BILKENT UNIVERSITY

PRINCIPLES OF ENGINEERING MANAGEMENT Quiz 1 - Solutions		
Code : *IE 400* Acad.Year : *2023-2024* Semester : *SPRING* Date : *16.02.2024* Time : *9:30* Duration : *50 min*	Last Name: Name : Student # : Signature :	Section #: 1
	2 QUESTIONS ON 2 PAGES TOTAL 10 POINTS	
1. (5)	2. (5)	

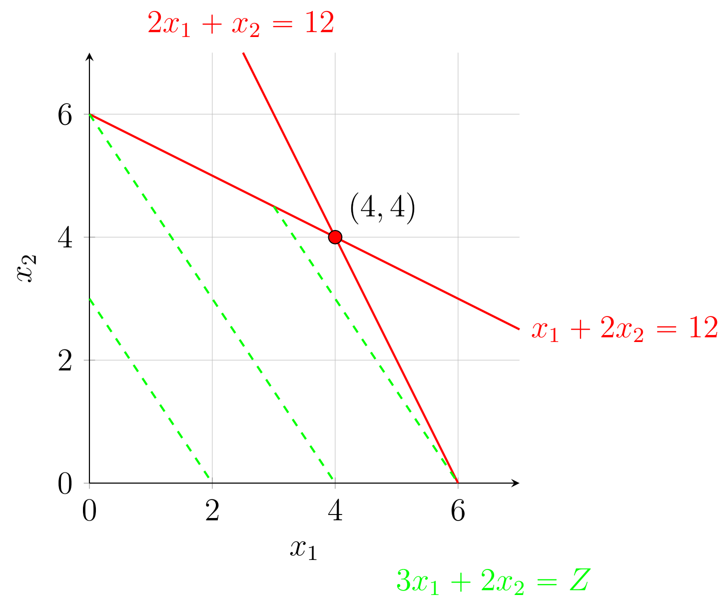
1. *(5 pts)* Consider the following LP problem:

$$\begin{aligned} \text{maximize} \quad & 3x_1 + 2x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \leqslant 12, \\ & x_1 + 2x_2 \leqslant 12, \\ & x_1, x_2 \geqslant 0. \end{aligned} \tag{P}$$

a) Solve this problem using the graphical method.

b) Let the optimal solution you find at **a)** be the pair $(\hat{x}_1, \hat{x}_2)$. Suppose that your objective function is replaced with $\alpha x_1 + 2x_2$ where α is some real number. Find all possible values of α, such that the unique optimal solution for this updated problem is $(\hat{x}_1, \hat{x}_2)$.

Sol:



a) Optimal solution is $(4, 4)$ with optimal value 20.

b) This point is the intersection lines $2x_1 + x_2 = 12$ and $x_1 + 2x_2 = 12$. Slopes of these lines are -2 and $-1/2$. If the slope of $\alpha x_1 + 2x_2$ is in the interval $(-2, -1/2)$, $(4, 4)$ will be the unique optimal solution for the updated problem. Slope of this new objective is $-\alpha/2$. Hence $\alpha \in (1, 4)$ is required.

2. *(5 pts)* A company makes two products (X and Y) using two machines (A and B). Each unit of X that is produced requires 50 minutes processing time on machine A and 30 minutes processing time on machine B. Each unit of Y that is produced requires 24 minutes processing time on machine A and 33 minutes processing time on machine B. At the start of the current week there are 30 units of X and 90 units of Y in stock. Available processing time on machine A is forecast to be 40 hours and on machine B is forecast to be 35 hours. The demand for X in the current week is forecast to be 75 units and for Y is forecast to be 95 units. Company policy is to maximise the combined sum of the units of X and the units of Y in stock at the end of the week.

a) Formulate the problem of deciding how much of each product to make in the current week as a linear program. Define the decision variables, constraints and the objective function explicitly.

b) Write the equation of an isoprofit line and write the improving direction for this problem.

Sol:

a) Let x be the number of units of X produced in current week and y be the number of units of Y produced in the week. Constraints are: machine A time $50x + 24y \leqslant 2400$, machine B time $30x + 33y \leqslant 2100$, minimum requirements $x \geqslant 75 - 30$ and $y \geqslant 95 - 90$. The objective is to maximize $(x + 30 - 75) + (y + 90 - 95)$. Hence we have the model

$$\begin{aligned} \max \quad & x + y - 50 \\ \text{s.t.} \quad & 50x + 24y \leqslant 2400, \\ & 30x + 33y \leqslant 2100, \\ & x \geqslant 45, \\ & y \geqslant 5, \\ & x, y \geqslant 0. \end{aligned}$$

b) $x + y - 50 = 0$ is an isoprofit line while $c = (1, 1)$ is the improving direction.