Topological Preliminaries:

Defination:

A (collection) family τ of subset of a set X is called a topology in X if τ contains the void set $\emptyset$ and set X, the union of every one of its subfamilies, and the intersection of every one of its finite subfamilies.

The pair (X, τ) is called a topological space.

→ X is called the topological space (where τ is understood)

→ A set X for which the topologi~~cal space~~ τ has been specified is called (topological space.)

Properties:

(1) $\emptyset$ and X are in τ.

(2) The union of the elements of any subcollection of τ is in τ.

(3) The intersection of the elements of any finite subcollection of τ is in τ.

* Topological space is an ordered pair (X, τ)
[(Consistin of set X) and (a topology on set X)] ie τ
(X, τ) where τ is defined on set (X)

If X is a topological space with topology τ
Subset of $(X) = U$
We say that if U Belongs to Collection of τ

Using this we say that: topological space is a set X togeather with a Collection of subset of X; "U" opensets. Such that $\emptyset$ and X are Both open from properties (ii) and (ii) we get → Arbitary unions and finite intersections of opensets are open.

Example:

$X = \{1, 2, 3\}$ we define the topology:

$\tau = \{\emptyset, \{1\}, \{1,2\}, X\}$

$\Downarrow$ $\Downarrow$

$\tau = \{\emptyset, \{1\}, \{1,2\}, \{1,2,3\}\}$;

(X, τ) Topologicalspace

$\tau =$
→ $\emptyset$ = Empty set
→ $\{1\}$ = Subset Containing only one element
→ $\{1, 2\}$ = " " two element
→ $X = \{1, 2, 3\}$ = Entire set

opensets for (X, τ)

In this topology Any set that belongs to τ is an open set. ie $\emptyset, \{1\}, \{1,2\}\ X$

Discrete topology:
let set $X = \{a, b, c\}$
Discrete topology (τ)
$= \{\emptyset, \{a\}, \{b\}, \{c\}$
$\{a,b\}\ \{a,c\}\ \{b,c\}$
$\{a,b,c\}\}$

If X is a set then collection of all subsets of X is a topology on X;

Similarly: Trival Topology: (Indiscrete)
Let $X = \{a, b, c\}$
The collection Consisting of X & $\emptyset$
$\tau = \{\emptyset, X\} = \{\emptyset, \{a, b, c\}\}$