Thesis Defense:

An Algorithmic Fault-Tolerant Control Architecture Without Actuator Redundancy

Alp Marangoz

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Supervisor: Asst. Prof. Dr. Ali Türker KUTAY

The Problem

Fault Tolerant Control

Detect & Identify Fault

Reconfigure the system in order to cope with the fault

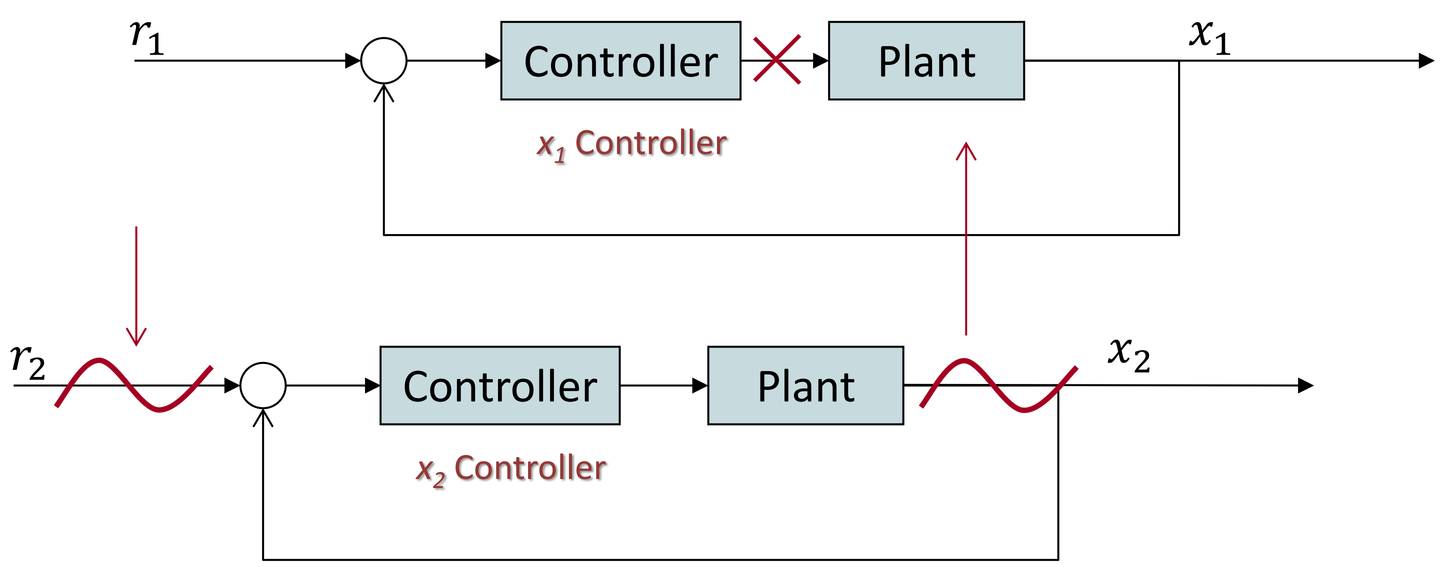
Using redundant components (Over-actuation)

Finding a controllable subspace (Analytical Redundancies)

Redundancy is not feasible in many systems and analytical redundancies are not easy to find!

Proposed Solution

- Inject perturbations on the controlled states that are connected to healthy actuators, in order to compensate for the failed components and maintain overall stabilization of the system.



Scope of The Thesis Work

- A Control methodology is developed that can be used as a fault-tolerant control strategy
  - Fault mitigation strategy is proposed and formulated as a control problem
  - A Nonlinear control system architecture is developed
  - Theoretical analysis is conducted for determination of design parameters and limits

- The problem is applied to robotic manipulator control problem
  - Applied the formulation to Euler-Lagrange equations
  - Numerical simulations are conducted on 2-Link and 3-Link robotic manipulator cases

- The problem is applied to quadrotor attitude control problem
  - Applied the formulation to attitude rate equations
  - Numerical simulations are conducted for complete loss of single and two propeller cases

Method

- Consider the MIMO System:

$$\dot{x} = f(x, u) \quad \Longrightarrow \quad \begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, u_1) \\ \dot{x}_2 &= f_2(x_1, x_2, u_2) \end{aligned}$$

Faulty Actuator(s)

- Grouping is rather arbitrary: $x \in \mathbb{R}^n, x_1 \in \mathbb{R}^{n-m}, x_2 \in \mathbb{R}^m, u_2 \in \mathbb{R}^m$

- With fault mitigation act of $u_1 = 0$, the problem becomes a stabilization problem for a general cascade system:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2, 0) \\ \dot{x}_2 &= f_2(x_1, x_2, u_2) \end{aligned}$$

$$\dot{x}_1 = f_1(x_1, x_2)$$
$$\dot{x}_2 = f_2(x_1, x_2, u)$$

Underactuated Control:

$x \in \mathbb{R}^{n+m}, x_1 \in \mathbb{R}^n,\ x_2 \in \mathbb{R}^m, u_2 \in \mathbb{R}^m$

Adaptive Control: $x_2 \to u,\ \cancel{u}$

$$\dot{x}_1 = f_1(x_1, u)$$
$$\dot{u} = f_2(x_1, u) \qquad \text{(adaptive law)}$$

Robust Control: $f_1(x_1, x_2) \to f_1(x_1)$ and $x_1 \to d$

$$\dot{d} = f_1(d\ ) \qquad \text{(Exogenous system)}$$
$$\dot{x}_2 = f_2(x_2, d, u)$$

- A basic question would be: if both f_1 and f_2 are stable, does the cascade system becomes stable?
- What if $f_1(x_1, 0)$ is unstable? (Non-minimum phase systems, Zero-dynamics)

Practical Approaches

$$\dot{x} = f(x,u)$$

Fully actuated

$$\dot{x} = f(x) + g(x,u)$$

Feedback linearization, adaptive control, SMC, ADI etc.

Interconnected

$$\boxed{\begin{aligned}\dot{x}_1 &= f_1(x_1,x_2)\\ \dot{x}_2 &= f_2(x_1,x_2,u)\end{aligned}}$$

Some form of back-stepping is applied

Interconnected, Control-affine form

$$\dot{x}_1 = f_1(x_1,x_2)$$
$$\dot{x}_2 = f_2(x_1,x_2) + g(x_1,x_2)\cdot u$$

Linear Cascade

$$\dot{x}_1 = f_1(x_1,x_2)$$
$$\dot{x}_2 = A\cdot x_2 + B\cdot u$$

Strict Feedback form

$$\begin{aligned}\dot{x}_1 &= f_1(x_1) + g_1(x_1)\cdot x_2\\ \dot{x}_2 &= f_2(x_2) + g_2(x_2)\cdot x_3\\ &\vdots\\ \dot{x}_n &= f_n(x_n) + g_n(x_n)\cdot u\end{aligned}$$

Normal Form

$$\begin{aligned}\dot{x}_0 &= f(x_0,x_1,\cdots,x_n)\\ \dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ &\vdots\\ \dot{x}_n &= u\end{aligned}$$

Results with open-loop unstable internal dynamics are available only in these forms

Ming-Li Chiang and Alberto Isidori. "Nonlinear output regulation with saturated control for a class of non-minimum phase systems" (2015)

Exogenous system

$\dot{w} = s(w)$

Internal states (Linearly related to e)

$\dot{z} = f_0(w,z) + g_0(w,z)e$

Controlled states (Linearly related to u)

$\dot{e} = q_0(w,z,e) + b(w,z,e) \cdot u$

$y_r = Cz, y_e = e$

$$\dot{\eta} = \Phi(\eta) + Ge$$

$$\dot{\xi} = (A - F_\kappa C)\xi + F_\kappa y_r$$

$$u = -k_c \left\{ e - \left(sat_{lc}[L_1\xi] + \lambda_s sat_{ls}\left[\frac{(L_2\eta + L_3\xi)}{\lambda_s}\right] \right) \right\}$$

Proposed Approach

$$\dot{x}_1 = f_1(x_1, x_2)$$
$$\dot{x}_2 = f_2(x_1, x_2, u)$$

Let the reference trajectory of $x_2$be r_2 and stabilizing perturbations be $\epsilon \cdot r$

Reference trajectory of x_2

Perturbations on x_2

$e_2 = x_2 - (r_2 + r)$: Tracking error of x_2

$$\dot{x}_1 = f_1(x_1, r_2 + r + e_2\ )$$
$$\dot{x}_2 = f_2(x_1, x_2, u)$$
$$\epsilon\dot{r} = \alpha \cdot [f_1(x_1, r_2 + r) - f_{r1}(x_1, r_2)] + g_1(r)$$

Small parameter ϵ

Dynamic law for calculation of the trajectory perturbation

Singular Perturbation Theory

Perturbation theory deals with solution of dynamic systems (ODEs and PDEs) that contain a small parameter ϵ

eg: van der Pol oscillator

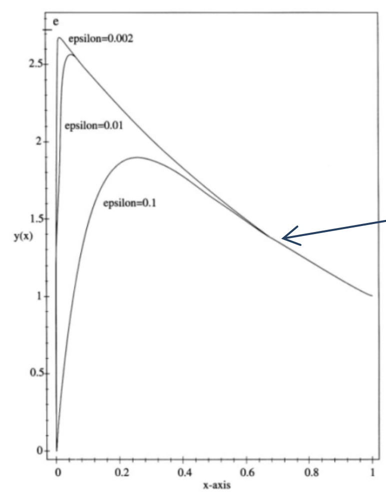
$$\frac{d^2y}{dt^2} + \epsilon \cdot (y^2 - 1) \cdot \frac{dy}{dt} + y = 0$$

Construct solutions, using the solution of the reduced problem $\frac{d^2y}{dt^2} + y = 0$

Singular pertubation problem: Omission of the small parameter reduces the order of the system:

$$\epsilon \cdot \frac{d^2y}{dt^2} + (1 + \epsilon) \cdot \frac{dy}{dt} + y = 0$$

$$y(x) = \frac{e^{-x} - e^{-x/\epsilon}}{e^{-1} - e^{-1/\epsilon}}$$



$$y(x) = e^{1-x}$$

Tikhonov's Theorem

Singular Perturbation problem
$$\left\{\boxed{\begin{aligned}\dot{x} &= f(t,x,z,\epsilon)\\ \epsilon\dot{z} &= g(t,x,z,\epsilon)\end{aligned}}\right. \longrightarrow \quad g(t,x,z^* = h(t,x),0) = 0$$

$$\dot{x} = f(t,x,h(t,x),0) \longrightarrow \quad \text{Reduced problem}$$

$$\frac{dy}{d\tau} = g(t,x,y+h(t,x),0) \longrightarrow \quad \text{Boundary Layer Model}$$

Assumptions of the Tikhonov's Theorem

1. The functions *f*, *g* and their first partial derivatives with respect to *(x, z, ϵ)* are continuous. The function *h(t, x)* and the Jacobian $\partial g(t, x, z, 0)/\partial u$ have continuous first partial derivatives with respect to their arguments. The initial data are smooth functions of ϵ.
2. The reduced system has a unique solution $\bar{x}(t)$, defined on $[t_0, t_1]$, and $\|\bar{x}(t)\| \leq r_1 < r$ for all $t \in [t_0, t_1]$
3. The origin of the boundary layer system is <u>exponentially stable</u>, uniformly in *(t, x)*.

Then,

$$x(t,\epsilon) - \bar{x}(t) = \mathcal{O}(\epsilon)$$
$$z(t,\epsilon) - h\big(t,\bar{x}(t)\big) - \hat{y}(t/\epsilon) = \mathcal{O}(\epsilon)$$

Tikhonov's Theorem

eg: Actuator dynamics

$$\dot{x} = A \cdot x + B \cdot v$$
$$\epsilon \cdot \dot{v} = A_a \cdot v + B_a \cdot u$$

Representing fast actuator time constant

Typical Practice: $\dot{x} = A \cdot x + K \cdot u$ $\Rightarrow$ $\dot{x} = A \cdot x + B \cdot \underbrace{(-A_a^{-1} \cdot B_a \cdot u)}_{\text{DC gain of the actuator}}$

Replace it with gain

Actually: solution of $A_a \cdot v^* + B \cdot u = 0$

Inspiring Idea: Adaptive Dynamic Inversion

Hovakimyan, Lavretsky and Sasane proposed to define control input as a singularly perturbed dynamic system (2007)

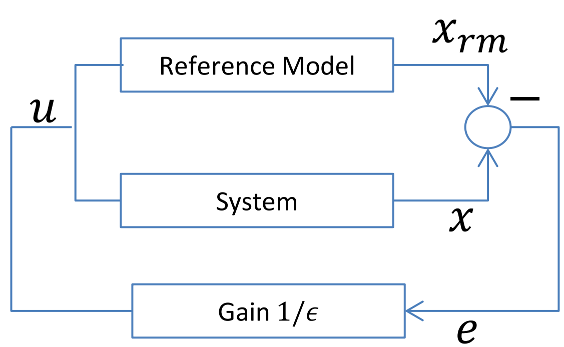
$$\dot{x} = f(t, x, u)$$

$$\epsilon \cdot \dot{u} = -sign\left(\frac{\partial f}{\partial u}\right) \cdot \underbrace{[f(t, x, u) - (\overbrace{A_r \cdot x_r + B_r \cdot r}^{\text{Reference model}})]}_{\text{Tikhonov's theorem states that this term will converge to zero!}}$$

> Later studies revealed that, this form is equivalent to high gain proportional control, no further results after that (2010):

$$u = -\frac{1}{\epsilon} \cdot sign\left(\frac{\partial f}{\partial u}\right) \cdot \int [f(t, x, u) - (A_r \cdot x_r + B_r \cdot r)] \cdot dt$$

$$\Rightarrow \sim -\frac{1}{\epsilon} \cdot sign\left(\frac{\partial f}{\partial u}\right) \cdot (x - x_{rm})$$



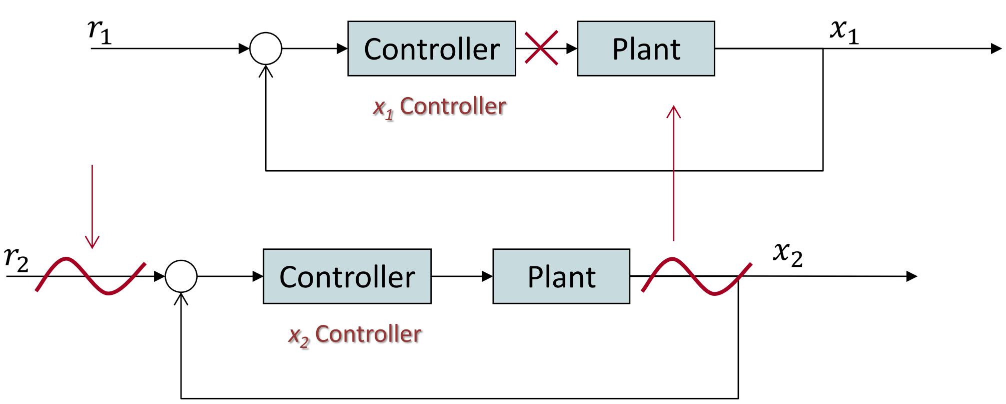
Proposed Method

$$\dot{x}_1 = f_1(x_1, x_2)$$
$$\dot{x}_2 = f_2(x_1, x_2, u)$$

$$\dot{x}_1 = f_1(x_1, r_2 + r + e_2)$$
$$\dot{x}_2 = f_2(x_1, x_2, u)$$
$$\epsilon\dot{r} = \alpha[f_1(x_1, r_2 + r) - f_{r1}(x_1, r_2)] + g_1(r)$$

Existing controller

$$\dot{x}_1 = f_1(x_1, r_2 + r + e_2)$$
$$\dot{e}_2 = g_2(x_1, e_2, u)$$
$$\epsilon\dot{r} = \alpha\underbrace{[f_1(x_1, r_2 + r) - f_{r1}(x_1, r_2)]}_{f_1(x_1, r_2 + r) \to f_{r1}(x_1, r_2)} + g_1(r)$$



Stable reference dynamics for x_1

System Dynamics

$$\dot{x}_1 = f_1(x_1, x_2)$$
$$\dot{x}_2 = f_2(x_1, x_2, u)$$

Controled Dynamics

$$\dot{x}_1 = f_1(x_1, r_2 + r + e_2) \rightarrow e_2 = x_2 - (r_2 + \epsilon \cdot r)$$
$$\dot{e}_2 = g_2(e_2, x_1, u)$$
$$\epsilon \cdot \dot{r} = \alpha \cdot [f_1(x_1, r_2 + r + e_2) - f_{r1}(x_1, r_2)] + g_1(r)$$

Reduced Problem

$$\dot{x}_1 = f_1(x_1, r_2 + r^* + e_2) = f_{r1}(x_1, r_2) + \cancel{g_1(r^*)}$$
$$\dot{e}_2 = g_2(e_2, x_1, u)$$

Boundary Layer Problem

$$\frac{dy}{d\tau} = \alpha \cdot [f_1(x_1, r_2 + r^* + y + e_2) - f_{r1}(x_1, r_2)] + g_1(r^* + y)$$

Assumptions of the Proposed Controller

1. The homogeneous system $\dot{x}_2 = f_2(x_1, x_2, u)$ is small-time locally controllable from $x_{2,0} = 0 \; \forall \; x_1 \in \mathbb{R}^{n-m}$ *and* $x_2 \in \mathbb{R}^m$
2. The system $\dot{x}_1 = f_1(x_1, x_2)$ is controllable for the virtual control input x_2

Assumptions of the Tikhonov's Theorem

1. The functions *f, g* and their first partial derivatives with respect to *(x, z, ϵ)* are continuous. The function *h(t, x)* and the Jacobian ∂*g*(*t, x, z,* 0)/∂*u* have continuous first partial derivatives with respect to their arguments. The initial data are smooth functions of ϵ.

2. The reduced system has a unique solution $\bar{x}(t)$, defined on $[t_0, t_1]$, and $\|\bar{x}(t)\| \leq r_1 < r$ for all $t \in [t_0, t_1]$

3. The origin of the boundary layer system is exponentially stable, uniformly in *(t, x)*.

Stability of Boundary Layer Equation

Most restrictive assumption is the one related to the exponential stability of the boundary layer

A dynamic system is exponentially stable if there exist positive constants c, k and λ

$$\|x(t)\| \leq \underbrace{k \cdot \|x(t_0)\| \cdot e^{-\lambda \cdot (t-t_0)}}, \forall \|x(t_0)\| < c$$

States are bounded by exponentially decaying function

Stability of Boundary Layer Equation

Design constraints on $f_{r1}(x_1, x_2)$ and $g_1(r)$ can be derived from the analysis of the boundary layer equation:

$$\frac{dy}{d\tau} = \alpha \cdot [f_1(x_1, r_2 + r^* + y) - f_{r1}(x_1, r_2)] + g_1(r^* + y)$$

Parameters (x_1, r_2, r^*) can be considered as time dependent signals and stability theorems on Linear Time Varying systems can be used for derivation of design constraints.

Exponential Stability of LTV Systems

Theorem: *Let the origin $x = 0$ be an equilibrium point of $\dot{x} = f(t,x)$ and $D \subset \mathbb{R}$ be a domain containing $x = 0$. Suppose $f(t,x)$ is piecewise continuous in t and locally Lipschitz in x for all $t \geq 0$ and $x \in D$. Let $V(t,x)$ be a continuously differentiable function such that*

$$k_1\|x\|^a \leq V(t,x) \leq k_2\,\|x\|^a$$

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x) \leq -k_3\,\|x\|^a$$

for all $t \geq 0$ and $x \in D$, where k_1, k_2, k_3 and a are positive constants. Then, the origin is exponentially stable. If the assumptions hold globally, the origin will be globally exponentially stable.

Exponential Stability of LTV Systems
Perturbed System

Corollary: Consider the perturbed system:

$$\dot{x} = f(x) + g(t, x)$$

Exponentially stable with Lyapunov function $V(x)$.

$$c_1 \|x\|^2 \leq V(x) \leq c_2 \|x\|^2$$

$$\frac{\partial V}{\partial x} f(t, x) \leq -c_3 \|x\|^a$$

$$\left\| \frac{\partial V}{\partial x} \right\| \leq c_4 \|x\|$$

Perturbed System is exponentially stable if

$$\|g(t, x)\| \leq \gamma \|x\|$$

$$0 \leq \gamma < \frac{c_3}{c_4}$$

Exponential Stability of Boundary Layer Equation

$$\frac{dy}{d\tau} = \underbrace{\alpha \cdot [f_1(x_1, r_2 + r^* + y) - f_{r1}(x_1, r_2)]}_{\textit{Perturbation}} + \underbrace{g_1(r^* + y)}_{A_r(r)}$$

$$\|g(t,x)\| \leq \gamma \|x\|$$

Exponentially stable with Lyapunov function:

$$V(x) = x^T P x$$

Exponential Stability of Boundary Layer Equation

$$\frac{dy}{d\tau} = \alpha \cdot [f_1(x_1, r_2 + r^* + y) - f_{r1}(x_1, r_2)] + \underbrace{g_1(r^* + y)}_{A_r(r)}$$

$$f_1(x_1, x_2) = f_{11}(x_1) + f_{12}(x_1) \cdot x_2$$

$$f_1(x_1, x_2) = \underbrace{f_1(x_1, r_2)}_{f_{11}(x_1)} + \underbrace{\left.\frac{\partial f_1}{\partial x_2}\right|_{x_1, r_2}}_{f_{12}(x_1)} \cdot x_2$$

$$\frac{dy}{d\tau} = [A_r + \alpha \cdot f_{12}(x_1)] \cdot y + \alpha \cdot f_{11}(x_1)$$

$\|f_{11}(t)\| \leq \delta$ $\qquad$ $PA_r + A_r^T P + Q = 0$

$\|f_{12}\| \leq \gamma_f$ $\qquad$ $\dfrac{1}{2\lambda_{max}(P)} > \gamma_f$

A Simple Example

$$\begin{aligned}\dot{x}_1 &= x_1^2 + x_2^3 \\ \dot{x}_2 &= u\end{aligned} \;\Rightarrow\; \begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 0 \\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix} u$$

Internal Dynamics is Unstable

Linearized System is Uncontrollable

Let $V(x) = \frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 > 0$

$$\dot{V}(x) = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1^3 + x_1x_2^3 + x_2u < 0$$

$$u = -x_1x_2 - \frac{x_1^2}{x_2} \longrightarrow \underbrace{-x_1^2(1 - x_1) + -x_2^2(1 - x_2) < 0}_{\text{True for } x_1, x_2 < 1}$$

Problem!

A Simple Example

$$
\begin{aligned}
\dot{x}_1 &= x_1^2 + x_2^3 \\
\dot{x}_2 &= u
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
\dot{x}_1 &= x_1^2 + x_2^3 \\
\epsilon\dot{r} &= \alpha[x_1^2 + (r_2 + r)^3 - A_{rm}x_1] + A_r r \\
u &= K(r_2 + r - x_2)
\end{aligned}
$$

$$
f_1(x_1, x_2) = \underbrace{f_1(x_1, r_2)}_{f_{11}(x_1) = x_1^2} + \underbrace{\left.\frac{\partial f_1}{\partial x_2}\right|_{x_1, r_2}}_{f_{12}(x_1) = 3r_2^2} \cdot x_2
$$

A Simple Example

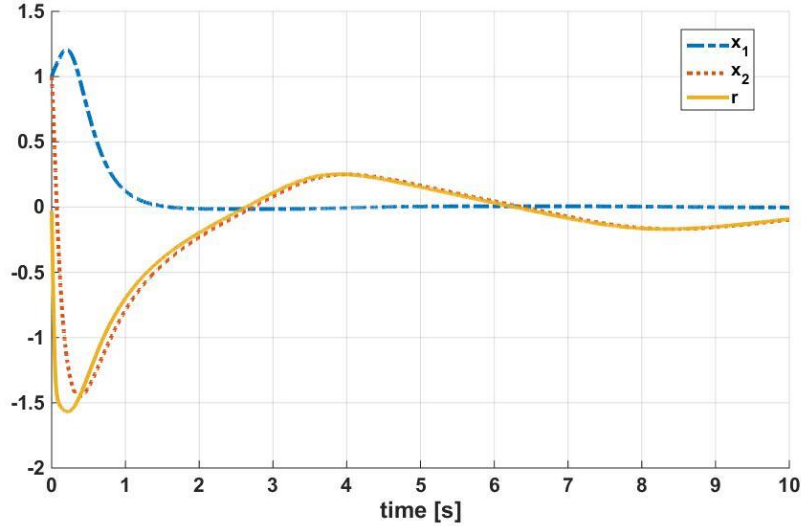
$$\begin{aligned}
\dot{x}_1 &= x_1^2 + x_2^3 \\
\epsilon \dot{r} &= \alpha[x_1^2 + (r_2 + r\ )^3 - A_{rm} x_1] + A_r r \\
u &= K(r_2 + r - x_2)
\end{aligned}$$

Boundary Layer Equation:

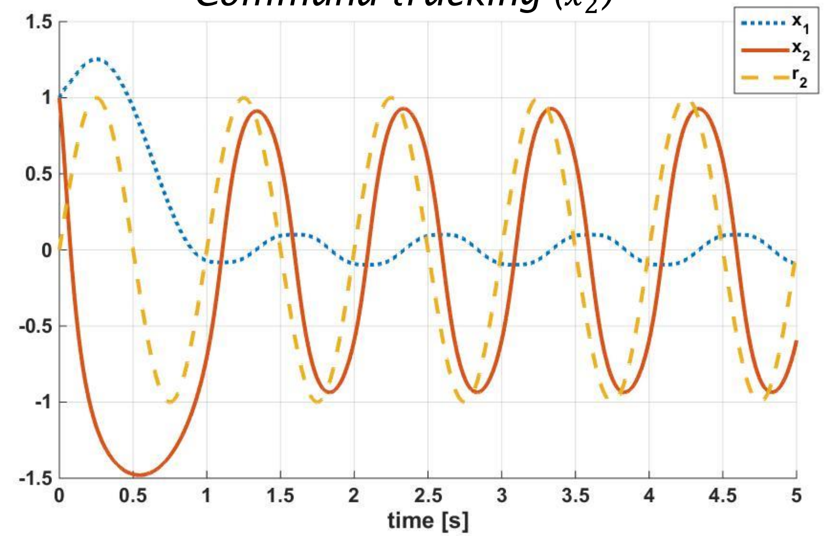
$$\frac{dy}{d\tau} = [A_r + \underbrace{\alpha \cdot 3r_2^2}_{\textit{Always stable for } \alpha < 0}] \cdot y + \alpha \cdot x_1^2$$

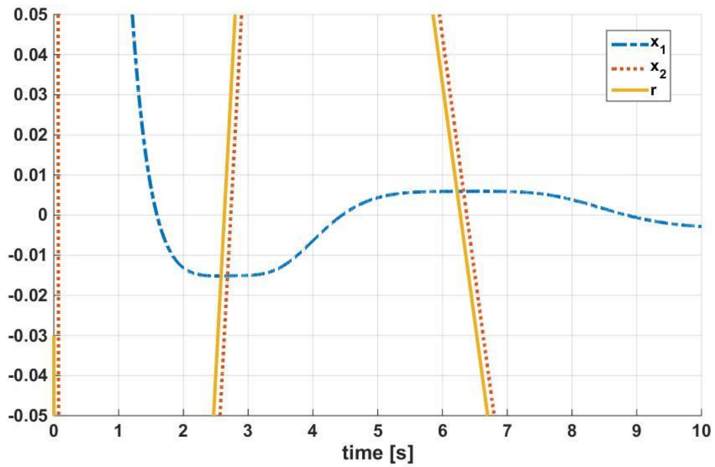
A Simple Example

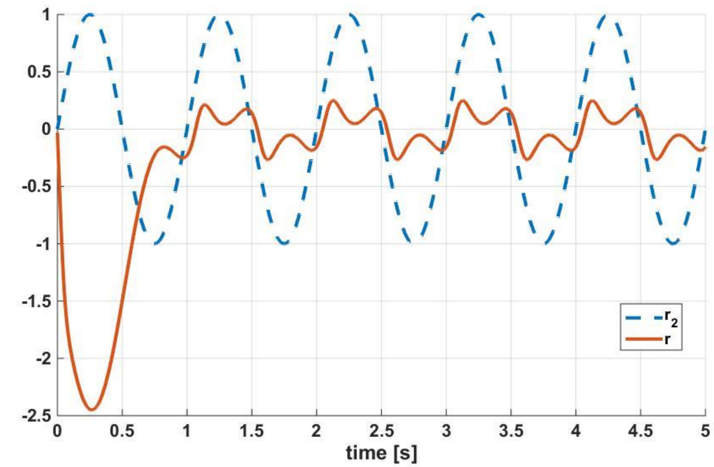
Stabilization around origin



Command tracking (x_2)

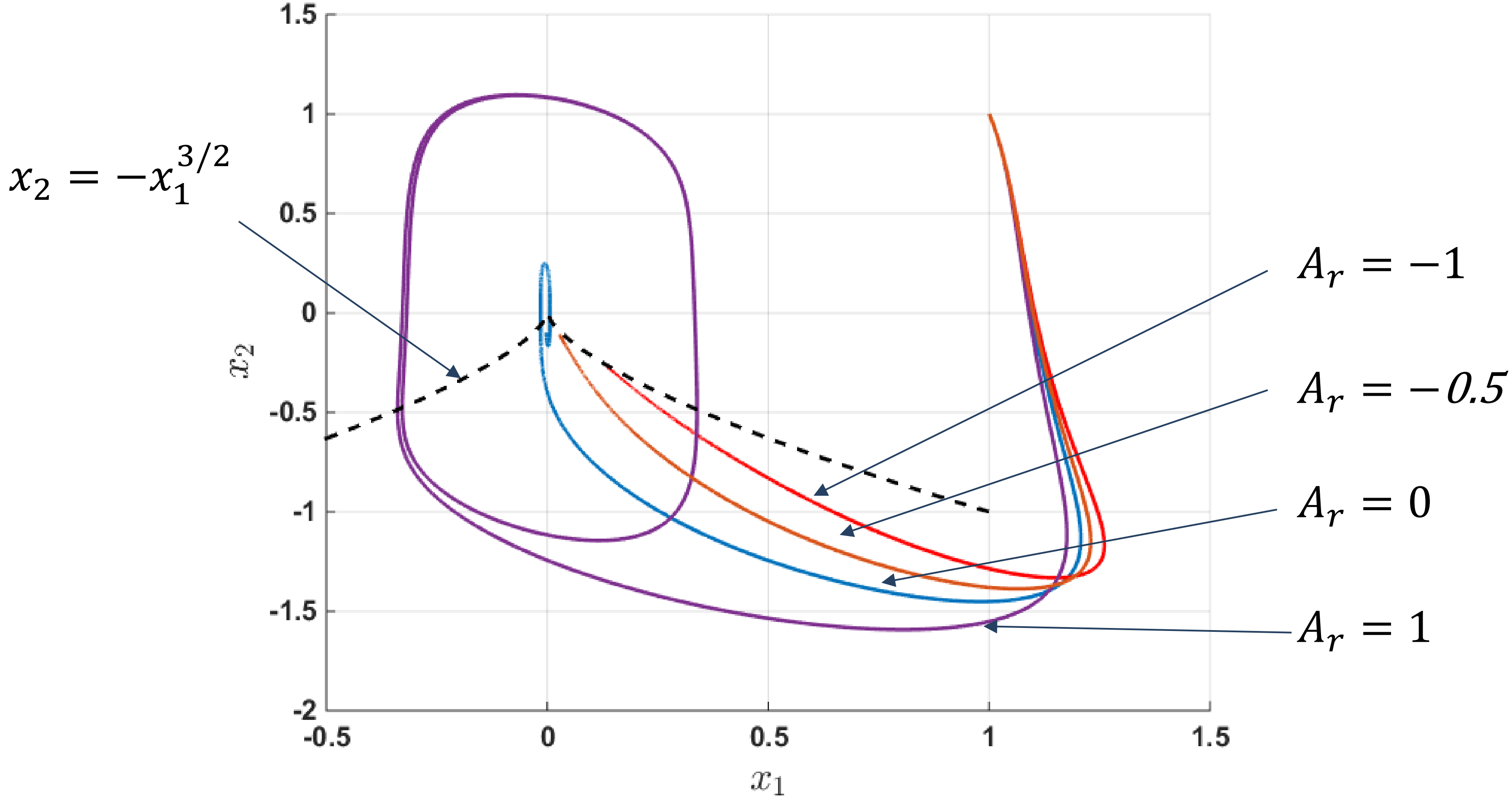






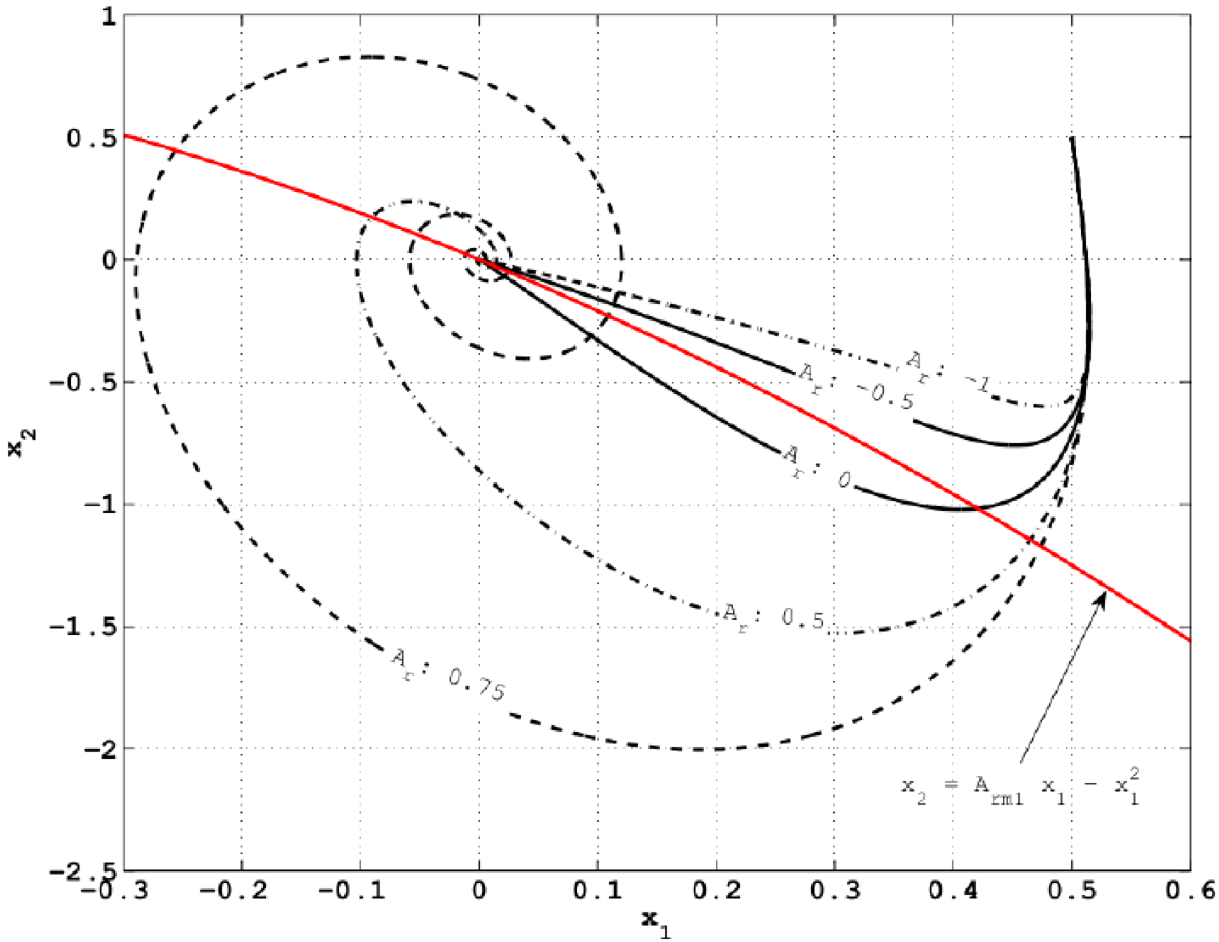
A Simple Example

$$\begin{aligned}\dot{x}_1 &= x_1^2 + x_2^3 \\ \dot{x}_2 &= u\end{aligned}$$



$$
\begin{aligned}
\dot{x}_1 &= x_1^2 + x_2 \\
\dot{x}_2 &= u
\end{aligned}
$$

A Simple Example



Summary

Proposed methodology is quite straightforward:

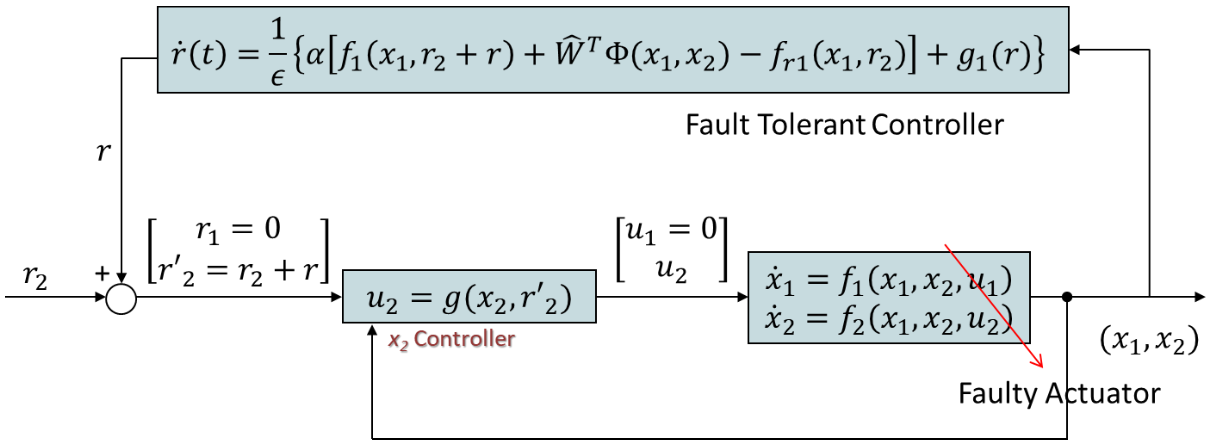
- Reformulate the problem:

$$\dot{x} = f(x,u) \quad \Rightarrow \quad \begin{aligned} \dot{x}_1 &= f_1(x_1,x_2) \\ \dot{x}_2 &= f_2(x_1,x_2,u) \end{aligned}$$

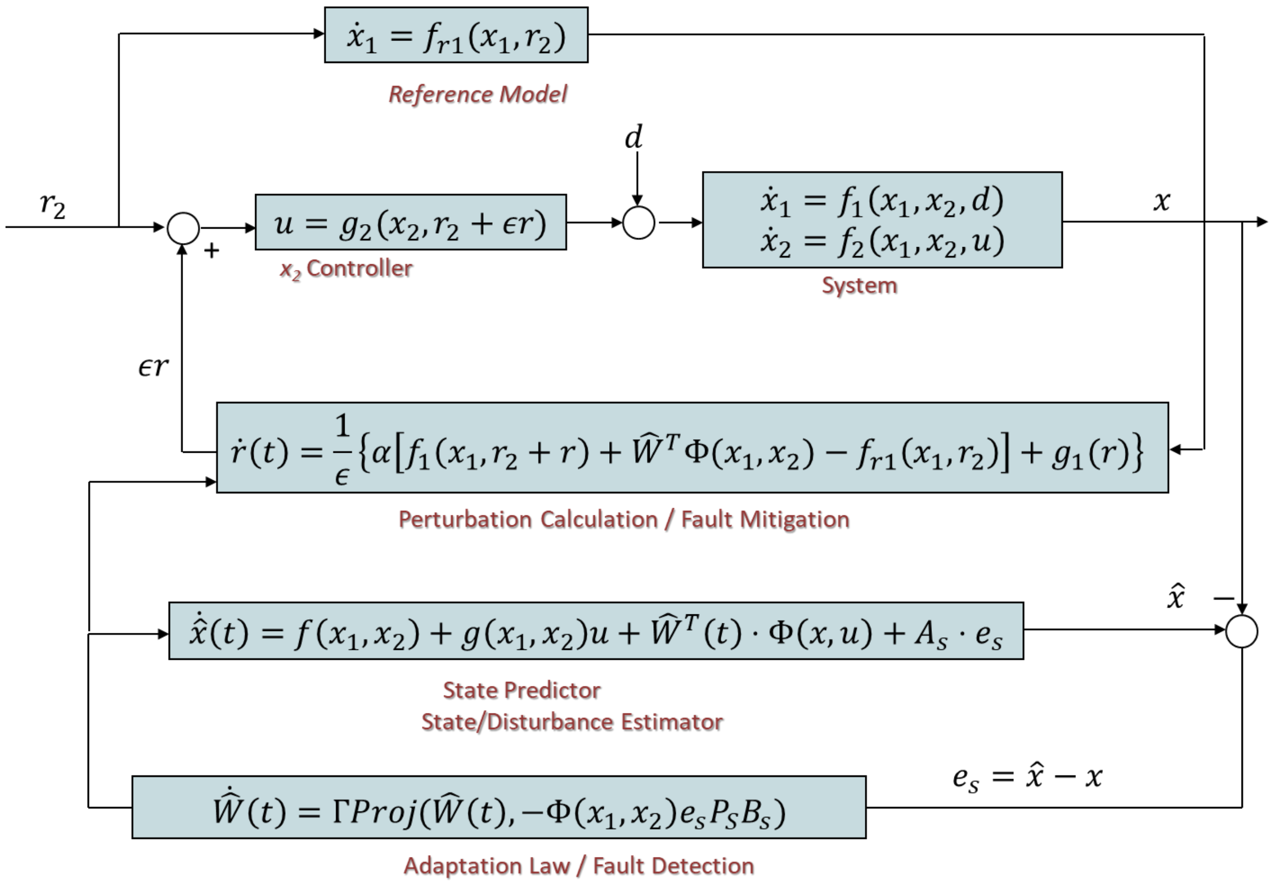
- Calculate the bounds

$$\|f_{12}\| \le \gamma_f \qquad PA_r + A_r^T P + Q = 0 \qquad \frac{1}{2\lambda_{max}(P)} > \gamma_f$$

- Apply it paralel to an existing controller



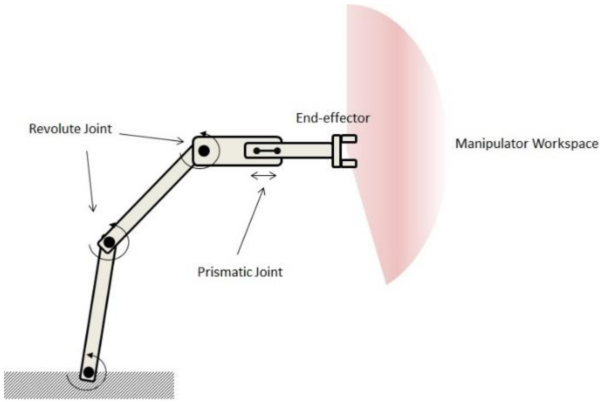
Summary

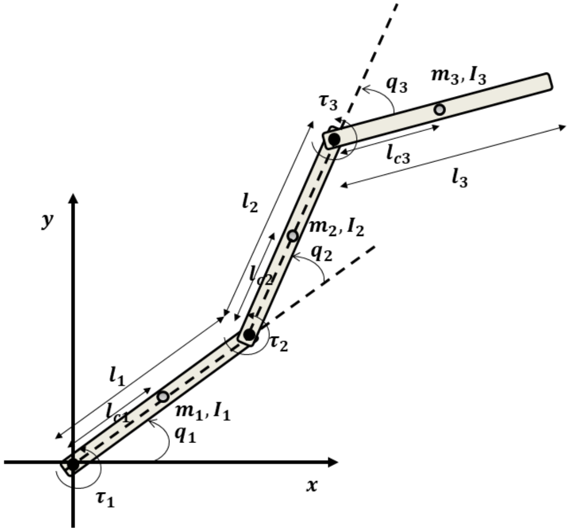


- Of course, many problem specific manipulations should be done
- As usual, some problems are more suitable than others

Application to Robotic Manipulators

$$H\ddot{q} + C\dot{q} + G = \tau$$





(Denmark Technical University: https://www.youtube.com/watch?v=sMZRnE3q72c)

Euler-Lagrange Equations

$$H\ddot{q} + C\dot{q} + G = \tau \quad \Rightarrow \quad \tau = H \cdot \left(\ddot{q}_d - K_v \dot{e} - K_p e\right) + C\dot{q} + G$$

Underactuated Systems: $q \to (q_1, q_2)$

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \qquad \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = C\dot{q} + G \qquad \begin{matrix} H_{11} \cdot \ddot{q}_1 + H_{12} \cdot \ddot{q}_2 + \phi_1 = 0 \\ H_{21} \cdot \ddot{q}_1 + H_{22} \cdot \ddot{q}_2 + \phi_2 = \tau_2 \end{matrix}$$

"Collocated partial linearized form:"

$$\tau_2 = (H_{22} - H_{21} \cdot H_{11}^{-1} \cdot H_{12}) \cdot v - H_{21} \cdot H_{11}^{-1} \cdot \phi_1 + \phi_2$$

$$\boxed{\begin{matrix} \ddot{q}_1 = -\mathrm{H}_{11}^{-1} \cdot (H_{12} \cdot v + \phi_1) \\ \ddot{q}_2 = v \end{matrix}}$$

v, control input

Fault Diagnosis

- Faults in robotic manipulators are usually modeled as additive faults

$$H\ddot{q} + C\dot{q} + G = \tau - F$$

$$F_i = \begin{cases} \tau_i & \text{for free−swing faults} \\ \tau_i - \gamma t & \text{for ramp fault} \\ \tau_i - \tau_{max} & \text{for saturated actutors} \end{cases}$$

- First order form of this equation couples the fault signals

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & -H^{-1}C \end{bmatrix} x + \begin{bmatrix} 0 \\ -H^{-1}G \end{bmatrix} + \begin{bmatrix} 0 \\ -H^{-1}F \end{bmatrix} + \begin{bmatrix} 0 \\ H^{-1}\tau \end{bmatrix} \quad \text{with} \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

Fault Diagnosis

- A change of variables is proposed that would decouple the disturbance signal

$$x = \begin{bmatrix} q \\ H\dot{q} \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} 0 & I \\ 0 & -C'H^{-1} \end{bmatrix} x + \begin{bmatrix} 0 \\ -G \end{bmatrix} + \begin{bmatrix} 0 \\ -F \end{bmatrix} + \begin{bmatrix} 0 \\ \tau \end{bmatrix}$$

$$C_{ij}(q, \dot{q}) = \frac{1}{2}\left(H_{ij,k} + H_{ik,j} - H_{kj,i}\right)\dot{q}_k$$

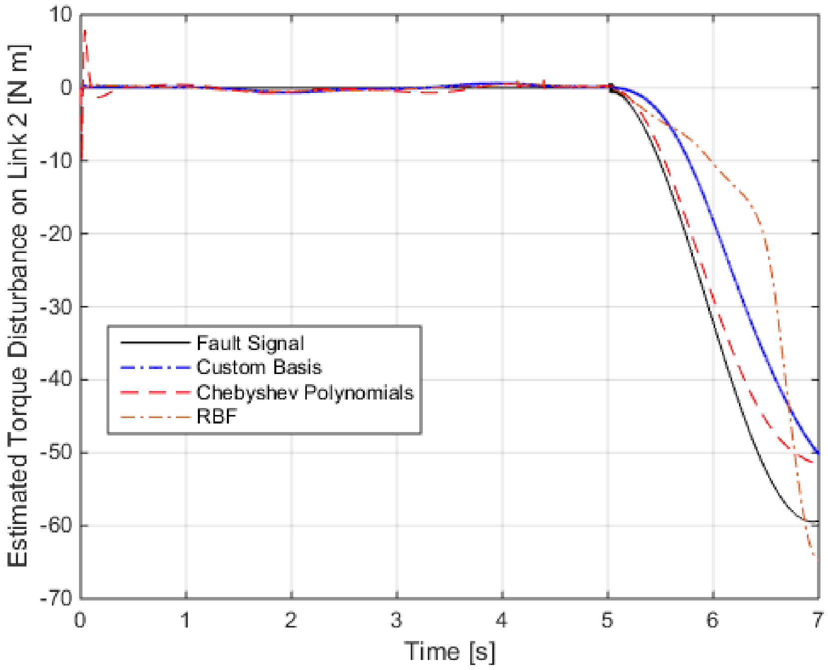
vs.

$$C'_{ij}(q, \dot{q}) = \frac{1}{2}\left(-H_{ij,k} + H_{ik,j} - H_{kj,i}\right)\dot{q}_k$$

Fault Diagnosis

$$\dot{\hat{x}} = \begin{bmatrix} 0 & I \\ 0 & -C'H^{-1} \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ -G \end{bmatrix} + \begin{bmatrix} 0 \\ -\hat{F} \end{bmatrix} + \begin{bmatrix} 0 \\ \tau \end{bmatrix} + A_s e_s$$

with $$\hat{F}_i = \sum_{j=1}^{N} W_{ij} \psi_j(\hat{x})$$



(This result is submitted to IEEE Transactions on Control Systems)

Fault Mitigation

$$\ddot{q}_1 = -\mathrm{H}_{11}^{-1} \cdot (H_{12} \cdot v + \phi_1)$$
$$\ddot{q}_2 = v$$

$$v = -K_v\underbrace{(\dot{q}_2 - \dot{q}_{2d})}_{\dot{e}} - K_p\underbrace{(q_2 - q_{2d})}_{e}$$

Closed loop system dynamics:

$$\begin{bmatrix}\dot{q}_1\\ \ddot{q}_1\end{bmatrix} = \begin{bmatrix}0 & I\\ 0 & -H_{11}^{-1}C_{11}\end{bmatrix}\begin{bmatrix}q_1\\ \dot{q}_1\end{bmatrix} + \begin{bmatrix}0\\ -H_{11}^{-1}(G_1 + C_{12}\dot{q}_2)\end{bmatrix} + H_{11}^{-1}H_{12}\begin{bmatrix}0 & 0\\ K_p & K_v\end{bmatrix}\begin{bmatrix}e\\ \dot{e}\end{bmatrix}$$

$$\epsilon\begin{bmatrix}\dot{r}\\ \ddot{r}\end{bmatrix} = \alpha\left\{\begin{bmatrix}0 & I\\ 0 & -H_{11}^{-1}C_{11}\end{bmatrix}\begin{bmatrix}q_1\\ \dot{q}_1\end{bmatrix} + \begin{bmatrix}0\\ -H_{11}^{-1}(G_1 + C_{12}\dot{q}_2)\end{bmatrix}\right\} -$$
$$\left(\begin{bmatrix}0 & I\\ A_{rm21} & A_{rm22}\end{bmatrix}\begin{bmatrix}q_1\\ \dot{q}_1\end{bmatrix} + \begin{bmatrix}0\\ B_{rm21}\end{bmatrix}r_1\right) + A_r\begin{bmatrix}r\\ \dot{r}\end{bmatrix}$$

Boundary Layer Equation

$$x_1 = \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} \qquad x_2 = \begin{bmatrix} q_2 \\ \dot{q}_2 \end{bmatrix}$$

$$\epsilon \begin{bmatrix} \dot{r} \\ \ddot{r} \end{bmatrix} = \alpha \left\{ \underbrace{\begin{bmatrix} 0 & I \\ 0 & -H_{11}^{-1} C_{11} \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix}}_{f_{11}(x_1)} + \underbrace{\begin{bmatrix} 0 \\ -H_{11}^{-1}(G_1 + C_{12}\dot{q}_2) \end{bmatrix}}_{f_{12}(x_1, x_2)} \right\} - \left(\begin{bmatrix} 0 & I \\ A_{rm21} & A_{rm22} \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ B_{rm21} \end{bmatrix} r_1 \right) + A_r \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$$

After some calculations:

$$\boxed{\frac{dy}{d\tau} = \left(A_r + \alpha \begin{bmatrix} 0 & 0 \\ \left. \left(H_{11}^{-1} \frac{\partial H_{11}}{\partial q_2} H_{11}^{-1} G_1 - H_{11}^{-1} \frac{\partial G_1}{\partial q_2} \right) \right|_{q_2 = r_2} & 0 \end{bmatrix} \right) y + \alpha \begin{bmatrix} 0 \\ - \left. H_{11}^{-1} G_1 \right|_{q_2 = r_2} \end{bmatrix}}$$

Valid for all robotic manipulator systems!

Design Constraints

$$\epsilon \begin{bmatrix} \dot{r} \\ \ddot{r} \end{bmatrix} = \alpha \left\{ \begin{bmatrix} 0 & I \\ 0 & -H_{11}^{-1} C_{11} \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -H_{11}^{-1}(G_1 + C_{12}\dot{q}_2) \end{bmatrix} - \left(\begin{bmatrix} 0 & I \\ A_{rm21} & A_{rm22} \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ B_{rm21} \end{bmatrix} r_1 \right) \right\} + A_r \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$$

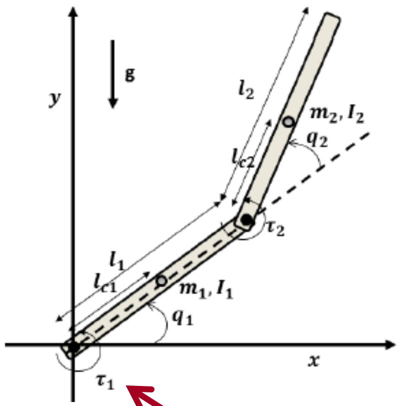
Choose $G_1(r_1, r_2) = 0$

$$\left. \frac{\partial f_1}{\partial x_2} \right|_{x_2=(r_2,0)} = \begin{bmatrix} 0 & 0 \\ \left. \left(H_{11}^{-1} \frac{\partial H_{11}}{\partial q_2} H_{11}^{-1} G_1 - H_{11}^{-1} \frac{\partial G_1}{\partial q_2} \right) \right|_{q_2=r_2} & 0 \end{bmatrix}$$

$\left\| \left. \frac{\partial f_1}{\partial x_2} \right|_{x_2=(r_2,0)} \right\| < \gamma_f$ Choose A_r such that $PA_r + A_r^T P + Q = 0$ $\quad \frac{1}{2\|P\|_F} > \gamma_f$

$$\|A\|_F = \sqrt{tr(AA^T)}$$

Example :Vertical Two-Link Robot Arm



Faulty link

$$\boldsymbol{H_{11}} = I_1 + I_2 + m_1 \cdot l_{c1}^2 + m_2 \cdot (l_1^2 + l_{c2}^2 + 2 \cdot l_1 \cdot l_{c2} \cdot cos(q_2))$$
$$\boldsymbol{H_{12}} = \boldsymbol{H_{21}} = I_2 + m_2 \cdot (l_{c2}^2 + l_1 \cdot l_{c2} \cdot cos(q_2))$$
$$\boldsymbol{H_{22}} = I_2 + m_2 \cdot l_{c2}^2$$
$$\boldsymbol{C_{11}} = -2 \cdot m_2 \cdot l_1 \cdot l_{c2} \cdot \sin(q_2) \cdot \dot{q}_2$$
$$\boldsymbol{C_{12}} = -m_2 \cdot l_1 \cdot l_{c2} \cdot \sin(q_2) \cdot \dot{q}_2$$
$$\boldsymbol{C_{21}} = m_2 \cdot l_1 \cdot l_{c2} \cdot \sin(q_2) \cdot \dot{q}_1$$
$$\boldsymbol{C_{22}} = 0$$
$$\boldsymbol{G_1} = (m_1 \cdot l_{c1} + m_2 \cdot l_1) \cdot g \cdot \cos(q_1) + m_2 \cdot l_{c2} \cdot g \cdot \cos(q_1 + q_2)$$
$$\boldsymbol{G_2} = m_2 \cdot l_{c2} \cdot g \cdot \cos(q_1 + q_2)$$
$$\boldsymbol{u} = [\tau_1 \quad \tau_2]^T$$

$$\epsilon \begin{bmatrix} \dot{r} \\ \ddot{r} \end{bmatrix} = \alpha \left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -H_{11}^{-1} G_1 \end{bmatrix}_{q_2 = r_2 + r} - \left(\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} r_1 \right)_{r_1 = 90^\circ} \right\} + A_r \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$$

$$r_2 = 0$$

$$\frac{dy}{d\tau} = \left(A_r + \begin{bmatrix} 0 & 0 \\ -m_2 l_{c2} g \sin q_1 & 0 \end{bmatrix} \right) y + \begin{bmatrix} 0 \\ -H_{11}^{-1}(m_1 l_{c1} + m_2 l_1 + m_2 l_{c2}) g \cos q_1 \end{bmatrix}$$

Vertical Two-Link Robot Arm Problem

$$\epsilon \begin{bmatrix} \dot{r} \\ \ddot{r} \end{bmatrix} = \alpha \left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -H_{11}^{-1} G_1 \end{bmatrix}_{q_2 = r} - \left(\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} r_1 \right)_{r_1 = 90°} \right\} + A_r \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$$

Constraints on A_r:

- $A_r = \begin{bmatrix} 0 & c \\ a & b \end{bmatrix}$
- $a, b < 0$ and $c > 0$ for $\mathrm{A_r}$ Hurwitz
- $PA_r + A_r^T P + Q = 0$ with $\underbrace{\dfrac{1}{2m_2 l_{c2} g}}_{\sim 0.1} > \|P\|_F$

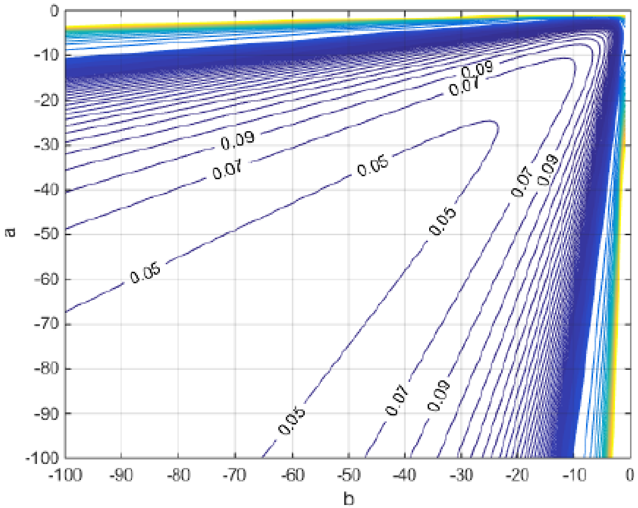


Figure 3.6: Contour plot of $\|P\|_F$ for different values of a and b, where P is the solution to the Lyapunov equation $PA_r + A_r^T P + Q = 0$ with Q as the identity matrix and A_r in the form of $A_r = \begin{bmatrix} 0 & c \\ a & b \end{bmatrix}$ with $c = 5$.

Vertical Two-Link Robot Arm Problem

Numerical Parameters

l_1	l_{c1}	m_1	I_1
1 m	0.5 m	1 kg	0.0833 kg m^2
l_2	l_{c2}	m_2	I_2
1 m	0.5 m	1 kg	0.0833 kg m^2

Reference Model for q_1
($\xi = 0.7,\ \omega_n = 10Hz$)

$$A_{rm} = \begin{bmatrix} 0 & 1 \\ -3947.84 & -87.97 \end{bmatrix}$$

$$B_{rm} = \begin{bmatrix} 0 \\ 3947.84 \end{bmatrix}$$

Controller gains for q_2
($\xi = 1.4,\ \omega_n = 5Hz$)

$$K_1 = 87.97$$

$$K_2 = 986.96$$

Perturbation Parameters

$$A_r = \begin{bmatrix} 0 & 5 \\ -30 & -30 \end{bmatrix}$$

$$\alpha = 1$$

$$\epsilon = 0.5$$

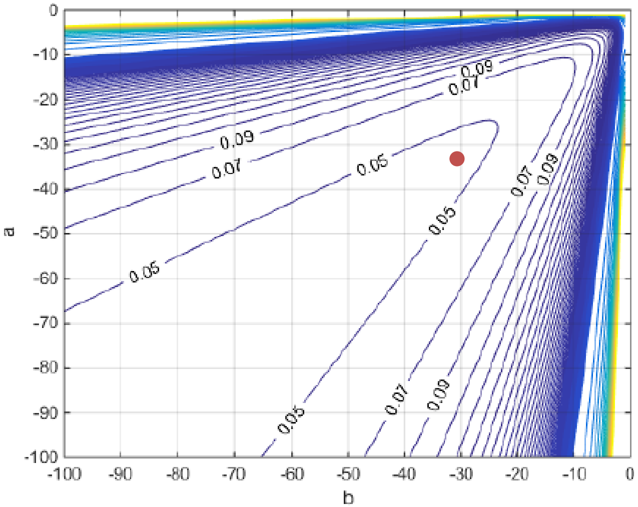


Figure 3.6: Contour plot of $\|P\|_F$ for different values of a and b, where P is the solution to the Lyapunov equation $PA_r + A_r^T P + Q = 0$ with Q as the identity matrix and A_r in the form of $A_r = \begin{bmatrix} 0 & c \\ a & b \end{bmatrix}$ with $c = 5$.

Vertical Two-Link Robot Arm Problem

$$\epsilon \begin{bmatrix} \dot{r} \\ \ddot{r} \end{bmatrix} = \alpha \left\{ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ -H_{11}^{-1} G_1 \end{bmatrix}_{q_2 = r} - \left(\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix} \begin{bmatrix} q_1 \\ \dot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} r_1 \right)_{r_1 = 90^\circ} + D \right\} + A_r \begin{bmatrix} r \\ \dot{r} \end{bmatrix}$$

Adaptive Term

$$D_i = \sum_{j=1}^{N} W_{ij} \psi_j(q_1, q_2)$$

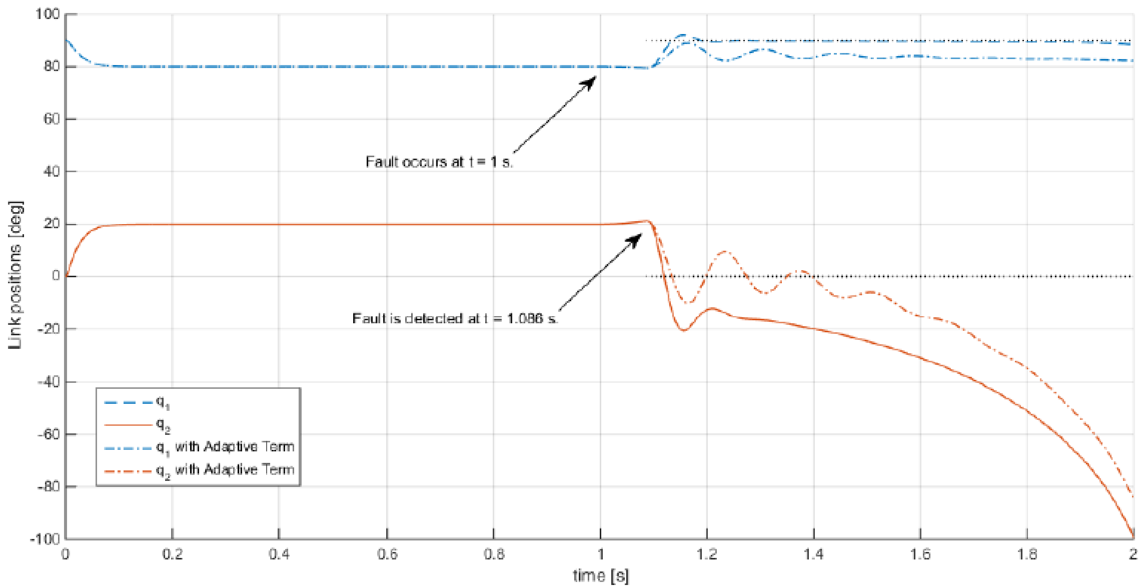
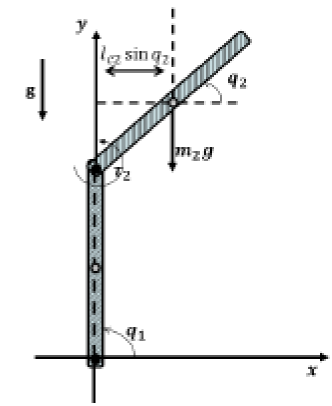
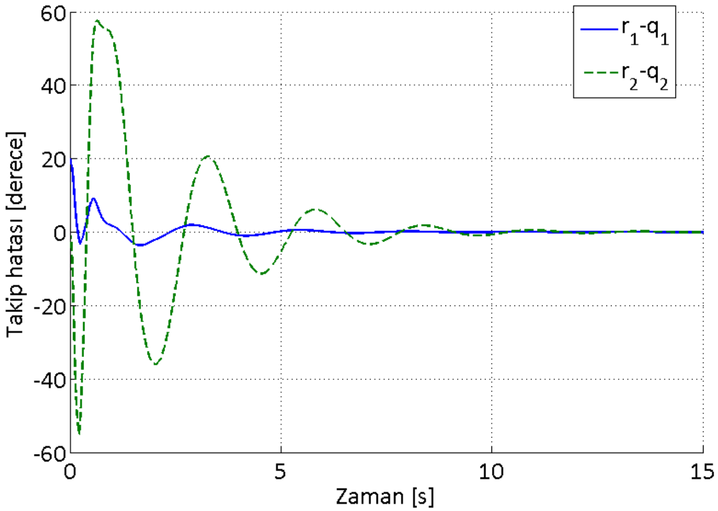


Figure 3.11: Link position for the two-link robot arm problem simulation



Vertical Two-Link Robot Arm Problem

It is possible to find a stabilizing law:



"Non-collocated partial linearized form:"

$$\ddot{q}_1 = v$$

$$\ddot{q}_2 = -\mathrm{H}_{12}^{+} \cdot (H_{11} \cdot v + \phi_1)$$

$$\tau = (H_{21} - H_{22} \cdot H_{12}^{+} \cdot H_{11}) \cdot v - H_{22} \cdot H_{12}^{+} \cdot \phi_1 + \phi_2$$

$$\ddot{q}_1 = -K_1\dot{q}_1 - K_2(q_1 - r - r_1)$$

$$\ddot{q}_2 = H_{12}^{+}H_{11}K_1\dot{q}_1 + H_{12}^{+}H_{11}K_2(q_1 - r_1) - K_3\dot{q}_2 - K_4 q_2$$

(This results is presented in TOK2017, İstanbul)

Example: Horizontal Three-Link Robot Arm

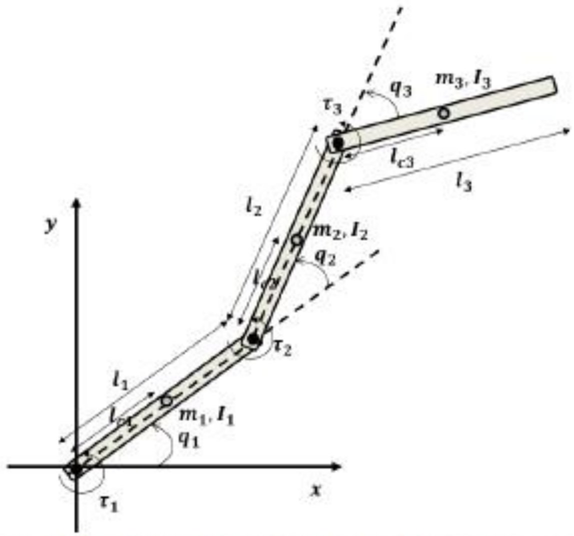
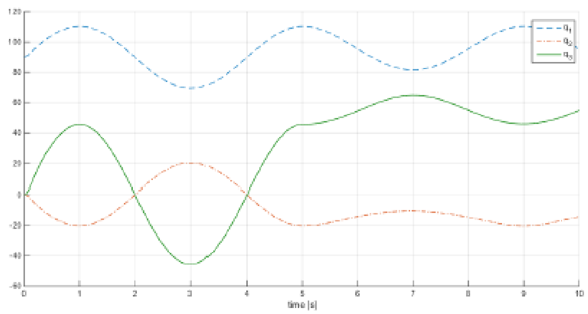


Figure 3.12: Vertical two-link robot manipulator system

l_1	l_{c1}	m_1	I_1
1 m	0.5 m	1 kg	0.0833 kg m^2
l_2	l_{c2}	m_2	I_2
1 m	0.5 m	1 kg	0.0833 kg m^2
l_3	l_{c3}	m_3	I_3
1 m	0.5 m	1 kg	0.0833 kg m^2



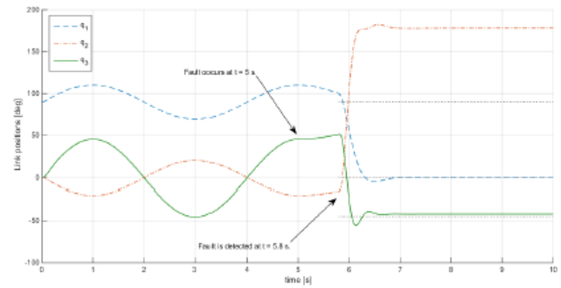


Figure 3.14: Link position for the three-link robot arm problem simulation

Horizontal Three-Link Robot Arm Problem

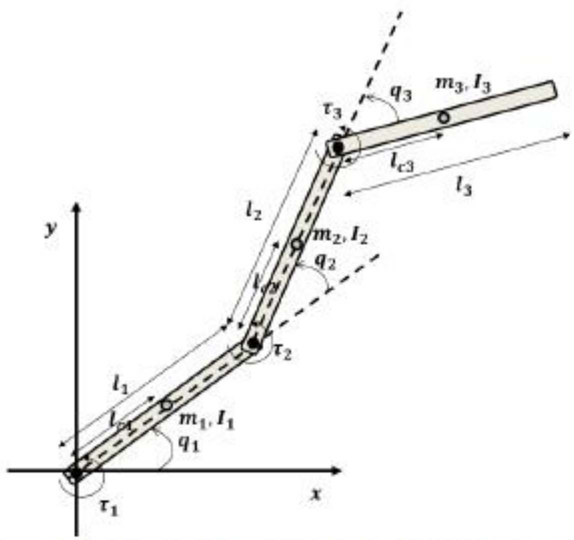
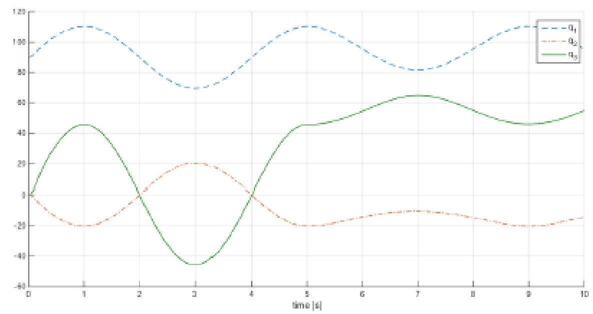


Figure 3.12: Vertical two-link robot manipulator system



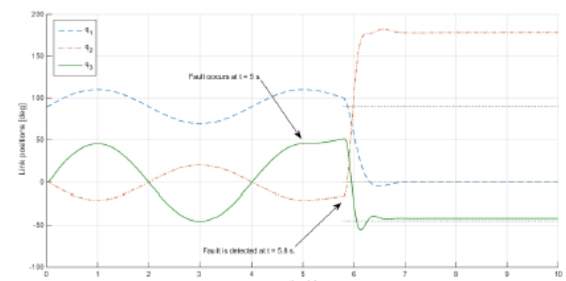
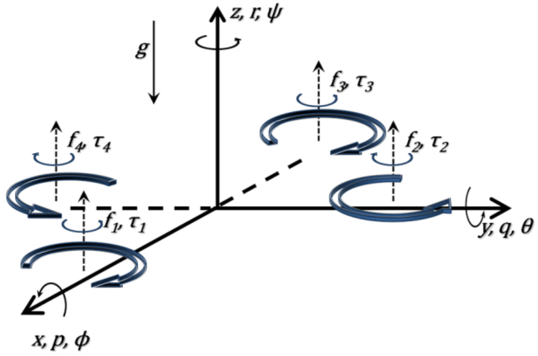


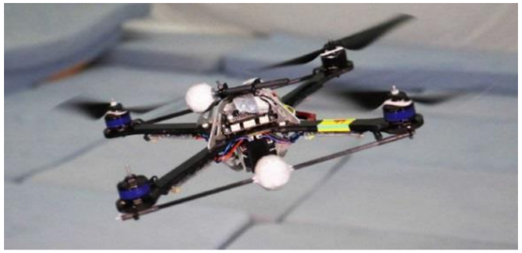
Figure 3.14: Link position for the three-link robot arm problem simulation

l_1	l_{c1}	m_1	I_1
1 m	0.5 m	1 kg	0.0833 kg m^2
l_2	l_{c2}	m_2	I_2
1 m	0.5 m	1 kg	0.0833 kg m^2
l_3	l_{c3}	m_3	I_3
1 m	0.5 m	1 kg	0.0833 kg m^2

Application to Quadrotor Attitude Control

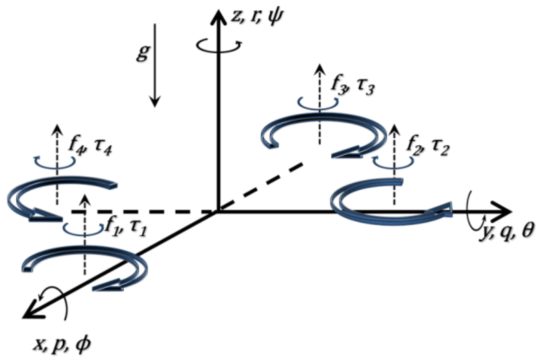
$$\dot{\omega} = (\omega \times I^{-1} \cdot \omega) + I^{-1} \cdot u$$
$$a^B = \sum f - \boldsymbol{R}^{-1} \cdot m \cdot g$$





Equations of Motion

$$I\dot{\omega} = -(\omega \times I \cdot \omega) + M$$
$$a^B = \sum f - \boldsymbol{R}^{-1} \cdot m \cdot g$$



$$f_i = \kappa\omega_i^2 \qquad \tau_i = \kappa_\tau f_i = \kappa\omega_i^2$$

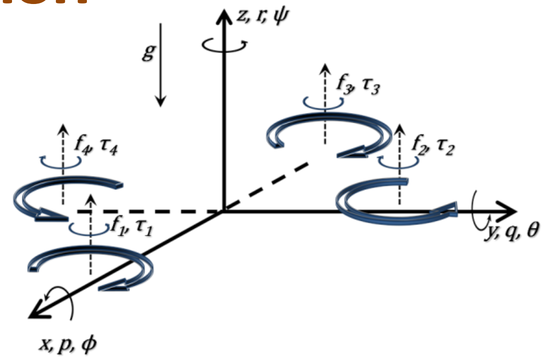
$$I_{xx}^T \cdot \dot{p} = -(I_{zz}^T - I_{xx}^T) \cdot q \cdot r + l \cdot (f_2 - f_4) - I_{zz}^P \cdot q \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4) - \kappa_{dxx} \cdot p \cdot \|[p \quad q \quad r]\|$$

$$I_{xx}^T \cdot \dot{q} = (I_{zz}^T - I_{xx}^T) \cdot p \cdot r + l \cdot (f_3 - f_1) + I_{zz}^P \cdot p \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4) - \kappa_{dyy} \cdot q \cdot \|[p \quad q \quad r]\|$$

$$I_{zz}^T \cdot \dot{r} = \kappa_\tau \cdot (f_1 - f_2 + f_3 - f_4) - \kappa_{dzz} \cdot r \cdot \|[p \quad q \quad r]\|$$

$$a^B = (f_1 + f_2 + f_3 + f_4) - \boldsymbol{R}^{-1} \cdot m \cdot g$$

Equations of Motion



Simplified form of Attitude Rate Equations:

$$I_{xx}^T \cdot \dot{p} = -(I_{zz}^T - I_{xx}^T) \cdot q \cdot r + l \cdot (f_2 - f_4)$$

$$I_{xx}^T \cdot \dot{q} = (I_{zz}^T - I_{xx}^T) \cdot p \cdot r + l \cdot (f_3 - f_1)$$

$$I_{zz}^T \cdot \dot{r} = -\gamma r + \kappa_\tau \cdot (f_1 - f_2 + f_3 - f_4)$$

$$\dot{p} = -a \cdot q \cdot r + c \cdot (f_2 - f_4)$$

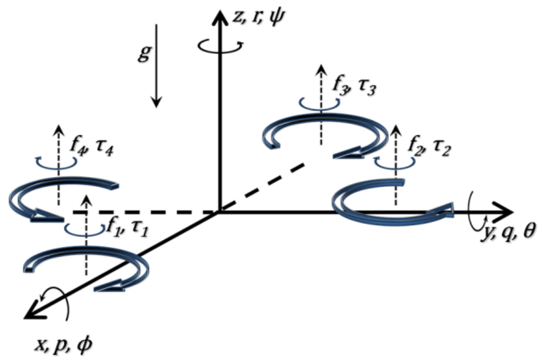
$$\dot{q} = a \cdot p \cdot r + c \cdot (f_3 - f_1)$$

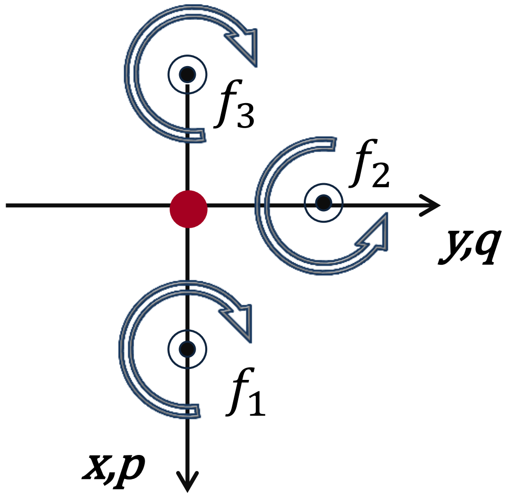
$$\dot{r} = -\gamma r + b \cdot (f_1 - f_2 + f_3 - f_4)$$

$$a, b, c, \gamma > 0$$

$$a = \frac{I_{zz}^T - I_{xx}^T}{I_{xx}^T} \qquad b = \frac{\kappa_\tau}{I_{zz}^T} \qquad c = \frac{l}{I_{xx}^T}$$

Flight With Missing Propellers





$$\dot{p} = -a \cdot q \cdot r + c \cdot (f_2 - f_4)$$

$$\dot{q} = a \cdot p \cdot r + c \cdot (f_3 - f_1)$$

$$\dot{r} = -\gamma r + b \cdot (f_1 - f_2 + f_3 - f_4)$$

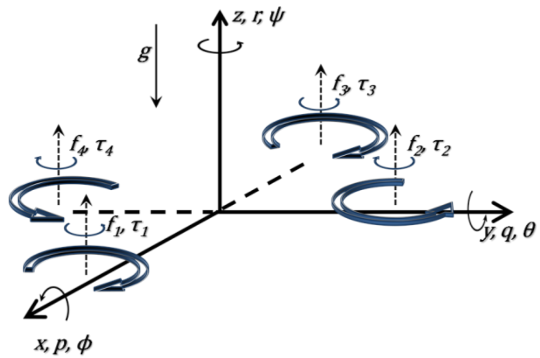
Unbalanced Torque

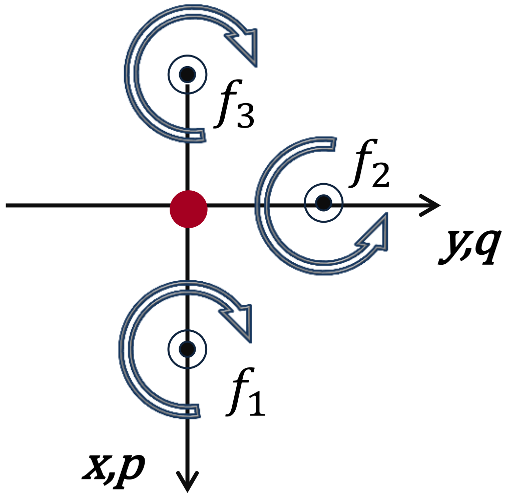
$$\dot{p} = -a \cdot q \cdot r + c \cdot f_2$$

$$\dot{q} = a \cdot p \cdot r + c \cdot (f_3 - f_1)$$

$$\dot{r} = -\gamma r + b \cdot \underbrace{(f_1 - f_2 + f_3)}_{\textit{Unbalanced Torque}}$$

Flight With Missing Propellers





$$\dot{p} = -a \cdot q \cdot r + d$$

$$\dot{q} = a \cdot p \cdot r + c \cdot (f_3 - f_1)$$

$$\dot{r} = -\gamma r + b \cdot \left(f_1 + f_3 - \frac{d}{c}\right)$$

Two Alternatives:

- $f_2 = 0$ *so that* $d = 0$ *(Two-propeller case)*
- $f_2, q, r, p \neq 0$ *so that* d *is canceled out.*

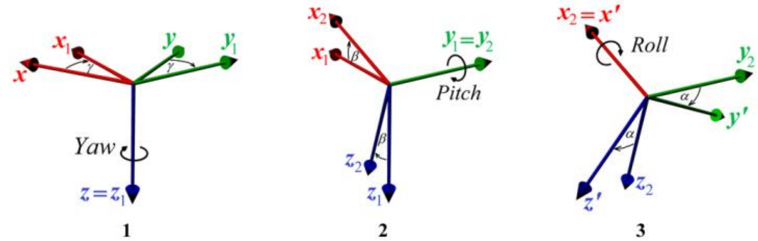
**Third alternative:* $q = q' \sin \omega t$*,* $r = r' \sin \omega t$ *and* $aq'r' = d$

Attitude Stabilization

$$u_1 = c \cdot (f_2 - f_4) \qquad \dot{p} = -a \cdot q \cdot r + u_1$$

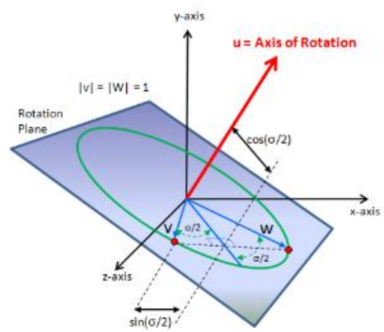
$$u_2 = c \cdot (f_3 - f_1) \qquad \dot{q} = a \cdot p \cdot r + u_2$$

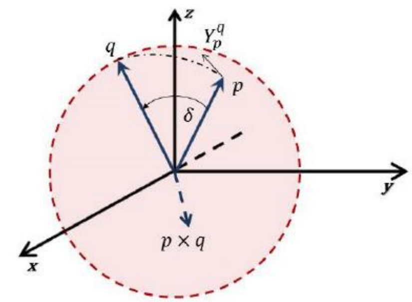
$$u_3 = b \cdot (f_1 - f_2 + f_3 - f_4) \qquad \dot{r} = -\gamma r + b \cdot u_3$$



(p, q, r)

$$\boxed{u = -k_R e_R - k_\omega e_\omega}$$





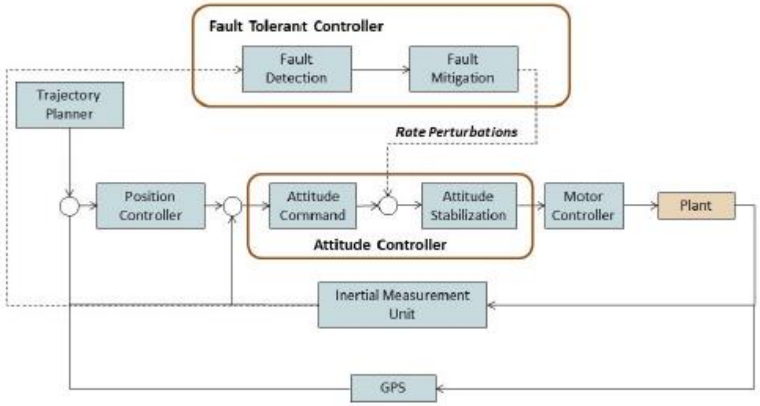
$$e_R = [\Delta\phi \quad \Delta\theta \quad \Delta\psi]$$

$$e_R \times = \frac{1}{2}\left(R_d^T R_B^E - {R_B^E}^T R_d\right)$$

$$e_R = \Gamma \times \Gamma_d$$

$$Y_p^q = vers(p \times q) \times p \qquad \delta = \arccos(\langle p, q \rangle) \qquad \frac{d\delta}{dt} = -\langle \dot{p}, Y_p^q \rangle \qquad e_R = -\delta \begin{bmatrix} -\langle Y_p^q, p_2 \rangle \\ \langle Y_p^q, p_1 \rangle \end{bmatrix}$$

Proposed Method



$$\dot{p} = -a \cdot q \cdot r + d$$

$$\dot{q} = a \cdot p \cdot r + c \cdot (f_3 - f_1)$$

$$\dot{r} = -\gamma r + b \cdot \left(f_1 + f_3 - \frac{d}{c} \right)$$

Perturbation Equation

$$\dot{x}_1 = \begin{bmatrix} \dot{\delta} \\ \dot{\omega}_1 \end{bmatrix} = \begin{bmatrix} 0 & p_2' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \omega_1 \end{bmatrix} + \begin{bmatrix} -p_1' \\ -a\omega_3 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ d \end{bmatrix}$$

$$\dot{x}_2 = \dot{\omega}_2 = a\omega_3 \cdot \omega_1 + u$$

$$\begin{aligned} \epsilon \dot{r} &= \alpha \cdot [f_1(x_1, B_r(r_2 + r)) - f_{r1}(x_1, B_r r_2)] + A_r r \\ q' &= B_r r \end{aligned}$$

$$\begin{aligned} \epsilon \dot{r} &= \alpha \cdot \left\{ \begin{bmatrix} 0 & p_2' \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \\ \omega_1 \end{bmatrix} + \begin{bmatrix} -p_1' \\ -a\omega_3 \end{bmatrix} B_r r + \begin{bmatrix} 0 \\ d \end{bmatrix} - A_{rm} \begin{bmatrix} \delta \\ \omega_1 \end{bmatrix} \right\} + A_r r \\ q' &= B_r r \end{aligned}$$

$$B_r = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Boundary Layer Equation

$$\epsilon\dot{r} = \alpha\cdot\left\{\begin{bmatrix}0 & p_2'\\0 & 0\end{bmatrix}\begin{bmatrix}\delta\\\omega_1\end{bmatrix}+\begin{bmatrix}-p_1'\\-a\omega_3\end{bmatrix}B_r r+\begin{bmatrix}0\\d\end{bmatrix}-A_{rm}\begin{bmatrix}\delta\\\omega_1\end{bmatrix}\right\}+A_r r$$
$$q' = B_r r$$

$$f_{11}(x_1)=\begin{bmatrix}0 & p_2'\\0 & 0\end{bmatrix}x_1 \qquad f_{12}=\begin{bmatrix}-p_1'\\-a\omega_3\end{bmatrix}B_r=\begin{bmatrix}1 & 0\end{bmatrix}=\begin{bmatrix}-p_1' & 0\\-a\omega_3 & 0\end{bmatrix}$$

$f_{11}(x_1)$ defines a neutrally stable system and therefore $\|f_{11}(x_1)\|\leq\delta$ condition is automatically satisfied for such kind of system.

The other condition is related to the bounds of $\|f_{12}\|$. Since f_{12} is rank deficit, Frobenius norm is more appropriate for calculation:

$$\|f_{12}\|_P=\sqrt{tr\left(f_{12}f_{12}^T\right)}=\sqrt{p_1'^2+(a\omega_3)^2}$$

Since p_1' is an element of a unit vector, it is bounded by 1. Also $a\approx 1$ numerically. Therefore following inequality holds:

$$\|f_{12}\|_P\leq\sqrt{1+\omega_3^2}=\gamma_f$$

A_r can be chosen for the expected maximum yaw rate ω_3 such that A_r that solves the Lyapunov equation $PA_r+A_r^TP+Q=0$ with Q as the identity matrix with appropriate dimensions and $\frac{1}{2\lambda_{max}(P)}>\gamma_f$.

Maximum eigenvalue of A_r for different maximum yaw rates are calculated and the results are shown in Figure4.5.

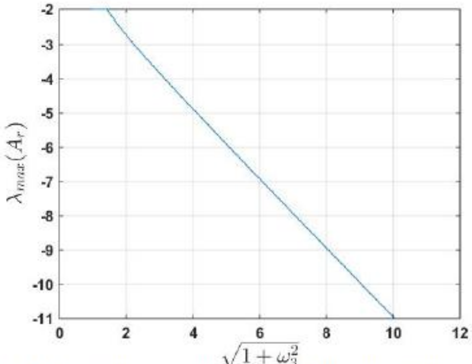
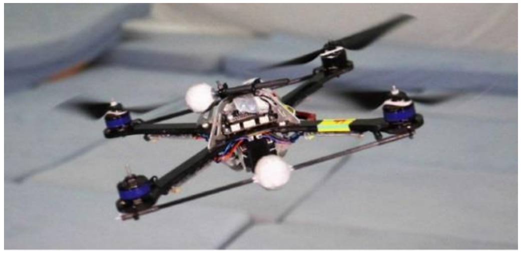


Figure 4.5: Maximum eigenvalue of A_r for different maximum yaw rates.

Numerical Results



Property	Value
Moment of Inertia Tensor:	$I^T_{xx} = I^T_{yy} = 2.7 \times 10^{-3}\,\mathrm{kgm}^2$ $I^T_{zz} = 5.2 \times 10^{-3}\,\mathrm{kgm}^2$
Moment of Inertia of a Propeller:	$I^P_{zz} = 1.5 \times 10^{-5}\,\mathrm{kgm}^2$
Mass:	$m = 0.5\,\mathrm{kg}$
Rotor distance	$l = 0.17\,\mathrm{m}$
Propeller force coefficient	$\kappa_f = 6.41 \times 10^{-6}\,\mathrm{Ns^2rad^{-2}}$
Propeller torque coefficient	$\kappa_\tau = 1.72 \times 10^{-2}\,\mathrm{Nms^2rad^{-2}}$
Motor time constant	$\sigma_M = 15\,\mathrm{ms}$
Rotational drag coefficients	$\kappa_{dxx} = \kappa_{dyy} = 0.7 \times 10^{-5}\,\mathrm{Nms^2rad^2}$ $\kappa_{dzz} = 1.4 \times 10^{-4}\,\mathrm{Nms^2rad^2}$

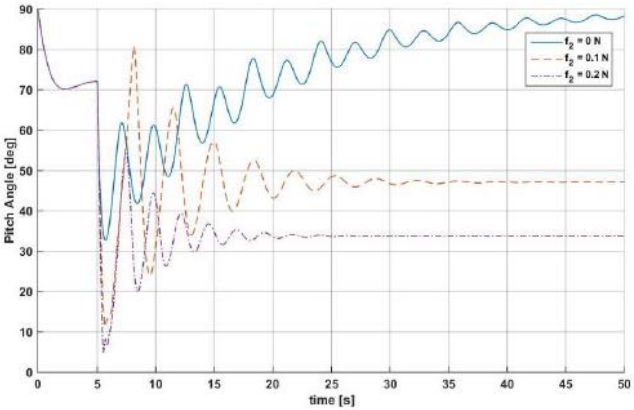


Figure 4.10: Angle between the positive z axis of the propeller and the local level plane

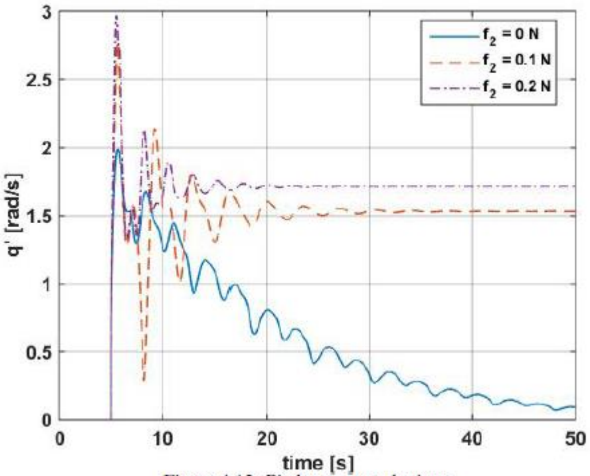


Figure 4.12: Pitch rate perturbations

Conclusion

- The main output of the thesis work is a nonlinear control architecture that can be used as an algorithmic Fault Tolerant Control System

- Theoretical analyses are conducted and applications on complex problems are shown

- 2 Conference papers are presented and 1 journal article is submitted

- Proposed methodology can also be applied to other interesting problems.