

Assignment 9

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Outline

1 Papoulis Solutions

Problem

Example 7.11

The random variables x_1 and x_2 are jointly normal with zero mean. Determine their conditional density $f(x_2 | x_1)$.

Solution

We know that if the random variables s, x_1, \dots, x_n are jointly normal with zero mean, the linear and nonlinear estimators of s are equal:

$$\hat{s} = a_1 x_1 + \dots + a_n x_n = g(X) = E\{s | X\} \quad (1)$$

$$f(s | x_1, \dots, x_n) = \frac{1}{\sqrt{2\pi P}} e^{\frac{-[s - (a_1 x_1 + \dots + a_n x_n)]^2}{2P}} \quad (2)$$

therefore,

$$E\{x_2 | x_1\} = a x_1 \quad (3)$$

where, $a = \frac{R_{12}}{R_{11}}$

Also,

$$\sigma_{x_2|x_1}^2 = P = E\{(x_2 - a x_1)x_2\} = R_{22} - a R_{12} \quad (4)$$

Solution

Therefore,

$$f(x_2 | x_1) = \frac{1}{\sqrt{2\pi P}} e^{\frac{-(x_2 - ax_1)^2}{2P}} \quad (5)$$