Story Spring Derivation

Modified Ibarra-Medina-Krawinkler Bilinear material

ASCE 7-10 Equivalent Lateral Force Procedure

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1 Design story stiffness

The deflection of a given story x is given by equation 12.8-15 as:

$$\delta_x = \frac{C_d \delta_{xe}}{I_e} \tag{1}$$

Where δ_{xe} is the calculated elastic deflection. This is taken to be:

$$\delta_{xe} = \frac{F_x}{K_x} \tag{2}$$

Where F_x is the design story shear (§12.8.3) and K_x is the story stiffness. The story drift is the difference between story deflections:

$$\Delta_x = \delta_x - \delta_{x-1} \tag{3}$$

Equations (1) and (2) can be substituted into this to get:

$$\Delta_x = \frac{C_d}{I_e}\left(\frac{F_x}{K_x} - \frac{F_{x-1}}{K_{x-1}}\right) \tag{4}$$

K_x is then solved for:

$$K_x = \frac{F_x}{\frac{\Delta_x I_e}{C_d} + \frac{F_{x-1}}{K_{x-1}}} \tag{5}$$

2 Design story strength

The maximum strength of each story x, M_c, is taken as:

$$M_{cx} = \Omega_o F_x \tag{6}$$

Where Ω_o is the system overstrength factor and F_x is the design story shear.

3 Pre-capping rotation

The equivalent lateral force procedure returns the maximum strength, M_c, which is used to back-calculate the yield strength, M_y, of the story. This also requires an assumption of the ratio of the pre-capping rotation, θ_p, to the rotation at yield, θ_y. This is treated as a variable in the analysis, C_{py} or `py_factor`. Using linear relations and this proportional assumption, the following system of equations is developed:

$$\theta_{px} - C_{py}\theta_{yx} = 0 \tag{7a}$$

$$a_s K_x \theta_{px} + M_{yx} = M_{cx} \tag{7b}$$

$$\theta_{yx} - K_x^{-1} M_{yx} = 0 \tag{7c}$$

This system is solved for each floor x. a_s and C_{py} are assumed to be the same for each story.

4 Post-capping and ultimate rotation

The post-capping rotation is currently defined as proportional to θ_p:

$$\theta_{pc} = C_{pcp}\theta_p \tag{8}$$

The ultimate rotation is currently defined simply in terms of the previously calculated rotations, taking some amount of θ_{pc}:

$$\theta_u = \theta_y + \theta_p + C_{pcu}\theta_{pc} \tag{9}$$