IE400 - Study Set 3

July 15, 2024

1) NASA has 4 astronauts to be employed at a space mission for 4 posts on a spaceship. The posts require different qualifications (competencies) but each astronaut can perform each one of these tasks, albeit with varying degrees of proficiency. The proficiency rating of astronaut i assigned to post j is measured as p_{ij}. It is asked to find the optimal assignment of astronauts to posts which gives the highest total proficiency rating while meeting the following requirements:

- Each astronaut should be assigned to a single post.
- Each post should have a single astronaut.

2) Ford has four automobile plants. Each is capable of producing the Taurus, Lincoln or Escort but it can only produce one of these cars. Suppose that Ford should produce all car types. The fixed cost of operating each plant for a year and the variable cost of producing a car of each type at each plant are given in the table below:

	Fixed Cost	**Variable Cost**		
		Taurus	Lincoln	Escort
Plant 1	7 billion	12000	16000	9000
Plant 2	6 billion	15000	18000	11000
Plant 3	4 billion	17000	19000	12000
Plant 4	2 billion	19000	22000	14000

a) Ford faces the following restrictions:

i. Each plant can produce one type of car.

ii. The total production of each type of car must be at a single plant; that is, for example, if any Tauruses are made at plant 1, then all Tauruses must be made there.

iii. If plants 3 and 4 are used, then plant 1 must also be used.

Formulate this problem in order to minimize the total cost.

b) Suppose now that instead of the restriction ii. in part a), Ford faces the following restriction: Either at most one of the plants 1 and 2 or at least one of the plants 3 and 4 should be used. Add the necessary constraint to your model.

3) Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \max \quad & 40x_1 + 5x_2 + 60x_3 + 8x_4 \\ \text{s.t.} \quad & 18x_1 + 3x_2 + 20x_3 + 5x_4 \leq 25, \\ & x_1, x_2, x_3, x_4 \in \{0, 1\} \end{aligned}$$

4) Use the branch-and-bound method to find the optimal solution to the following IP:

$$\begin{aligned} \min \quad & 9x_1 + 13x_2 + 10x_3 + 8x_4 + 8x_5 \\ \text{s.t.} \quad & 6x_1 + 3x_2 + 2x_3 + 4x_4 + 7x_5 \geq 40, \\ & x_1 \leq 1, x_2 \geq 1, x_3 \geq 2, x_4 \geq 1, x_5 \leq 3, \\ & x_1, x_2, x_3, x_4, x_5 \text{ integer.} \end{aligned}$$