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Dynamic Collision Avoidance of Mobile Robot Based on Velocity Obstacles

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***Abstract*—The dynamic avoiding collision with moving obstacles of mobile robot has been studied. For the existing problems of the past velocity obstacles methods used for dynamic collision avoidance, the collision distance and collision time were considered. Furthermore, in the velocity variation space, we translated the dynamics collision avoidance problem into an optimization problem, defined a new objective function, and the corresponding dynamic collision avoidance arithmetic was designed. Simulation experiments show that the proposed methods effectively overcome conservative strategy of robot's motion of collision avoidance, achieve rapid movement towards the goal when robot avoiding collision with moving obstacles.**

Keywords- mobile robot; velocity obstacles; dynamic collision avoidance; motion planning

I. INTRODUCTION

The dynamic avoiding collision with moving obstacles is a basic problem of mobile robot motion planning, many scholars in recent years have been studying this important subject. Where, a velocity obstacles method was proposed [1,2], which translated the Configuration Obstacles into Velocity Obstacles according to relative velocity information in the velocity space. Then based on velocity obstacle, the conception of nonlinear velocity obstacle was proposed [3,4], too. Furthermore, references [5,6] through the establishment of the relative coordinates, used the relative velocity to make motion planning in the acceleration space. References [7-9] seen collision avoidance behavior as the interactive dynamic process between the robot and moving obstacles, translated the collision avoidance problems into a control problems or optimization problems in robot's acceleration space.

However, these methods based on the velocity obstacles were only make planning in robot's velocity space achieved in one control period, did not fully take into account the impact of distance and time before collision, and the reasonable selection issues of achievable velocity space under kinetics constraints were not considered.

In this paper, first, we describe and analyze the dynamic collision avoidance method based on velocity obstacles, then depending on the existed problems, develop a corresponding improvement methods, in the velocity variation space, defined a new optimization objective function, make the robot's dynamic collision avoidance planning.

II. PRINCIPLE OF VELOCITY OBSTACLES

To the plane moving object P, suppose at time t its speed is $\boldsymbol{V}_P$, then can use its reference position and velocity vector to express its state. Therefore, the motion of object P can be defined as

$$\boldsymbol{P}(t)=(\boldsymbol{x}_P(t),\ \boldsymbol{V}_P(t)) \tag{1}$$

Where, $\boldsymbol{x}_P(t)=[x_P,\ y_P]^T$ is position coordinates, $\boldsymbol{V}_P(t)$ is velocity vector. In the following analysis, the sensor detects the movement of obstacles which to be reduced to round, set the current time is t, the planning and control period is T, the velocity of robot and all moving obstacles within period T remains the same, only vary at next moment, i.e. $t + T$.

Shown as Fig. 1, at current time t, in global coordinates $\{X,\ Y\}$, mobile robot R positioning at $\boldsymbol{x}_R=(x_R,\ y_R)$, its velocity is $\boldsymbol{V}_R$; moving obstacle O positioning at $\boldsymbol{x}_O=(x_O,\ y_O)$, its velocity is $\boldsymbol{V}_O$, the radius of the obstacle O "expand" to R_O according to the robot's size, so the mobile robot can be modeled as a mass point, and call the "expanded" obstacle O as configured Position Obstacle (i.e. PO). l_{MO} and l_{NO} are the rays of tangent between the R and O, D_{RO} is the measured distance between R and O in the ray direction l_{RO}.

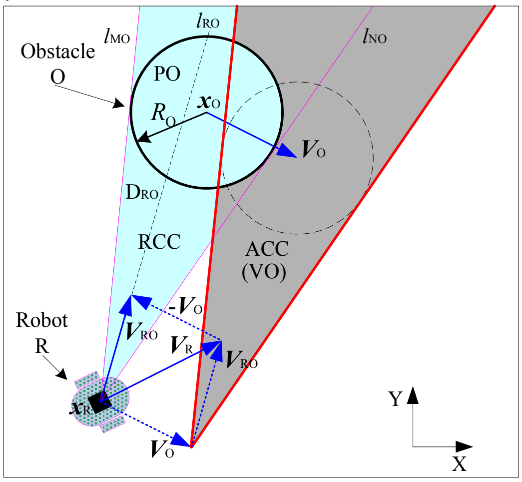

Figure 1. Collision avoidance planning based on relative velocity

(1) Relative Collision Cone

Define the relative velocity of mobile robot and obstacle

$$\boldsymbol{V}_{RO} = \boldsymbol{V}_R - \boldsymbol{V}_O \tag{2}$$

Then we can translate the dynamic collision avoiding problem into a static problem by relative velocity, i.e. the O seen as a static obstacle, the robot's speed is seen as $\boldsymbol{V}_{\mathrm{RO}}$. The collision avoidance planning between motion ($\boldsymbol{x}_{\mathrm{R}}(t)$, $\boldsymbol{V}_{\mathrm{R}}(t)$) and motion ($\boldsymbol{x}_{\mathrm{O}}(t)$,$\boldsymbol{V}_{\mathrm{O}}(t)$) is equivalent to the planning between the robot with velocity $\boldsymbol{V}_{\mathrm{RO}}(t)$ and the static obstacle with radius R_{O}.

If the $\boldsymbol{V}_{\mathrm{RO}}$ stay the same, l_{RO} is its direction-ray, the collision conditions for robot and the obstacle O is

$$l_{RO} \cap \mathrm{PO} \neq \varnothing \tag{3}$$

So that equation (3) set up a collection of relative velocity, defined as the Relative Collision Cone (RCC) in the velocity space.

$$\mathrm{RCC} = \{\, \boldsymbol{V}_{RO} \mid l_{RO} \cap \mathrm{PO} \neq \varnothing \,\} \tag{4}$$

That is the area between l_{MO} and l_{NO}, as shown in Fig. 1. To the robot's relative velocity $\boldsymbol{V}_{\mathrm{RO}}$, if $\boldsymbol{V}_{\mathrm{RO}} \in$ RCC, then the robot will collide with the obstacle O.

(2) Velocity obstacles

RCC expressed the collision caused by relative velocity $\boldsymbol{V}_{\mathrm{RO}}$, indirectly define the set of $\boldsymbol{V}_{\mathrm{R}}$ which would result in collision between R and O. Shown as in Fig. 1, translate RCC along $\boldsymbol{V}_{\mathrm{O}}$ obtain the region called as ACC(Absolute Collision Cone)[2].

$$\mathrm{ACC}=\mathrm{RCC} \oplus \boldsymbol{V}_O \tag{5}$$

Where $\oplus$ is the Minkowski vector sum operator.

It can be seen that, the tip of $\boldsymbol{V}_{\mathrm{R}}$ inside ACC equivalent to $\boldsymbol{V}_{\mathrm{RO}} \in$ RCC, so when the tip of $\boldsymbol{V}_{\mathrm{R}}$ inside ACC the collision may happen. ACC directly expressed the set of $\boldsymbol{V}_{\mathrm{R}}$ that the robot may collide with the obstacle O, which called VO (Velocity Obstacle).

$$\mathrm{VO} = \{\boldsymbol{V}_R \mid (\boldsymbol{V}_R - \boldsymbol{V}_O) \in RCC\} \tag{6}$$

Fig. 2 is an example of multi-moving obstacles, where the velocity of O_1 is VO_1, its velocity obstacle is VO_1; the velocity of O_2 is VO_2, its velocity obstacle is VO_2. It can be seen that the tip of $\boldsymbol{V}_R$ inside ACC_1 and ACC_2, i.e. $\boldsymbol{V}_{\mathrm{R}} \in \mathrm{VO}_1$ and $\boldsymbol{V}_{\mathrm{R}} \in \mathrm{VO}_2$, if robot and O_1 and O_2 maintain their current velocities in the next times, the robot may collide with O_1 and O_2. Otherwise, Selecting $\boldsymbol{V}_{\mathrm{R}}$ outside of VO_1 and VO_2 in the next times would avoid collision.

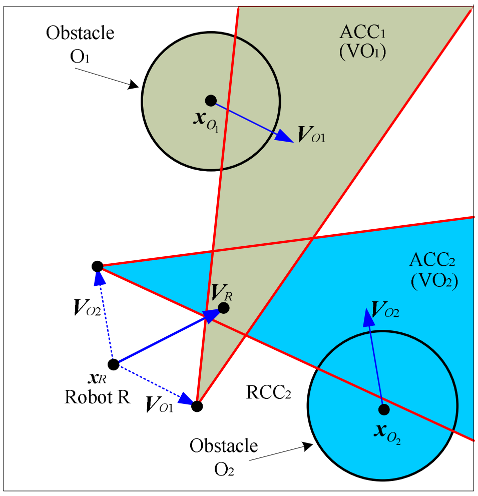

Figure 2. Velocity obstacles method illustrations

By above analysis, the following conclusions can be drawn.

Lemma 1: To the mobile robot and a single moving obstacle O, calculate the PO, RCC, and VO, respectively, then the collision avoidance conditions are:

$\boldsymbol{P}_R \notin \mathrm{PO}$ and $\boldsymbol{V}_R \notin \mathrm{VO}$ (i.e. the tip of $\boldsymbol{V}_{\mathrm{R}}$ outside ACC)

Lemma 2: To the mobile robot and multi-moving obstacles $\{\mathrm{O}_1, \mathrm{O}_2, \dots, \mathrm{O}_N\}$, calculate the PO_i, ACC_i and VO_i correspond to $\mathrm{O}_i (i=1,2,\dots,N)$, respectively, then the collision avoidance conditions are:

$\boldsymbol{P}_R \notin \mathrm{MPO}$ and $\boldsymbol{V}_R \notin \mathrm{MVO}$ (i.e. the tip of $\boldsymbol{V}_{\mathrm{R}}$ outside MACC)

Where $\mathrm{MPO} = \bigcup_{i=1}^{N} \mathrm{PO}_k$, $\mathrm{MVO} = \bigcup_{i=1}^{N} \mathrm{VO}_k$, $\mathrm{MACC} = \bigcup_{i=1}^{N} \mathrm{ACC}_k$.

III. Improved Velocity Obstacles Method

A. Distance and Time before Collision

In order to eliminate the conservative collision avoidance strategy of robot [10], considering the distance and time before collision, modify the radius of moving obstacles as follow.

$$\hat{R}_{\mathrm{O}_k} = (h(\gamma_t) + h(\gamma_D)) R_{\mathrm{O}_k} \tag{7}$$

Where $\gamma_t = \dfrac{t_{\mathrm{C}_k}}{T}$, $\gamma_D = \dfrac{D_{\mathrm{RO}_k}}{L_{\mathrm{saf}}}$, $t_{\mathrm{c}_k} = \dfrac{D_{\mathrm{RO}_k} - R_{\mathrm{rob}}}{V_{\mathrm{RO}_k}}$ is called as collision time, D_{RO_k} is called as collision distance. $h(\gamma_t)$ and $h(\gamma_D)$ are weighting function[11], if $h(\gamma_t) + h(\gamma_D) > 1$ then set $h(\gamma_t) + h(\gamma_D) = 1$.

B. Velocity Variation Space

If mobile robot does not collide with the moving obstacles during time Δt, and the variation of the robot's velocity during Δt is expressed as

$$x = \Delta\theta_R, \quad y = \Delta V_R \tag{8}$$

The space defined by (8) is the velocity variation space.

As seen in Fig. 1, the collision area of obstacle O in velocity variation space corresponding to RCC is

$$D = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \Re^2 \;\middle|\; \begin{array}{l} \varphi_N \le \angle(([V_R + y, \theta_R + x]^T - [V_O, \theta_O]^T), X_R) \le \varphi_M, \varphi_M \ge \varphi_N \\ \begin{cases} \angle(([V_R + y, \theta_R + x]^T - [V_O, \theta_O]^T), X_R) \ge \varphi_N \\ \angle(([V_R + y, \theta_R + x]^T - [V_O, \theta_O]^T), X_R) \le \varphi_M \end{cases}, \quad \varphi_M < \varphi_N \end{array} \right\} \tag{9}$$

Where φ_M and φ_N are the angle of l_{MO} and l_{NO}, respectively. The complement of D denoted by $\overline{D}$, then the non-collision area is $D_{void} = \overline{D}$.

Consider the kinematics and dynamics constraints of robot, here we set $\Delta v_{R\max}$ and $\Delta v_{R\min}$ as the maximum and minimum variation of amplitude of $\boldsymbol{V}_{\mathrm{R}}$ during T; set $\Delta\alpha_{R\max}$ and $\Delta\alpha_{R\min}$ as the maximum and minimum variation of angle of $\boldsymbol{V}_{\mathrm{R}}$ during T. Then the achievable variation area of $\boldsymbol{V}_{\mathrm{R}}$ during Δt is

$$\Omega_{fea}=\left\{\begin{bmatrix}x\\y\end{bmatrix}\in\Re^2 \;\middle|\; \begin{array}{c}\Delta\alpha_{R\min}\dfrac{\Delta t}{T}\le x\le\Delta\alpha_{R\max}\dfrac{\Delta t}{T}\\ \max(-V_R,\Delta v_{R\min}\dfrac{\Delta t}{T})\le y\le\min(v_{R\max}-V_R,\Delta v_{R\max}\dfrac{\Delta t}{T})\end{array}\right\} \tag{10}$$

Generally, have $\Delta t \ge T$, so we can call Ω_{fea} as multi-step achievable dynamic window in the velocity variation space.

For example, in the current some velocities of robot and single obstacle O, the area of D and Ω_{fea} as shown in Fig. 3.

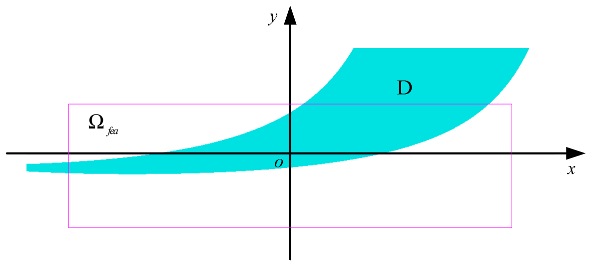

Figure 3. The ***D*** and Ω_{fea} of obstacle **O** in velocity variation space

When the robot detects a dynamic obstacles set $Dob(N)$, to every $O_i \in Dob(N)$, has one D_i accordingly. So, in the velocity variation space, the non-collision area of robot is

$$D_{void}=\bigcap_{i\in N} D_{void}^{i}=\overline{\bigcup_{i\in N} D_i} \tag{11}$$

And by the collision analysis in velocity variation space, if robot may collide with {O_1, O_2, … , O_m} of $Dob(N)$ (m≤N), calculate the corresponding collision time t_{c_k} (k=1,2,…,m) between the robot and obstacle O_k, let Δt =min($t_{c_1}, t_{c_2}, \ldots, t_{c_m}$) in order to determine Ω_{fea}.

Meanwhile, the robot towards the goal can be seen as an anti-problem of collision avoiding. If the velocity of goal point G is $V_G=[V_G,\theta_G]^T$, then the velocity goal of robot is $\boldsymbol{V}_g = \boldsymbol{V}_{RG} + \boldsymbol{V}_G$, where $\boldsymbol{V}_{RG} = [V_{RG},\theta_{goal}]^T$ ($V_{RG}=\min(V_{R\max},\sqrt{\|x_G-x_R\|})$, $\theta_{goal}=\angle((x_G-x_R),X_R)$). So, when calculate $\boldsymbol{V}_g=[V_g,\theta_g]^T$, the goal point p_g in velocity variation space corresponds to G is

$$x_g=\theta_g-\theta_R,\quad y_g=V_g-V_R$$

C. Optimal Collision Avoidance Strategies

In the velocity variation space, the Optional non-collision area is $D_{void}\cap\Omega_{fea}$, so the problem of dynamic collision avoidance can be transformed into the optimal selection of decision-making point inside $D_{void}\cap\Omega_{fea}$. And the decision-making point p* should be as close to the origin, while away from the goal point p_g as close as possible. To this end, optimization objective function is defined as

$$J(x,y)=k_1(x^2+y^2)+k_2(x-x_g)^2+k_3(y-y_g)^2 \tag{12}$$

But when $D_{void}\cap\Omega_{fea}=\Phi$, choose

$$p^*=\{(x,y)\,|\,x=\min(|\Delta\alpha_{R\min}\frac{\Delta t}{T}-x_g|,|\Delta\alpha_{R\max}\frac{\Delta t}{T}-x_g|),y=0\} \tag{13}$$

D. Dynamic Planning Algorithm for Collision Avoiding

Assuming the mobile robots detects N moving obstacles using perception sensors, i.e. $Dob(N)=\{O_1,O_2,\cdots,O_N\}$, where every $O_i=\{x_{O_i},R_{O_i},\boldsymbol{V}_{O_i}\}$ maybe collide with robot. Design the algorithm for Dynamic collision avoidance as follow.

- Input: $Dob(N)=\{O_1,O_2,\cdots,O_N\}$, V_R, θ_R, $P_R(x_R,y_R)$, $P_G(x_G,y_G)$
- Output: $\dot V_R$, $\dot\theta_R$

Step 1: initialize ca=1, J_{temp}=J_{max} (a large positive number), $up=0$

Step 2: For i=1 to N calculate $\varphi_{M_i}, \varphi_{N_i}, t_{ci}$;

Get $\Delta t=\min(t_{c1},t_{c2},\cdots,t_{cN})$

Step 3: based on $V_R,\theta_R,\Delta t$, calculate Ω_{fea}

Step 4: For x=$\underline{\Delta\theta}$ to $\overline{\Delta\theta}$

{For y=$\underline{\Delta V}$ to $\overline{\Delta V}$

{ca=1;

For i=1 to N

{

Calculate

$$\theta_{RO_i}=\angle(([V_R+y,\theta_R+x]^T-[V_{O_i},\theta_{O_i}]^T),X_R);$$

If $\varphi_{M_i}\ge\varphi_{N_i}$ then

{If $\varphi_{N_i}\le\theta_{RO_i}\le\varphi_{M_i}$ then ca=0;}

If $\varphi_{M_i}<\varphi_{N_i}$ then

{If $\varphi_{N_i}\le\theta_{RO_i}$ or $\theta_{RO_i}\le\varphi_{M_i}$ then ca=0}

}

If ca=1 then

{Calculate $J(x,y)$;

If $J(x,y)<J_{temp}$ then {J_{temp}= $J(x,y)$; p* = {x,y};}

$up=1$;}

}

}

Step 5: If $x==\overline{\Delta\theta}$ and $y==\overline{\Delta V}$

If($up==0$) then

$$p^*=\{(x,y)\,|\,x=\min(|\underline{\Delta\theta}-x_g|,|\overline{\Delta\theta}-x_g|),y=0\}$$

Step 6: calculate $\dot V_R=y^*/T$, $\dot\theta_R=x^*/T$

Step 7: If $\dot V_R\ge\Delta v_{R\max}$ then $\dot V_R=\Delta v_{R\max}$;

If $\dot V_R\le\Delta v_{R\min}$ then $\dot V_R=\Delta v_{R\min}$;

If $\ddot\theta_R\ge\Delta\alpha_{R\min}$ then $\ddot\theta_R=\Delta\alpha_{R\max}$;

If $\ddot\theta_R\le\Delta\alpha_{R\min}$ then $\ddot\theta_R=\Delta\alpha_{R\min}$

IV. SIMULATION EXPERIMENTS AND ANALYSIS

Make simulation experiments under Visual C + + soft environment, set $v_{R\max} = 1.0$ m/s, $\Delta v_{R\max} = 0.4$ m/s^2, $\Delta v_{R\min} = -1.2$ m/s^2, $\omega_{R\max} = 2.0$ rad/s, $\Delta\alpha_{R\max} = 0.5$ rad/s^2, $\Delta\alpha_{R\min} = -0.5$ rad/s^2, and assuming the size and velocity of moving obstacles have been detected.

Robot toward the goal can be seen as a dynamic collision problem, Fig. 5 shows the process of mobile robot avoiding moving obstacle and catching the moving goal.

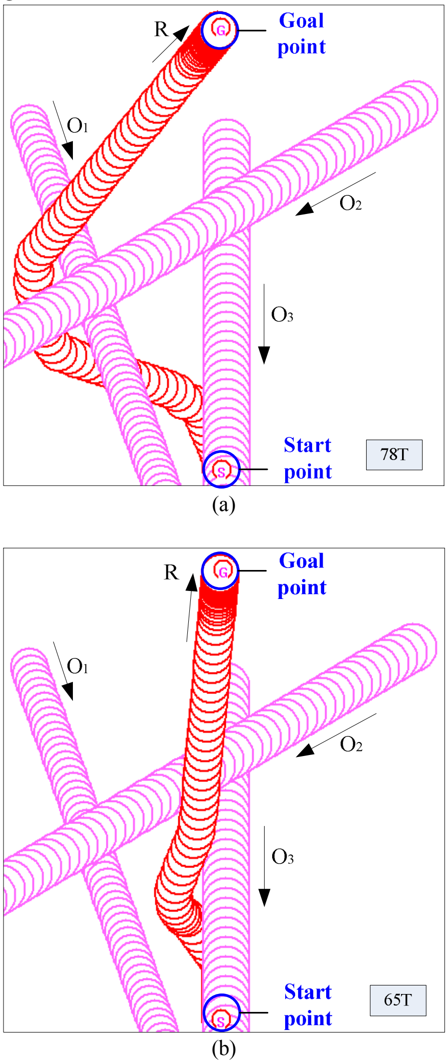

Figure 4. Avoiding collisions with multiple moving obstacles

As shown in Fig. 5.18, there are three moving the obstacles O1, O2, O3, figure (a) and (b) are the simulation results of mobile robot dynamic collision avoiding. Where, figure (a) is the motion path Did not consider the collision time and collision distance; figure (b) is the motion path considered the collision time and collision distance, using (7) to modify the radius of obstacle. It can be seen by comparing that Figure (b) avoid the risk of collision but also eliminates the conservative planning of figure (a), achieving better path, the time spent to reach the Goal is 65T (i.e. 65 periods), less than the figure (a), 78T.

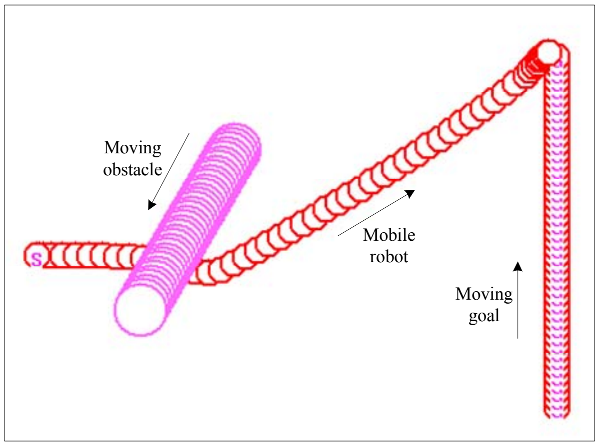

Figure 5. Avoiding moving obstacle and catching the moving goal

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