Appendix A

Proof. Theorem 1. It can be explained by observing that, for $K+1$ concepts containing K existed concepts $c_1, \cdots c_K$ and a new added concept γ, we can produce the first level trees combinations as below. Notice that each atomic element o can be one of the $c_1, \cdots c_K$ concepts. In order to compute the total number of trees combinations, we show what is the number of tree combinations by assigning the K concepts to each item:

- $(\gamma(\overbrace{o \cdots o}^{K \text{concepts}}))$: the number of trees combinations by taking the concept labels into the account are: $\binom{K}{0} L(1) \times 2 \times L(K)$; the reason for multiplying the number of trees combinations for K concepts to 2 is because while the left side contains an atomic γ concept, there are two choices for the right side of the tree in the first level: either we compute the total number of trees for K concepts from the first level or we keep the first level as a $\overbrace{o \cdots o}^{K \text{concepts}}$ atomics and keep all K concepts together, then continue the number of K trees combinations from the second level of the tree.
- $((\gamma o)(\overbrace{o \cdots o}^{K-1 \text{concepts}}))$: similar to the previous part we have $\binom{K}{1} L(2) \times 2 \times L(K-1)$ trees combinations by taking the concepts labels into the account. $\binom{K}{1}$ indicates the number of combinations for choosing a concept from the K concept and put it with the new concept separately. While $L(2)$ is the number of trees combinations for the left side of tree separated with the new concept γ.
- $((\gamma o o)(\overbrace{o \cdots o}^{K-2 \text{concepts}})), \cdots$
- $((\gamma \overbrace{o \cdots o}^{K-1 \text{concepts}})o)$: $\binom{K}{K-1} L(K) L(1)$ in this special part, we follow the same formula except the single concept in the right side has only one possible combination in the first level equal to $L(1)$.

All in all, the sum of these items calculates the total number of tree hierarchies for $K+1$ concepts.

The first few number of total number of trees combinations for $1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \cdots$ concepts are: 1 , 1, 4, 26, 236, 2752, 39208, 660032, 12818912, $282137824, \cdots$. In the case of the SHL dataset that we use in the empirical evaluation, we have 8 different concepts and thus, the number of different types of hierarchies for this case is $L(8) = 660,032$.