

In general, the constraints on the administration rate u and the concentrations c can be of the form:

$$G \cdot c_k + H u_k \leq M; \forall k \in \mathbb{N}_{[0, N]}$$

where $G \in \mathbb{R}^{n_c \times n}$, $H \in \mathbb{R}^{n_c \times m}$ and $M \in \mathbb{R}^{n_c}$.

For the composite variable

$$z = (u_0, c_1, u_1, c_2, \dots, c_{N-1}, u_{N-1}, c_N)'$$

the constraints are written as:

$$\Phi z \leq \phi$$

where:

$$\Phi = \begin{bmatrix} H & 0 \\ G & H \\ & G & H \\ & & \ddots & \ddots \end{bmatrix} \text{ and } \phi = \begin{bmatrix} M - G c_0 \\ M \\ \vdots \\ M \end{bmatrix}$$

with $\Phi \in \mathbb{R}^{(n_c \cdot N) \times N(n+m)}$ and $\phi \in \mathbb{R}^{N n_c}$.

Constraints may not be imposed on all concentrations but only on some c_j for $j \in J$ where $J = \{j_1, j_2, \dots, j_K\}$ with $K \leq n$. Let

$$\Delta_J = \begin{bmatrix} e(j_1)' \\ e(j_2)' \\ \vdots \\ e(j_K)' \end{bmatrix}$$

where $e(j)$ is a vector of \mathbb{R}^n whose j -th entry is 1 and all other entries are 0. The constraints on c are written as:

$$\begin{cases} \Delta_J \cdot c_k \leq c_{\max}^J; \forall k \in \mathbb{N}_{[0, N]} \\ 0 \leq c_k \end{cases}$$

and in terms of \mathbb{Z} they become:

$$\begin{bmatrix} I_m & & & \\ & \Delta_{\bar{\mathcal{J}}} & & \\ & & I_m & \\ & & & \ddots \\ & & & & I_m \\ & & & & & \Delta_{\bar{\mathcal{J}}} \end{bmatrix} \cdot \mathbb{Z} \leq \begin{bmatrix} U_{\max} \\ \bar{\mathcal{J}} \\ C_{\max} \\ U_{\max} \\ \bar{\mathcal{J}} \\ C_{\max} \\ \vdots \\ U_{\max} \\ \bar{\mathcal{J}} \\ C_{\max} \end{bmatrix}$$

$\bar{\mathcal{J}} C_{\max}$ refers to the constraints (bounds) imposed on C^j with $j \in \bar{\mathcal{J}}$, i.e., if

$$\bar{\mathcal{J}} C_{\max} = \begin{bmatrix} C_{\max}^{\bar{\mathcal{J}}}(1) \\ C_{\max}^{\bar{\mathcal{J}}}(2) \\ \vdots \\ C_{\max}^{\bar{\mathcal{J}}}(k) \end{bmatrix}$$

Then:

$$\begin{aligned} C^{j_1} &\leq C_{\max}^{\bar{\mathcal{J}}}(1) \\ &\vdots \\ C^{j_k} &\leq C_{\max}^{\bar{\mathcal{J}}}(k). \end{aligned}$$