IE400 - STUDY SET 2

July 1, 2024

1) Kocalar Delivery Company must make deliveries to 20 customers whose respective demands are d_j, for $j = 1, \ldots, 20$. Each customer can be visited by more than one truck to satisfy the demands. The company has seven trucks available with capacities C_k and daily operating costs q_k for $k = 1, \ldots, 7$.

   a) Formulate the model to determine which trucks to use so as to minimize the cost of delivering to all the customers.

   b) Add the following requirements to your model:

- A single truck cannot deliver to more than six customers.

- Customer 1 and customer 7 cannot be visited by the same truck.

- Both customer 2 and customer 12 should be visited by the same truck(s).

- If a truck visits customer 4 then it has to visit customer 19 or customer 20. (Assume demands of these customers do not exceed truck capacities.)

- Either at least two of the customers 5, 6 and 7 or at most three of customers 11, 12, 13 and 14 should be visited by truck 4.

2) A paper manufacturer produces rolls of standard fixed width w and of standard length l. Customers order rolls of width w but with varying lengths. In particular, d_k rolls with length l_k and width w are ordered for customer $k = 1, \ldots, n$. (assume $l_k \le l$ for all $k = 1, \ldots, n$). Model the problem to find the minimum number of rolls that should be cut to meet the demand?

Hint: Let M be a large number such that M number of rolls are enough to satisfy all demand (e.g. $M = \sum_{k=1}^{n} d_k$ is such value) Assume the manufacturer has M number of unmanufactured rolls in the stock. Try to use different decision variables than those we used in class.

3) Reatix must determine how much investment and dept to undertake during the next year. Each TL invested reduces the net present value (NPV) of the company by 10kr, and each TL of debt increases the NPV by 50kr (due to deductibility of interest payments). Reatix can invest at most 1 million TL during the coming year. Dept can be at most 40% of the investment. Reatix now has 800,00 TL in cash available. All investment must be paid from current cash or borrowed money. Set up an LP that maximizes the NPV of Reatix. Then, solve this LP by Simplex method.

4) Consider the following simplex tableau of a given minimization LP problem:

	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
z	1	0	0	A	-10	-1	B	32
s_3	0	0	0	C	4	-2	1	D
x_2	0	0	1	-2	F	-1	0	6
x_1	0	E	0	-1	1	2	0	4

Give general conditions on each of the unknowns A-F such that each of the following statements is true. Even if the statement holds independently of the values of a specific variable, you should still mention that variable.

a) The tableau is final and there exists a unique optimal solution.

b) The simplex method determines an unbounded solution from this tableau.

c) The current bfs is degenerate (not necessarily optimal).

5) Consider the following LP:

$$
\begin{aligned}
\min \quad & -x_1 - x_2 \\
\text{s.t.} \quad & x_1 - x_2 - x_3 = 1, \\
& -x_1 + x_2 + 2x_3 - x_4 = 1, \\
& x_1, x_2, x_3, x_4 \geq 0
\end{aligned}
$$

a) Construct the Big-M artificial LP. Construct the tableau which is ready to apply Simplex during Big-M method.

b) Does it correspond to an optimal table? If not, solve it by Simplex method.

c) What can you say about the problem after solving it by the Big-M method?