## **Story Spring Derivation**

Modified Ibarra-Medina-Krawinkler Bilinear material ASCE 7-10 Equivalent Lateral Force Procedure

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#### 1 Design story stiffness

The deflection of a given story x is given by equation 12.8-15 as:

$$\delta_x = \frac{C_d \delta_{xe}}{I_e} \tag{1}$$

Where  $\delta_{xe}$  is the calculated elastic deflection. This is taken to be:

$$\delta_{xe} = \frac{V_x}{K_x} + \delta_{(x-1)e} \tag{2}$$

Where  $V_x$  is the design story shear (§12.8.3),  $K_x$  is the story stiffness, and  $\delta_{(x-1)e}$  is the elastic deflection of the story below x. The story drift is the difference between story deflections:

$$\Delta_{x} = \delta_{x} - \delta_{x-1} \tag{3}$$

Taking  $\Delta_x$  to be the allowable story drift,  $\Delta_a$ , and substituting equations (1) and (2) into (3) obtains:

$$\Delta_a = \frac{C_d}{I_e} \left( \frac{V_x}{K_x} + \delta_{(x-1)e} - \delta_{(x-1)e} \right) \tag{4}$$

 $K_x$  is then solved for:

$$K_{x} = \frac{C_{d}V_{x}}{I_{e}\Delta_{a}} \tag{5}$$

#### 2 Design story strength

The maximum strength of each story x,  $V_c$ , is taken as:

$$V_c = \Omega_o V_x \tag{6}$$

Where  $\Omega_o$  is the system overstrength factor and  $V_x$  is the design story shear.

### 3 Pre-capping deflection

The design strength,  $V_c$ , is connected to the yield strength,  $V_y$ , by the parameter  $C_{vc}$ :

$$V_{v} = C_{vc}V_{c} \tag{7}$$

The deflection at yield and pre-capping deflection can now be calculated. Using linear relations and equation (7), the following equations are developed:

$$\Delta_{y} = \frac{V_{y}}{K_{x}} \tag{8}$$

$$\Delta_p = \frac{V_c - V_y}{a_s K_x} \tag{9}$$

These equations are evaluated for each floor x.  $a_s$  and  $C_{yc}$  are assumed to be the same for each story.

# 4 Post-capping and ultimate deflection

The post-capping deflection is currently defined as proportional to  $\Delta_p$ :

$$\Delta_{pc} = C_{pcp} \Delta_p \tag{10}$$

The ultimate deflection is currently defined as proportional to the sum of the previously calculated deflections:

$$\Delta_u = C_{upc} \left( \Delta_y + \Delta_p + \Delta_{pc} \right) \tag{11}$$