

What is Finite Element Analysis?

Vibrations of a Jet Engine Turbine

Cyclic Mode 1, Harmonic Index 1

What is Finite Element Analysis?

Vibrations of a Jet Engine Turbine

Cyclic Mode 1, Harmonic Index 9

What is Finite Element Analysis?

Vibrations of a Jet Engine Turbine

Cyclic Mode 3, Harmonic Index 9, 5x Auto Scale

Why Finite Element Analysis?

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- Enables faster prototyping

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- Enables faster prototyping
- Identify weak spots and potential failure points
- Test and adjust assumptions about geometry and materials

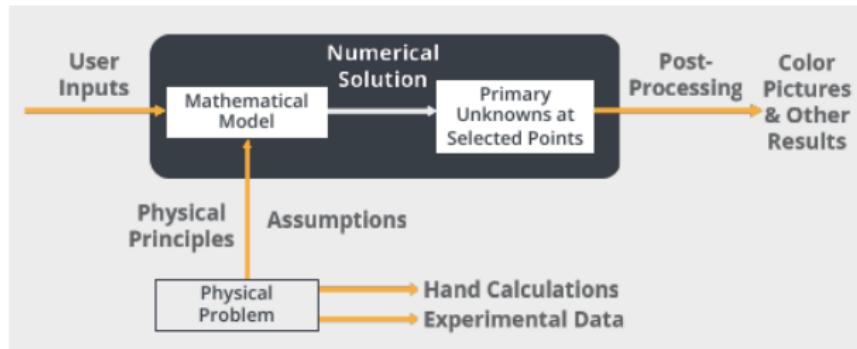
How Does Finite Element Analysis Work?

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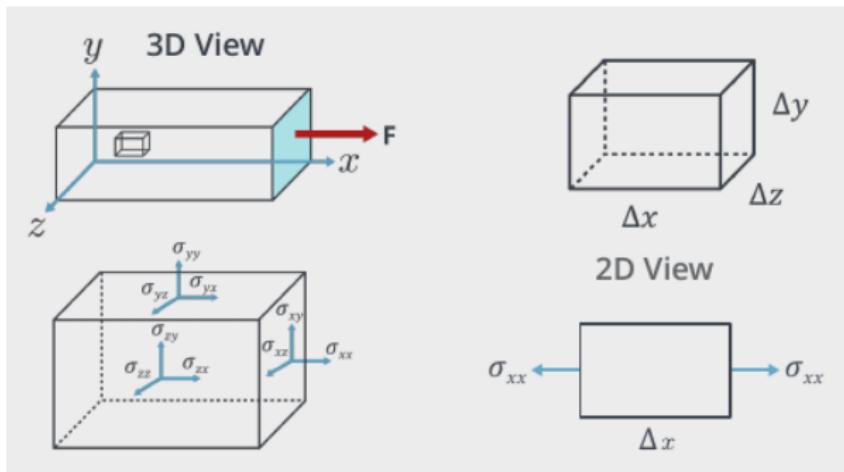
The Black Box of Finite Element Analysis Software



How Does Finite Element Analysis Work?

Consider an Infinitesimal Element of the Geometry

(For a simple 1D Bar example)



How Does Finite Element Analysis Work?

Derive Governing Equations:

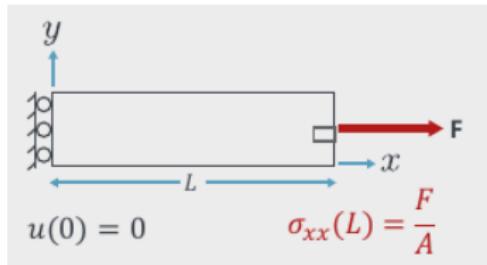
$$E \frac{d^2u}{dx^2} + f_x = 0$$

How Does Finite Element Analysis Work?

Derive Governing Equations:

$$E \frac{d^2u}{dx^2} + f_x = 0$$

Identify Boundary Conditions:



How Does Finite Element Analysis Work?

Keep Track of Assumptions:

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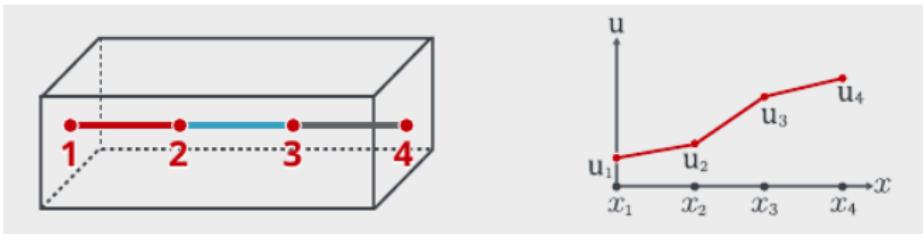
Keep Track of Assumptions:

- Small displacement
- Stress is linearly proportional to strain (good for up to 15°)
- Small strain

(Each of these assumptions simplifies the model for the analysis!)

How Does Finite Element Analysis Work?

Discretization (ie, Mesh) and Interpolation



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From the “Strong Form”

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To the “Weak Form”

$$\int_0^L \frac{dw}{dx} E \frac{du}{dx} dx = w E \frac{du}{dx} \Big|_0^L + \int_0^L w f_x dx$$

How Does Finite Element Analysis Work?

To Algebraic Equations . . .

$$\begin{aligned} & w_1 \left[\left(\frac{E}{\Delta x} \right) (u_1 - u_2) + \left(\frac{F}{A} \right) - \left(\frac{f_y \Delta x}{2} \right) \right] \\ & + w_2 \left[\left(\frac{E}{\Delta x} \right) (-u_1 + 2u_2 - u_3) - f_y \Delta x \right] \\ & + w_3 \left[\left(\frac{E}{\Delta x} \right) (-u_2 + 2u_3 - u_4) - f_y \Delta x \right] \\ & + w_4 \left[\left(\frac{E}{\Delta x} \right) (-u_3 + u_4) - \left(\frac{F_R}{A} \right) - \left(\frac{f_y \Delta x}{2} \right) \right] = 0. \end{aligned}$$

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And then solve for u_2, u_3, u_4 and F_R (u_1 is given by our Boundary Condition).

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Scary? Well, yes, it is! But this is why we use computers!

Verification and Validation

Two Last Things: Verification and Validation!

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Verification:

- Check and double-check assumptions
- Review Boundary, Essential, and Periodic Conditions
- Refine the mesh and compare with previous results

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- Review Boundary, Essential, and Periodic Conditions
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Validation: Compare results with experimental data.

Finite Element Analysis Software

A (Very Small) Sampling of Finite Element Analysis Software

Proprietary

- ANSYS
- SolidWorks
- Mathematica
- Coreform
- Abaqus
- Aquaveo

Open Source

- BRL-CAD
- CalculiX
- Elmer FEM
- FreeFEM
- FreeCAD

Resources

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- eCornell Engineering Certificate Programs
 - Finite Element Analysis
<https://ecornell.cornell.edu/certificates/engineering/finite-element-analysis/>
 - Fluid Dynamics
<https://ecornell.cornell.edu/certificates/engineering/fluid-dynamics-simulations-using-ansys/>
- SimCafe
<https://confluence.cornell.edu/display/simulation>

Questions

Any Questions?

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`https://www.linkedin.com/in/alpheus-madsen`

`https://www.github.com/alpheus-madsen`

Slides:

`https://github.com/alpheus-madsen/presentation-slides/2025-provo-linux-users-group-intro-tofea.pdf`