Lect 11 3/12/18

Extrapolation:

X rooms are not all real nbers. for exple $X = \{0.5, 30\}$
It is a set of lines not $f(x)$. Extrapolation => dangerou
inside points: interpolation.
Outside: points: Extrapolations. or out of sample.

EXTRAPOLATION

$$X^* \notin \text{Range}[X] := [X_{1,\max}, X_{1\max}] \times [X_{.2,\max} \times$$
$$] \times \ldots \times [X_{.p,\max}, X_{.p,\max}]$$

$X_{.1} = f(x)$, $X_2 \sim f_1(x)$

if X^* is to n +the "tails" of this distr => Extra

$\vec{y} \in \mathbb{R}^n$
$X \in \mathbb{R}^{n \times (p+1)}$ s.t rank $[X] = p+1$
: linear, multivariants,

Linear Ind means duplicate info

$$\vec{b} = (X^T X)^{-1} X^T \vec{y}$$

$g(\vec{x}^*) = \vec{x}^* \vec{b} = b_0 + b_1 x_1^* + \ldots + b_p x_p^*$: is p+1 dimen space.

$\hat{\vec{y}} = X\vec{b}$ this is $n \times 1$.

$$\Rightarrow \hat{\vec{y}} = \underbrace{X(X^T X)^{-1} X^T}_{H} \vec{y}$$

H "hat max" is extimate.

$\hat{\vec{y}} = H\vec{y}$. Rank(H)?

$\vec{y} = g(\vec{x}) + \vec{e} \Rightarrow \vec{y} = \hat{\vec{y}} + \vec{e} \Rightarrow \vec{e} = \vec{y} - \hat{\vec{y}}$. what dim($\vec{e}$

$\dim(\vec{e})$ is "n by 1" $n \times 1$.

$$SSE = \sum_{i=1}^{10} e_i^2 \Rightarrow \|\vec{e}\|^2$$

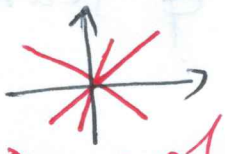
$$MSE = \frac{1}{n-(e+1)} SSE$$

MSE → mean Square

$n-(e+1)$ → called degree of freedom in model.

Why the model $(e+1)$ is $\Rightarrow$ there are "p+1" things that I can choose. Degree of freedom also means rank of H.

Having equation only with with slope



Degree of freedom is "1".

What is $RMSE = \sqrt{MSE} = \sqrt{\frac{1}{n-(e+1)} SSE}$

$$= \frac{\|\vec{e}\|}{\sqrt{n-(e+1)}}.$$

$$R^2 := \frac{S_y^2 - S_e^2}{S_y^2}$$

Data frame ≠ diff matrix (must be nbers)

If Data from x matrix $\Rightarrow$ error.

In real data, what do you think? Problem b/c co dim is incorrect.

Linear indpdt $\Rightarrow$ perfect

"1 col is exactly combination of others.

Rank (S) = 14 because 13 + 1

"Degree of freedom $\Rightarrow$ better fit.

If nom ↗ by 1 $\Rightarrow$ y ↗ by 1 (chart-projection). R-Studio.

RMSE $\simeq$ S_e → standard error. I $\simeq [\hat{y} \pm 2S_e]$

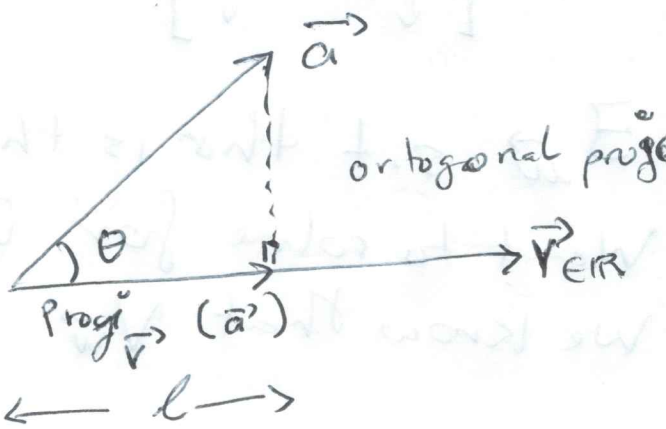
What "R" is doing inside!

R convert data frame into matrix.

Analysis of variance: it is a just a linear model on category predictor.

What is "p" with one predictor ⟹

LINEAR ALGEBRA



Imagine vectors $\vec{a}$ & $\vec{v}$.

BY LAW of COSINE

Law of

$$\cos(\theta) = \frac{\vec{a}\cdot\vec{v}}{\|\vec{a}\|\cdot\|\vec{v}\|}$$

$$\ell = \|\vec{a}\|\cos(\theta) = \frac{\vec{a}\cdot\vec{v}}{\|\vec{v}\|}$$

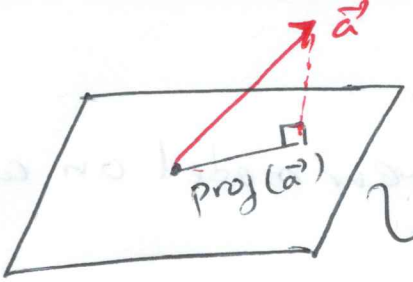
Projection $\text{Proj}(\vec{a}) = \ell \cdot \vec{v}_0 = \frac{\vec{a}\cdot\vec{v}}{\|v\|^2}\vec{v} = \frac{a^T\cdot\vec{v}\vec{v}}{\vec{v}^T\vec{v}} = \frac{v\vec{v}^T}{\vec{v}^T\vec{v}}\vec{a}$ ($\vec{v}^T\cdot\vec{a}$)

Let $\vec{a} \in \mathbb{R}^n$, $\vec{v} \in \mathbb{R}^n$

$\frac{\vec{v}\,\vec{v}^T}{\vec{v}^T\vec{v}}$ ($\vec{v}\,\vec{v}^T$: $m\times 1$) is called $H_{\vec{a}}$

$$HH = \left(\frac{\vec{v}\,\vec{v}^T}{\|\vec{v}\|^2}\right)\left(\frac{\vec{v}\,\vec{v}^T}{\|\vec{v}\|^2}\right) = \frac{\vec{v}\|\vec{v}\|^2\vec{v}^T}{(\|\vec{v}\|^2)^2} = \frac{\vec{v}\,\vec{v}^T}{\|\vec{v}\|^2} = H$$

$\|\vec{v}\|^2\cdot\|\vec{v}\|^2$



linearly indpdt

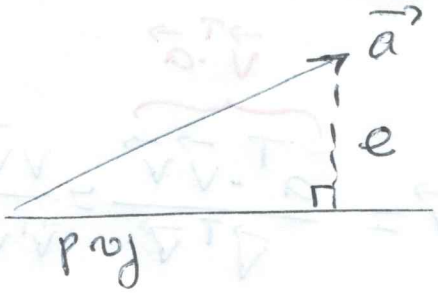
$U = \text{colsp}[\vec{u_1}, \cdots, \vec{u_k}]$

$V = \begin{bmatrix} \downarrow & & \downarrow \\ \vec{v} & \cdots & \vec{v}_k \\ \downarrow & & \downarrow \end{bmatrix}$ full rank.

$\exists \vec{w}$ s.t this is the $\text{proj}_U(\vec{a}) = V\vec{w}$.

Want to solve for $\vec{w}$.

We know that $V\vec{w}$



$\vec{e} + \text{proj}(\vec{a}) = \vec{a}$