Assignment 2

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***Abstract*—This document contains the solution for Assignment 2 (ICSE Class 12 Maths 2018 Q.22)**

22 [ICSE 12 2018]: A manufacturing company manufactures two types of teaching aids A and B. Both the products require fabrication time as well as finishing time in the making. The number of hours required for producing 1 unit of each and the corresponding profit is given in the following table:

Item	Fabrication Time	Finishing Time	Profit(in INR)
A	9 hours	1 hour	80
B	12 hours	3 hours	120

TABLE I

In a week, the company has availability of not more than 180 hours of fabrication time and 30 hours of finishing time.
Assuming that all items manufactured are sold, how should the manufacturer schedule his weekly production in order to maximise the profit?
Formulate it as an LPP and solve it graphically.
Solution: Let x be the number of teaching aid A and y be the number of teaching aid B produced. From the given information, the problem can be formulated as

$$P = \max_{x,y} 80x + 120y \tag{1}$$

$$9x + 12y \leq 180 \tag{2}$$

$$x + 3y \leq 30 \tag{3}$$

which can be expressed in vector form as

$$P = \max_{\mathbf{x}} \begin{pmatrix} 80 & 120 \end{pmatrix} \mathbf{x} \tag{4}$$

$$\begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix} \mathbf{x} \preceq \begin{pmatrix} 60 \\ 30 \end{pmatrix} \tag{5}$$

$$\mathbf{x} \succeq \mathbf{0} \tag{6}$$

1) *Graphical solution:* From Fig. 1, the feasible region is a quadrilateral with vertices

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 20 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \end{pmatrix}, \begin{pmatrix} 12 \\ 6 \end{pmatrix} \tag{7}$$

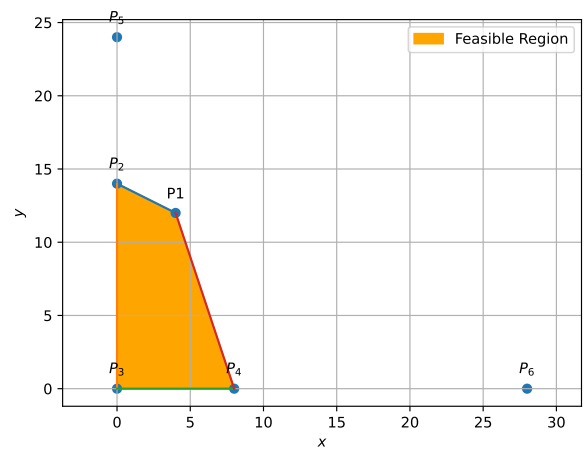

Fig. 1.

with respective profit

$$\begin{pmatrix} 80 & 120 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0 \tag{8}$$

$$\begin{pmatrix} 80 & 120 \end{pmatrix} \begin{pmatrix} 20 \\ 0 \end{pmatrix} = 1600 \tag{9}$$

$$\begin{pmatrix} 80 & 120 \end{pmatrix} \begin{pmatrix} 0 \\ 10 \end{pmatrix} = 1200 \tag{10}$$

$$\begin{pmatrix} 80 & 120 \end{pmatrix} \begin{pmatrix} 12 \\ 6 \end{pmatrix} = 1680 \tag{11}$$

Thus, the manufacturer should produce 12 teaching aids of type A and 6 teaching aids of B weekly.

2) *Lagrange Multipliers:* The given problem is expressed in the form

$$P = -\min_{\mathbf{x}} \begin{pmatrix} 80 & 120 \end{pmatrix} \mathbf{x} \tag{12}$$

$$\begin{pmatrix} 3 & 4 \\ 1 & 3 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} \preceq \begin{pmatrix} 60 \\ 30 \\ 0 \\ 0 \end{pmatrix} \tag{13}$$

The Lagrangian, defined as the linear combination of the loss function and the

constraints is defined as

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\lambda}) = &- \begin{pmatrix} 80 & 120 \end{pmatrix} \mathbf{x} \\ &+ \lambda_1 \left[\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} - 60 \right] \\ &+ \lambda_2 \left[\begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} - 30 \right] + \lambda_3 \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] \\ &+ \lambda_4 \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \end{aligned} \tag{14}$$

Taking the derivative

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = 0, \tag{15}$$

we obtain

$$3\lambda_1 + \lambda_2 - \lambda_3 = 80 \tag{16}$$
$$4\lambda_1 + 3\lambda_2 - \lambda_4 = 120 \tag{17}$$
$$3x_1 + 4x_2 = 60 \tag{18}$$
$$x_1 + 3x_2 = 30 \tag{19}$$
$$x_1 = 0 \tag{20}$$
$$x_2 = 0 \tag{21}$$

It is obvious that $x_1 = 0, x_2 = 0$ are infeasible. Hence, considering only λ_1, λ_2 as the active multipliers, the above equations can be expressed as

$$\begin{pmatrix} 0 & 0 & 3 & 1 \\ 0 & 0 & 4 & 3 \\ 3 & 4 & 0 & 0 \\ 1 & 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 80 \\ 120 \\ 60 \\ 30 \end{pmatrix} \tag{22}$$

yielding the optimal solution as

$$\begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} 12 \\ 6 \\ 24 \\ 8 \end{pmatrix} \tag{23}$$