Space Rotation

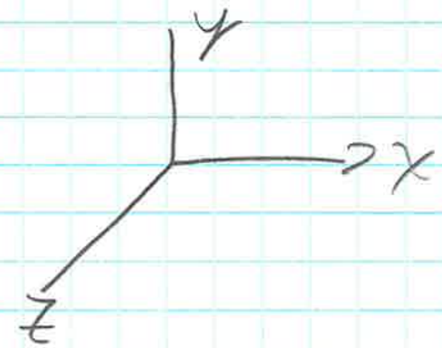
Catia Relative Position

transpormation

into Absolute position

3D space Rotation

Consider the space as an euclidean space



Rotations are Counter clockwise

Order of rotation Matters

$$\text{new } P = \text{old } P \times R_x \times R_y \times R_y$$

where

$$R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_y(\theta_y) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_z(\theta_z) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cccc} & x & y & z \\ \text{If} \quad V = [& 1, & 1, & 1] \end{array}$$

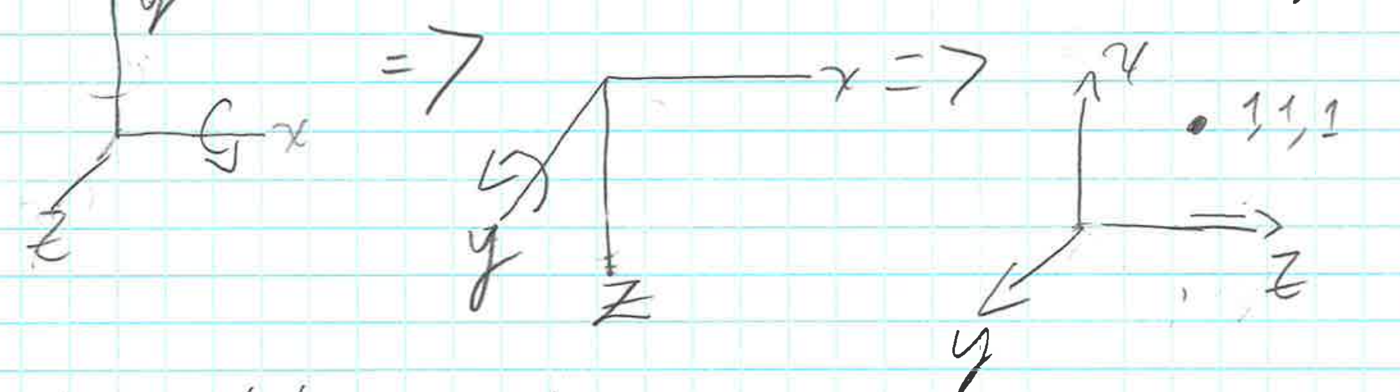
$$R_x(\theta_x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta_y) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta_z) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate axis x an y 90° Counter-Clockwise

rotate x rotate y



when rotating x y $\theta x = 90$ $\theta_y = 90$ $\theta_z = 0$

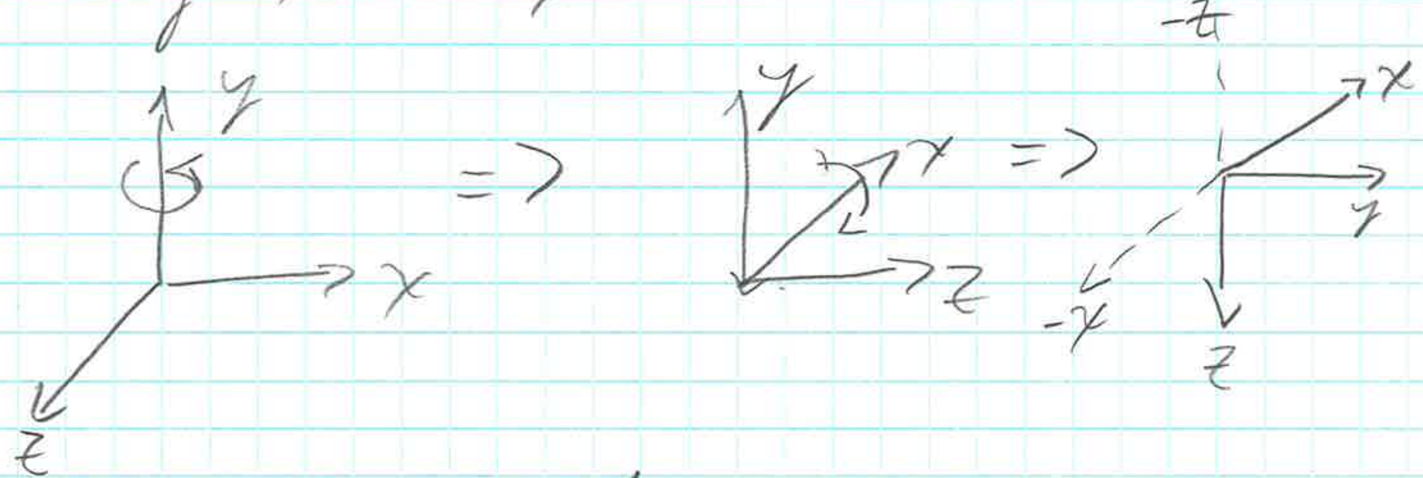
$$V_{new} = V\, R(x)\, R(y)\, R_z$$

$$[1, 1, 1] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$= [1, 1, -1] \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} = [1\ 1\ 1]$$

Change the rotation order,

y then x



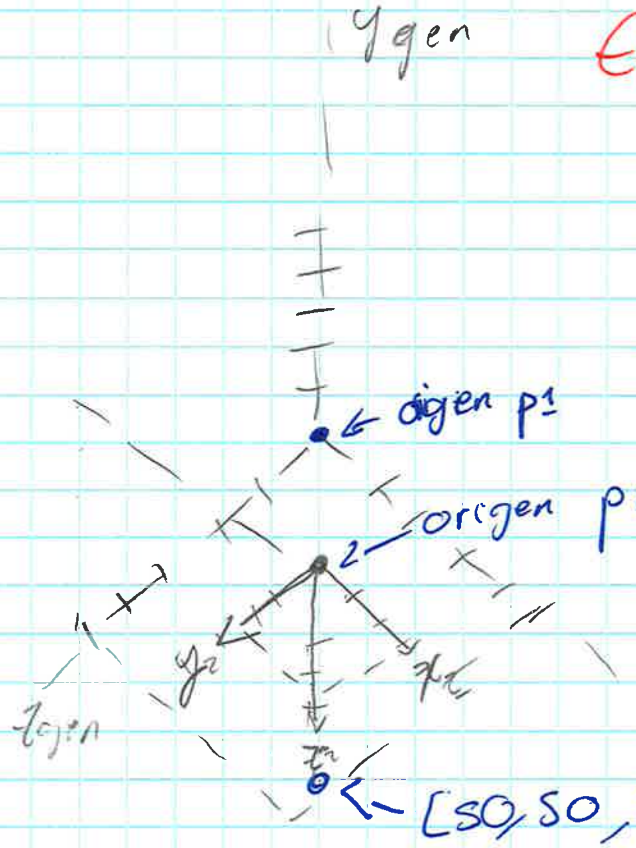
$$V_{new} = V R_y R_x \cancel{R_z}^{\to 1}$$

$$= \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1, & 1, & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 & -1 \end{bmatrix} \checkmark$$

Example 1



origen p1

origen p2 x rotation 90° [50,50,50]

[50,50,50] with respect to its parent

you grow in xgen

you grow in zgen

but you cancel on ygen due to rotation

hence loc s/B [100, 0, 100]

Confirmed with Catia

E1 Continues...

what we need for each level from particular to general rotate and compensate

top level with respect to parent
parent is rotated anticlockwise 90°
hence need to get point with respect to un-rotated parent

unrotated parent is obtained by substracting 360 from angle of rotation

If $rx = 90$ $\quad Rx = 360 - 90 = 270$
$ry = 0$ $\quad Ry = 360 - 0 = 0$
$rz = 0$ $\quad Rz = 360 - 0 = 0$

$270° = \frac{3\pi}{2}$ rad

then rotate the vector

$$V = [50 \;\; 50 \;\; 50] \overset{\text{rot } X}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\frac{3\pi}{2}) & -\sin(\frac{3\pi}{2}) \\ 0 & \sin(\frac{3\pi}{2}) & \cos(\frac{3\pi}{2}) \end{bmatrix}} =$$

$$= [50 \;\; 50 \;\; 50] \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = [50 \;\; -50 \;\; 50]$$

now the parent is translates by 50 50 50

then $V = V_{\text{untrans}} + \text{translation}$

$$V = [50 \quad -50 \quad 50] + [50 \quad 50 \quad 50]$$

$$V = [100 \quad 0 \quad 100] \checkmark$$

To rotate a Vector in space or ~~on~~ a matrix of axis Vectors that describe a new space we need to multiply such Vector or matrix by a rotation matrix

Rotation Matrix R

It is the sequence of rotation on each of the axis in x, y, z order

$$R = R_x * R_y * R_z$$

Counter-clockwise rotations

where

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix} \times \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ 0 & 1 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y \end{bmatrix} \times \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

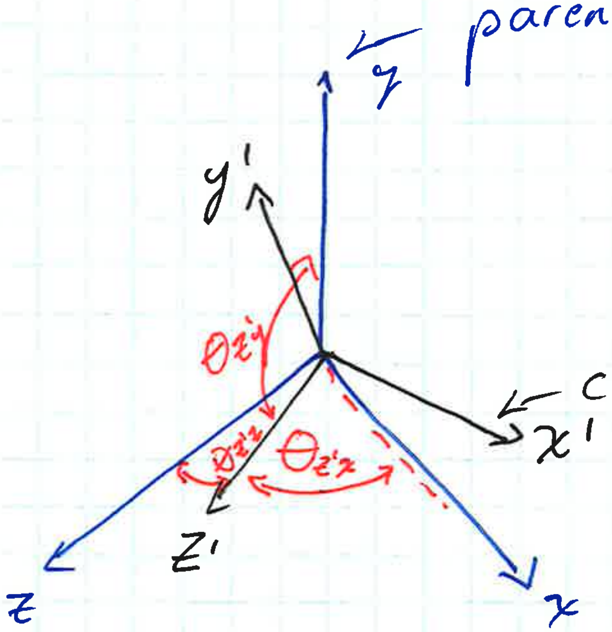
$$= \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y \\ \sin\theta_x \sin\theta_y & \cos\theta_x & -\sin\theta_x \cos\theta_y \\ -\cos\theta_x \sin\theta_y & \sin\theta_x & \cos\theta_x \cos\theta_y \end{bmatrix} \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 \\ \sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{R = \begin{bmatrix} \cos\theta_y \cos\theta_z & -\cos\theta_y \sin\theta_z & \sin\theta_y \\ \sin\theta_x \sin\theta_y \cos\theta_z + \cos\theta_x \sin\theta_z & -\sin\theta_x \sin\theta_y \sin\theta_z + \cos\theta_x \cos\theta_z & -\sin\theta_x \cos\theta_y \\ -\cos\theta_x \sin\theta_y \cos\theta_z + \sin\theta_x \sin\theta_z & \cos\theta_x \sin\theta_y \sin\theta_z + \sin\theta_x \cos\theta_z & \cos\theta_x \cos\theta_y \end{bmatrix}}$$

to transform a space rotating it:

Space Axis Matrix

is a matrix that represents the x, y, z Components of a space x', y', z' axis with respect to the parets x, y z axis



Space Matrix

$$\begin{bmatrix} \cos\theta_{x'x} & \cos\theta_{x'y} & \cos\theta_{x'z} \\ \cos\theta_{y'x} & \cos\theta_{y'y} & \cos\theta_{y'z} \\ \cos\theta_{z'x} & \cos\theta_{z'y} & \cos\theta_{z'z} \end{bmatrix}$$

The space Axis Matrix provided by Catia is the Axis information of the already transformed space M' where M is the ~~axis~~ space Matrix of the top level space and R is the transformation matrix

hence

$$M' = M \times R$$

Since the top level space has no rotation

$$M = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

hence

$$M' = I \times R$$

$$M' = R$$

The Space Axis Matrix provided by Catia is nothing but the transformation matrix itself.

```
#USER INPUT
  #Relative Coordinates of the origin point in space
    V=[[illegible],[illegible],[illegible]];
  #Counter-Clockwise rotation of the parts/products parent relative to the toplevel space
    Xa=[illegible]; #Degrees
    Ya=[illegible]; #Degrees
    Za=[illegible]; #Degrees
#-------------------------------------------------------------------------------------------------------
#FIXED CODE, DONT MODIFY THE CODE BELOW
#Invert the direction of rotation, we want to un-rotate the rotated element to get the equivalent position
#The matrix below works for counter-clockwise rotations
  Xa=360-Xa;
  Ya=360-Ya;
  Za=360-Za;
#Convert to Radians
  Xa=Xa*pi/180;
  Ya=Ya*pi/180;
  Za=Za*pi/180;
#Method 1 Individual Rotations
  #Rx=[1, 0, 0 ; 0, cos(Xa), -sin(Xa); 0, sin(Xa), cos(Xa)];
  #Ry=[cos(Ya), 0, sin(Ya); 0, 1, 0 ; -sin(Ya), 0, cos(Ya)];
  #Rz=[cos(Za), -sin(Za), 0 ; sin(Za), cos(Za), 0; 0, 0, 1] ;
  #R=Rx*Ry*Rz
#Method 2 Create Rotation Matrix with Rx, Ry, Rz already multiplied
  R1=[cos(Ya)*cos(Za), -cos(Ya)*sin(Za), sin(Ya)];
  R2=[sin(Xa)*sin(Ya)*cos(Za)+cos(Xa)*sin(Za),
-sin(Xa)*sin(Ya)*sin(Za)+cos(Xa)*cos(Za), -sin(Xa)*cos(Ya)];
  R3=[-cos(Xa)*sin(Ya)*cos(Za)+sin(Xa)*sin(Za),
cos(Xa)*sin(Ya)*sin(Za)+sin(Xa)*cos(Za), cos(Xa)*cos(Ya)];
  R=[R1;R2;R3]
#Result Show the absolute
  Out=V*R
```

Octave

```
R =

  -1.1807e-016   6.4279e-001  -7.6604e-001
  -7.0711e-001   5.4168e-001   4.5452e-001
   7.0711e-001   5.4168e-001   4.5452e-001

Out =

  -3.5388e-016   8.6307e+001   7.1497e+000
```