

SVD

THEORY

- 1) SVD (Singular Value Decomposition) is basically a matrix factorization technique, which decomposes any matrix into 3 generic and familiar matrices. It has some applications in PCA, Machine Learning and Image Processing.
- 2) $A \in Mat_{m \times n}(\mathbb{R})$ can be decomposed as:

$$A = U\Sigma V^T$$

where, $V \in Mat_{n \times n}$, $U \in Mat_{m \times m}$, $\Sigma \in Mat_{m \times n}$

- 3) Here, Σ is a rectangular diagonal matrix & columns of U,V are orthogonal unit vectors.
- 4) In terms of this factorization,

$$X^T X = W\Sigma^T U^T U \Sigma W^T = W\Sigma^T \Sigma W^T = W\Sigma^T \Sigma W^T \quad (1)$$

QUIZ

- 1) Find the singular values of the matrix $B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$
- 2) Find the SVD of the matrix $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

ANSWER

- 1) $BB^T = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$. Singular values are 3 & 1.