Assignment 8

Varun Gupta
cs21btech11060

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LaTeX

Outline

Problem

Ex 6.33

Let x and y be jointly normal random variables with parameters μ_x, μ_y, σ_x^2, σ_y^2 and r. Find a necessary and sufficient condition for x + y and x - y to be independent.

Solution

We know,

$$\Sigma_x = E[(x-\mu)(x-\mu)^T] = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \tag{1}$$

where,

$$X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2}$$

$$\mu = E(X) \tag{3}$$

Σ_x is defined as the covariance matrix of x.

Solution

Let, z_1 = x+y & z_2 = x-y

$$z = \begin{pmatrix} x+y \\ x-y \end{pmatrix} \tag{4}$$

$$\Sigma_z = E[(z-\mu)(z-\mu)^T] \tag{5}$$

$$\Sigma_z = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{6}$$

$$\Sigma_z = \begin{pmatrix} \sigma_x^2 + 2\rho\sigma_x\sigma_y + \sigma_y^2 & \sigma_x^2 - \sigma_y^2 \\ \sigma_x^2 - \sigma_y^2 & \sigma_x^2 - 2\rho\sigma_x\sigma_y + \sigma_y^2 \end{pmatrix} \tag{7}$$

Solution

As, x+y & x-y are independent, Σ_z is a diagonal matrix,

$$\Rightarrow \sigma_x^2 - \sigma_y^2 = 0 \Rightarrow \sigma_x = \sigma_y \tag{8}$$