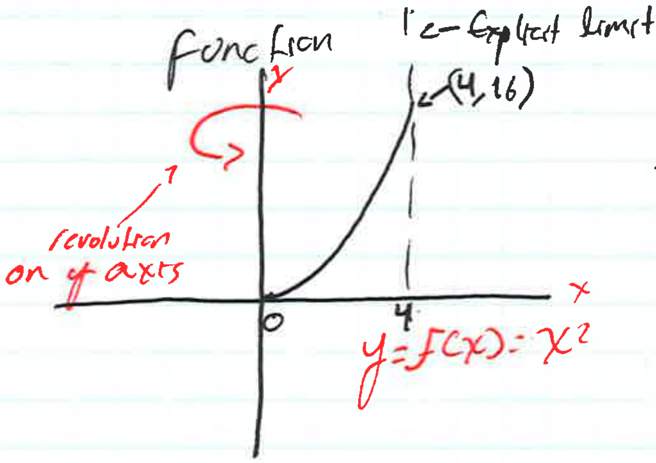
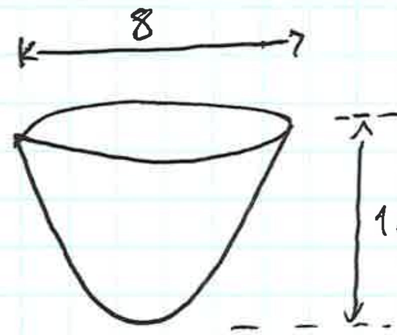
Revolution Of Solids

Method to determine the Volume of a solid based on the revolution of a function around one of the axis using an explicit integration i.e.





$V = 128\pi$

Procedure:

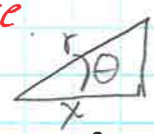
given the 2D function

$y = f(x)$

$$\text{Volume} = 2\pi \int_a^b f(x)\cdot x \cdot dx$$

where a and b are the lower and higher limits of the function to integrate

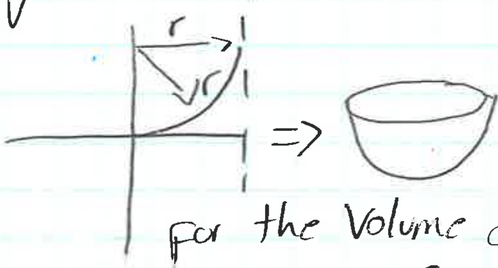
Example: Determine Volume of a sphere



$\sqrt{r^2-x^2}$

$x^2+y^2=r^2$ [function of the Circle]

$y = f(x) = \sqrt{r^2-x^2}$



for the Volume of half an sphere, $V_{1/2} = 2\pi \int_0^r x\sqrt{r^2-x^2}\,dx$

$x = r\cos\theta \qquad \sqrt{r^2-x^2} = r\sin\theta$

$dx = -r\sin\theta\, d\theta$

$V_{1/2} = -2\pi r^3 \int_0^r \sin^2\theta \cos\theta\, d\theta$

$V_{1/2} = -2\pi r^3 \frac{\sin^3\theta}{3}\Big|_0^r = -2\pi \cancel{r^3} \frac{(r^2-x^2)^{3/2}}{\cancel{r^3}}\Big|_0^r$

$V_{1/2} = +\frac{2\pi}{3}\left[(\cancel{r^2-r^2})^{3/2} + (r^2-0)^{3/2}\right]$

$V = 2 \cdot V_{1/2} = \boxed{\frac{4}{3}\pi r^3}$