AI1110 Assignment 1

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Q.)
In the Figure 1, O is the centre of the circle. $\angle DAE$ = 70°. Find giving suitable reasons, the measure of:

1) $\angle BCD$
2) $\angle BOD$
3) $\angle OBD$

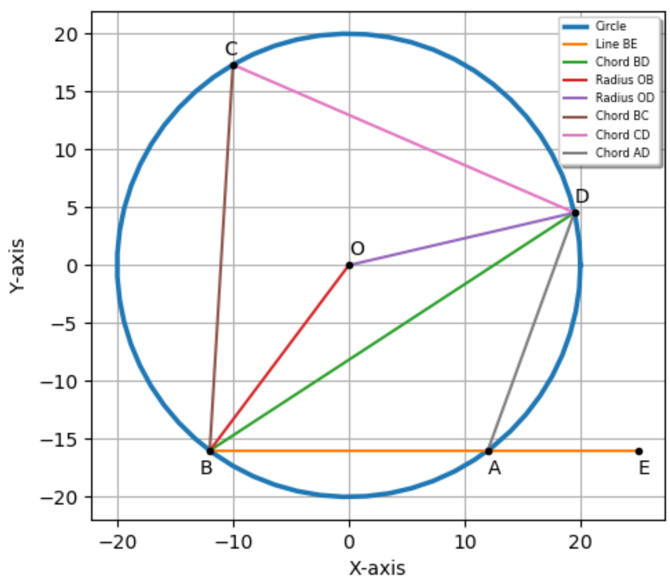

Fig. 1. Problem figure

Solution:
Given,

$$\angle DAE = \theta = \frac{7\pi}{18} \tag{1}$$

Assume, $\mathbf{O}$ (centre of circle) be origin. Assuming $\mathbf{A}$, $\mathbf{B}$ is reflection of $\mathbf{A}$ about Y-axis. The standard basis vectors are defined as

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3}$$

The general formula for image of any point P about line with equation:

$$\mathbf{n}^\top \mathbf{x} = c \tag{4}$$

is given by:

$$\mathbf{R} = \mathbf{P} + 2\frac{c - \mathbf{n}^\top \mathbf{P}}{\|\mathbf{n}\|^2}\mathbf{n} \tag{5}$$

where $\mathbf{n}$ is the normal vector of the line. For Y-axis,

$$\mathbf{n} = \mathbf{e}_1 \tag{6}$$

$$c = 0 \tag{7}$$

Hence,

$$\mathbf{B} = \mathbf{A} + 2\frac{c - \mathbf{n}^\top \mathbf{A}}{\|\mathbf{n}\|^2}\mathbf{n} \tag{8}$$

$$= \mathbf{A} + 2\frac{0 - \mathbf{e}_1^\top \mathbf{A}}{1^2}\mathbf{e}_1 \tag{9}$$

$$= \mathbf{A} - 2(\mathbf{e}_1 \cdot \mathbf{A})\mathbf{e}_1 \tag{10}$$

$\mathbf{D}$ is obtained by rotating $\mathbf{B}$ by 2θ anti-clockwise about $\mathbf{O}$. Rotation matrix $\mathbf{Q}$ is given by:

$$\mathbf{Q} = \begin{pmatrix} \cos 2\beta & -\sin 2\beta \\ \sin 2\beta & \cos 2\beta \end{pmatrix} \tag{11}$$

where a point vector is rotated anti-clockwise by β. Hence,

$$\mathbf{D} = \mathbf{Q} \cdot \mathbf{B} \tag{12}$$

$\because$ BE is a straight line,

$$\Rightarrow \angle BAD = \pi - \theta \tag{13}$$

$\because$ Sum of opposite angles in a cyclic quadrilateral is 180°,

$$\Rightarrow \angle BAD + \angle BCD = \pi \tag{14}$$

$$\Rightarrow \angle BCD = \theta \tag{15}$$

In general,

$$\angle BOD = 2\angle BCD = 2\theta \tag{16}$$

$\because$ OB = OD = R

$$\Rightarrow \angle OBD = \angle ODB \tag{17}$$

$\because$ The sum of angles of any triangle equals the straight angle.

$$\Rightarrow \angle BOD + \angle ODB + \angle OBD = \pi \tag{18}$$

$$\Rightarrow 2\theta + 2\angle OBD = \pi \tag{19}$$

$$\Rightarrow \angle OBD = \frac{\pi}{2} - \theta \tag{20}$$

Using, θ = 70°, we can say

1) $\angle BCD$ = 70°
2) $\angle BOD$ = 140°
3) $\angle OBD$ = 20°

The input parameters for drawing the figure are available in table shown below.

Symbol	**Value**	**Description**
R	20	Radius of the Circle
$\mathbf{O}$	$\mathbf{0}$	Centre of the circle (Origin)
$\mathbf{A}$		
$\mathbf{B}$	$\mathbf{A} - 2(\mathbf{e}_1 \cdot \mathbf{A})\mathbf{e}_1$	
$\mathbf{C}$		
$\mathbf{D}$	$\mathbf{Q} \cdot \mathbf{B}$	
$\mathbf{E}$		
θ	$\frac{7\pi}{18}$	$\angle DAE$