**CIDM 6320- HOMEWORK ONE**

***PROOF EXERCISES***

***Proof One***

Show that **Solution**

We start the solution with a simplified linear regression model, where we do not include the y-intercept of the straight line

(1)

Where = dependent variable, =gradient, = independent variable, =random error term

Remember that ……………. (1.5)

In this case:

(2)

This implies that:

(3)

Remember that, if then, ………………... (3.5)

This implies that:

(4)

Remember

……………… (4.5)

This implies that:

(5)

Take the derivative of (5)

(6)

Methodology for the derivative (Chain rule)

Let

Let u=

Then (7)

Now …………… (7.5)

Then ………………. (8)

Multiply (7.5) by (8)

(9)

We differentiate in order to find out what values of minimize the sum of squared residuals. By differentiating and setting it to 0, we can find the minimum or maximum turning point. By taking a derivative of the first derivative, we can also determine whether it is a maximum or minimum turning point.

We set the derivative in (6) above to 0, like we were looking for the turning points

(10)

We divide both sides by 2:

(11)

We expand equation (11)

(12)

Remember that:

………………. (12.5)

This implies that (12) can be written as:

(13)

In equation (13), the summation law of differences as illustrated in equation (12.5) is used to break up the summation across the difference

We carry the terms to opposite sides:

(14)

Remember that is a constant\*\*\*

This implies that:

(15)

Therefore:

(16)

***QED***

***Proof Two***

Show that: ………. (16.5)

Where = Expected value of our fitted value and = dependent variable

We assume that and are constants at value \*\*\*\*

has components of in its equation, which makes it a random variable. Because values of are dependent on the random variable , by association is a random variable.

Because we are trying to prove equation (16.5), it is only logical to arrange (16) in a way that shows a relationship between and . To achieve this, we substitute in the equation (16).

Remember is given in equation (1):

Therefore:

(17)

We distribute in the numerator, that means we multiply through the bracket.

(18)

By the law of addition, we can separate the variables above into additive pairs

(19)

is a common factor and so it can be placed separately under both sides of the additive pairs.

(20)

The distributive property is used above to factor out the constant from

We assume that and are constants at value \*\*\*\*

The equation becomes:

(21)

We use the statistical concept of expected value (the average value of large number of realizations of a random variable) to show that is unbiased.

(22)

The expected value of a fixed number is that number, meaning

The expected value of an expected number remains an expectation, meaning

This implies that:

(23)

Remember that:

for constant k and random function .

From (22), is our constant and is a function of random variables.

(24)

The expectation of a sum is the sum of its expectations. Based on this statement, we move the expectation sign into the summation.

(25)

For the proof to occur, and should be uncorrelated. We would show that this condition is equivalent to

If and are uncorrelated then the covariance of and equals 0.

(26)

We use the definition of covariance and set it to 0:

(27)

We multiply out the covariance equation:

(28)

We use the fact that the expectation of a sum is the sum of its expectations:

0 (29)

We will use the fact that and are fixed numbers and pull them out of the expectations:

0 (30)

We assume that ,which simply means that the mean of our error term is simply not relevant. This allows us to cancel any terms with and .

This implies that:

(31)

We arrived at (30) by substituting 0 for any variables that had and .

We substitute 0 back in (25) and it becomes:

…… (25)

(32)

Therefore:

(Proven)

***QED***

***Replication Exercises***

***Chapter 6, Question 2***

***Part A***

*Democrats*

Here we attempt to do a scatter plot of Federal Funds rate (dependent Variable-Y-axis) to the quarters since the previous election for Democrats (independent Variable-X-axis).

**Figure 1: Fed Funds Rate(FFR) to Quarters since Previous Elections for Democrats**

**Figure 2:Fed Funds Rate(FFR) to Quarters since Previous Elections for Democrats (Jitter Effect)**

The scatterplot in Figure 1 shows the relationship between the FFR and the Quarters since the previous election for Democrats. The X-axis is labelled from -2 to 15.

The scatterplot in Figure 2 shows the same relationship but with a modified version of the data for the Quarters since the previous election for Democrats.

*Republicans*

Here we attempt to do a scatter plot of Federal Funds rate (dependent Variable-Y-axis) to the quarters since the previous election for Republicans (independent Variable-X-axis).

**Figure 3: Fed Funds Rate(FFR) to Quarters since Previous Elections for Republicans**

**Figure 4::Fed Funds Rate(FFR) to Quarters since Previous Elections for Republicans (Jitter Effect)**

The scatterplot in Figure 3 shows the relationship between the FFR and the Quarters since the previous election for Republicans. The X-axis is labelled from -0 to 15.

The scatterplot in Figure 2 shows the same relationship but with a modified version of the data for the Quarters since the previous election for Republicans.

*Comment on Relationship*

The data for Democrats indicates a positive gradient for the regression line while the data for republicans indicates a negative gradient.

The republican data shows a better correlation coefficient () of 0.0126 compared to the Democratic data coefficient of 0.0053; meaning that the data observed for republicans is closer to the fitted line than the data observed for democrats.

Qualitatively, the Democrats (Elected Democrats) have a higher FFR (~18) in the first quarter containing the election (Election=0) and the higher FFR (~16) in the quarter before the next election (Election=15) when compared to the Republicans (Elected Republicans).

***Part B***

Interaction variable = Party Affiliation (Party\_Aff)

New Equation:

(1)

Where FFR= Federal Funds Rate, , ,, and

Our dummy variable party affiliation can only take on 2 numbers which are 1 (Democratic president) and 0 (Republican president).

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| SUMMARY OUTPUT | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| *Regression Statistics* | |  |  |  |  |  |  |  |
| Multiple R | 0.105622 |  |  |  |  |  |  |  |
| R Square | 0.011156 |  |  |  |  |  |  |  |
| Adjusted R Square | 0.00252 |  |  |  |  |  |  |  |
| Standard Error | 3.464904 |  |  |  |  |  |  |  |
| Observations | 232 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |  |  |
|  | *df* | *SS* | *MS* | *F* | *Significance F* |  |  |  |
| Regression | 2 | 31.01668 | 15.50834 | 1.291764 | 0.27678 |  |  |  |
| Residual | 229 | 2749.273 | 12.00556 |  |  |  |  |  |
| Total | 231 | 2780.289 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0%* | *Upper 95.0%* |
| Intercept | 5.33067 | 0.428852 | 12.4301 | 1.88E-27 | 4.48567 | 6.175669 | 4.48567 | 6.175669 |
| elec\_dem | 0.049296 | 0.060812 | 0.810635 | 0.418417 | -0.07053 | 0.169119 | -0.07053 | 0.169119 |
| elec\_rep | -0.03791 | 0.052898 | -0.71668 | 0.474302 | -0.14214 | 0.066318 | -0.14214 | 0.066318 |
|  |  |  |  |  |  |  |  |  |

The model above is given by the equation:

(2)

We use our dummy/interactive variable to determine the differences in the FFR when a Democrat is elected and when a Republican is elected.

1. Republican President means party affiliation = 0; FFR =
2. Democratic President means party affiliation=1; FFR =

***Part C***

Under Republicans the effect of election is statistically insignificant because the coefficient of an elected Republican is -0.03791 and an absolute t-stat value of 0.71668. The t-stat value is low and the coefficient is negative making it insignificant.

Under Democrats the effect of election is statistically insignificant because the coefficient of an elected Democrat is 0.049296 and an absolute t-stat value of 0.810635, which are not very high values. The coefficient of Democrats is more significant than that of Republicans but overall those values are not very significant.

Both coefficients have good p-values (0.4 range), which means that our coefficients are less likely to take 0 values.

*Methodology*

The answer can be determined by looking at the coefficients (value) and determine its effect on the variable measured. The next step involves looking at the t-stat absolute value because this value provides insight on the likelihood of the coefficient being zero. A high t-stat absolute means a less likelihood that the coefficient will be zero and low value means a higher likelihood that the coefficient will be zero.

We can also look at the p-value of the results. A low p-value means the probability of the coefficients being zero is low and often times the p-value is really all we need to know.

We can also run a Difference of Means Test to see the relationship between election variable for Democrats and Republicans to the FFR.

**Figure 5: Difference of Means Test**

***Part D***

**Figure 6:Fitted Line\_Democrats**

**Figure 7:Fitted Line\_Republicans**

The fitted line for Democrats shows a relationship with a positive gradient meaning that every 0.0547 increase in the Elec\_Dem variable yields 0.0547 increase in the FFR.

The fitted line for Republicans shows a relationship with a negative gradient meaning that every 0.0734 decrease in the Elec\_Rep variable yields 0.0734 increase in the FFR.

***Part E***

(3)

Where FFR= Federal Funds Rate, , ,,, and

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| SUMMARY OUTPUT | |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| *Regression Statistics* | |  |  |  |  |  |  |  |
| Multiple R | 0.969783 |  |  |  |  |  |  |  |
| R Square | 0.940478 |  |  |  |  |  |  |  |
| Adjusted R Square | 0.939429 |  |  |  |  |  |  |  |
| Standard Error | 0.853828 |  |  |  |  |  |  |  |
| Observations | 232 |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
| ANOVA |  |  |  |  |  |  |  |  |
|  | *df* | *SS* | *MS* | *F* | *Significance F* |  |  |  |
| Regression | 4 | 2614.801 | 653.7003 | 896.6812 | 9.1E-138 |  |  |  |
| Residual | 227 | 165.488 | 0.729022 |  |  |  |  |  |
| Total | 231 | 2780.289 |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
|  | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0%* | *Upper 95.0%* |
| Intercept | 0.173028 | 0.136696 | 1.265782 | 0.206889 | -0.09633 | 0.442384 | -0.09633 | 0.442384 |
| inflation | 0.114304 | 0.024274 | 4.708859 | 4.34E-06 | 0.066472 | 0.162135 | 0.066472 | 0.162135 |
| lag\_FEDFUNDS | 0.89409 | 0.021725 | 41.15475 | 3.1E-107 | 0.851282 | 0.936899 | 0.851282 | 0.936899 |
| elec\_dem | 0.01844 | 0.015037 | 1.226296 | 0.221358 | -0.01119 | 0.048069 | -0.01119 | 0.048069 |
| elec\_rep | -0.01732 | 0.013041 | -1.32805 | 0.185496 | -0.04302 | 0.008378 | -0.04302 | 0.008378 |

1. Effect of elections for Republicans shows an insignificant statistical coefficient of -0.01732 and a t-stat absolute value of 1.32805.
2. The coefficient for Democrats shows an insignificant statistical coefficient of 0.01844 and a t-stat value of 1.226296.
3. The coefficient for Lag\_FEDFUNDS shows a significant statistical coefficient of 0.089409 and a high t-stat value of 41.15475.
4. The coefficient for Inflation shows an insignificant statistical coefficient of 0.114304 and t-stat value of 4.708859.