# Choquet Integral

An Example:

- Suppose a school is more scientifically than literary oriented.
- How can we compare these 3 students?

| *Student* | *Math* | *Physics* | *Literature* |
|---|---|---|---|
| a | 18 | 16 | 10 |
| b | 10 | 12 | 18 |
| c | 14 | 15 | 15 |

- A candidate set of weights can be $\{\frac{3}{8}, \frac{3}{8}, \frac{2}{8}\}$.
- But what if the school wants to favor well equilibrated students without weak points?
  - Then the student $c$ should be considered better than student $a$ and $b$.
  - This cannot be simply done by simple weighting sum procedure!!
- So how are we going rank them?!
  - Choquet integral can address such problems.

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- Choquet Integral

$$\mathcal{C}_\mu(x) = \sum_{i=1}^{n}(x_{\tau(i)} - x_{\tau(i-1)})\mu(\{\tau(i), \ldots, \tau(n)\})$$

$x_{\tau(1)} \leq x_{\tau(2)} \leq \ldots \leq x_{\tau(n)}, \;\; x_{\tau(0)} = 0$

# Choquet Integral

## Continued...

| *Student* | *Math(1)* | *Physics(2)* | *Literature(3)* | $\{\tau\}$ | $\{x_\tau\}$ |
|---|---|---|---|---|---|
| a | 18 | 16 | 10 | $\{0,\ 3,\ 2,\ 1\}$ | $\{0,\ 10,\ 16,\ 18\}$ |
| b | 10 | 12 | 18 | $\{0,\ 1,\ 2,\ 3\}$ | $\{0,\ 10,\ 12,\ 18\}$ |
| c | 14 | 15 | 15 | $\{0,\ 1,\ 2,\ 3\}$ | $\{0,\ 14,\ 15,\ 15\}$ |

$$\mathcal{C}_\mu(x) = \sum_{i=1}^{n} (x_{\tau(i)} - x_{\tau(i-1)}) \mu(\{\tau(i), \ldots, \tau(n)\})$$

$$\mu(\{1,\ 2,\ 3\}) = 1, \qquad \mu(\{\emptyset\}) = 0$$

$$\mu(\{1\}) = \mu(\{2\}) = 0.45, \qquad \mu(\{3\}) = 0.3$$

$$\mu(\{1,\ 3\}) = \mu(\{2,\ 3\}) = 0.9, \qquad \mu(\{1,\ 2\}) = 0.5$$

# Choquet Integral

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| c | 14 | 15 | 15 | $\{0,\ 1,\ 2,\ 3\}$ | $\{0,\ 14,\ 15,\ 15\}$ |

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$$\mathcal{C}_\mu(a) = (10-0) \times \mu(\{3,2,1\}) + (16-10) \times \mu(\{2,1\}) + (18-16) \times \mu(\{1\}) = 13.9$$
$$\mathcal{C}_\mu(b) = (10-0) \times \mu(\{1,2,3\}) + (12-10) \times \mu(\{2,3\}) + (18-12) \times \mu(\{3\}) = 13.6$$
$$\mathcal{C}_\mu(c) = (14-0) \times \mu(\{3,2,1\}) + (15-14) \times \mu(\{2,3\}) + (15-15) \times \mu(\{3\}) = 14.9$$

# Choquet Integral
## Continued...

| *Student* | *Math* | *Physics* | *Literature* | Weighted sum | Choquet integral |
|---|---|---|---|---|---|
| a | 18 | 16 | 10 | 15.25 | 13.9 |
| b | 10 | 12 | 18 | 12.75 | 13.6 |
| c | 14 | 15 | 15 | 14.62 | 14.9 |