$$V_N(\pi, \hat{c}_j, \hat{d}_j) = \|c_N - \bar{c}_j\|_P^2 + \sum_{k=0}^{N-1} \left(\|c_k - \bar{c}_j\|_Q^2 + \|u_k - \bar{u}_j\|_R^2 \right),$$

Calculated by (32) offline.

Calculated by (33)

where $\|c\|_P^2 = c' P c$.

$u_k \in \mathbb{R}^m$
$c_k \in \mathbb{R}^n$
$d_k \in \mathbb{R}^{n_d}$ — $n_d \leq$ number of outputs (here: 1)

The optimisation problem is:

$$V_N^*(\hat{c}_j, \hat{d}_j) = \min_{\pi = \{u_i\}_{i=0}^{N-1}} V_N(\pi, \hat{c}_j, \hat{d}_j),$$

subj. to:

1) $c_{k+1} = Ac_k + Bu_k + B_d \cdot d_k$; $k \in \mathbb{N}_{[0, N-1]}$
2) $d_{k+1} = d_k$; $k \in \mathbb{N}_{[0, N-1]}$
3) $Gc_k + Hu_k \leq M$; $k \in \mathbb{N}_{[0, N]}$

But, in our case we may consider simpler constraints:

$$0 \leq c_k \leq c_{max}$$
$$0 \leq u_k \leq u_{max}$$

4) $c_0 = \hat{c}_j$
5) $d_0 = \hat{d}_j$

Define $z = [u_0 \ \cancel{c_1} \ u_1 \ \cancel{c_2} \ \cdots \ u_{N-1} \ \cancel{c_N}] \in \mathbb{R}^{N(n+m)}$

$$V_N(\pi; \hat{c}_j, \hat{d}_j) = (c_N - \bar{c}_j)' P (c_N - \bar{c}_j) + \sum_{k=0}^{N-1} (c_k - \bar{c}_j)' Q (c_k - \bar{c}_j)$$
$$+ (u_k - \bar{u}_j)' R (u_k - \bar{u}_j) =$$

$$= c_N' P c_N + \underbrace{\bar{c}_j' P \bar{c}_j}_{\text{constant}} - \overbrace{\bar{c}_j' P c_N - \underbrace{c_N' P \bar{c}_j}_{=\bar{c}_j' P c_N,\ \text{because } P = P'}}^{=-2\bar{c}_j' P c_N} +$$

$$+ \sum_{k=0}^{N-1} c_k' Q c_k + \bar{c}_j' Q c_j - 2\bar{c}_j' Q c_k + u_k' R u_k + \bar{u}_j' R \bar{u}_j - 2\bar{u}_j' R u_k$$

$$= \underbrace{c_N' P c_N + \sum_{k=0}^{N-1} [c_k' \;\; u_k'] \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} c_k \\ u_k \end{bmatrix}}_{\text{Quadratic terms}} +$$

$$+ \underbrace{\left(-2\bar{c}_j' P c_N + \sum_{k=0}^{N-1} [-2\bar{c}_j Q \;\; -2\bar{u}_j R] \begin{bmatrix} c_k \\ u_k \end{bmatrix} \right)}_{\text{Linear terms}}$$

$$+ \underbrace{\text{const}}_{\text{just omit it.}}$$

Then, the cost function V_N can be written down as a function of z as follows

$$V_N(z; \hat{c}_j, \hat{u}_j) = \frac{1}{2} z' H z + q' z$$

where

$$\frac{1}{2} H = \begin{bmatrix} Q & R & & & 0 \\ & & Q & R & \\ 0 & & & \ddots & Q \; R \; P \end{bmatrix} \quad \text{and} \quad q = -2 [\bar{c}_j' Q \;\; \bar{u}_j' R \;\; \bar{c}_j' Q \;\; \bar{u}_j' R \cdots \bar{c}_j' Q \;\; \bar{u}_j' R \;\; \bar{c}_j' P]'$$

The constraints have the form: $0 \le z \le z_{max}$ where $z_{max} = [u_{max} \; c_{max} \; \cdots \; u_{max} \; c_{max}] \in \mathbb{R}^{N(n+m)}$

Now notice that the equality constraints $c_{k+1} = Ac_k + Bu_k + B_d d_k,\ \forall k \in \mathbb{N}_{[0,N-1]}$, given that $d_{k+1} = d_k,\ \forall k \in \mathbb{N}_{[0,N-1]}$ can be written as follows:

$$
\begin{aligned}
I \cdot c_1 - Bu_0 &= B_d d_0 + Ac_0, \\
I \cdot c_2 - Ac_1 - Bu_1 &= B_d d_0, \\
I \cdot c_3 - Ac_2 - Bu_2 &= B_d d_0, \\
&\vdots \\
I \cdot c_N - Ac_{N-1} - Bu_{N-1} &= B_d d_0,
\end{aligned}
$$

which is then written in the form $K \cdot z = L$, i.e.,

$$
z = \begin{bmatrix} u_0 & c_1 & u_1 & c_2 & \cdots & c_{N-1} & u_{N-1} & c_N \end{bmatrix}'
$$

$$
\begin{bmatrix}
-B & I & & & & & & \\
 & -A & B & I & & & 0 & \\
 & & & -A & B & I & & \\
 & 0 & & & \ddots & \ddots & \ddots & \\
 & & & & & -A & B & I
\end{bmatrix} z = \begin{bmatrix} B_d \cdot d_0 + A \cdot c_0 \\ B_d d_0 \\ B_d \cdot d_0 \\ \vdots \\ B_d \cdot d_0 \end{bmatrix}
$$

where $c_0 = \hat{c}_j$

where $d_0 = \hat{d}_j$