ]_{1/2^-} + \dots$$

- $\alpha, \beta, \dots$ : spectroscopic amplitudes
- $|\alpha|^2, |\beta|^2, \dots$ : spectroscopic factors

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

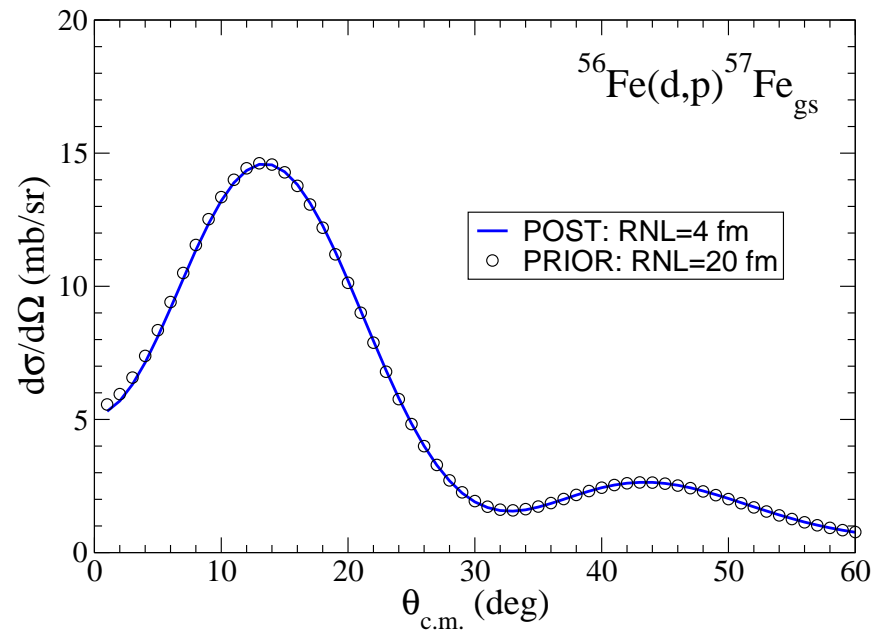
### Spectroscopic amplitudes:

```
&CFP in=1 ib=1 ia=1 kn=1 a=1.0 /  
&CFP in=2 ib=1 ia=1 kn=2 a=1.0 /
```

- **in=1**: projectile state  
**in=2**: target state
- **ib**: index for state of composite  
**ia**: index for state of core
- **a**: spectroscopic amplitude

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Post and prior equivalence:

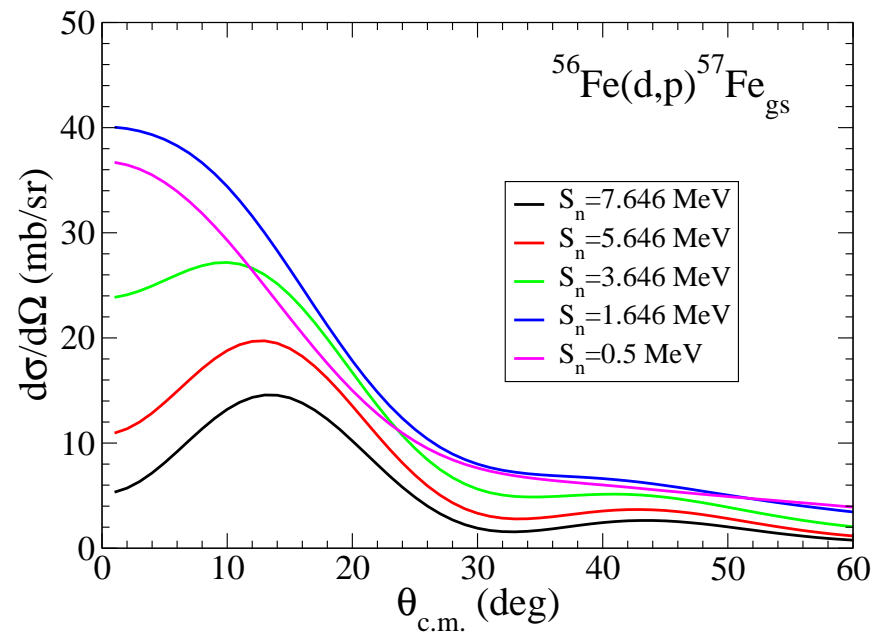


➡ *Post and prior give identical results, provide that the parameters are adequate for convergence.*

## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Dependence with binding energy:

```
&OVERLAP kn1=2 (...) be=7.646 /  
&OVERLAP kn1=2 (...) be=5.646 /  
&OVERLAP kn1=2 (...) be=5.646 /  
&OVERLAP kn1=2 (...) be=3.646 /  
&OVERLAP kn1=2 (...) be=1.646 /  
&OVERLAP kn1=2 (...) be=0.100 /
```



## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

Dependence with beam energy:

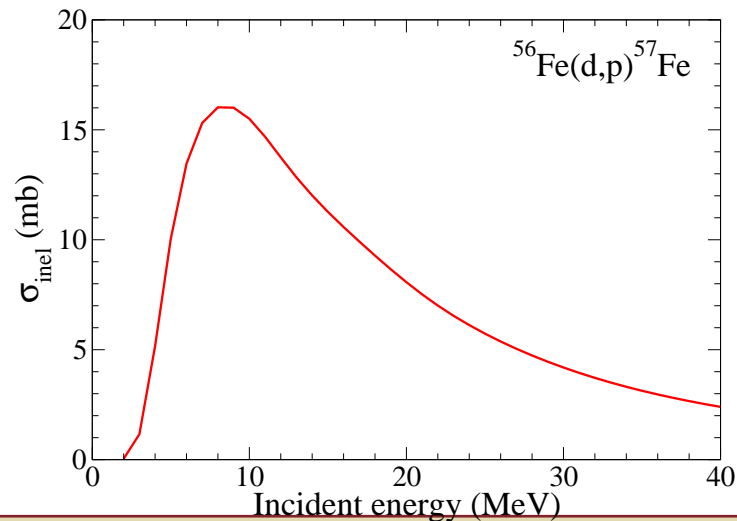
- Calculating several energies in one single run. In `fe56dp_ebeam.in`:

```
&FRESKO (...) elab(1:4)= 2 40 0 0 nlab(1:3)= 38 0 0 /
```

will run FRESKO for  $E_{\text{lab}}=2, 3, 4, \dots, 40$  MeV.

- Extracting total inelastic cross sections from main output:

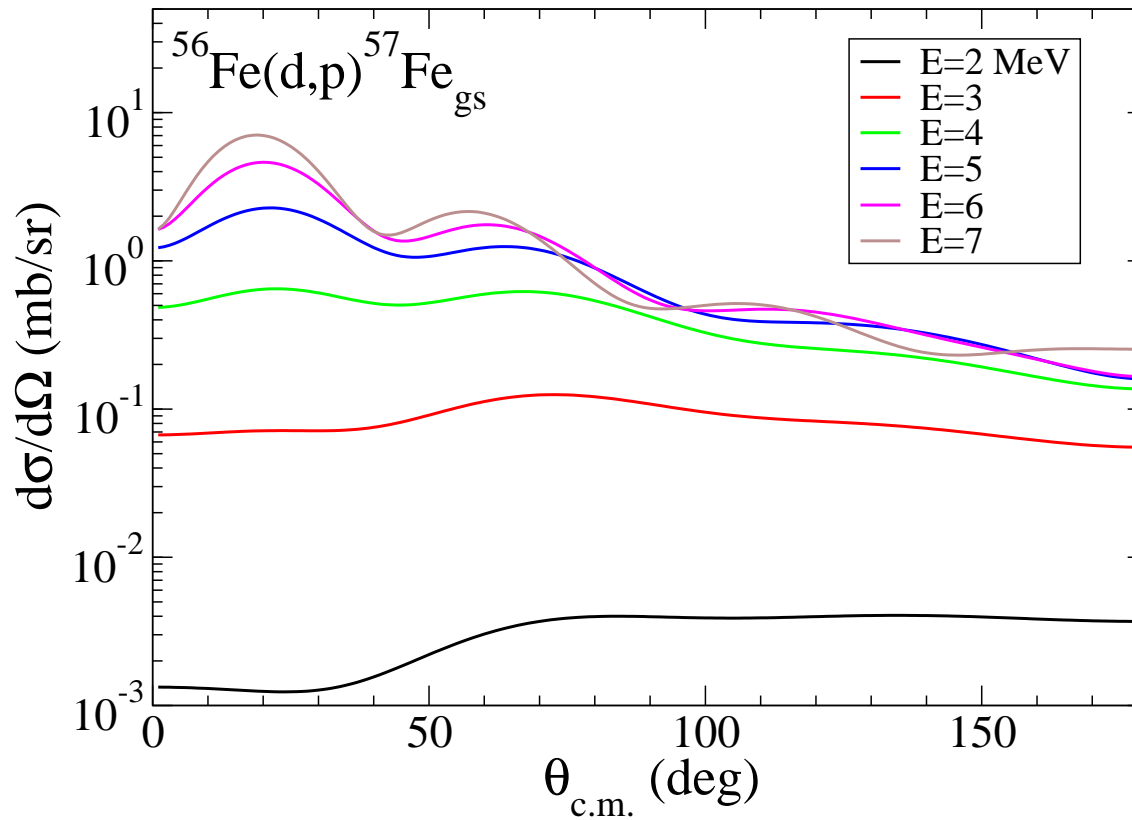
```
grep "partition 2" fe56dp_ebeam.out | awk '{print NR+1, $9}' > xsec_vs_elab.dat
```



## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### Dependence with beam energy

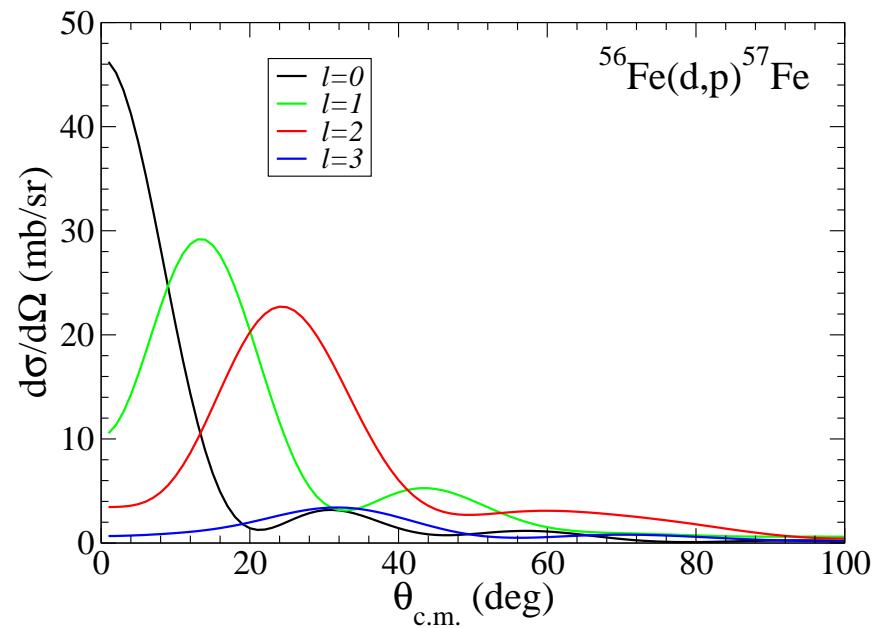
- $E \gg V_b$ : diffractive structure, forward peaked.
- $E \ll V_b$ : smooth dependence with  $\theta$ , backward peaked.



## Transfer example: $^{56}\text{Fe}(d,p)^{57}\text{Fe}$

### Selectivity of $\ell$ :

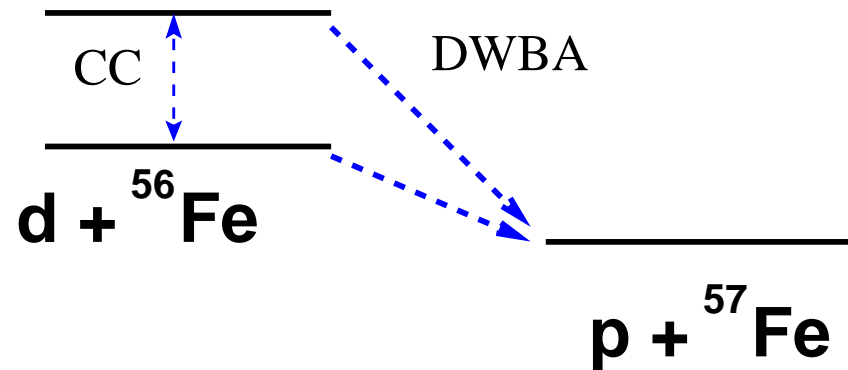
```
&OVERLAP (...) nn=2 l=0 sn=0.5 j=0.5 kbpot=3 be=7.646 /  
&OVERLAP (...) nn=2 l=1 sn=0.5 j=1.5 kbpot=3 be=7.646 /  
&OVERLAP (...) nn=2 l=2 sn=0.5 j=2.5 kbpot=3 be=7.646 /  
&OVERLAP (...) nn=1 l=3 sn=0.5 j=2.5 kbpot=3 be=7.646 /  
&OVERLAP (...) nn=1 l=4 sn=0.5 j=4.5 kbpot=3 be=7.646 /
```





## Beyond DWBA: CCBA formalism

When there are strongly coupled excited states in the initial or final partition, the CC and DWBA formalisms can be combined → CCBA



## **Proposed exercise:** $^{208}\text{Pb}(^8\text{Li}, ^7\text{Li})^{209}\text{Pb}$

**Proposed exercise:** Calculate the transfer differential cross section angular distribution for 1 neutron transfer reaction  $^{208}\text{Pb}(^8\text{Li}, ^7\text{Li})^{209}\text{Pb}$ , leading to the ground state of the  $^{209}\text{Pb}$  nucleus ( $J^\pi = 9/2^+$ ).

Ingredients:

- for the optical potentials, you may use those of the previous exercises.
- for the bound wavefunctions, use a WS potential with  $r_0 = 1.25$  fm and  $a_0 = 0.65$  fm.
- assume unit spectroscopic factors for the initial and final states.

## **Proposed exercise:** $^{208}\text{Pb}(^8\text{Li}, ^7\text{Li})^{209}\text{Pb}$

**Input example for  $^{208}\text{Pb}(^8\text{Li}, ^7\text{Li})^{209}\text{Pb}$ :** `li8pb_trans.in`

```
208Pb(8Li,7Li)209Pb(gs) input example
NAMELIST
&FRESKO hcm=0.05 rmatch=60.000 rintp=0.2
      hnl=0.025 rnl=3 centre=0.00 jtmin=0.0 jtmax=80.
      thmin=2.00 thmax=180.00 thinc=2.00 it0=1 iter=1 smats=2
      xstabl=1 elab=34.404 /

&PARTITION namep='8Li' massp=8.0225 zp=3
      namet='208Pb' masst=207.9766 zt=82 qval=0.0000 pwf=T
      nex=1 /
&STATES jp=2.0 bandp=1 ep=0.0000 kkp=1.0 cpot=1
      jt=0.0 bandt=1 et=0.0000 kkt=0.0 fexch=F /

&PARTITION namep='7Li' massp=7.0160 zp=3
      namet='209Pb' masst=208.9810 zt=82 qval=1.9040 pwf=T
      nex=1 /
&STATES jp=1.5 bandp=-1 ep=0.0000 cpot=2 jt=4.5
      bandt=1 et=0.0000 fexch=F /

&partition /
```

## Proposed exercise: $^{208}\text{Pb}(^8\text{Li}, ^7\text{Li})^{209}\text{Pb}$

(continued ...)

```
&POT kp=1 ap=8 at=208 rc=1.25 /
&POT kp=1 type=1 itt=F p1=15.4 p2=1.3 p3=0.65 p4=58.3 p5=1.3 p6=0.7 /

&POT kp=2 itt=F at=209.000 ap=7.000 rc=1.250 /
&POT kp=2 type=1 itt=F p1=15.4 p2=1.300 p3=0.650
      p4=13.2 p5=1.3 p6=0.65 p7=0.000 /

&POT kp=3 type=0 shape=0 itt=F ap=7 at=0.000 /
&POT kp=3 type=1 shape=0 itt=F p1=44.675
      p2=1.25 p3=0.65 /

&POT kp=4 itt=F ap=208.000 at=0.000 rc=1.250 /
&POT kp=4 type=1 itt=F p1=60.000 p2=1.250
      p3=0.650 /

&POT kp=5 itt=F at=208.000 ap=7.000 rc=1.250 /
&POT kp=5 type=1 itt=F p1=15.4 p2=1.300 p3=0.650
      p4=13.2 p5=1.300 p6=0.65 p7=0.000 /
&pot /
```

## Proposed exercise: $^{208}\text{Pb}(^8\text{Li}, ^7\text{Li})^{209}\text{Pb}$

(continued ...)

```
&OVERLAP kn1=1 ic1=1 ic2=2 in=1 nn=1 l=1 sn=0.5  
  ia=1 j=1.5 ib=1 kbpot=3 be=2.0330 isc=1  
  ampl=0.0000 /  
&OVERLAP kn1=5 ic1=1 ic2=2 in=2 nn=2 l=4 sn=0.5  
  j=4.5 kbpot=4 be=3.9440 isc=1 nam=1 ampl=0.0000 /  
&overlap /  
  
&COUPLING icto=-2 icfrom=1 kind=7 ip2=-1 ip3=5 /  
&CFP in=1 ib=1 ia=1 kn=1 a=1 /  
&CFP in=2 ib=1 ia=1 kn=5 a=0.8450 /  
&cfp /  
  
&coupling /
```

## Lecture 4: Calculations including continuum states

- ❖ Halo and borromean nuclei
- ❖ Continuum discretization
- ❖ CDCC formalism
- ❖ Bin wavefunction
- ❖ Application to deuteron scattering
- ❖ CDCC applied to exclusive breakup
- ❖ Application to  ${}^6\text{He}$  and  ${}^6\text{Li}$  scattering
- ❖ Four body CDCC calculations
- ❖ Spectroscopy to unbound states
- ❖ The  ${}^{10}\text{Li}$  and  ${}^{11}\text{Li}$  systems
- ❖  ${}^9\text{Li} + d \rightarrow {}^{10}\text{Li} + p$

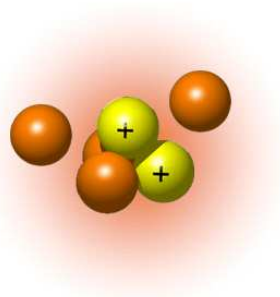
# Lecture 4: Calculations including continuum states

## Exotic nuclei, halo nuclei and Borromean systems

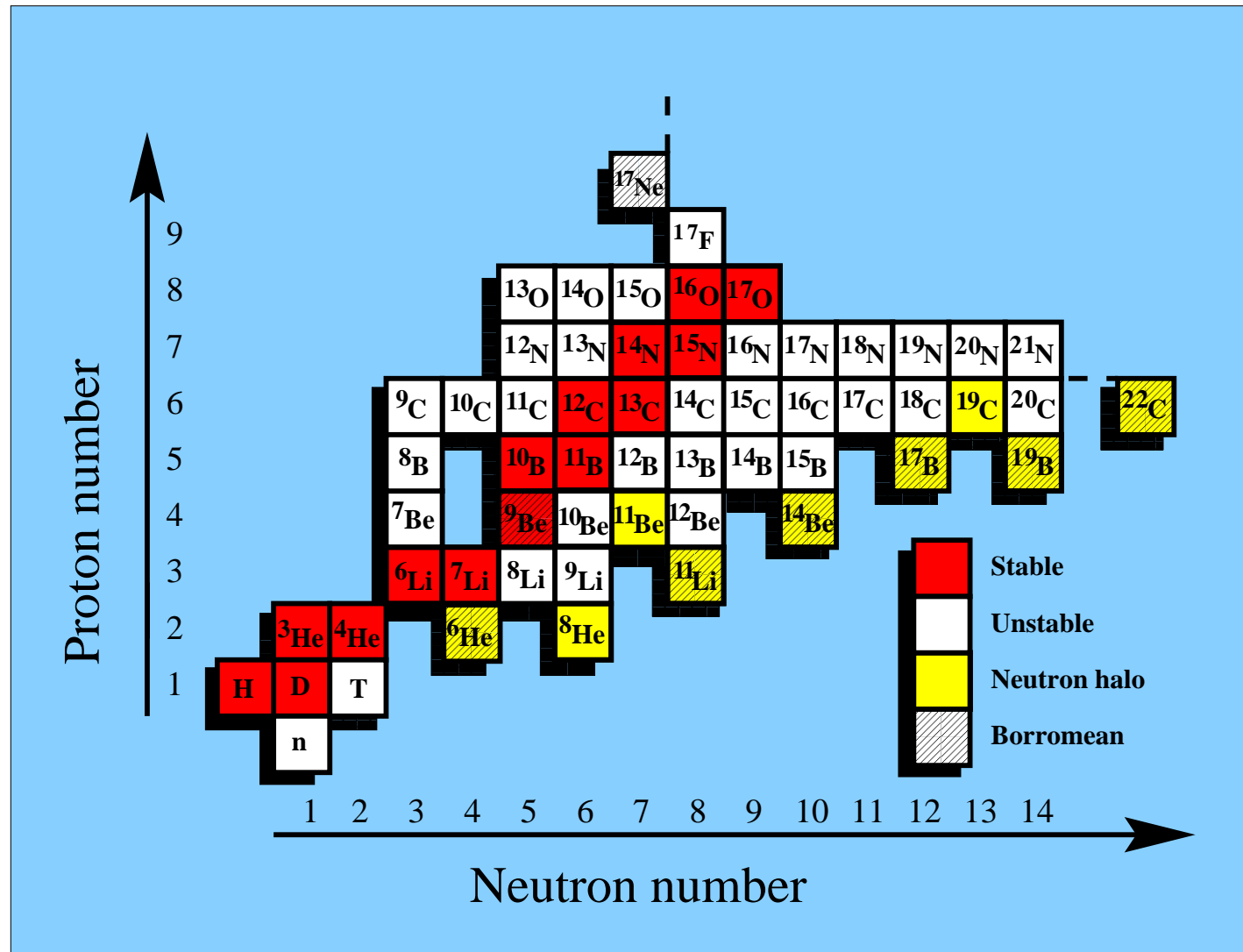
- **Radioactive nuclei:** they typically decay by  $\beta$  emission.

**E.g.:**  ${}^6\text{He} \xrightarrow{\beta^-} {}^6\text{Li} \quad (t_{1/2} \simeq 807 \text{ ms})$

- **Weakly bound:** typical separation energies are around 1 MeV or less.
- **Spatially extended**
- **Halo structure:** one or two weakly bound nucleons (typically neutrons) with a large probability of presence beyond the range of the potential.
- **Borromean nuclei:** Three-body systems with no bound binary sub-systems.



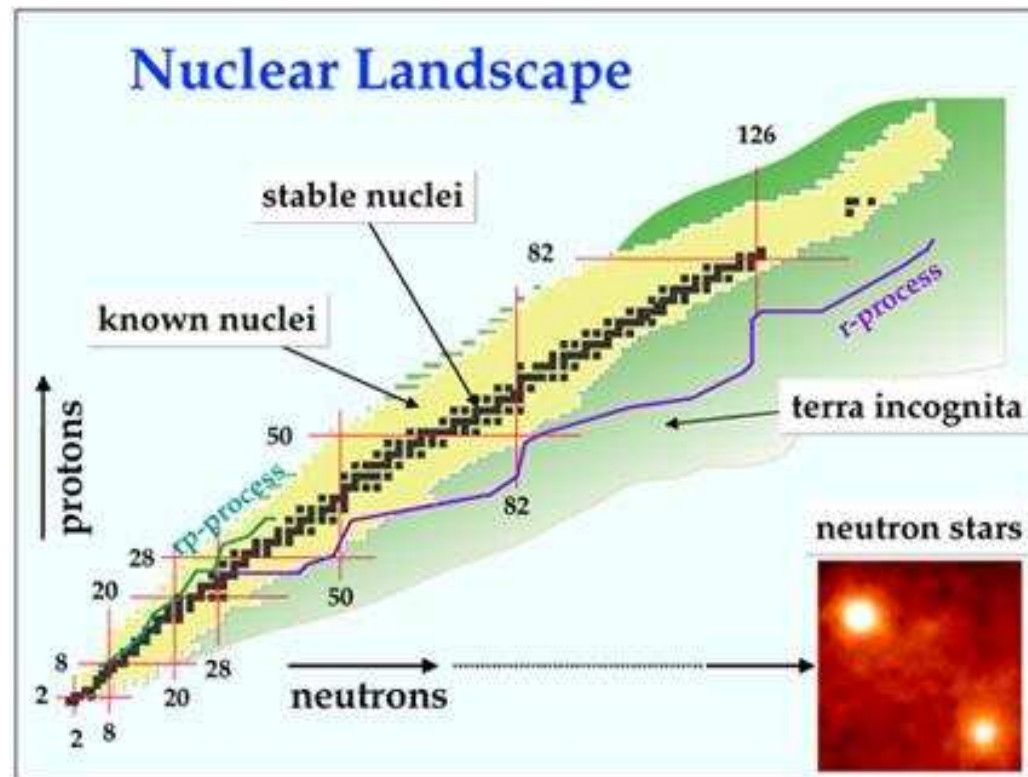
# Exotic nuclei, halo nuclei, and Borromean systems





## Why study reactions with exotic nuclei?

- ➡ Many properties of nuclear structure (level spacing, magic numbers, etc) could be different from normal nuclei.
- ➡ Many reactions of astrophysical interest are known to involve nuclei far from the stability valley.



## *Some difficulties inherent to the study of reactions with exotic beams*

### Experimentally:

- Exotic nuclei are short-lived and difficult to produce. Beam intensities are typically small.

### Theoretically:

- Exotic nuclei are easily broken up in nuclear collisions  $\Rightarrow$  coupling to the continuum plays an important role.
- Effective NN interactions, level schemes, etc are different from stable nuclei.
- Many exotic nuclei exhibit complicated cluster (few-body) structure.

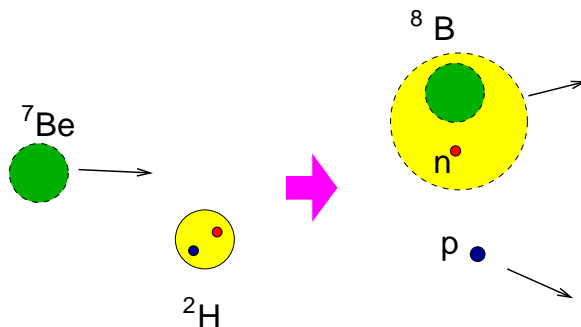
## Indirect measurements for nuclear astrophysics

- Many reactions of astrophysical interest at energies too low to be measured e at present experimental facilities.
- Coulomb breakup and transfer reactions provide indirect information for these processes.

**Example:** The rate for the  ${}^7\text{Be} + p \rightarrow {}^8\text{B} + \gamma$  reaction depends mainly on the overlap ( ${}^8\text{B} | {}^7\text{Be}$ ), which can be investigated by means of other direct reactions:

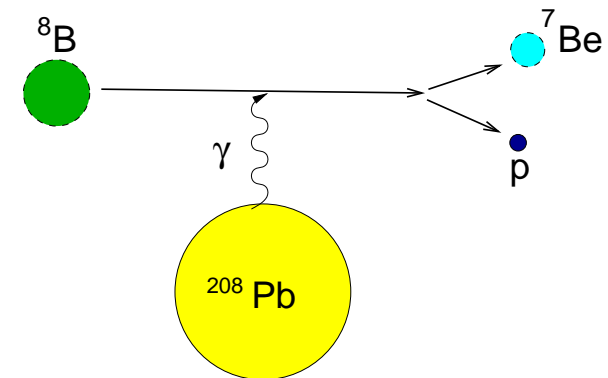
Transfer reactions:

E.g.:  $d({}^7\text{Be}, {}^8\text{B})n$



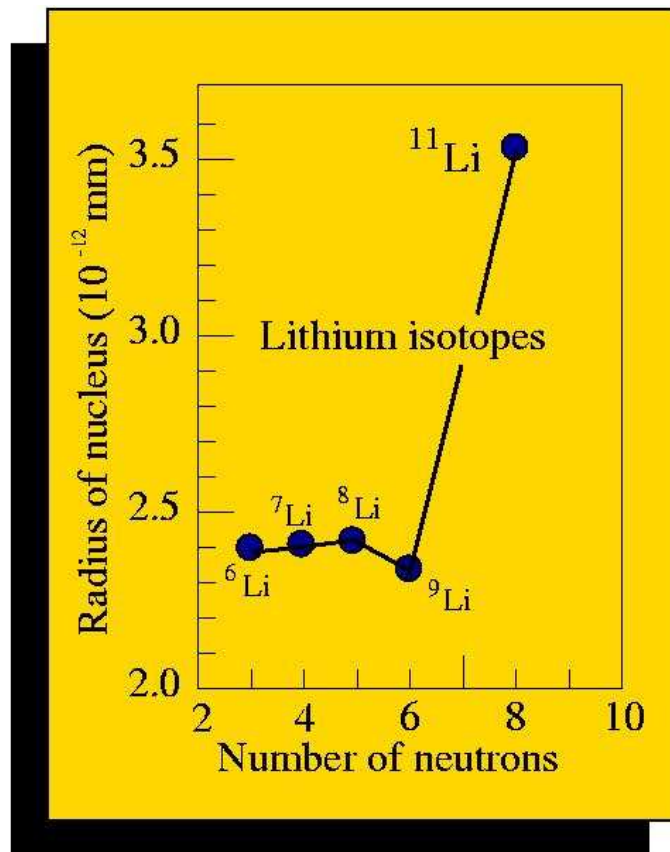
Coulomb breakup reactions:

E.g.:  ${}^8\text{B} + {}^{208}\text{Pb} \rightarrow {}^7\text{Be} + p + {}^{208}\text{Pb}$



## Some difficulties inherent to the study of reactions with exotic beams

➡ First evidences of the existence of halo nuclei came from reaction cross sections measurements.

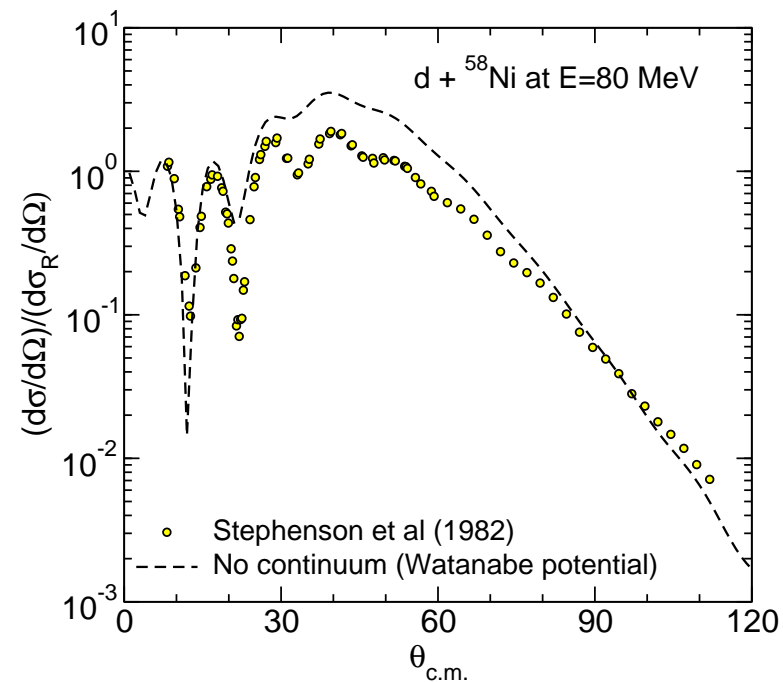


Tanihata *et al*, PRL55, 2676 (1985)

## Failure of the calculations without continuum

Three-body calculation (p+n+<sup>58</sup>Ni) with Watanabe potential:

$$V_{00}(\mathbf{R}) = \int d\mathbf{r} \phi_{gs}(\mathbf{r}) (V_{pt} + V_{nt}) \phi_{gs}(\mathbf{r})$$



☞ *Three-body calculations omitting breakup channels fail to describe the experimental data.*

# *The role of the continuum in the scattering of weakly bound nuclei*

## The origins of the CDCC method:

- Pioneering work of Johnson & Soper for deuteron scattering: PRC1,976(1970)  $\Rightarrow$  p-n continuum represented by a single s-wave state.

PHYSICAL REVIEW C

VOLUME 1, NUMBER 3

MARCH 1970

### **Contribution of Deuteron Breakup Channels to Deuteron Stripping and Elastic Scattering**

R. C. JOHNSON

*Department of Physics, University of Surrey, Guildford, Surrey, England*

AND

P. J. R. SOPER\*

*International Centre for Theoretical Physics, Trieste, Italy*

(Received 10 November 1969)

We present a model of deuteron stripping and elastic scattering which treats explicitly the contributions from channels in which the deuteron is broken up into a relative  $S$  state and the target is in its ground state. An adiabatic treatment of these channels leads to a description of deuteron stripping which resembles

# The role of the continuum in the scattering of weakly bound nuclei

- More realistic formulation by G.H. Rawitscher [ PRC9, 2210 (1974)] and Farrell, Vincent and Austern [Ann.Phys.(New York) 96, 333 (1976)].

PHYSICAL REVIEW C

VOLUME 9, NUMBER 6

JUNE 1974

## Effect of deuteron breakup on elastic deuteron-nucleus scattering

George H. Rawitscher\*

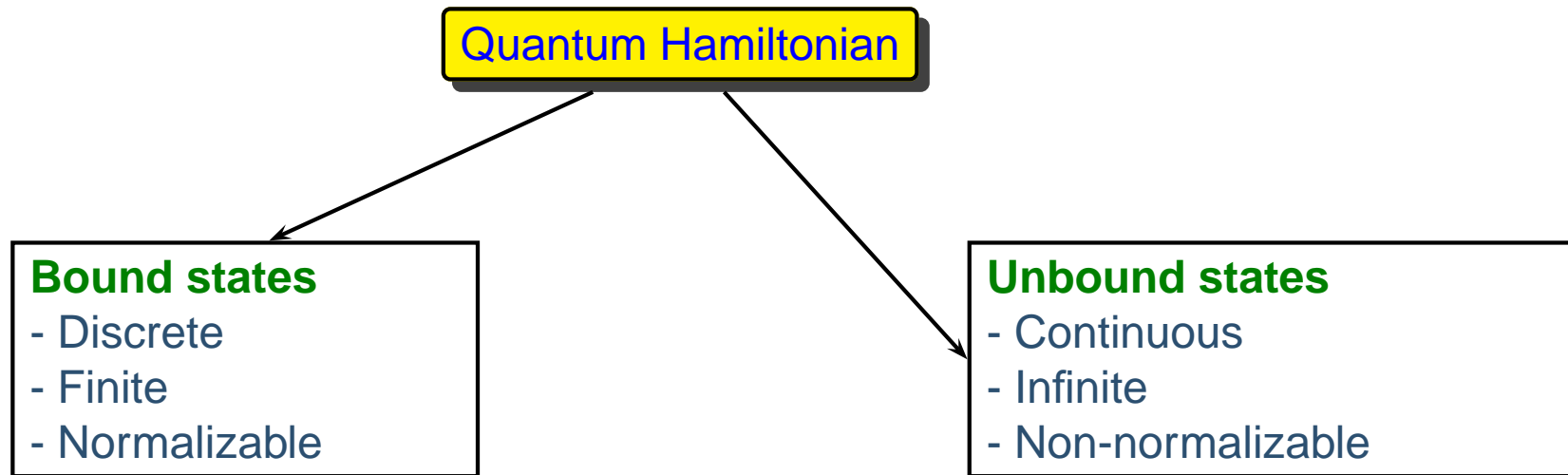
*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139,<sup>†</sup>  
and Department of Physics, University of Surrey, Guildford, Surrey, England*

(Received 1 October 1973; revised manuscript received 4 March 1974)

The properties of the transition matrix elements  $V_{ab}(R)$  of the breakup potential  $V_N$  taken between states  $\phi_a(\vec{r})$  and  $\phi_b(r)$  are examined. Here  $\phi_a(\vec{r})$  are eigenstates of the neutron-proton relative-motion Hamiltonian, and the eigenvalues of the energy  $\epsilon_a$  are positive (continuum states) or negative (bound deuteron);  $V_N(\vec{r}, \vec{R})$  is the sum of the phenomenological proton nucleus  $V_{p-A}(|\vec{R} - \frac{1}{2}\vec{r}|)$  and neutron nucleus  $V_{n-A}(|\vec{R} + \frac{1}{2}\vec{r}|)$  optical potentials evaluated for nucleon energies equal to half the incident deuteron energy. The bound-to-continuum transi-

- Full numerical implementation by Kyushu group (Sakuragi, Yahiro, Kamimura, and co.): Prog. Theor. Phys.(Kyoto) 68, 322 (1982)

## *Inclusion of the continuum in CC calculations: continuum discretization*



**Continuum discretization:** represent the continuum by a finite set of square-integrable states

<b>True continuum</b>	→	<b>Discretized continuum</b>
Non normalizable	→	Normalizable
Continuous	→	Discrete

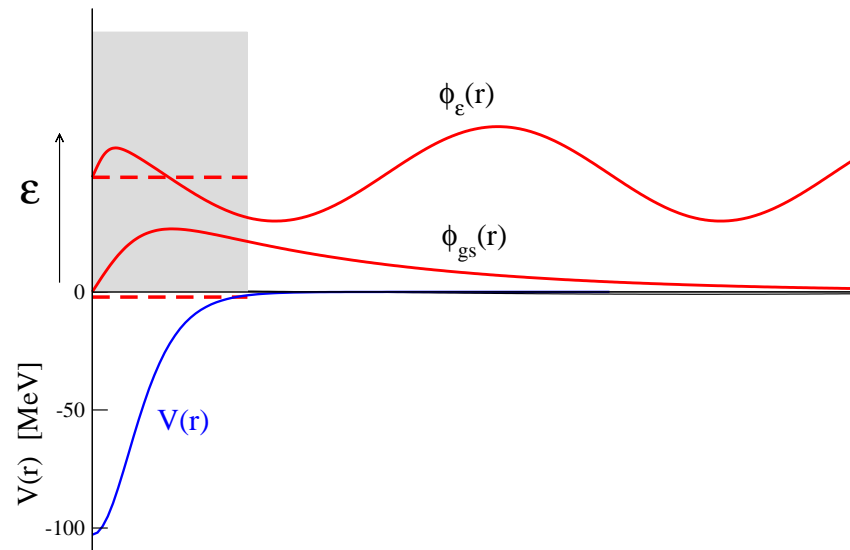


# *Continuum discretization*

## SOME POPULAR METHODS OF CONTINUUM DISCRETIZATION:

- **Box method:**  
Eigenstates of the  $H$  in a large box.
- **Sturmian basis**
- **Gamow states:** complex-energy eigenstates of the Schrödinger equation.
- **Pseudostate method:**  
Expand continuum states in a complete basis of square-integrable states (eg. HO)
- **Bin method:**  
Square-integrable states constructed from scattering states.

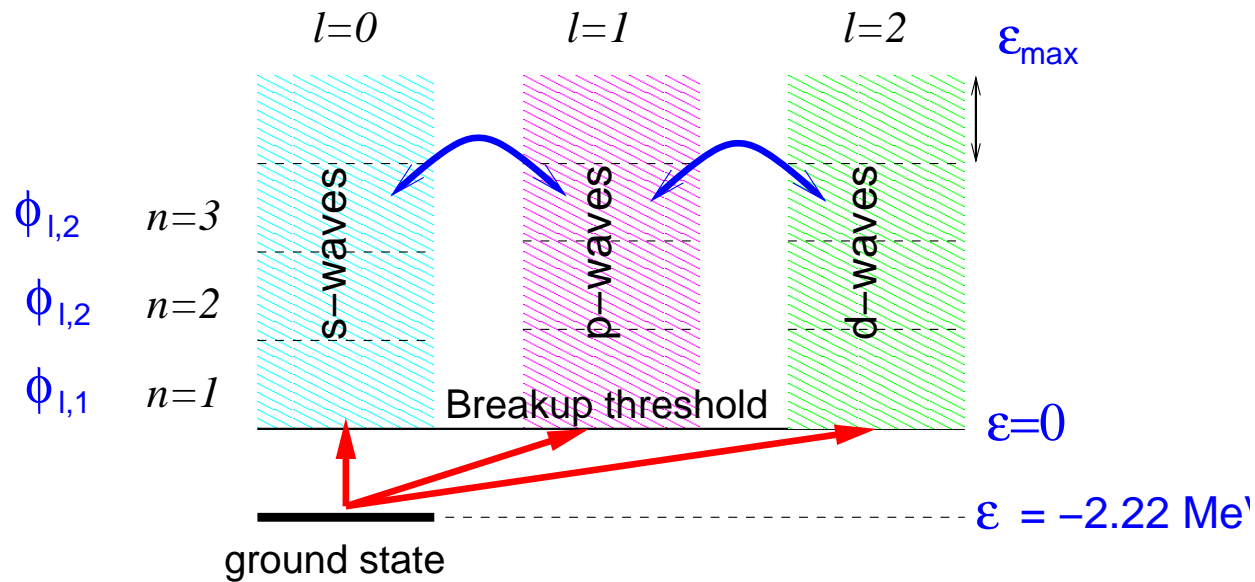
## Bound versus scattering states



Unbound states are not suitable for CC calculations:

- Continuous (infinite) distribution in energy.
- Non-normalizable:  $\langle \phi_k(r)^* | \phi_{k'}(r) \rangle \propto \delta(k - k')$

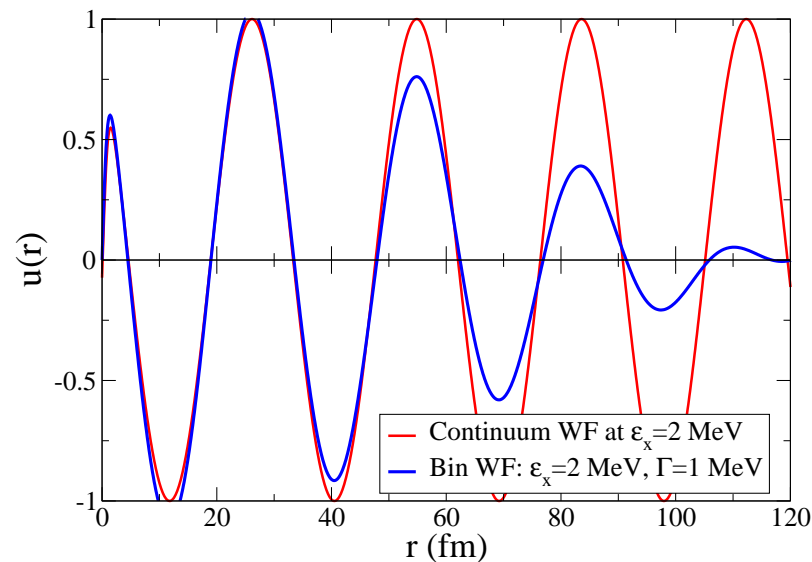
**Example:** discretization of the deuteron continuum in terms of energy bins.



Bin wavefunction:

$$u_{\ell sj,n}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{\ell sj,k}(r) dk$$

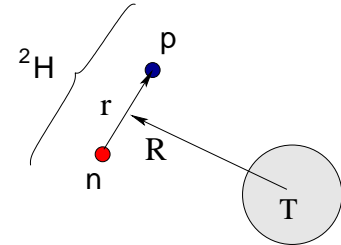
- $k$ : linear momentum
- $u_{\ell sj,k}$ : scattering states (radial part)
- $w(k)$ : weight function



# CDCC formalism for deuteron scattering

- **Hamiltonian:**  $H = T_R + h_r + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$
- **Model wavefunction:**

$$\Psi(\mathbf{R}, \mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^N \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$



- **Coupled equations:**  $[H - E]\Psi(\mathbf{R}, \mathbf{r})$

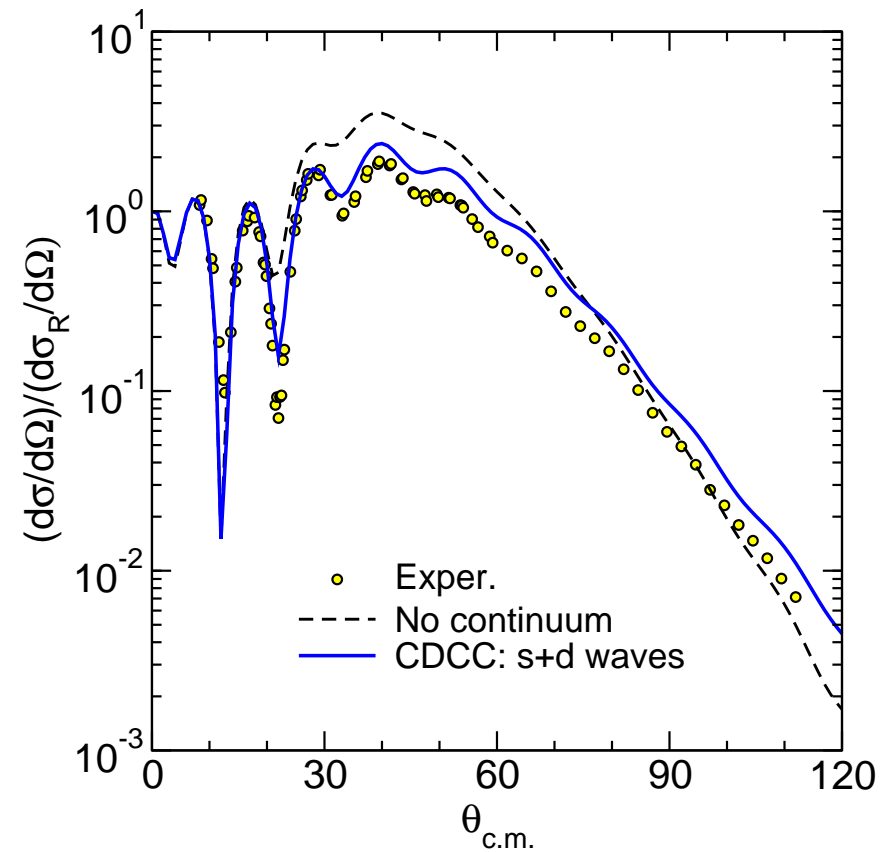
$$\left[ -\frac{\hbar^2}{2\mu} \left( \frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + \epsilon_n - E \right] f_{\alpha J}(R) + \sum_{\alpha'} i^{L'-L} V_{\alpha:\alpha'}^J(R) f_{\alpha' J}(R) = 0$$

$$\alpha = \{L, \ell, s, j, n\}$$

- **Transition potentials:**

$$V_{\alpha;\alpha'}^J(\mathbf{R}) = \int d\mathbf{r} \phi_{\alpha}(\mathbf{r})^* \left[ V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2}) \right] \phi_{\alpha'}(\mathbf{r})$$

## Application of the CDCC formalism: $d+^{58}\text{Ni}$

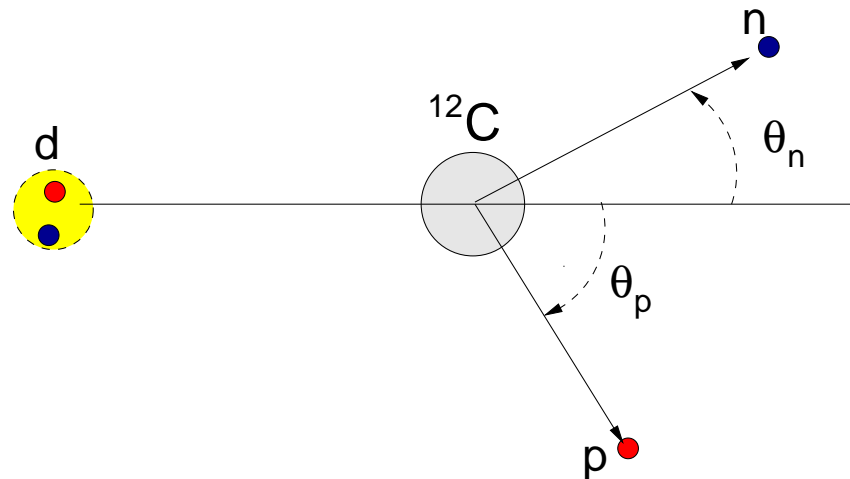


➡ Inclusion of the continuum is important to describe the data

## Application of the CDCC formalism: $d+{}^{58}\text{Ni}$

Observables for exclusive breakup can be obtained from the S-matrix elements:

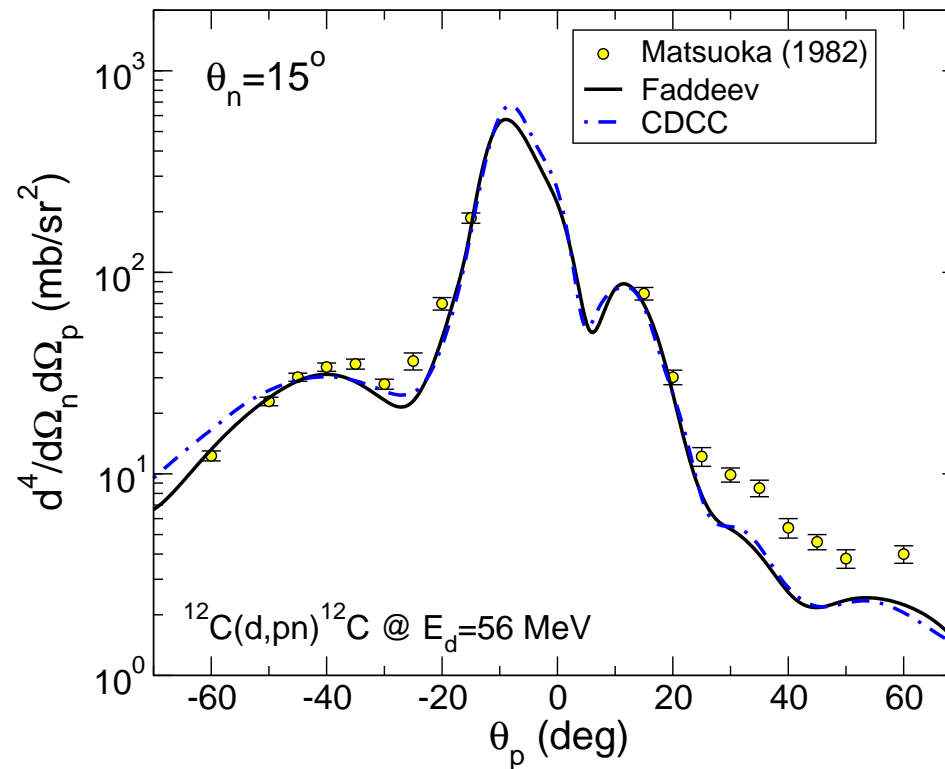
$$\Psi^{\text{CDCC}}(\mathbf{r}, \mathbf{R}) \rightarrow S_{\text{gs},n} \rightarrow \frac{d\sigma^3}{d\Omega_p d\Omega_n dE_p}$$



N. Matsuoka *et al.*, Nucl. Phys. **A 391**, 357 (1986).

## Application of the CDCC formalism: $d+^{58}\text{Ni}$

Observables for exclusive breakup: proton angular distribution



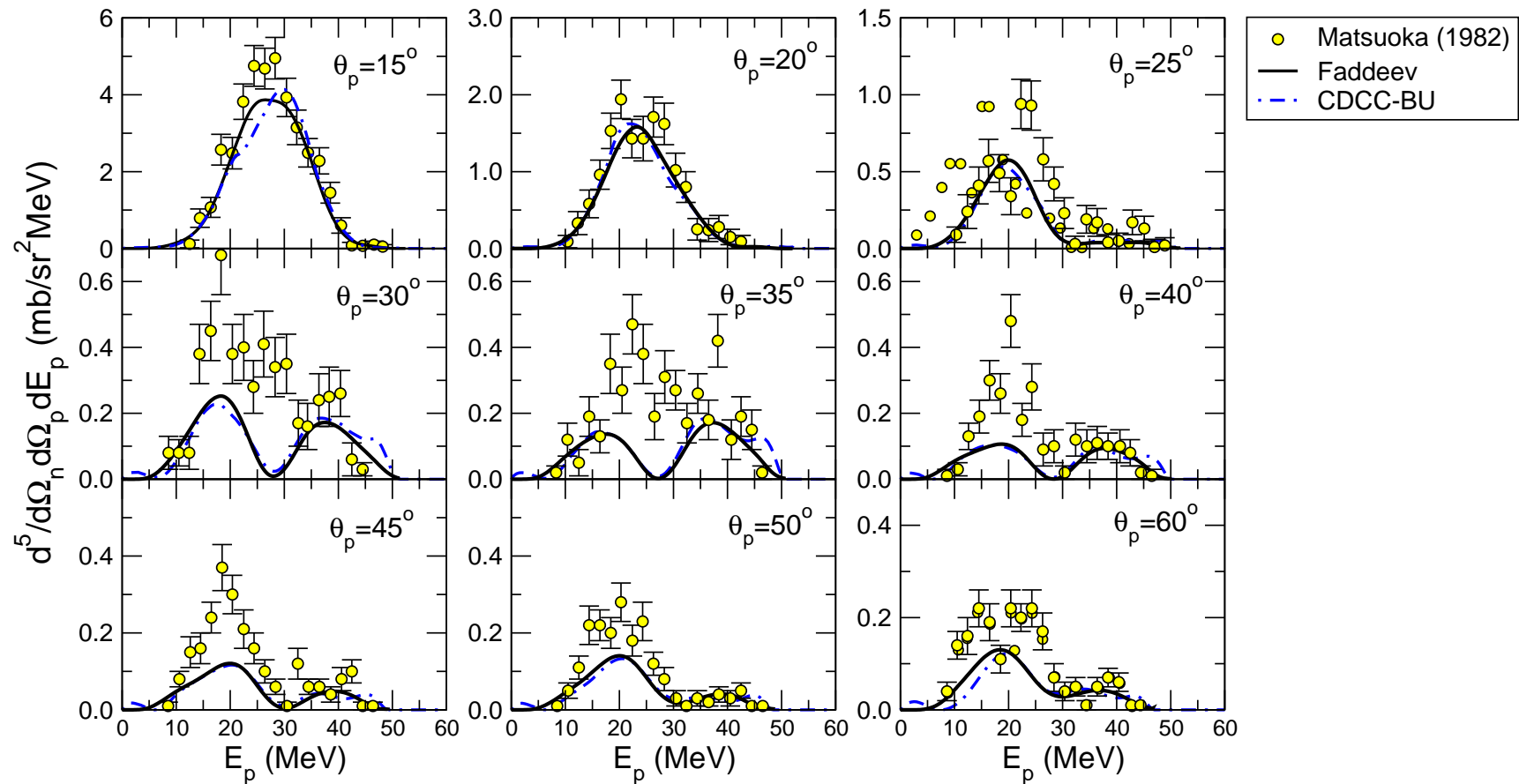
A.Deltuva, A.M.M., E.Cravo, F.M.Nunes, A.C.Fonseca, Phys.Rev. C 76, 064602 (2007)

☞ Very good agreement with Faddeev calculations!



# Application of the CDCC formalism: $d+^{58}\text{Ni}$

Observables for exclusive breakup: proton energy distribution for fixed  $\theta_n$  and  $\theta_p$

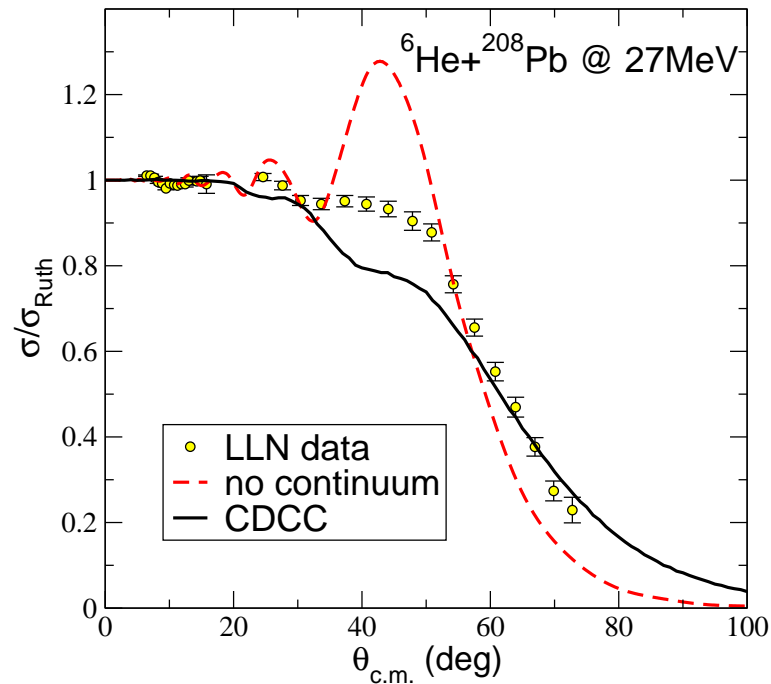
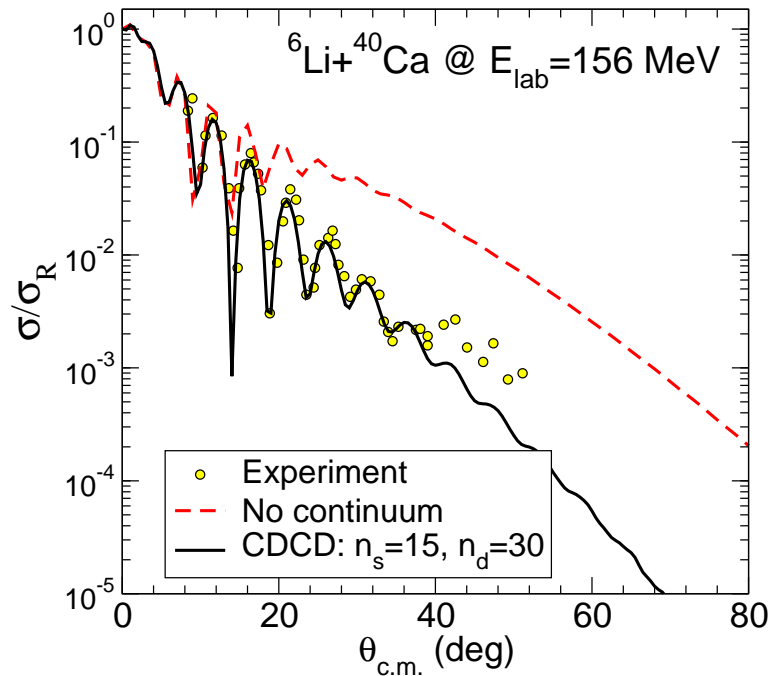


A.Deltuva et al, Phys.Rev. C 76, 064602 (2007)

# Application of the CDCC method: ${}^6\text{Li}$ and ${}^6\text{He}$ scattering

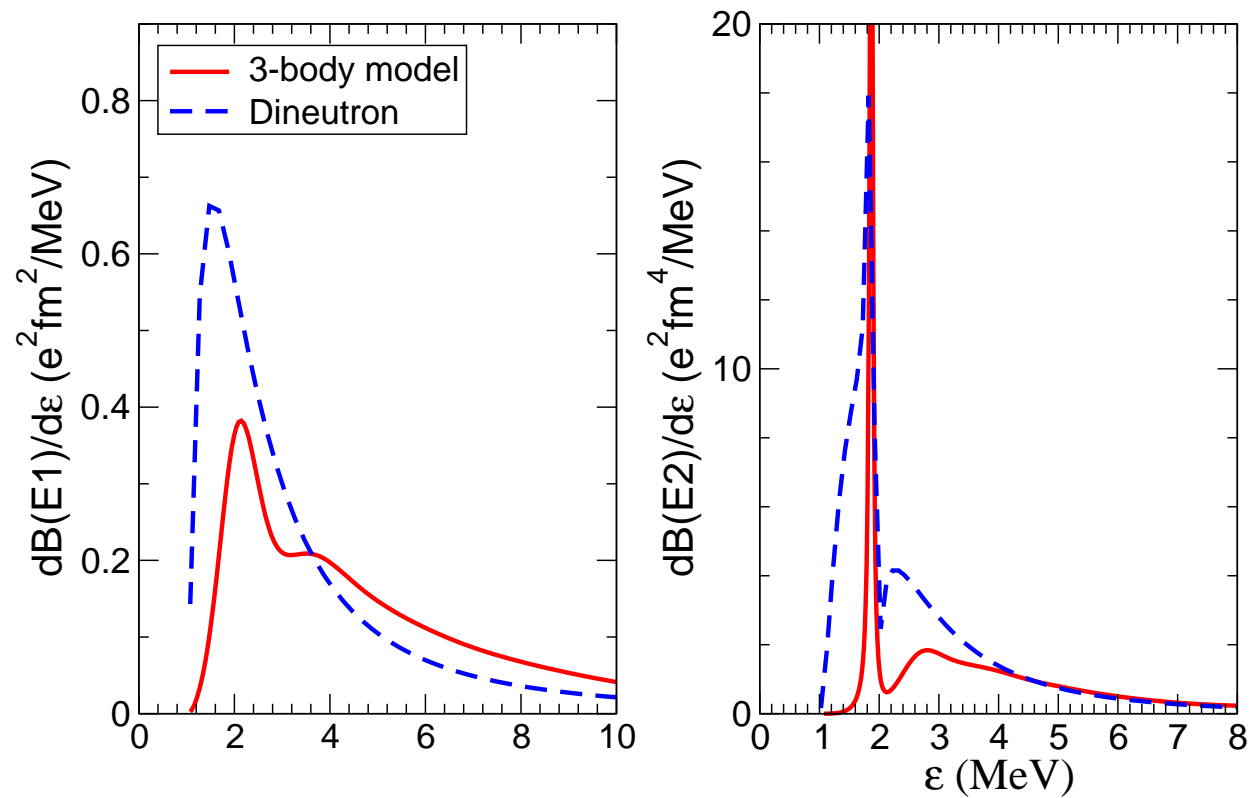
➡ The CDCC has been also applied to nuclei with a cluster structure:

- ${}^6\text{Li} = \alpha + d$
- ${}^6\text{He} = \alpha + {}^2\text{n}$



➡ **CDCC works for  ${}^6\text{Li}$  but fails for  ${}^6\text{He}$ !**

## ${}^6\text{He} + {}^{208}\text{Pb}$ within a di-neutron model



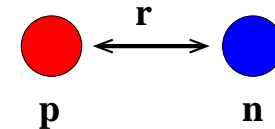
⇒ The dineutron model tends to overestimate the coupling to the continuum: we need a more sophisticated model for  ${}^6\text{He} \Rightarrow \alpha + n + n$

**Extension of the CDCC method to 3-body projectiles:  
four-body CDCC calculations**

# Four-body CDCC calculations for ${}^6\text{He}+{}^{208}\text{Pb}$

**2-body case:** 1 single degree of freedom (inter-cluster relative coordinate)

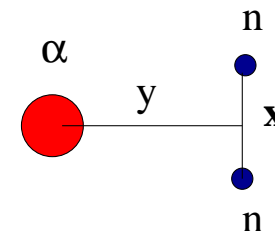
$$\phi_{n,\ell m}(\vec{r}) = R_{n\ell}(r)Y_{\ell,m}(\Omega),$$



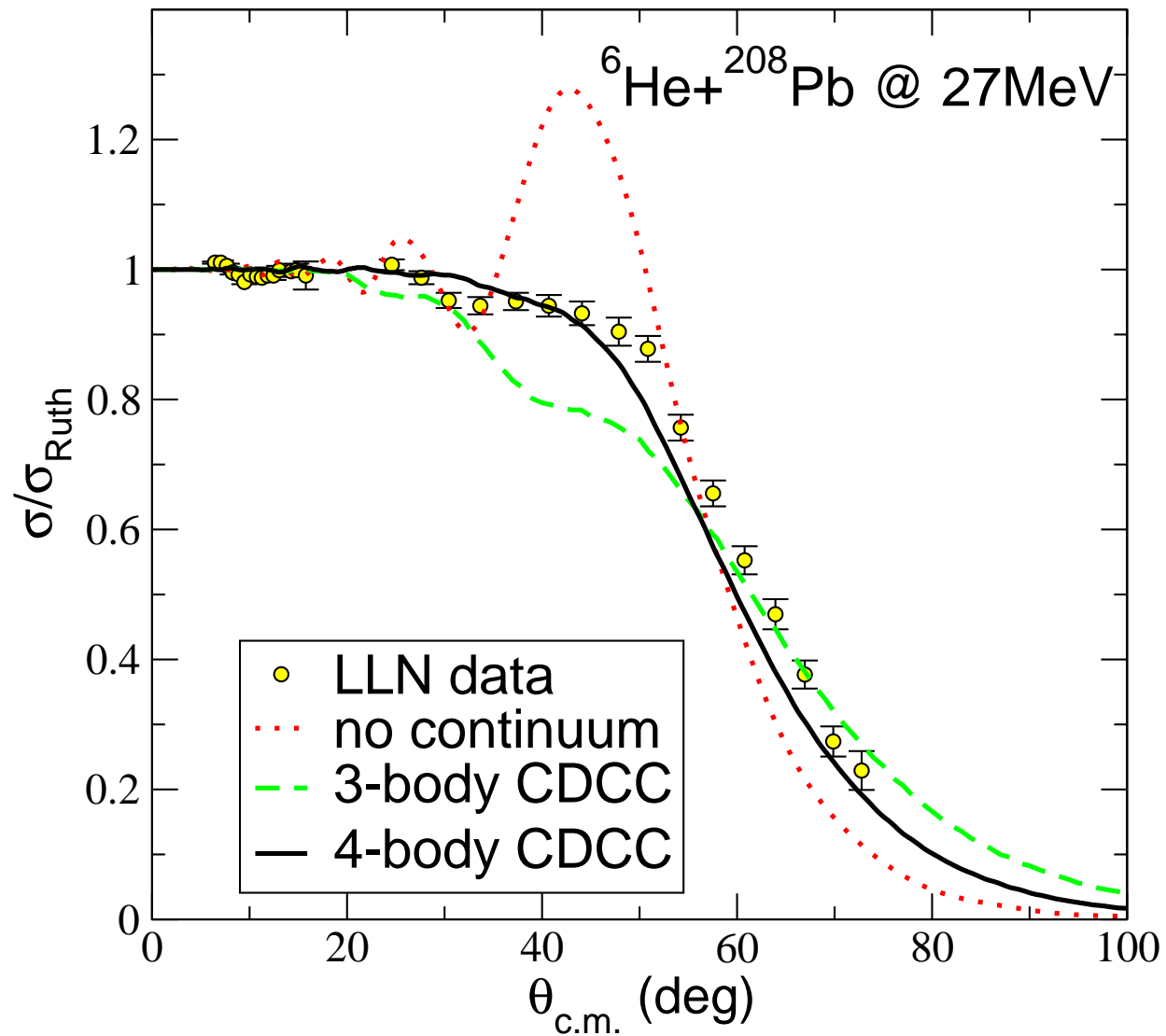
**3-body case:**

- 2 degrees of freedom (6 coordinates)
- 3-body Hamiltonian:  $H = T + V_{nn} + V_{n\alpha} + V_{n\alpha} + V_{nn\alpha}$
- The 3-body wavefunction can be expressed in different coordinate systems:
  - ❖ Jacobi coordinates:  $\{\mathbf{x}, \mathbf{y}\}$
  - ❖ Hyperspherical coordinates:  $\{\rho, \Omega_x, \Omega_y, \alpha\}$      $\rho^2 \equiv x^2 + y^2$      $\tan \alpha = \frac{x}{y}$

$$\Psi_{jm}(\rho, \Omega) = \frac{1}{\rho^{5/2}} \sum_{\beta=1}^{N_\beta} R_{n\beta}(\rho) \left[ \Upsilon_{Kl}^{l_x l_y}(\Omega_x, \Omega_y, \alpha) \otimes X_S \right]_{jm}$$



## Four-body CDCC calculations: ${}^6\text{He}+{}^{208}\text{Pb}$



## *Present and future developments*

- Calculation of nucleon-nucleon and nucleon-core correlations from three-body breakup
- Application to more complicated Borromean nuclei:  $^{11}\text{Li}$ ,  $^{14}\text{Be}$ , etc
- Core excitation:  $^{11}\text{Be} = ^{10}\text{Be}^* + n$   
Summers, Nunes and Thompson, Phys.Rev. C 74, 014606 (2006)

Spectroscopy to unbound states: the  ${}^9\text{Li}(\text{d},\text{p}){}^{10}\text{Li}$  case



# Spectroscopy to unbound states

## The $^{10}\text{Li}$ and $^{11}\text{Li}$ systems

- $^{11}\text{Li}$  radioactive:

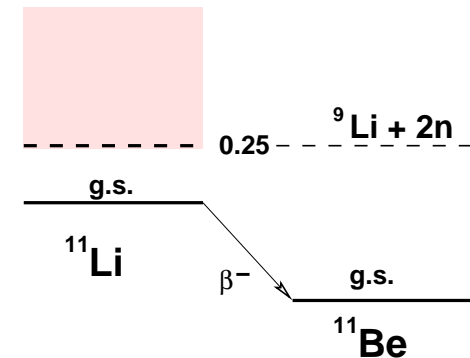


- $^{11}\text{Li}$  key example of Borromean nucleus:

- ❖  $n+n$  and  $n+^9\text{Li}$  unbound but,
- ❖  $n+n+^9\text{Li}$  has a (weakly) bound state.

- The structure of  $^{11}\text{Li}$  depends critically on:

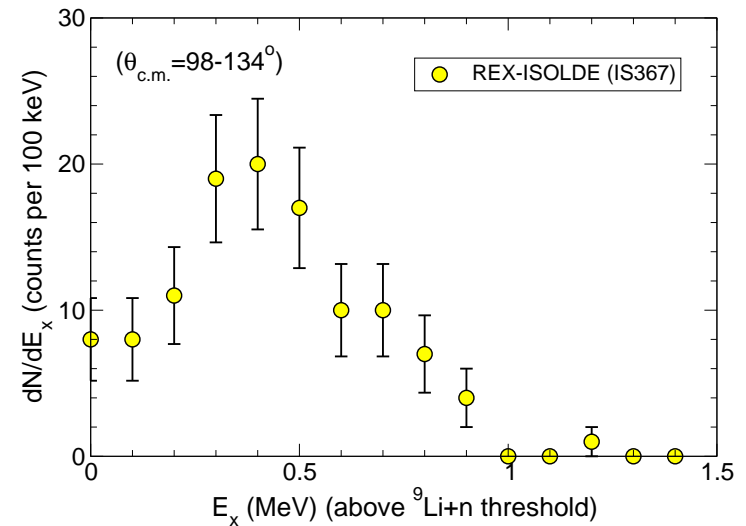
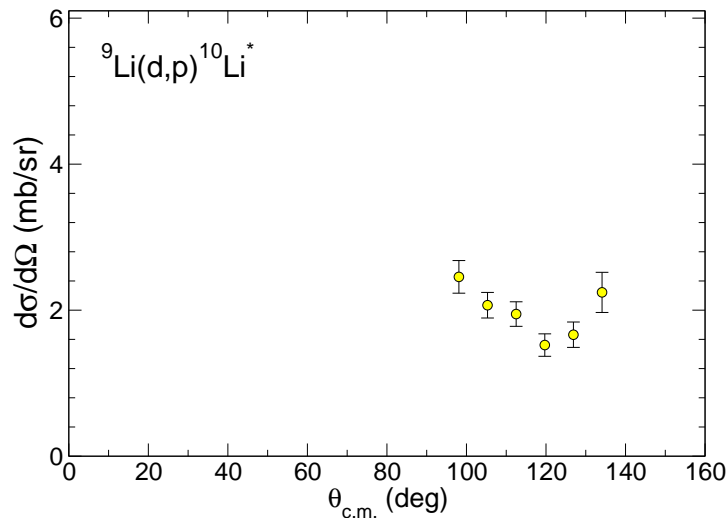
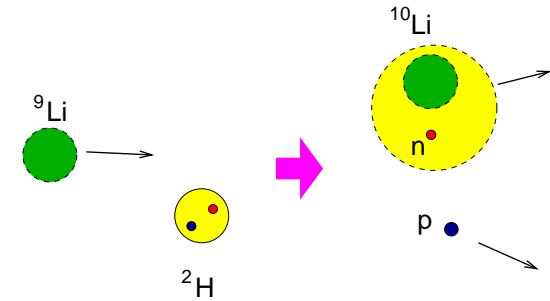
- ❖  $n+n \rightarrow$  well understood.
- ❖  $n+^9\text{Li} \rightarrow$  dominated by:  $\left\{ \begin{array}{l} \text{p-wave resonance} \\ \text{s-wave virtual state} \end{array} \right\} \rightarrow \text{not well understood}$



# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

## The Experiment:

- REX-ISOLDE (2002)
- ${}^9\text{Li}$  beam on D target at  $E = 2.75$  MeV/u
- The experiment provided angular and energy distributions for protons  $\Rightarrow {}^{10}\text{Li}$ .



## ***Spectroscopy to unbound states: $^9\text{Li}(d,p)^{10}\text{Li}$ case***

**MOTIVATION:** *What spectroscopic information can be obtained from the  $^{10}\text{Li}$  distributions?*

## Spectroscopy to unbound states: $^9\text{Li}(d,p)^{10}\text{Li}$ case

MOTIVATION: *What spectroscopic information can be obtained from the  $^{10}\text{Li}$  distributions?*

### IMPORTANT QUESTIONS TO ADDRESS:

- Q: What is the mechanism producing protons?  
A: Direct (compound estimated to be very small)
- Q: How does the reaction mechanism affect the  $^{10}\text{Li}$  energy spectrum?  
(eg. relationship between observed bump and continuum structures)  
A: Requires a proper reaction calculation; not just a fit to data

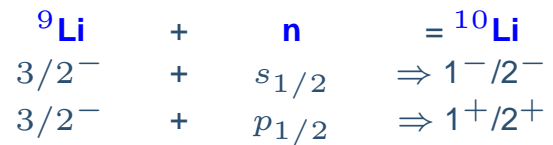
## *Spectroscopy to unbound states: $^9\text{Li}(d,p)^{10}\text{Li}$ case*

STRUCTURE + REACTION

# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

## STRUCTURE + REACTION

${}^{10}\text{Li}$



$p_{3/2}$  : bound state ( $\epsilon_b \simeq 4.1$  MeV)

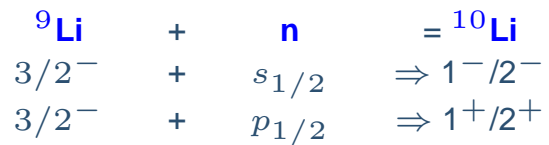
$s_{1/2}$  : deeply 0s bound state  
virtual 1s virtual state

$p_{1/2}$  : low energy resonance

# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

## STRUCTURE + REACTION

${}^{10}\text{Li}$

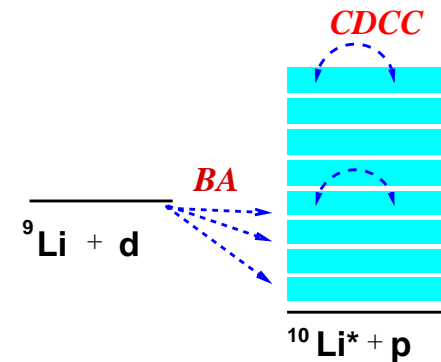


$p_{3/2}$  : bound state ( $\epsilon_b \simeq 4.1$  MeV)

$s_{1/2}$  : deeply 0s bound state  
virtual 1s virtual state

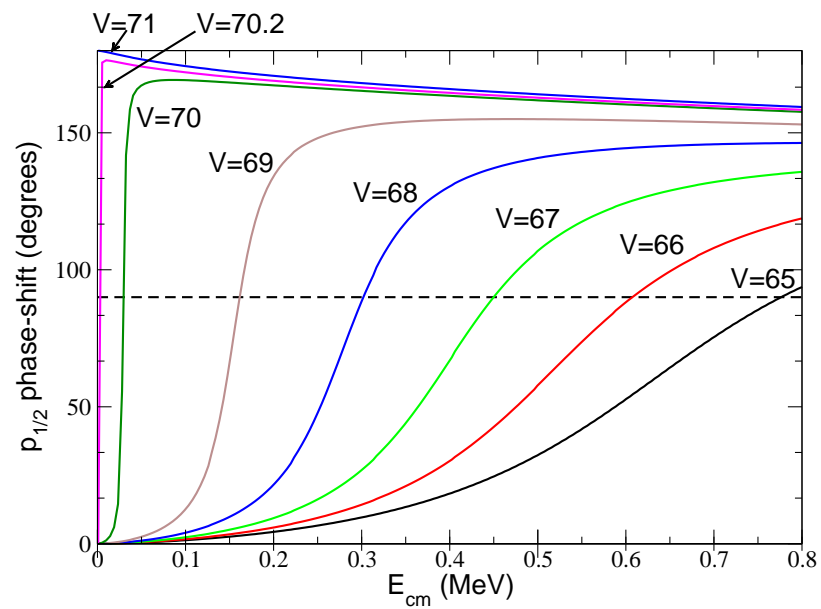
$p_{1/2}$  : low energy resonance

Transfer to the continuum  
(direct mechanism)



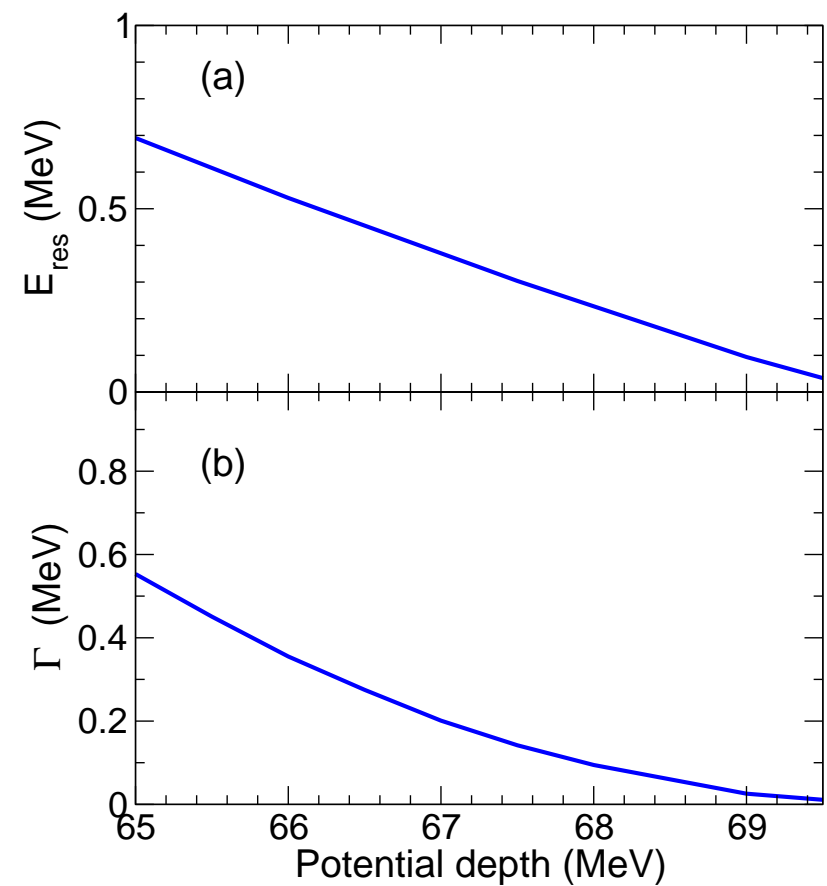
# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

$p_{1/2}$  resonance



- $E_{\text{res}} \rightarrow \delta(E_{\text{res}}) = \frac{\pi}{2}$

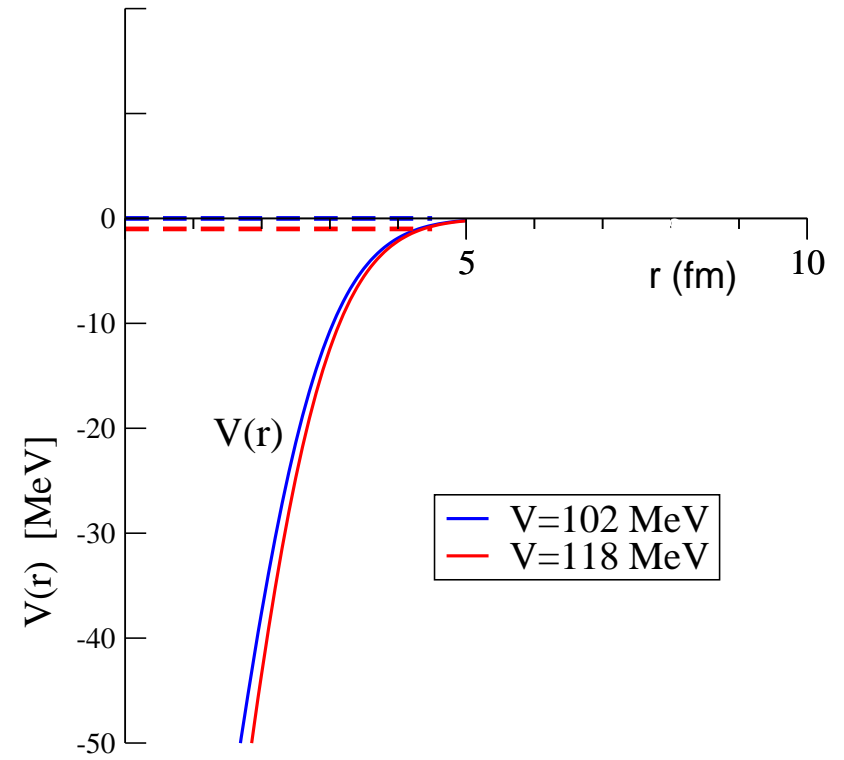
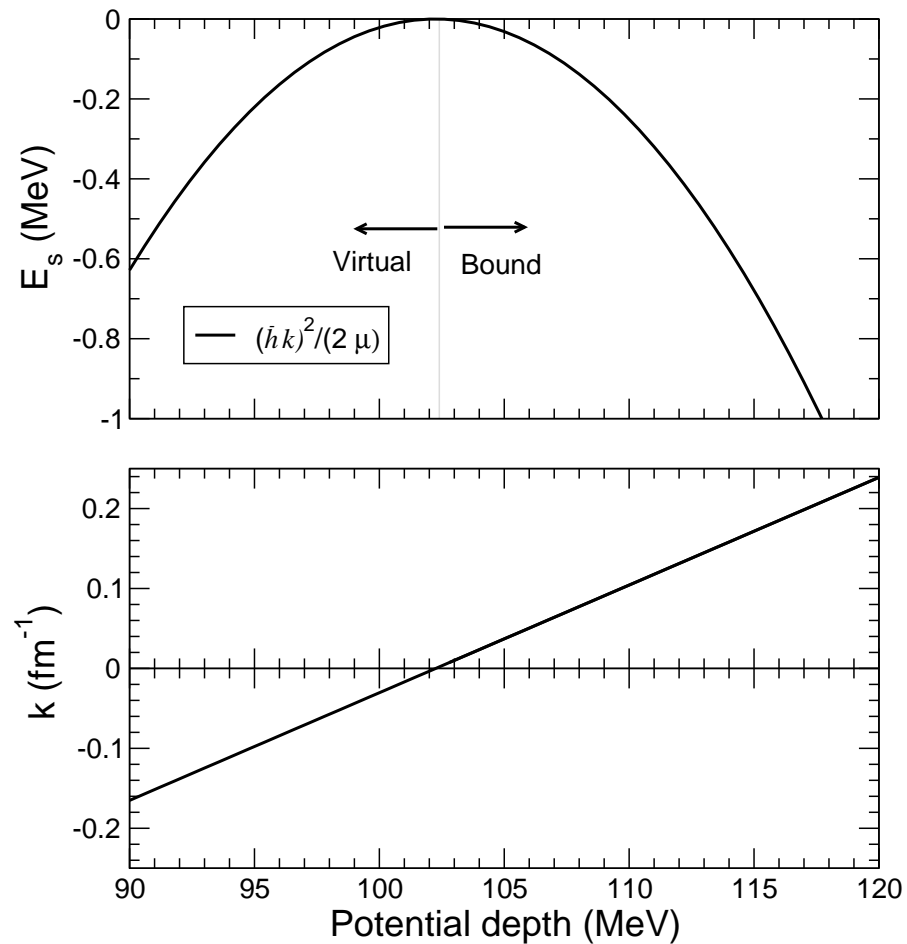
- $\frac{2}{\Gamma} = \frac{d\delta}{dE}$





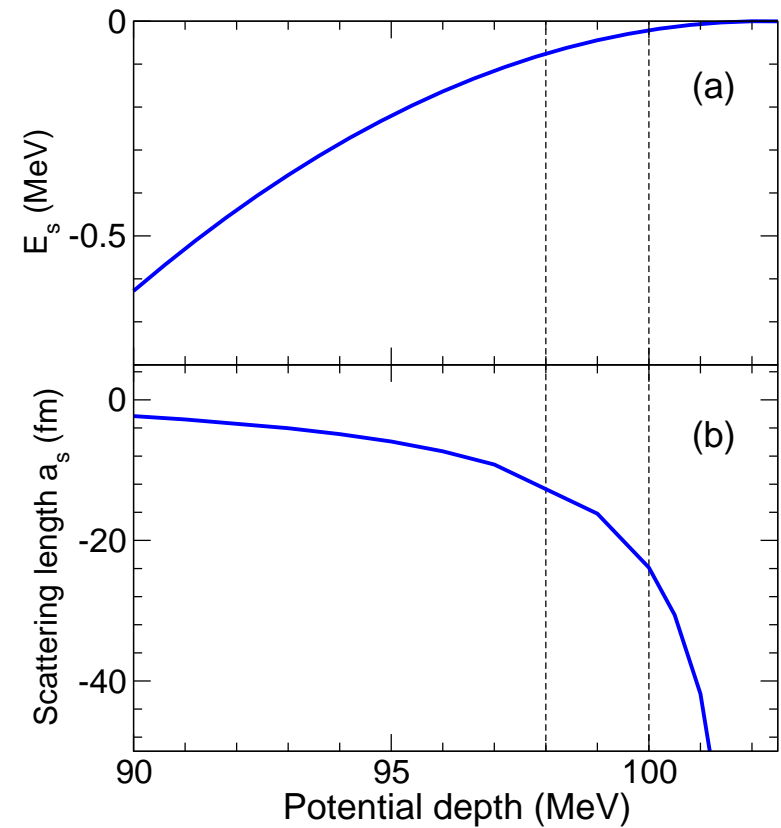
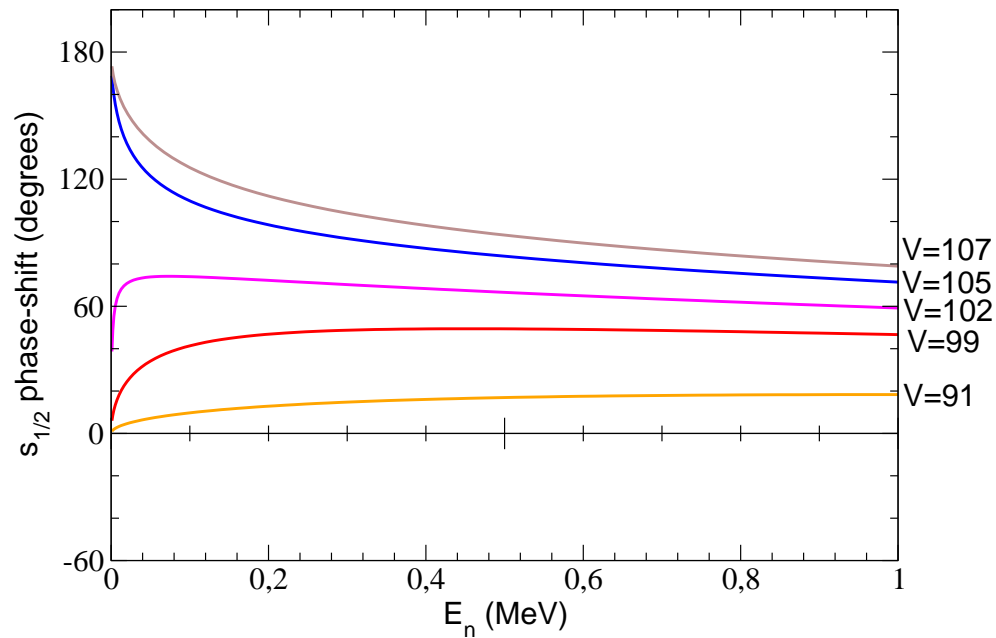
# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Appearance of a virtual state in  ${}^{10}\text{Li} = {}^9\text{Li} + n$ :



# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

$V_s$  (virtual state)



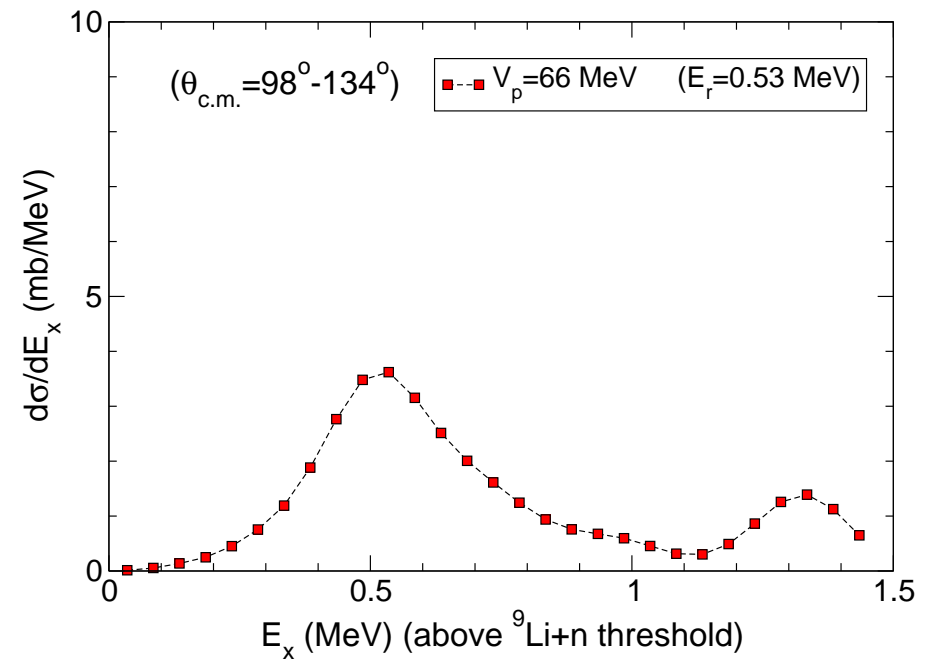
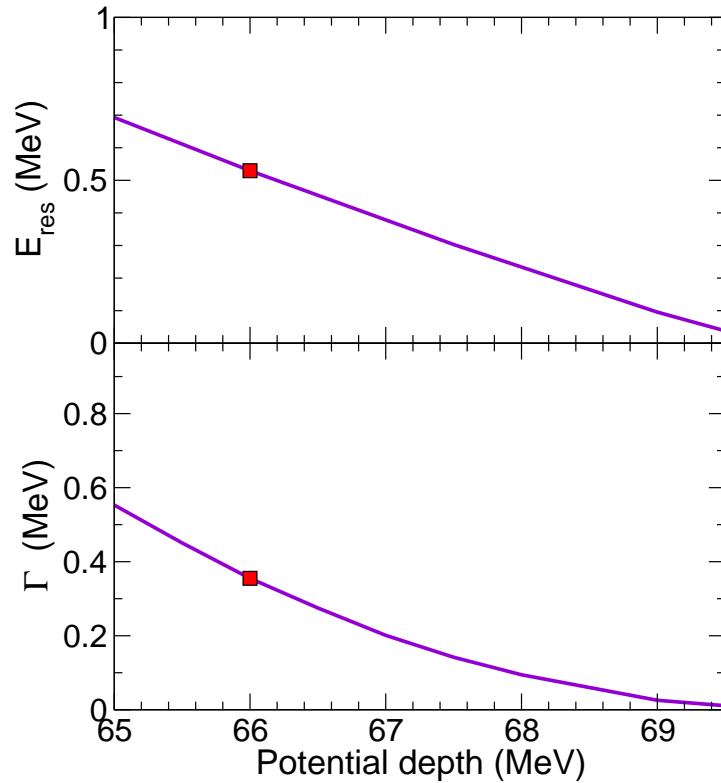
Scattering length:

$$a_s = - \lim_{k \rightarrow 0} \tan \frac{\delta(k)}{k}$$

# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Structure:  $p_{1/2}$  resonance

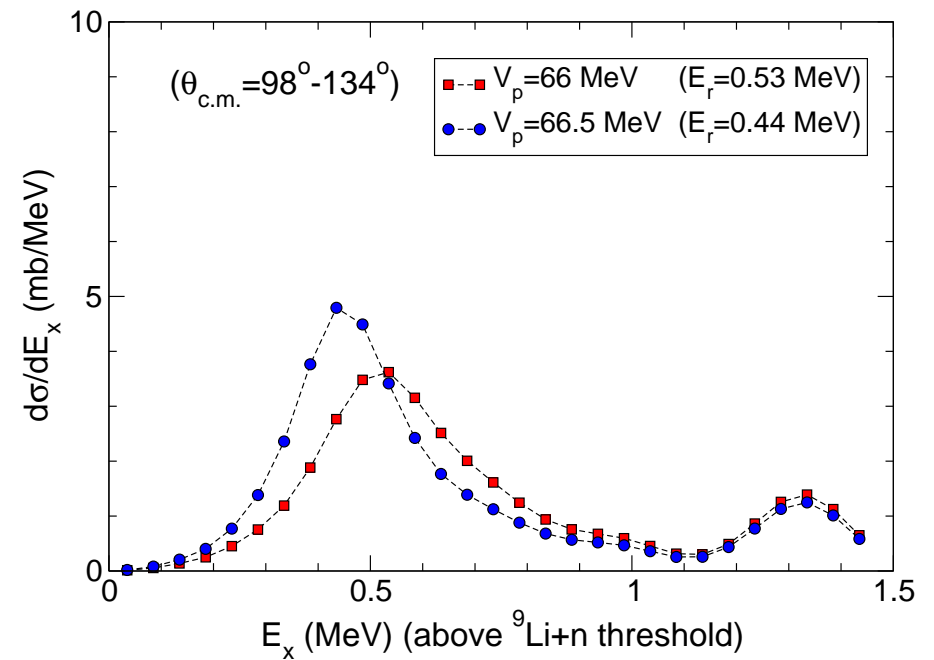
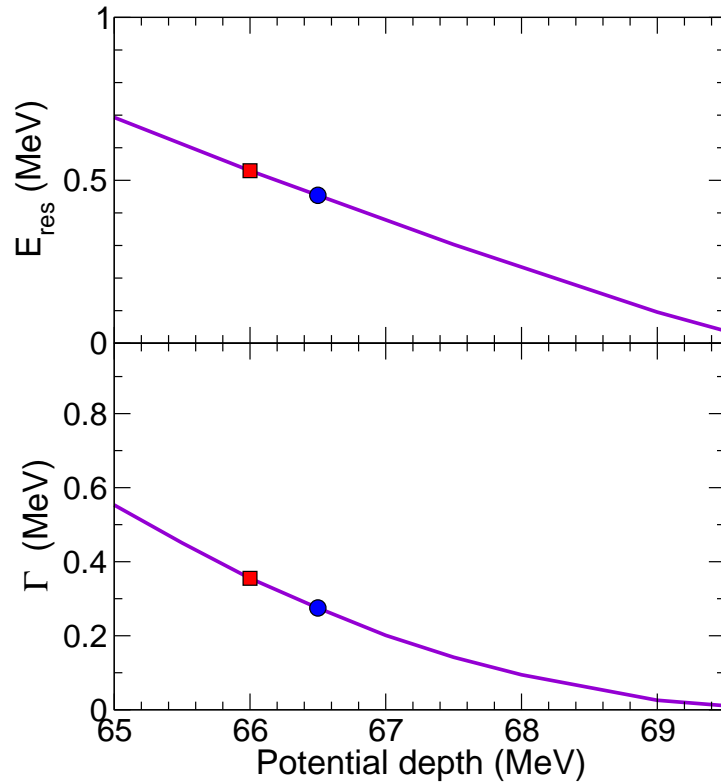
Reaction



# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Structure:  $p_{1/2}$  resonance

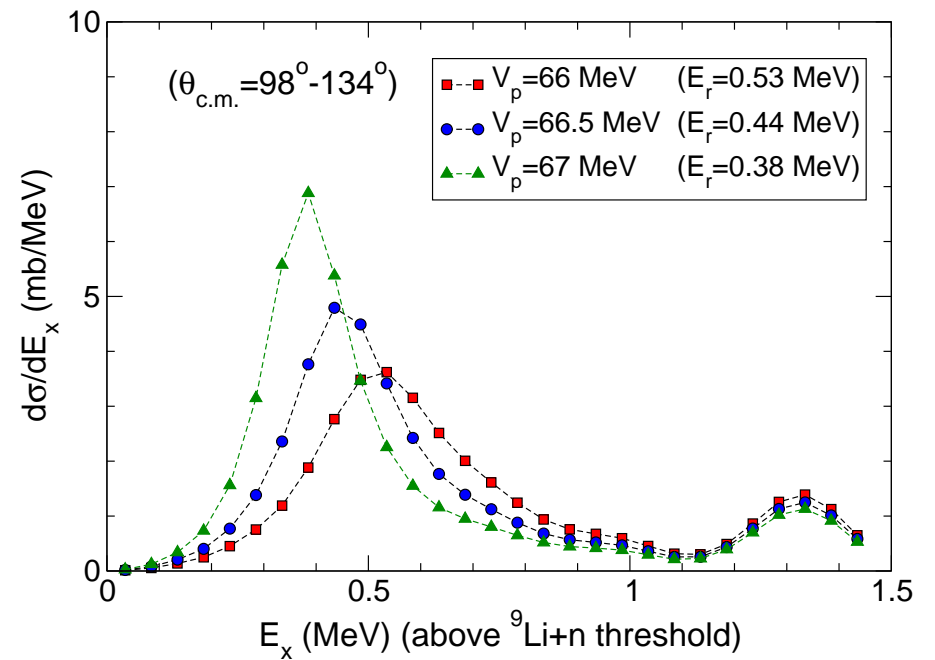
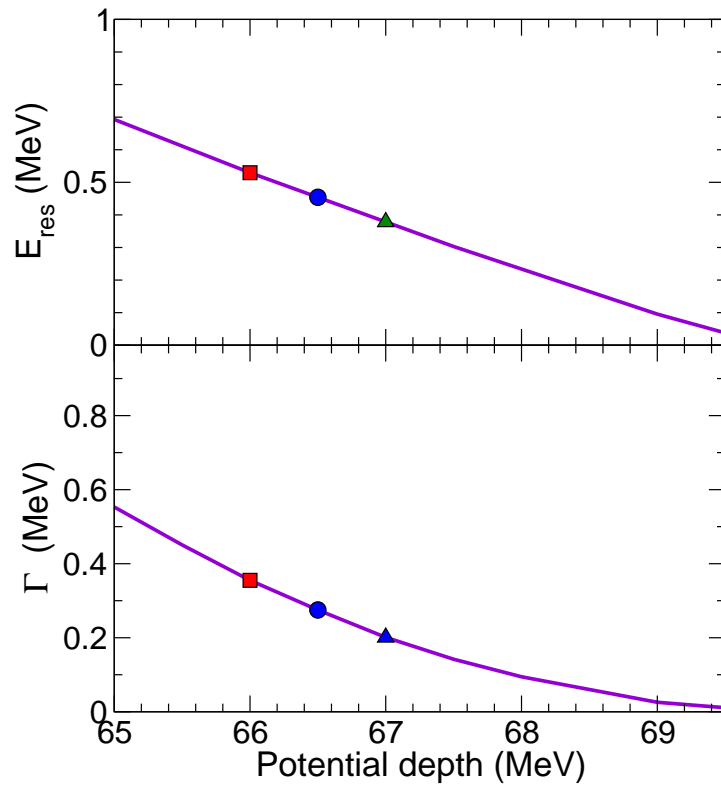
Reaction



# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Structure:  $p_{1/2}$  resonance

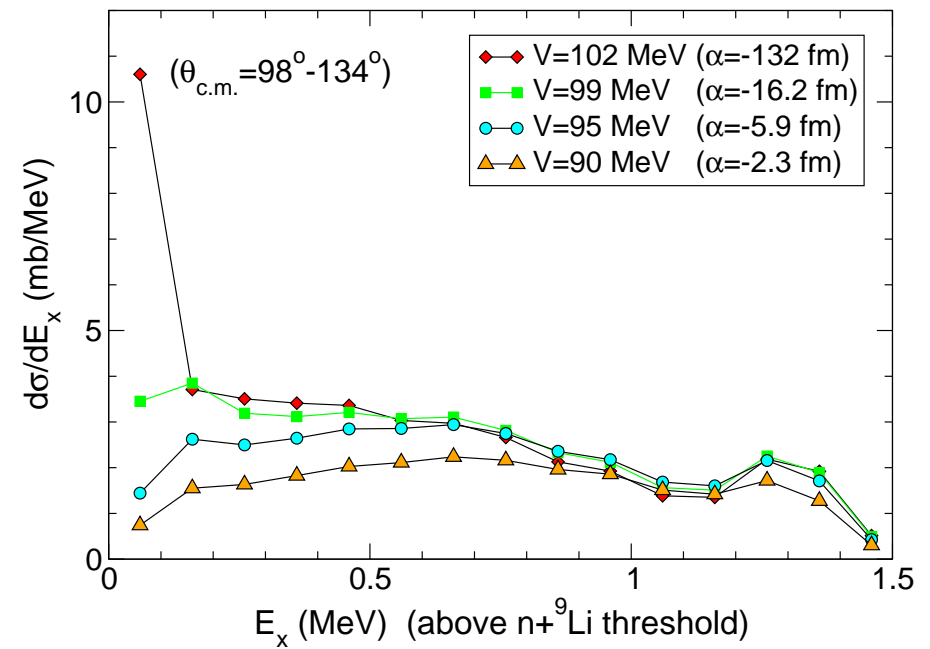
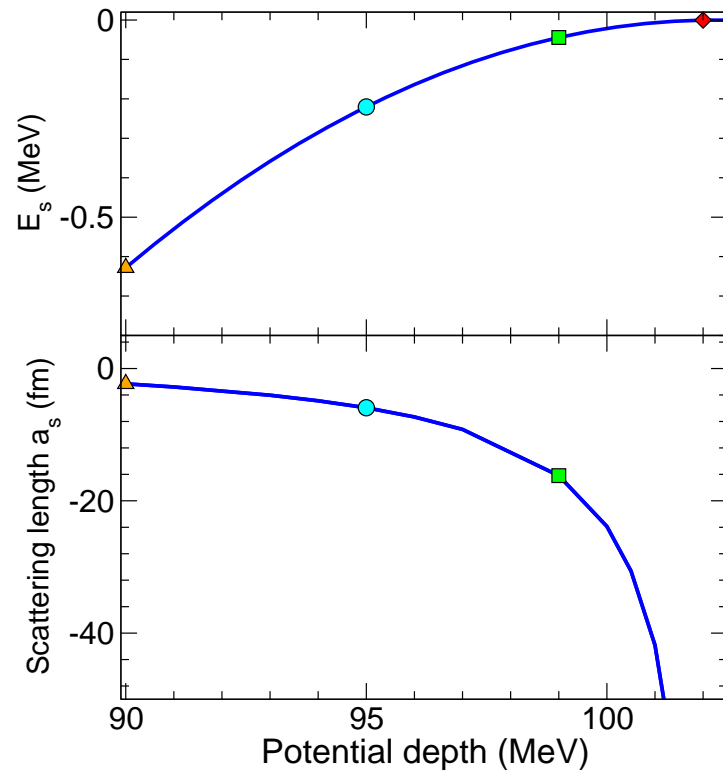
Reaction



# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$ case

Structure:  $s_{1/2}$  v.s.

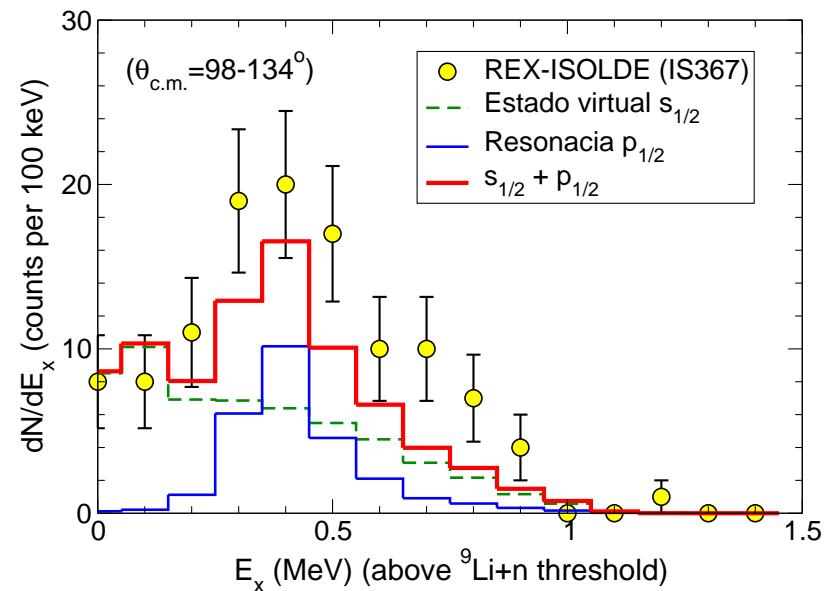
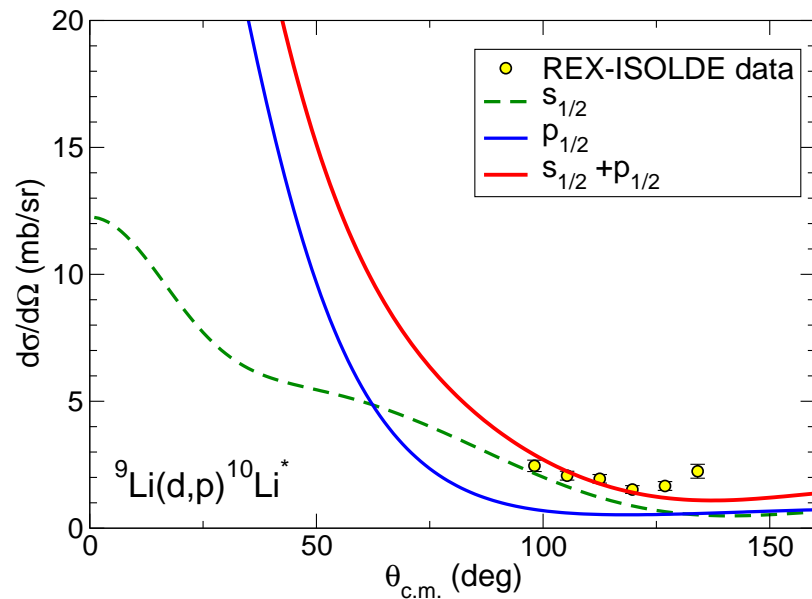
Reaction



# Spectroscopy to unbound states: ${}^9\text{Li}(d,p){}^{10}\text{Li}$

**BEST FIT RESULTS:** HP.Jeppesen et al, PLB642 (2006) 449

- $p_{1/2}$  resonance ( $1^{+}/2^{+}$  doublet):  $E_r \simeq 0.38 \text{ MeV}$ ,  $\Gamma = 0.2 \text{ MeV}$
- $s_{1/2}$  virtual state ( $1^{-}/2^{-}$  doublet):  $a_s \simeq -24 \text{ fm}$



Thanks!

A hand-drawn illustration in black ink. It features a simple, round face with a wide, curved smile and two small dots for eyes. A hand with five fingers is reaching up towards the word 'Thanks!'. The entire drawing is positioned below a thick, curved line that underlines the word. A small '©' symbol is visible at the bottom right of the face.



## Appendix: commented example for CDCC calculations with FRESCO

## CDCC formalism: physical ingredients

- Physical example:  $d+^{58}\text{Ni}$  at  $E = 80 \text{ MeV}$
- We need to provide FRESKO with the following ingredients:
  - **Participants:** projectile (core+valence) and target:  $p, n, ^{58}\text{Ni}$
  - **Potentials:**
    - Valence-target and core-target (complex)
    - $p$ - $n$  potential (for gs wavefunction and continuum states)
  - **Multipolarities (Q)** for coupling potentials
  - **Binning scheme** for each spin/parity combination:
    - Average bin energy:  $\bar{\epsilon}_n$
    - Width for each bin:  $\Delta\epsilon_n$

## CDCC formalism: $d+^{58}\text{Ni}$

### CDCC input example for $d+^{58}\text{Ni}$ : dni\_e80\_cdc.in

```
d+ 58Ni at E=80 MeV (CDCC) s+d waves
&CDCC
  hcm=0.1 rmatch=60
  elab=80 pel=1 lab=1 jbord=100
  thmax=180 thinc=1 chans=1 smats=2 xstabl=1
  nk=200 ncoul=0 reor=0 q=2 hat=F /

&NUCLEUS part='Proj' name='d' charge=1 mass=2.0 spin=0. parity=1 be=2.225 n=1 l=0 j=0. /
&NUCLEUS part='Core' name='n' charge=0 mass=1.0 spin=0 parity=+1 /
&NUCLEUS part='Valence' name='p' charge=1 mass=1 spin=0.0 /
&NUCLEUS part='Target' name='58Ni' charge=28 mass=58.0 spin=0. parity=1 /

&BIN spin=0.0 parity=+1 start=0.01 step=5.0 end=30 energy=F l=0 j=0.0 /
&BIN spin=2.0 parity=+1 start=0.01 step=5.0 end=50 energy=F l=2 j=2.0 /
&BIN /

&POTENTIAL part='Proj' al=58. rc=1.25 /

# Becchetti-Greenless potential for p +58Ni
&POTENTIAL part='Valence' al=58 rc=1.
  V=44.9 vr0=1.17 a=0.75 w=6.1 wr0=1.32 aw=0.53
  wd=2.21 wdr0=1.320 awd=0.53 /

# Becchetti-Greenless potential for p +58Ni
&POTENTIAL part='Core' al=58 rc=1.25
  V=42.6 vr0=1.17 a=0.75 w=7.24 wr0=1.26 aw=0.58
  wd=2.590 wdr0=1.26 awd=0.58 /

# Gaussian potential for p-n
&POTENTIAL part='Gs' shape= 2 V=72.150 a=1.484 /
```

## General variables: CDCC namelist

```
&CDCC hcm=0.1 rmatch=60  
      elab=80 jbord=100  
      thmin=0 thmax=180 thinc=1  
      smats=2 xstabl=1  
      nk=200 ncoul=0 reor=0 q=2 hat=F /
```

- $\text{ncoul}=0$ : Coulomb + nuclear couplings  
 $\text{ncoul}=1$ : only nuclear couplings  
 $\text{ncoul}=2$ : only Coulomb couplings
- $q$ : number of multipoles
- $nk$ : number of continuum states to construct the bin.

## CDCC formalism: $d+^{58}\text{Ni}$

### NUCLEUS namelist:

```
&NUCLEUS part='Proj' name='d' charge=1 mass=2.0 spin=0. parity=1  
        be=2.225 n=1 l=0 j=0. /  
&NUCLEUS part='Valence' name='n' charge=0 mass=1.0 spin=0 parity=+1 /  
&NUCLEUS part='Core' name='p' charge=1 mass=1 spin=0.0 /  
&NUCLEUS part='Target' name='58Ni' charge=28 mass=58.0 spin=0. parity=1 /
```

- **part**: specifies each cluster :  
part='proj' = projectile → d  
part='valence' → n  
part='core' → n  
part='target' →  $^{58}\text{Ni}$
- **name, charge, mass, spin, parity**
- **be**: binding energy
- **n, l, j**: quantum numbers

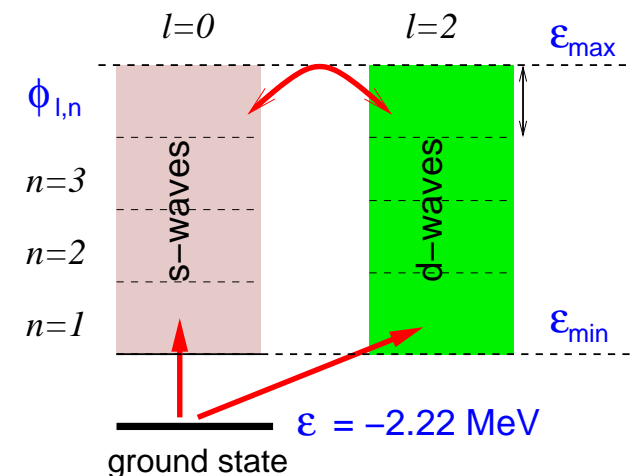
*(for simplicity, the neutron and proton spins are ignored)*

# CDCC formalism: $d+{}^{58}\text{Ni}$

## BIN namelist:

```
&BIN spin=0.0 parity=+1 start=0.0 step=5.0 end=30 energy=F l=0 j=0.0 /  
&BIN spin=2.0 parity=+1 start=0.0 step=5.0 end=50 energy=F l=2 j=2.0 /
```

- **spin, parity**: total angular momentum and parity
- **start**: minimum excitation energy  
**end**: maximum excitation energy  
**step**: make  $N = (\text{end} - \text{start}) / \text{step}$  bins
- **energy=T**: bins evenly spaced in energy  
**energy=F**: bins evenly spaced in  $k$
- **l, j**: orbital and total angular momentum of valence particle



## CDCC formalism: $d+^{58}\text{Ni}$

### Potentials:

- **projectile-target** potential (only Coulomb potential is needed here)

```
&POTENTIAL part='Proj' a1=58. rc=1.25 /
```

- **valence-target** and **core-target** potentials:

```
&POTENTIAL part='Valence' a2=58  
V=44.9 vr0=1.17 a=0.75 w=6.1 wr0=1.32 aw=0.53  
wd=2.21 wdr0=1.320 awd=0.53 /  
  
&POTENTIAL part='Core' a2=58 rc=1.25  
V=42.6 vr0=1.17 a=0.75 w=7.24 wr0=1.26 aw=0.58  
wd=2.590 wdr0=1.26 awd=0.58 /
```

- **p-n** potential (for gs and bins):

```
&POTENTIAL part='Gs' shape= 2 V=72.150 a=1.484 /
```

## ***CDDC formalism: $d+^{58}\text{Ni}$***

- Generate the FRESKO input (dni\_e80\_cdcc.in) from the CDC input (dni\_e80\_cdc.in):

```
cdc < dni_e80_cdc.in > dni_e80_cdcc.in
```

- Run fresco input:

```
fresco < dni_e80_cdcc.in > dni_e80_cdcc.out
```



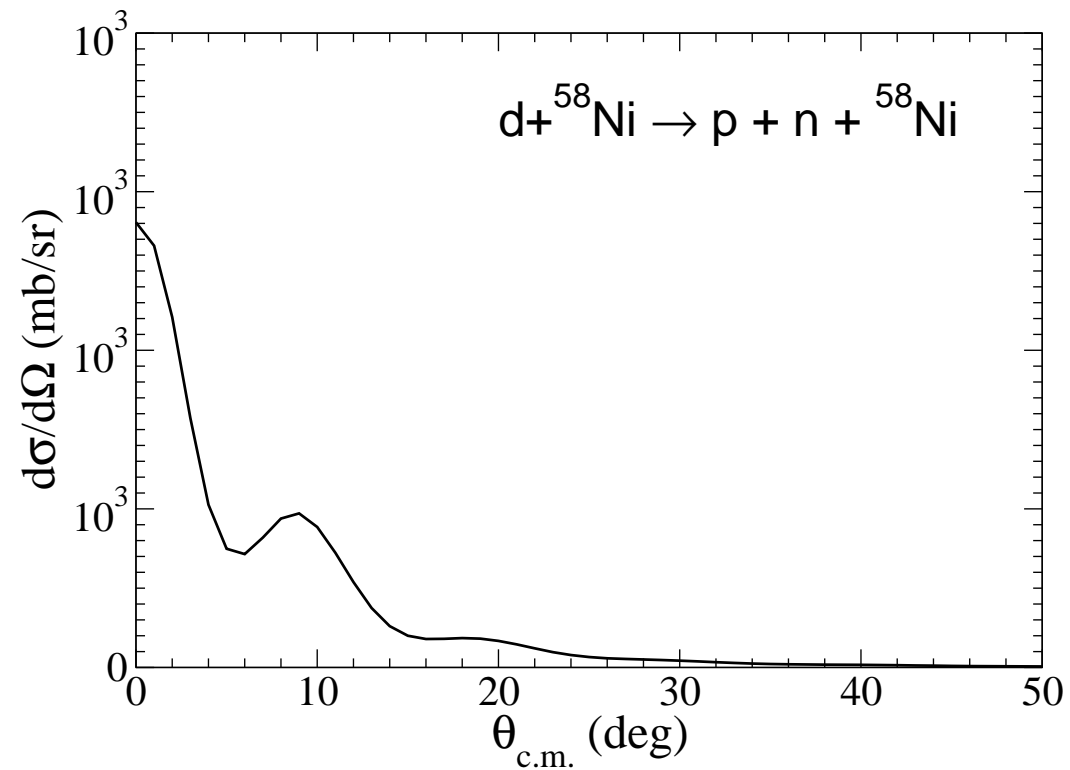
## Useful output files:

- `fort.16`: Angular distributions. Also separately in:
  - ❖ `fort.201`: elastic angular distribution
  - ❖ `fort.202`: breakup angular distribution for 1st bin
  - ❖ `fort.203`: breakup angular distribution for 2nd bin
  - ❖ ...
- `fort.13`: total (angle integrated) cross section for each bin.
- `fort.56`: Four columns: J , Fusion , Reaction , Breakup

## CDCC formalism: $d+{}^{58}\text{Ni}$

Total breakup angular distribution:

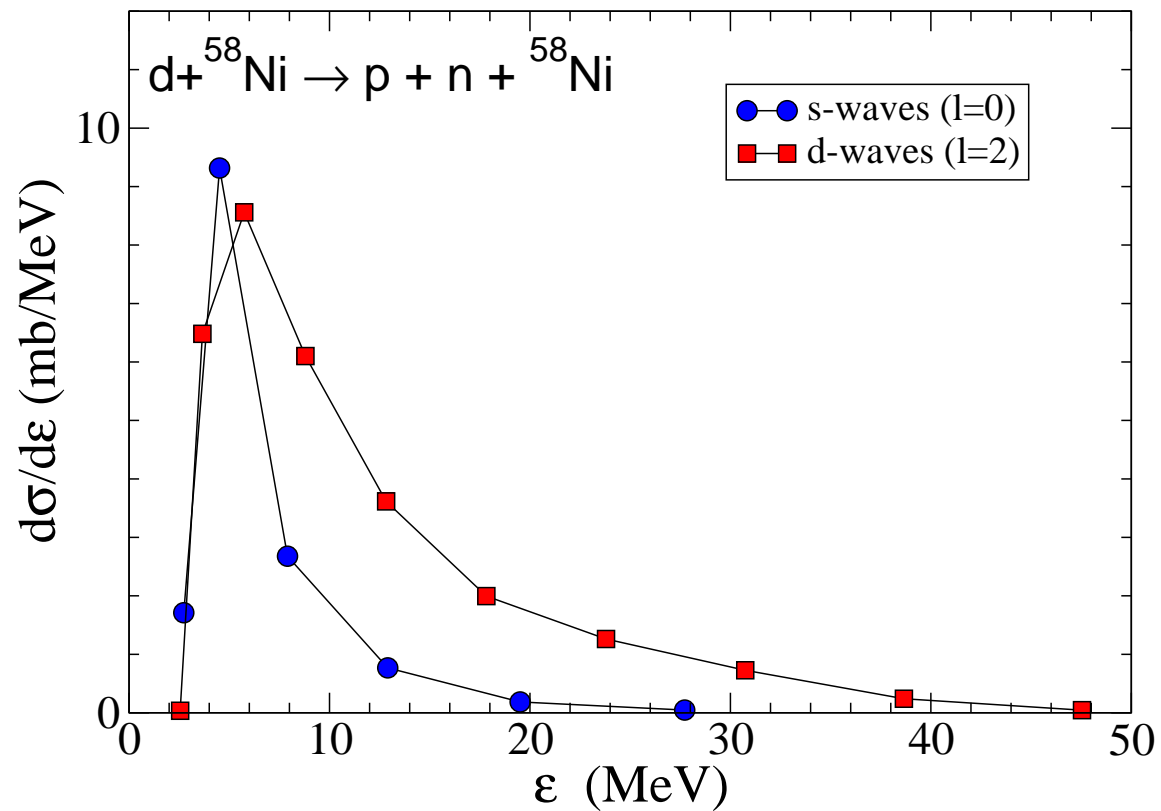
```
sumbins < fort.16 > sumbins.out
```



## CDCC formalism: $d + {}^{58}\text{Ni}$

Breakup energy distribution for each spin configuration

```
sumxen < fort.13 > sumxen.out
```



## *Useful scripts and post-processing codes:*

- `cdc.f`: Application to create FRESKO inputs for CDCC calculations.
- `getsmat.sh`: Extract S-matrix from `fort.7` file  
`getsmat < fort.7 > smat.out`
- `sumbins.f`: Adds all non-elastic angular distributions from `fort.16`  
`sumbins < fort.16> sumbins.out`
- `sumxen.f`: Energy distribution for each spin/parity from `fort.13`  
`sumxen < fort.13> sumxen.out`
- `frto2col.f`: Converts overlaps written by fresco in a two column format.