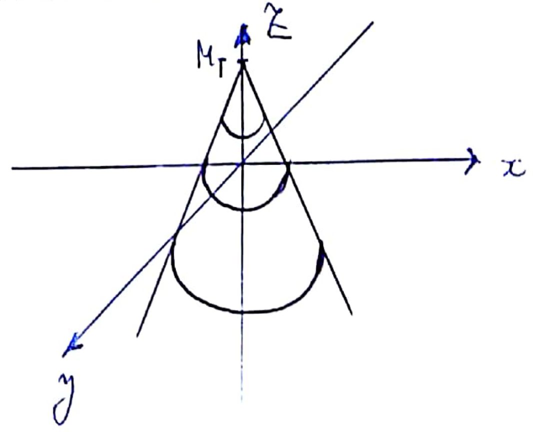
Create Cone:



$$M_T = c\,\frac{\cos\varphi}{\sin\varphi}$$

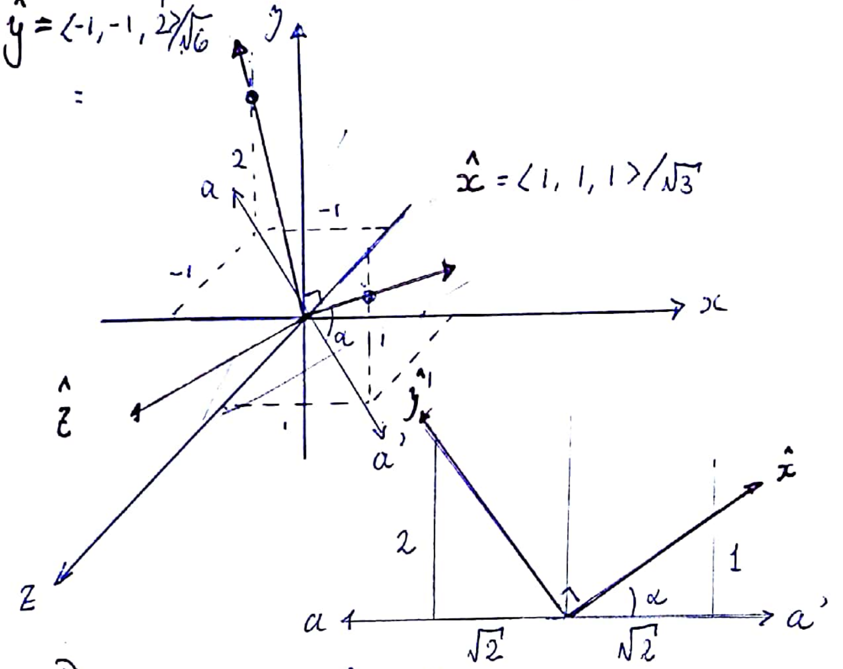
θ = linspace(0, 2π, 30)

v = linspace($-1.4\,M_T$, M_T, 30)

[θ, v] = meshgrid(θ, v)

$$\begin{array}{l} v \uparrow \\ \begin{bmatrix} (\theta_1, v_1) & (\theta_2, v_1) & \cdots \\ (\theta_1, v_2) & (\theta_2, v_2) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \xrightarrow[z = v]{\substack{x = (v - M_T)\,c\cos\varphi \\ y = (v - M_T)\,c\sin\varphi}} \begin{bmatrix} (x_{11}\, y_{11}\, z_{11}) & (x_{12}\, y_{12}\, z_{12}) & \cdots \\ (x_{21}\, y_{21}\, z_{21}) & (x_{22}\, y_{22}\, z_{22}) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \\ \longrightarrow \theta \end{array}$$

Transformation Matrix:



$\hat{x} = \langle 1, 1, 1 \rangle / \sqrt{3}$

$\hat{y} = \langle -1, -1, 2 \rangle / \sqrt{6}$

$\hat{z}$ = -cross($\hat{x}$, $\hat{y}$)

$\Gamma = [\hat{z}, \hat{y}, \hat{x}]$

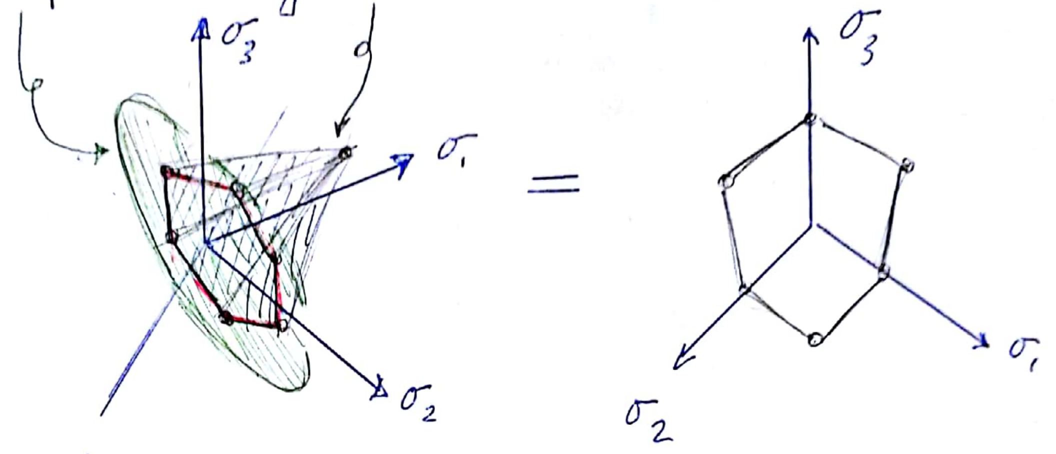
Project z onto $\hat{x}$

for row in x, y, z

$$xyz = [X(r,:), Y(r,:), Z(r,:)] = \begin{Bmatrix} x_{r1} & x_{r2} & x_{r3} & \cdots \\ y_{r1} & y_{r2} & y_{r3} & \cdots \\ z_{r1} & z_{r2} & z_{r3} & \cdots \end{Bmatrix}$$

$$\hat{x}\hat{y}\hat{z} = \Gamma \cdot xyz = \begin{Bmatrix} \hat{z}_1 x_r + \hat{y}_1 y_r + \hat{x}_1 z_r \\ \hat{z}_2 x_r + \hat{y}_2 y_r + \hat{x}_2 z_r \\ \hat{z}_3 x_r + \hat{y}_3 y_r + \hat{x}_3 z_r \end{Bmatrix}$$

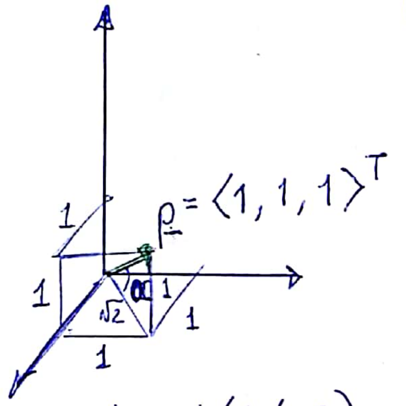
Eqn of plane + Yield fn = π-plot

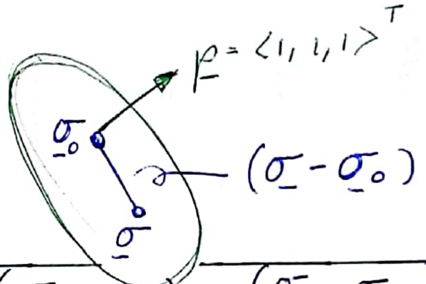


Eqn of plane : $\boxed{\underline{p} \cdot (\underline{\sigma} - \underline{\sigma_0}) = 0}$ (1)

normal vector

pt. in plane e.g. origin





$\alpha = \tan^{-1}(1/\sqrt{2})$

$$\boxed{(\sigma_1 - \sigma_{0_1}) + (\sigma_2 - \sigma_{0_2}) + (\sigma_3 - \sigma_{0_3}) = 0} \quad (3)$$

Yield fn : $\boxed{f(\sigma_1, \sigma_2, \sigma_3) = 0}$ (2)

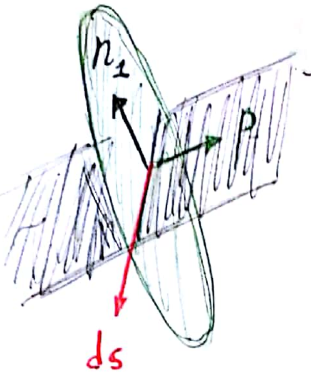
6 planes

$$f = \frac{1}{2}(\sigma_\alpha - \sigma_\beta) + \frac{1}{2}(\sigma_\alpha + \sigma_\beta)\sin\phi - c\cos\varphi = 0$$

or, $\boxed{n_\alpha \sigma_\alpha + n_\beta \sigma_\beta - c\cos\varphi = 0}$ (4)

where $n_\alpha = +\frac{1}{2}(1 + \sin\varphi)$, $n_\beta = -\frac{1}{2}(1 - \sin\varphi)$

Intersection btwn plane & yield fn:



f_1 $(\alpha = x, \ \beta = y)$

$$ds = p \times n_1$$

Parametric line eqn: $s_1 = s_0 + t\, ds$

Let $\sigma_3 = 0 \longrightarrow$ solve (plane $= 0$, $f_1 = 0$) $\rightarrow \sigma_1, \sigma_2$

$$s_0 = \langle \sigma_1, \sigma_2, 0 \rangle$$

Repeat for 5 remaining yield Surfaces.

$$p^T = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}, \quad n^T = \left[\begin{array}{c|c|c|c|c|c} n_\alpha & n_\alpha & 0 & n_\beta & n_\beta & 0 \\ n_\beta & 0 & n_\alpha & n_\alpha & 0 & n_\beta \\ 0 & n_\beta & n_\beta & 0 & n_\alpha & n_\alpha \end{array}\right]$$

```
for i in range(6):
    ds = cross (p, n[i])
    $f_i$ = n[i][0] $\sigma_1$ + n[i][1] $\sigma_2$ - c.cos$\varphi$      (4)
    plane = ($\sigma_1 - \sigma_{o1}$) + ($\sigma_2 - \sigma_{o2}$)      (3)
    Sol = solve ([$f_i$, plane], dict= True)
    So[0] = sol[0][$\sigma_1$] ; So[1] = sol[0][$\sigma_2$]
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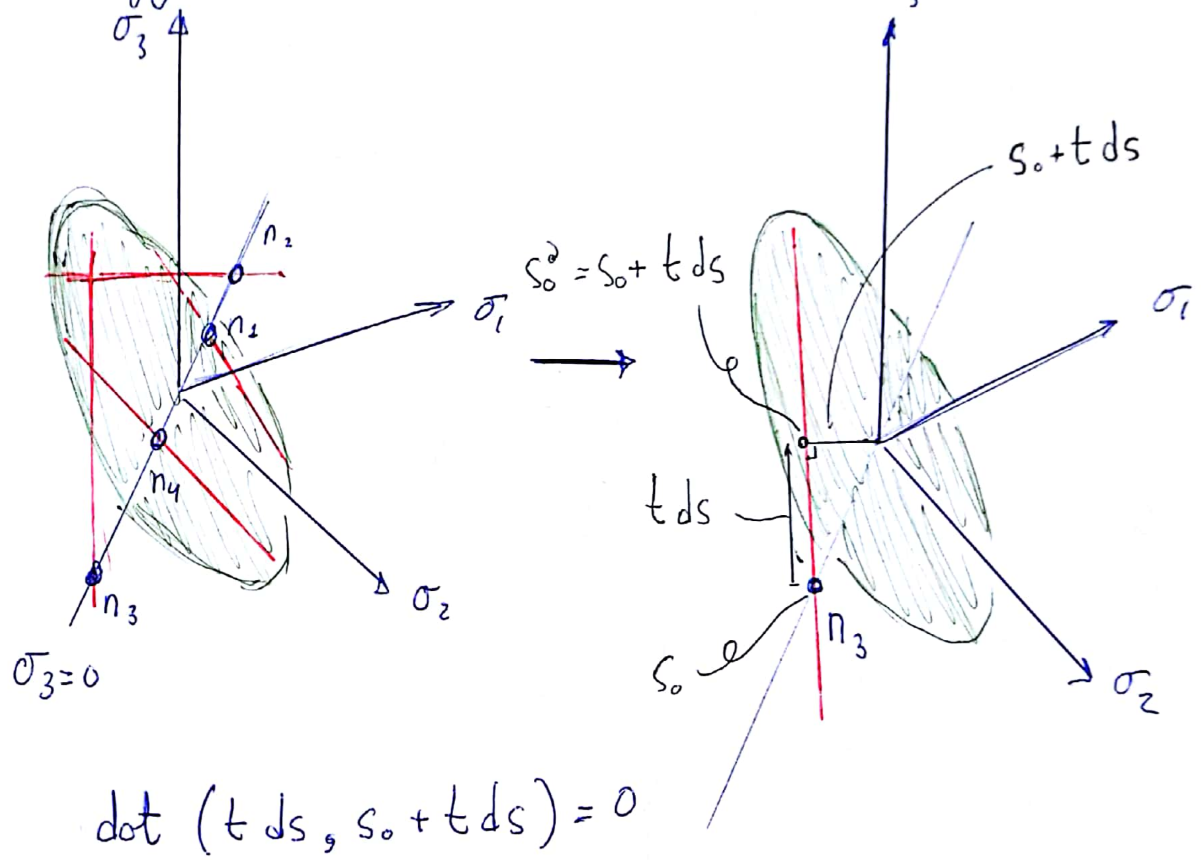
* * *

Point on line closest to origin:

Modify s_0 so it's closest



$$dot\,(t\,ds\,,\ s_0 + t\,ds) = 0$$

$$t(ds_1\,s_{0_1} + ds_2\,s_{0_2} + ds_3\,s_{0_3}) + t^2\,(ds_1^{\,2} + ds_2^{\,2} + ds_3^{\,2}) = 0$$

$$dot(ds, ds)\,t^2 + dot\,(ds_4, s_0)\,t = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b}{2a} \pm \frac{b}{2a}$$

$$t = -b/a = -\frac{dot(ds, s_0)}{dot\,(ds, ds)}$$

$$s_0 \mathrel{+}= t \cdot ds$$

This is used to position text indicating which f_i

Intersection btwn yield fn lines:

Store s_0 & ds in matrix form:

$$s_0^T = \left[\begin{array}{c|c|c} s_0(0,0) & s_0(1,0) & \\ s_0(0,1) & s_0(1,1) & \cdots \\ s_0(0,2) & s_0(1,2) & \end{array}\right]$$

$$ds^T = \left[\begin{array}{c|c|c} ds(0,0) & ds(1,0) & \\ ds(0,1) & ds(1,1) & \cdots \\ ds(0,2) & ds(1,2) & \end{array}\right]$$

Need to solve for x & y such that:

$$s_0[i] + x\, ds[i] = s_0[i+1] + y\, ds[i+1]$$

but since we only have 2 vars we can

$$s_0[i,0:2] + x\, ds[i,0:2] = s_0[i+1,0:2] + y\, ds[i+1,0:2]$$

```
for i in range (5)
    eqns = S0[i,0:2]+x ds[i,0:2] - S0[i+1,0:2] - y ds[i+1,0:2]
    sol = solve (eqns)
    S1[i] = S0[i]+ sol(x) ds[i]

# pt. between i=5 & i=0
# add S1[6]= S1[0]
```