

AI1110 Assignment 1

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Q.)

In the Figure 1, O is the centre of the circle. $\angle DAE = 70^\circ$. Find giving suitable reasons, the measure of:

- 1) $\angle BCD$
- 2) $\angle BOD$
- 3) $\angle OBD$

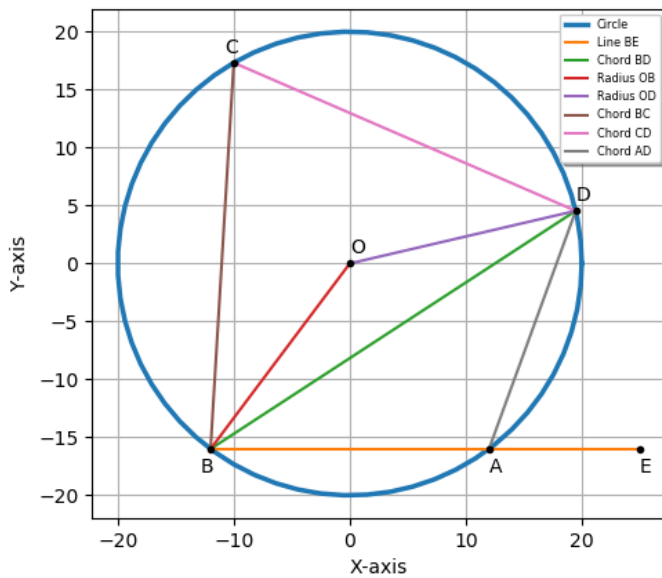


Fig. 1. Problem figure

Solution:

Given,

$$\angle DAE = \theta = \frac{7\pi}{18} \quad (1)$$

Assume, O (centre of circle) be origin. Assuming A, B is reflection of A about Y-axis. The standard basis vectors are defined as

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2)$$

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (3)$$

The general formula for image of any point P about line with equation:

$$\mathbf{n}^T \mathbf{x} = c \quad (4)$$

is given by:

$$\mathbf{R} = \mathbf{P} + 2 \frac{c - \mathbf{n}^T \mathbf{P}}{\|\mathbf{n}\|^2} \mathbf{n} \quad (5)$$

where \mathbf{n} is the normal vector of the line. For Y-axis,

$$\mathbf{n} = \mathbf{e}_1 \quad (6)$$

$$c = 0 \quad (7)$$

Hence,

$$\mathbf{B} = \mathbf{A} + 2 \frac{c - \mathbf{n}^T \mathbf{A}}{\|\mathbf{n}\|^2} \mathbf{n} \quad (8)$$

$$= \mathbf{A} + 2 \frac{0 - \mathbf{e}_1^T \mathbf{A}}{1^2} \mathbf{e}_1 \quad (9)$$

$$= \mathbf{A} - (2\mathbf{e}_1^T \mathbf{A}) \mathbf{e}_1 \quad (10)$$

D is obtained by rotating B by 2θ anti-clockwise about O. Rotation matrix Q is given by:

$$\mathbf{Q} = \begin{pmatrix} \cos 2\beta & -\sin 2\beta \\ \sin 2\beta & \cos 2\beta \end{pmatrix} \quad (11)$$

where a point vector is rotated anti-clockwise by β . Hence,

$$\mathbf{D} = \mathbf{Q}\mathbf{B} \quad (12)$$

\therefore BE is a straight line,

$$\Rightarrow \angle BAD = \pi - \theta \quad (13)$$

\therefore Sum of opposite angles in a cyclic quadrilateral is 180° ,

$$\Rightarrow \angle BAD + \angle BCD = \pi \quad (14)$$

$$\Rightarrow \angle BCD = \theta \quad (15)$$

In general,

$$\angle BOD = 2\angle BCD = 2\theta \quad (16)$$

\therefore OB = OD = R

$$\Rightarrow \angle OBD = \angle ODB \quad (17)$$

\therefore The sum of angles of any triangle equals the straight angle.

$$\Rightarrow \angle BOD + \angle ODB + \angle OBD = \pi \quad (18)$$

$$\Rightarrow 2\theta + 2\angle OBD = \pi \quad (19)$$

$$\Rightarrow \angle OBD = \frac{\pi}{2} - \theta \quad (20)$$

Using, $\theta = 70^\circ$, we can say

- 1) $\angle BCD = 70^\circ$
- 2) $\angle BOD = 140^\circ$
- 3) $\angle OBD = 20^\circ$

The input parameters for drawing the figure are available in table shown below.

Symbol	Value	Description
R	20	Radius of the Circle
O	0	Centre of the circle (Origin)
A	$\begin{pmatrix} 12 \\ -16 \end{pmatrix}$	Design Choice
B	$\mathbf{A} - (2\mathbf{e}_1^T \mathbf{A})\mathbf{e}_1$	Derived Vector
C	$\begin{pmatrix} -10 \\ 10\sqrt{3} \end{pmatrix}$	Design Choice
D	QB	Derived Vector
θ	$\frac{7\pi}{18}$	$\angle DAE$