IE400 - Study Set 1

June 12, 2024

1) Consider the following multiperiod, multiproduct Production-Inventory problem. Suppose that we are examining T periods $t = 1, \dots, T$, and some n products $i = 1, \dots, n$. There is an initial inventory of y_{i0} that is available at hand for each product i, $i = 1, \dots, n$. We need to principally determine the level of production for each product $i = 1, \dots, n$ during each of the periods $t = 1, \dots, T$, so as to meet the forecasted demand d_{it} for each product i during each period t when it occurs, at a minimal total cost. To take advantage of varying demand and costs, we are permitted to produce excess quantities to be stored for later use. However, each unit of product i consumes storage space s_i, and incurs storage cost of c_{it} during t, for $i = 1, \dots, n$, $t = 1, \dots, T$. The total storage space anticipated to be available for these n products during period t is S_t , $t = 1, \dots, T$. Furthermore, each unit of product i requires h_i hours of labor to produce, where the labor cost per hour when producing a unit of product i during period t is given by l_{it}, for $i = 1, \dots, n$, $t = 1, \dots, T$. The total number of labor hours available to manufacture these n products during period t is given by H_t. Formulate a linear program to solve this production-inventory control system problem, prescribing production as well as inventory levels for each product over each time period.

2) A company wishes to plan its production of two products with seasonal demands over a 12-month period from the beginning of January. The monthly demand of product 1 is 100,000 kg during the months of October, November, and December; 10,000 kg during the months of January, February, March and April; and 30,000 kg during the remaining months. The demand for product 2 is 50,000 kg during the months of October through February and 15,000 kg during the remaining months. Suppose that the cost of producing a kg of product 1 and 2 is \$5 and \$8, respectively, provided that these were produced prior to June. After June, the costs are reduced to \$4.5/kg and \$7/kg because of the installation of an improved production system. The total amount of products 1 and 2 that can be produced during any particular month cannot exceed 120,000 kg for Jan-Sept and 150,000 kg for Oct-Dec. Furthermore, at the end of each month products left in hand are carried to the inventory space in order to be used in the coming months. Each kg of product 1 occupies 2 cubic-feet and each kg of product 2, 4 cubic-feet of inventory. Suppose that the maximum inventory space allocated to these products is 150,000 cubic-feet and that the holding cost per cubic foot during any month is \$0.10. Formulate the production scheduling problem so that total production and inventory costs are minimized.

3) Sweet Taste produces two types of candies: Gummy Candy and Hard Candy, both of which consist of sugar, nuts, and chocolate. At present, Sweet Taste has in stock 15 kg of sugar, 3.5 kg of nuts, and 4.5 kg of chocolate. The mixture used to make Hard Candy must contain at least 22% nuts. The mixture used to make Gummy Candy must contain at least 8% nuts and 14% chocolate. Each gram of Hard Candy can be sold for $25, and each gram of Gummy Candy for $20. Formulate an LP that will maximize the revenue from candy sales.

4) Solve the following LP problem graphically.

$$\begin{array}{ll} \max & 11x + 3y \\ \text{s.t.} & 7x + 4y \leq 67, \\ & y \geq 6, \\ & -3x + 5y \leq 61, \\ & 6x - 4y \leq -12, \\ & x, y \geq 0. \end{array}$$

5) Solve the following LP problem graphically.

$$\begin{array}{ll} \min & 18a + 7b \\ \text{s.t.} & 17a + 9b \geq 87, \\ & 3a + 4b \geq 24, \\ & 3a - 9b \leq 30, \\ & 7a + 2b \geq 21, \\ & a, b \geq 0. \end{array}$$