Master Thesis
MSc 203 - Financial Markets
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Pricing and Optimization of Autocallables Structured Products

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Abstract

This paper was written in the context of an internship at Credit Suisse within the Cross Asset Structured Products Sales Team.

The purpose of this Master's Thesis is to review the existing literature on structured product and implement our findings with Computational Methods in order to be able to price, analyse, optimize and understand the development of new underlying strategies.

We will first provide an overview of the structured product's business and review the different types of product sold on the market. We will mainly be focussing on autocallables products during this Master 1 Thesis by first implementing a Path Dependent Monte Carlo Pricer on a Phoenix Note. We will also study the various expositions through the Greeks and understand how we can optimize autocallables to maximize our return.

Then we will work on a method to efficiently find the best basket combinations in the market, i.e. the one that allows for the highest coupon, using the Greeks of our product and review the rise of Fix Dividends Underlyings as a way for banks to overcome the current trading book saturation on the main indices. We will also propose a performance study of Decrement Index (Point and Percentage) vs. their normal version by backtesting these strategies in order to see if this extra complexity and risk for the investor has been paying out so far.

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Introduction

Structured Products and especially Autocallables are complex financial products that have allowed investors to get access to innovative and advanced payoffs even with the current low rate levels. These products offer investments solutions with “tailor-made risk-return profile” in a very cost-effective way and have led to the development of numerous structures, strategies, as well as new proprietary index.

The pandemic, the Ukrainian/Russian conflict and the high levels of implied volatility helped to make structured products immensely popular even with the current uncertain market. With the recent low entry points in the market and higher volatility and interest rates levels, the European structured product market has grown of 20% between February and April 2022[1]. This proves that structured products benefit from short-term market opportunities by exploiting them in a targeted manner. Unlike equities, structured products can be set up, so they not only benefit from market upturns but also take advantage of sideways or down- ward-moving prices. Product providers are constantly looking to develop products that will appeal to investors using the prevailing market conditions to provide a mix of return and protection. Investors can indeed appreciate the high level of flexibility and personalization of structured products, often found in regional trends. In fact, certificate providers have a wide range of possibilities in combining one or more of different options, and this allows them to prepare increasingly innovative products, capable of offering investment solutions complementary to the classic directional ones (which can be implemented through shares, bonds, and ETFs) and able to optimize the overall risk profile of the investor’s portfolio.

However finding the optimal product based on the client’s need can be a very long and tricky process. Many ”pricing” requests are in fact being done with arbitrary barrier values and underlyings that either the Sales or the client will set, mostly based on habit or gut feeling. This process can lead to many back and forth between the Sales / Structuring team and the client and can result in a sub-optimal product being offered to the final client. For instance it is common for a broker to simply make a request for quotation (RFQ) to different

[1]Structured Products Intelligence

banks and ask each of them to provide with the highest level of coupons within a pre-defined stock universe. This universe can be sometimes quite big especially once factoring Basket Combinations, and with each pricing runs taking up to a minute or two it can take a very long amount of time. We will also try to understand how the current trend over structured product has created an unbalanced market on some of the main indices and is resulting in the developement of new underlying. We will try to understand their pros and cons for the issuer and the the investor and try to review how this added complexity has resulted so far.

Chapter 1

Introduction to the Structured Products Business

1.1 The Bond Component

Structured products are packaged investments solutions that usually includes assets linked to interest rates plus one or more derivatives components. These products are created with a "tailor-made risk-return" profile for the investor and are usually sold over-the-counter between distributors or professional clients and Investment Banks. Structured products can be launched in different wrappers that determines their legal status. The wraper choice is used to meet the investor requirements regarding regulation and investment preferences. Notes and bonds are the most common wrappers used by financial institutions as they can be directly issued as senior unsecured debt. Most of structured products are created with a "European Medium Term Note" (EMTN) format where they replace usual investment-grade bonds payment features with non-traditional payoffs.

This payoff is linked to the chosen derivative strategy, and therefore to the performance of a single or multiple underlyings. The most known products are generally "Equity Linked Notes" (ELN) which values will depend on the performance of an Equity underlying such as stocks or indexes. However the possibilities are endless, and it is common to find credit, rates as well as hybrid products on the market. The idea is simple: issuers create a zero-coupon ("ZC") bond with a maturity matching the ELN maturity, and as no coupons are received during the product's life it allows the investor to buy this security at its discounted future value.

$$\boxed{Price\; of\; a\; Zero\; Coupon\; Bond\; = \frac{Face\; Value\; of\; the\; bond}{(1+r)^n}}$$

Using continuous compounding to discount cash flows, the price of a ZCB can be expressed as:

$$\boxed{B_{(t,T)} = e^{-r_{(t,T)}(T-t)}}$$

The difference between this present and future value is then used to fund the derivative component of ELN. Rates are therefore playing an important role as they directly impact the amount left to fund the derivative component. If this difference is big enough they allow for 100% capital protected products, where the derivative strategy cost matches the difference between present and future value of the bond.

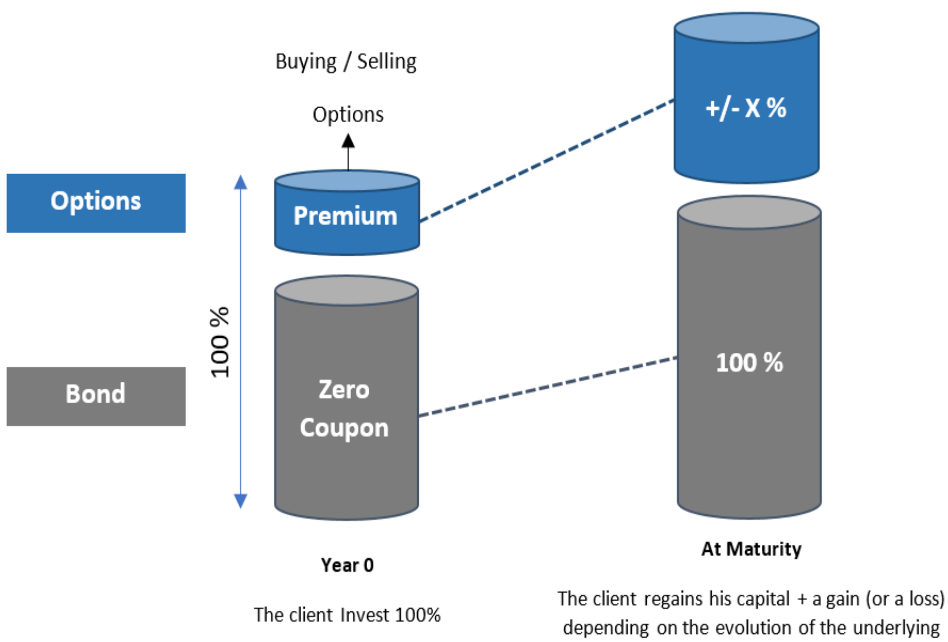

Figure 1.1: Components of a Capital Protected Structured Product

However, since 2008, rates have been constantly decreasing and even became negative. As a result, maturities of ELN have increased, due to the upward nature of the yield curve. As a proxy for the risk free rate Financial actors uses the interbank offered rate, matching the product's currency. In Europe, we generally use the Euribor 3 or 6 months (EUR3M/EUR6M). Euribor is short for Euro Interbank Offered Rate. Euribor rates are based on the average interest rates at which a large panel of European banks borrow funds from one another. For maturities exceeding one year we can use Constant Maturity Swap levels.

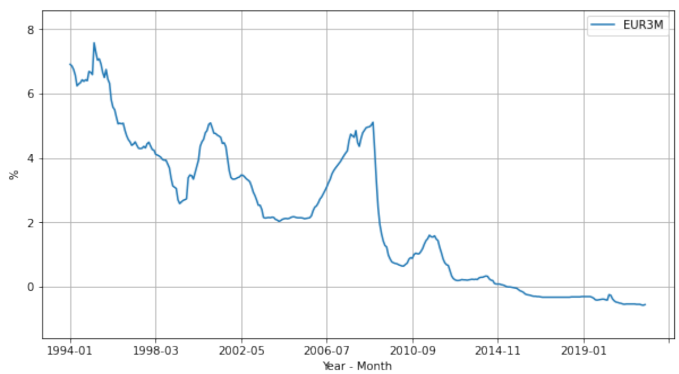

Figure 1.2: Evolution of the EUR3M, Source: Banque de France[1]

Once the deal is settled, the client has to provide the treasury department of the bank with 100% of the invested notional. In exchange for this cash the Asset Liability Department ("ALM") will remunerate the Equity Department by providing them with a rate and a spread. This treasury can then be put to use by the banks within its various activities. This spread, called the funding, is very important, since it will directly impact the prices provided to the client, which partly explains the difference in prices between different banks. This level of remuneration will vary according to maturity and the perception of the bank's risk by the market and by rating agencies (mainly SP, Moody's, Fitch). The higher the credit rating, the safer the bond's issuance, the lower the interest rate required by the investor to bear the credit risk, the higher the bond's price and finally the lower the amount of money left to invest in the derivative strategy. As a result, banks with higer default risk should, on paper, create products with more interesting features as the default becomes higher.

Each Bank uses a different funding grill that is being regularly updated. When pricing such products the issuer will use the matching funding level, but as we will detail in the next parts, some structured products do not have fixed maturities and can be redeemed prior to maturity. We then have to compute the fixed funding spread as the product of the probability of being redeemed at a specific date and the matching funding.

1.2 The Derivative Component

Even with the use of this funding spread, the current rate levels makes it difficult to structure attractive 100% capital guarantee products (KG products). In fact the short leg, i.e. the financing leg, needs to equal the long leg, used to benefit from our strategy. Therefore as the difference between the current and future price of the ZC decreases we have to participate in lesser attractive long strategies, which leads to less attractive products. In most cases, investors will need to put part of their capital at risk. To do so, we usually find different types of short put options embeded in the contract. Put options allow for tailor made loss participation and provide funding for the long leg strategy. This long leg is composed of various option packages that matches the desired/possible payoff.

An option is a contract between two parties that gives the buyer some form of rights, but not obligation, to buy or sell a particular asset in the future at a certain price. The most common options are said to be European, i.e. exercisable on the expiration date, American, i.e. exercisable at any time between the purchase and expiration dates and Asian, where the payoff is determined by the average underlying price over some pre-set period of time. When structuring ELN we tend to mostly use combinations of Calls, Puts and Barrier Options in order to match the desired payoff of the product. However, even if we can find an infinite number of possibilities within our pre-packaged investment solution, most of them are composed of call and puts with potentially added barrier levels, that activates/de-activates the options or gearing effects. In fact, if being Long a Call is a great way for the investor to be able to participate in the positive performance of the underlying, this strategy can be very expensive. The investor might then decide to put a cap to his earnings to limit the cost of this strategy. Depending on the cost of the strategy he might also need to gear his call in order to obtain a higher or lower participation in the performance of the underlying. Barrier levels on the other hand are also very usefull as they let us replicate payoffs with discontinuities.

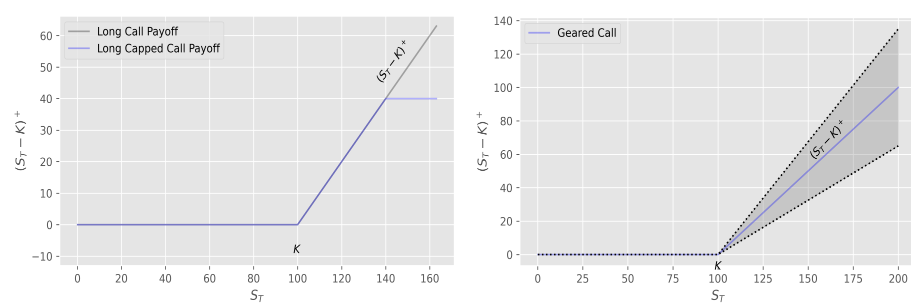

Figure 1.3: Long Call & Geared Call

Shorting (selling) options on the other hand allows the investor to finance his strategy. The Gearing and Capped features also comes in handy to manage the risk the investor is willing to take. Barrier Options and especially Knock In Puts are one of the most common types of puts sold by investors when investing in Structured Products. They allow for the capital to be protected until a specific threshold, the barrier level, and otherwise expose the investor to participation in the downside.

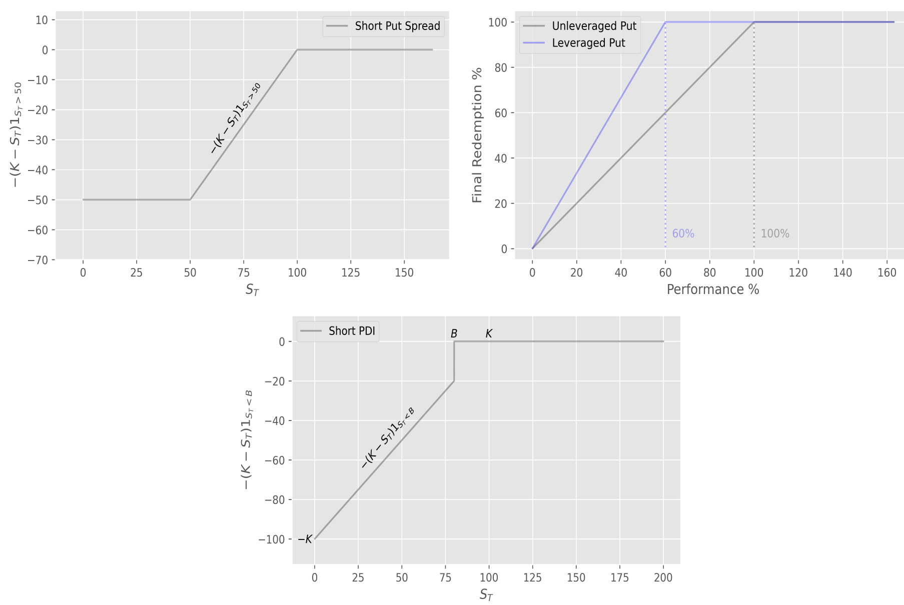

Figure 1.4: Leveraged Put & EKIP

Digital options are also very common in the structured products business. They are quite straigthforward. They are options that pay a fixed coupon if the underlying is below or above a predetermined level and does not give a payout at all in all other cases. They can be thought as call spreads with an extremely high gearing level. European-Knock-In-Puts, when combined with Binary options seems to form an excellent investment strategy as they allow for coupon payments and capital protection even in a flat or slightly decreasing market. However this type of option is not directly quoted. Both of these options are considered light exotic options as they cannot be perfectly replicated by a set of standard options. However it is possible to replicate them by using call spreads with a very high gearing level.

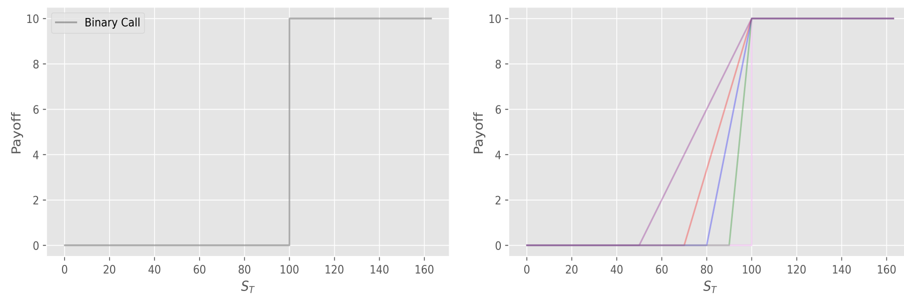

Figure 1.5: Binary Call and Call Spreads

One of the challenge of an issuer and a market maker such as an Investment Bank is to offer the most competitive price possible, in order not to lose the deal, while still being confident that it will be able to cover its hedging and issue costs and still make a profit. In fact, in exchange for the option price, i.e. the premium that the buyer is paying to benefit from the rights given by these products, the issuer has to support the risk of owing a cash flow to the owner (in case of a cash settlement). As shown above, this amount is defined by the payoff functions of the derivative strategy and can be viewed as the inner value of the option.

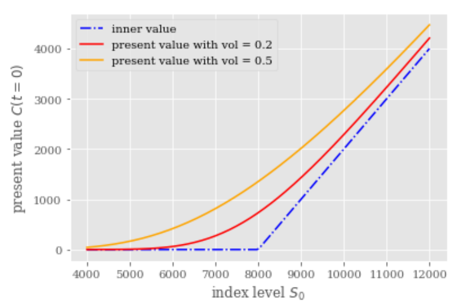

Figure 1.6: Inner and Present Value

However the inner value of the option should always be lower than it's option premium. It is the price at which a willing buyer and seller transact an option contract and is based on Supply & Demand. When it comes to valuing an option there are two main components to think of, the Intrinsic value and Time Value. Intrinsic value can be viewed as the amount of value in optionality right now (as if exercised today) while Time Value is the price of uncertainty. The option value and therefore the structured product value will vary during it's lifetime due to change in different parameters that can impact the derivative component. The Spot and Forward Price, the Strike, the Risk-Free interest rate, the

Time to expiry, Correlation of the underlying basket and even Dividends are all factors that impact the option price during the product's life-time, but one of the most important factor that impact the time value of optionality is volatility.

Since structured products are pre-packaged investment solutions it can sometimes be hard to directly understand how a shift in each parameters affect our products price. In fact we are both short and long options in most products. Thankfully, a change in option prices can be decomposed with a polynomial function where the coefficients for this polynomial are determined by the derivatives at a single point. We can approximate a change in options prices using a Taylor development of each main parameters.

$$\boxed{\Delta V = \frac{\delta V}{\delta S} * \Delta S + \frac{\delta V}{\delta \sigma} * \Delta \sigma + \frac{\delta V}{\delta t} * \Delta t + \frac{\delta V}{\delta r} * \Delta r + \frac{1}{2} * \frac{\delta^2 V}{\delta S^2} * (\Delta S)^2 + (Other)}$$

These exposures to each parameters are often reffered too as the "Greeks", they each indicate a change in X due to a move of Y. Below you will find a summary table of the main Greeks. We will use them in the second part to analyse our desired product's risk and sensitivities.

Greek	**Symbol**	**Measures**	**Change in option price due to ...**
Delta	$\Delta = \frac{\delta V}{\delta S}$	Equity Exposure	... a change in spot
Gamma	$\Gamma = \delta^2 V \overline{\delta S^2}$	Payout Exposure	... a change in δ
Theta	$\Theta = \frac{\delta V}{\delta t}$	Time Decay	... time passing
Vega	$\upsilon = \frac{\delta V}{\delta \sigma}$	Volatility Exposure	... a move in volatility
Rho	$\rho = \frac{\delta V}{\delta r}$	Interest Rate Exposure	... a move in interest rates
Dividend	$\epsilon = \frac{\delta V}{\delta q}$	Dividend Shift Exposure	... shifting dividend payments

There are many other N order exposures but we won't be focussing on these within this work.

1.3 The Main Categories of Structured Products

Equity Linked products can be divided in 4 types of products:

Capital Guaranteed Products offer partial or total nominal investment protection at maturity. These products usually offer upside participation in the underlying and/or conditional payments distributed periodically during the life of the product. One of the main shortfalls compared to a risk-free investment is that the performance is not guaranteed. An example of such product would be a shark note. For the below graphs, please note that B is the barrier level and K the strike level.

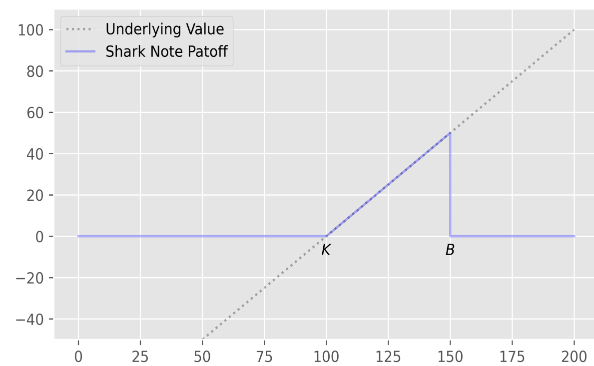

Figure 1.7: Shark Note Payoff with Knock Out Barrier at 150%

Performance Optimization products on the other hand are not capital protected at maturity. They generally offer a limited upside participation (i.e. capped performance) with a conditional protection of the capital or at least a limited potential downside participation in the underlying. Performance optimization products match perfectly moderate risk appetite investor's demand. A barrier bonus note is an example of such structure. This structure type is ideal for Neutral Market views.

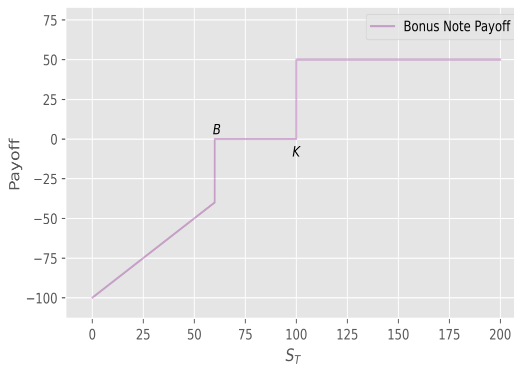

Figure 1.8: Barrier Bonus Note Payoff

Participation products offers underlying participation and lower capital protection. These products are tailored for investors with moderate to high-risk appetite and that are willing to participate in the movements of the underlying (both upside and downside movements). A Twin Win is an example of such structure.

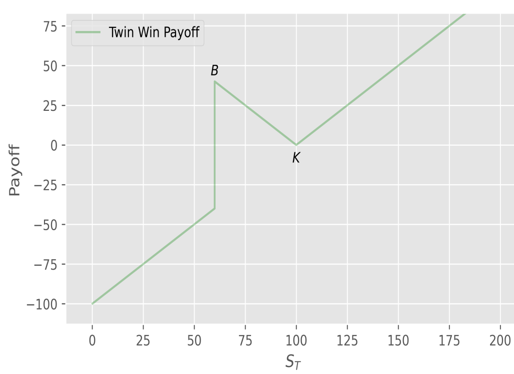

Figure 1.9: Twin Win Payoff

Finally, leveraged products allows for the investor to be exposed to more than 100% of his capital. They offer the investor the possibility to generate high returns with a low initial investment. These are suited for investors with a high-risk appetite since they could potentially lead to a total loss of their initial investment. They are often used on short term periods and when investor expect an upward movement in the underlying. This product are ideal for srong Bullish or Bearish views on the market as they amplifies the moves of the underlying.

As we can see the number of potential products is unlimited when it comes to structured products. It is therefore important for the investor to first decide what are the types of products better matching his risk appetite and to formulate his views on the market at different maturities. It is a tradeoff between risk and returns where capital guaranteed products are safer but features less return while leveraged products comes with more risk but potentially higher returns. All in all, structured products are particularly appealing thanks to their flexibility.

Structured Products further split into two distincts categories: Bullet and Callables Products. Bullet products, when not sold prior to maturity on the secondary market, only have one observation date at maturity. Previously plotted graphs are example of bullet products. A Callable on the other hand, in addition to the previously discussed features can be also "re-called" prior to its maturity due to a callable condition, eiter chosen by the issuer if te product is Issuer Callable or due to the own path of the underlying if the product is Autocallable. For the second one, and as the name suggests, the product automatically ends when an autocall condition is breached. This condition is a barrier condition verified on discrete "valuation dates". This usually happens when the underlying ends above the "autocall barrier" based on the "initial level" of the underlying but can also feature different levels expressed in % of the Initial Level. This feature also deactivate the short leg, which is in most cases a European Knock In Put. This discontinuity in the payoff makes it harder to price. They require different pricing techniques that we will describe in the next chapter of this Thesis. In fact the main difficulty with autocalls is their path dependent nature which directly affects the outcomes of the product.

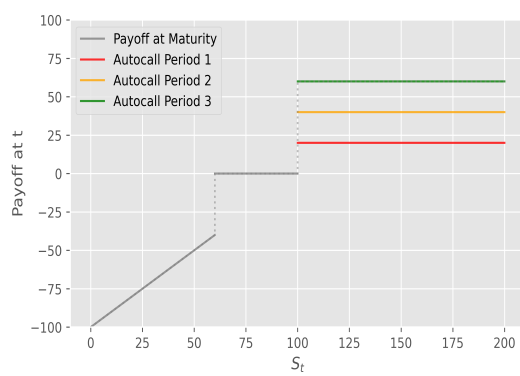

Figure 1.10: Autocall Payoff (Athena)

Please note that when pricing an Autocalls in addition to the choice of maturities (usually between 3 months and 12 years), the investor also has to choose his desired observation frequencies (daily, quarterly, semi-annually or annually) and period of "First Early Redemtion" and "Observation Tenor". These parameters can be tough to define as rising the first autocall observation start date and observation tenor will tend to lower autocall probability and or coupon observation dates but as a result generally leads to higher coupons. We can also note that it is in both the client and the issuer interest that the product is redeemed earlier. In fact as the product gets redeemed earlier there is a high chance that the client will decide to reinvest in a new product and therefore banks and intermediaries have the possibility to earn an Up front fee again. Please note that we usually have the following:

$$\boxed{AutocallBarrier \geq CouponBarrier \geq European - Knock - In - Barrier}$$

1.4 Assessing a Structured Products Risk

As explained in the previous part, structured products allow the client to invest with a pre-determined risk/return profile that both suits his view on the market and his current risk appetite. They are a great tool for portfolio diversification and risk management as they allow to shape the distribution of the portfolio returns. For instance capital protected product allow for no negative returns but with much lower average daily returns compared to it's underlying. Performance Optimization products on the other hand allows for an average return above the one from the underlying but with a very high tail skew in its distribution towards negative returns. And finally leveraged products enhance the overall shape of the distribution with higher average returns but with higher extreme positive and negative returns.

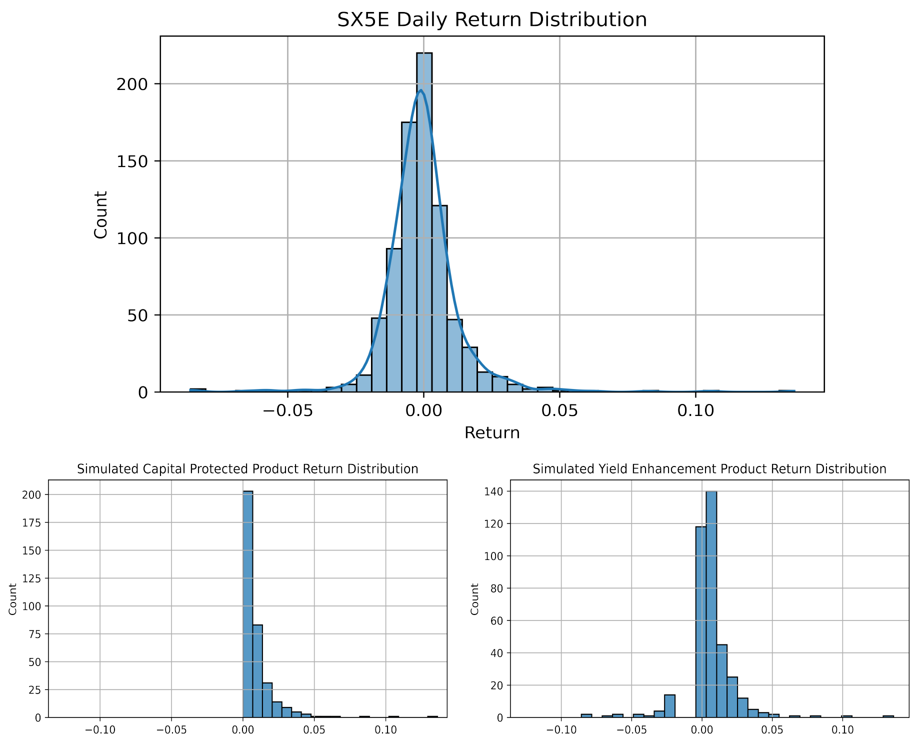

Figure 1.11: 3Y Return Distribution Plot

If the pros are numerous when investing in these structures we need to keep in mind that the risk linked to structured products can be hard to directly assess. The main risks are the following:

- **Market Risk**: The return from the investment can be zero or even negative due to adverse market conditions. Investment advice on future market trends is needed to ensure the payoff is understood and reflects that view.

- **Counterparty risk**: The risk that the issuer does not repay the principal or return. An assessment of the issuer's credit rating and of any other relevant information (credit default spreads, balance sheet strength...) are needed.

- **Liquidity risk**: There is only one market maker for the investment who does not provide an after-sales market or quotes a wide bid-offer spread. It is common to see spreads of around 1% with the "fair" value of the product.

- **Credit risk**: If the asset goes bankrupt, for example, the bond issuer within a portfolio does not repay the principal. Investors should assess the creditworthiness of the assets the solution is linked to (if disclosed) and their credit ratings.

Thankfully for the investor, when a product is in the process of being sold to a French client, the bank must communicate to the customer legal documents explaining the specific details of the product such as the evaluations and payment dates, the rating of the issuer and so on. All these characteristics can be viewed in the product's TermSheet. This document can be considered as a contract between the bank and the customer. In addition, the PRIIPS regulation requires the bank to provide the customer with a specific document about the risks incurred by the product structure. This document is the Key Information Document, more generally called "KID". It should be noted that the calculation of the risk indicators in the KID is based on the historical value. A minimum of 5 years of data is required for the results of the KID to be considered legal. To do so, the issuer needs to compute the measure of credit risk (CRM), based on the level of credit risk communicated by external credit rating agencies. Then, regarding Market Risk, we compute the Volatility Equivalent Value at Risk (VaR) which is also called the VEV and which is provided by the following formula:

$$\boxed{VEV = \frac{\sqrt{3.842 - 2 * ln(VaR_{97.5})} - 1.96}{\sqrt{T}}}$$

The following table is then used to determine the MRM:

MRM class	1	2	3	4	5	6	7
VEV	less than 0.5%	0.5% - 5%	5% - 12%	12% - 20%	20% - 30%	30% - 80%	more than 80%

Value at Risk (VaR) is used to quantify the financial risk of a portfolio by giving an estimate of the maximum loss over a specific period of time with various confidence levels. For instance, a portfolio with a VaR of 5 000 EUR over the next 5 days at 99% confidence means that we can say with a 99% confidence level that our portfolio loss will not exceed 5 000 EUR in 5 days. We can compute the VaR by either using a Monte Carlo Simulation, a method that will be developped in the next chapter, or by using the variance-covariance method. This second method uses historical price movements (standard deviation, mean price) over a specified lookback period, and then uses probability theory to calculate the maximum loss within a specific confidence interval. VaR however assumes that the returns of the underlying are standard normally distributed, which can be quite unrealistic for most assets.

The synthetic risk indicator is obtained by combining the two previously mentionned risk indicators and use the following table to define the risk level of the product.

ISR	MRM 1	MRM 2	MRM 3	MRM 4	MRM 5	MRM 6	MRM 7
MRC 1	1	2	3	4	5	6	7
MRC 2	1	2	3	4	5	6	7
MRC 3	3	3	3	4	5	6	7
MRC 4	5	5	5	5	5	6	7
MRC 5	5	5	5	5	5	6	7
MRC 6	6	6	6	6	6	6	7

Chapter 2

Structured Products Pricing and Optimization

In France, the vast majority of structured products sold are "performance enhancement" based. They usually expose the investor to higher risk than capital protected products as the capital loss can be total while the positive performance of the product is limited. This higher risk for the investor however results in higher possible expected returns. In this part I specify the product that I will study during this Thesis, discuss it's main characteristics and sensibilities and also propose a pricing algorithm for this product through the use of Monte Carlo methods. I will also present a way to quickly select the highest paying basket combinations within a universe of underlyings using greeks. Finally I will focuss on Decrement Indices and fix-dividends underlyings and try to review and understand if this added complexity for the investor is in the interest of both the issuer and the investor.

2.1 Presentation of a classic structure: The Phoenix Autocall

In this part we first focus on a famous payoff structure with discrete observation dates and a European capital protection barrier, the Phoenix Autocall. This product is characterised by three barrier conditions: a coupon barrier, the capital barrier level which dictates the capital protection threshold and finally the Autocall Level. These barriers are defined in percentage of the initial level of the underlying. Using the below table set up, each of these barriers have the following consequences:

Product	Underlying	Maturity / Frequency	Coupon Level	EKIP	Autocall Level	Coupon
Phoenix	Single Index	5Y / 6M	80%	60%	100% from Y1	5% p.s.

- If the underlying's price is above the Coupon Barrier at an observation date, the investor receives a pre-determined coupon.
- If the underlying's price is above the Autocall Barrier at an observation date, the product is autocalled and the investor receives it's initial investment.
- At maturity, if the product has not been already autocalled and if the underlying's price is below the European Knock In Put Barrier Level then the investor supports the negative performance of the underlying. Otherwise he receives back his initial investment.

The Phoenix Autocall offer a great investment opportunity for investors who anticipate a stagnating market as they will receives coupon payments even if the underlying stays flat or slightly declines. However coupon payments are capped and therefore investors with a strong Bullish view on the underlying might prefer different alternatives. Its potential outcomes are different both at and before maturity.

Before maturity, and at each observation date we have three possibilities:

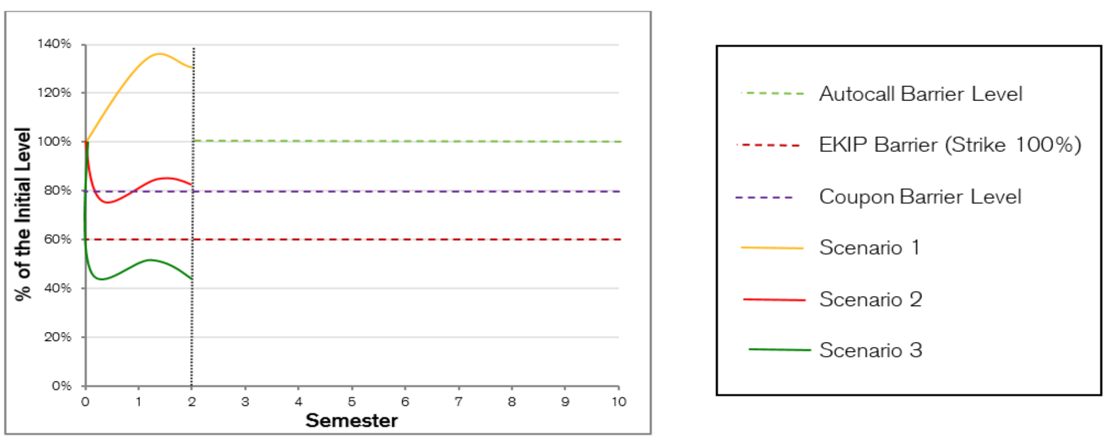

Figure 2.1: Possible Scenarios at Semester 2

- **Scenario 1:** At the first observation (Sesmester 1 - S1), the underlying's price is above the Coupon Barrier, investor receives a Coupon. At the second observation date, the underlying's price is above the Autocall and Coupon Barrier, the investor recovers 100% + Coupon + Product is redeemed before maturity.
- **Scenario 2:** The underlying's price is below the Coupon Barrier at S1 but at S2 the underlying's price it passes over it. However, the underlying

is under the Autocall Barrier, the investor receives Coupon and product continues.

- **Scenario 3:** The underlying's price is below the Coupon Barrier during S1 and S2, the product continues.

And at maturity we have the following:

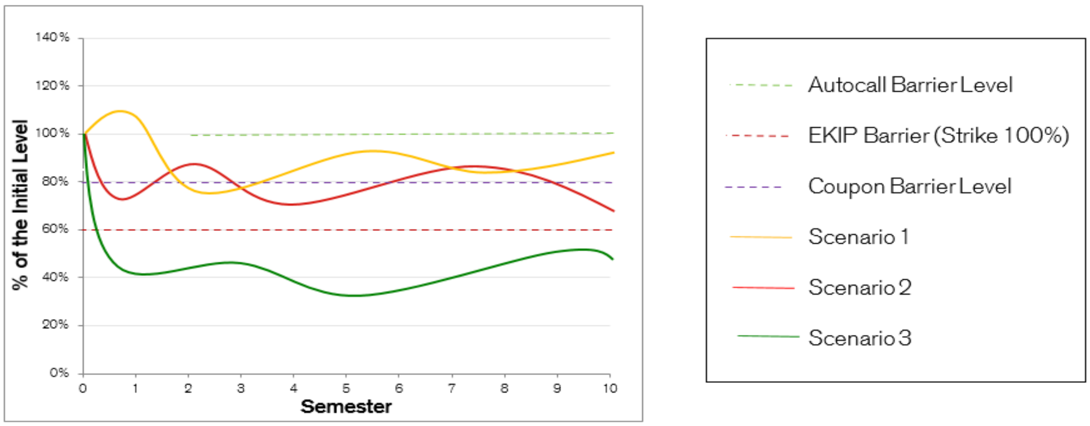

Figure 2.2: Possible Scenarios at Maturity

- **Scenario 1:** The underlying's price is above the Coupon Barrier, the investor recovers 100% + Coupon.

- **Scenario 2:** The underlying's price is below the Coupon Barrier but above the Knock-In Barrier, the investor recovers 100%.

- **Scenario 3:** The underlying's price is below the EKIP Barrier, the product receives 100% plus the performance of the underlying (negative).

Barrier levels therefore directly impact our product's lifetime and cashflows. Generally speaking, by lowering either the autocall or coupon barrier of our product we tend to increase the chance of being autocalled / receiving a coupon. In fact, as we can see on figure 2.3, most autocalls tend to have a very high probability of being autocalled during their first observation. It is most likely that if a product is not autocalled within the first autocall periods it will then last until maturity. As mentionned, one of the main goals of autocallables is to ensure a quick roll both for the client as it usually provides him with a higher return and secure his returns and for the issuer since they are able to sell another product and therefore take a margin fee again. This is where the concept of duration intervenes. The duration of an autocall is the average time interval at which it is going to be recalled.

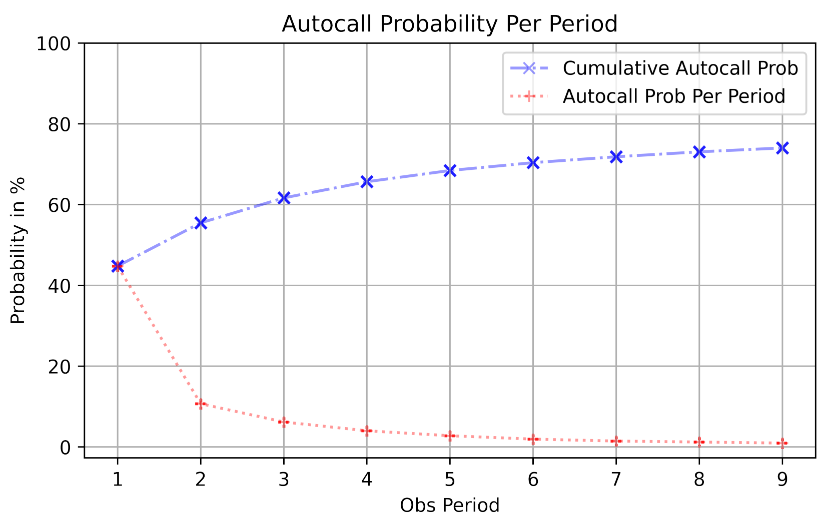

Figure 2.3: Autocall Probability with Autocall Barrier = 100%

2.2 Pricing Methodology

The price of an Autocall P_x can be viewed as it's expected discounted payoffs at each time periods CF_t, which can be rewritten in terms of recall probability. Bouzoubaa provides us with the proof of the below formula in his Book:

$$\boxed{P_x = E\left[e^{-rt}CF_t\right] = \sum_{t=1}^{n} p_t * CF_t e^{-rt}}$$

Another way to aproach a structured product pricing is to price each of its components separately. For instance, the Phoenix Autocall can be priced as short a EKIP, long an Autocallable Zero Coupon Bond Note and finally long a strip of digits at each observation dates. In practice this strip of digits is replaced by a Structured Barrier Note, which provides the coupon payments, with a Knock-In Feature at the coupon barrier level and a Knock-Out feature at the autocall level, as we don't want to pay for these options if the product gets redeemed.

2.2.1 Simulating Asset Paths

The first step to simulate any financial derivative is to choose a model which realistically represents its performance. One of the most known pricing approach to model an Equity Asset trajectory is the Black-Scholes model. It describes the variation of the underlying during a short period of time based on volatility and interest rates.

It was first introduced in 1973, and quickly became one of the most famous formula which gives an estimate of the theoretical price of an European option, ignoring any dividends paid during the option's lifetime. The model led to a boom in the option market, but has many flaws. In fact we need the following assumptions about assets and the market to be verified:

- Constant Risk-Free Rate
- Constant Volatility
- Efficient Markets: the instantaneous log returns of the asset price is following a random walk with drift, i.e. a geometric Brownian motion
- No Arbitrage Opportunity
- Ability to buy, sell, borrow and lend an infinie amount of asset
- Frictionless Market: no fees

The Geometric Brownian Motion, in the Black Scholes Model, is a stochastic process in which the logarithm of the randomly varying quantity follows a

Brownian motion with drift. The stochastic differential equation (SDE) is the following:

$$\boxed{dS_t = \mu S_t dt \ + \ \sigma S_t dW_t}$$

This equation can be decomposed in a deterministic part $\mu S_t dt$ that explains the path that the asset would usually follow and a stochastic part $\sigma S_t dW_t$ explaining the random moves of the stock where :

- W_t is a Brownian Motion
- We consider that $\mu =$ r due to the Forward relationship with the spot
- $\sigma \geq 0$ and is a deterministic value representing the underlying's volatility

Using this equation, and knowing S_0, a sample of the asset price at any time t can be generated :

$$\boxed{ln\frac{S_t}{S_0} = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t}$$

Iteration for a time step can be written as follow :

$$\boxed{S_t = S_{t-\Delta_t} * exp\left(\left(r - \frac{\sigma^2}{2}\right)\Delta_t + \sigma W_{\Delta_t}(t)\right)}$$

where $W_{\Delta_t}(t)$ is the increment of the Wiener process at time t and is therefore a normally distributed random variable.

Using this model we can now simulate independent random vectors that each represents potential paths of the underlying . Within those paths, we apply the underlying price at the observation dates and use the payoff function of the Phoenix Autocall to retrieve the price of the product at this period. We can then estimate today's price of the Phoenix Autocall by averaging its discounted cash flows. This method is called Monte-Carlo and it's efficiency rely on the Law of Large Number.

This method is extensively used in finance to compute prices and risk measurements on different derivatives that do not have direct solutions to their payoffs. It's especially usefull for payoffs with fixed observation dates and discontinuity such as Autocalls. It however relies on very large repeated sampling of random numbers to evaluate the Black and Scholes price. As shown in figure 2.5 for a simple call option pricing we need around 1 million + simulations to obtain reliable prices for our product (in this case matching the direct Call formula in the Black Scholes Model). Running this amount of scenarios can be a quite long process for complex structures like autocalls. Where we need to first generate the paths, then view for each simulation at what point and if the product is autocalled. Then compute for each scenarii the coupons received taking into

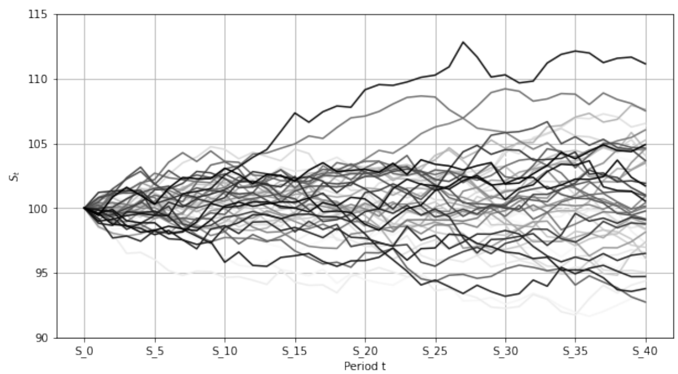

Figure 2.4: Simulating Geometric Brownian Motion

account if the product is not first autocalled and finally for non called paths compute the value at maturity.

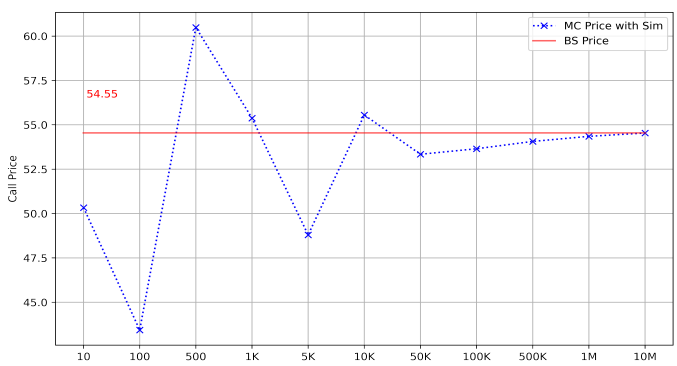

Figure 2.5: Call Price with MonteCarlo vs. B.S. formula

2.2.2 Volatility Models

There is however a big issue in our model choice. BS is a static model that relies on the constancy of its parameters, it would be more realistic if each variable could be modeled according to different conventions as a wrong choice in the parameters and their calibration could imply big differences in price with reality. One of the most important parameters directly impacting our option's time value is volatility. One more way to view it is that it increases the probability of the easiness to breach the barriers and especially the KIP barrier. Using a fixed and constant volatility model would not be in fact realistic as it implies a constant implied volatility (volatility infered from traded option prices) across all options listed on the market as well as during the lifetime of the product.

The phenomenon of varying implied volatility levels can be historically explained by the presence of potential market crash events and therefore a fat tail distribution in stock returns. As a consequence the demand for Out-of-The-Money Puts with low strike levels is generally stronger and directly impact the option prices, and therefore the implied volatility obtained. This Phenomenon of varying implied volatility accross options with different strikes levels is known as the Volatility Skew. Volatility anticipation also depends on the product's time to maturity, as different macro economic trends and events modifies market anticipation on the short vs long term. This creates a term structure of implied volatility with different possible shapes. This whole three-dimensional implied volatility surface (implied volatility, strike, maturity) can be generated with listed option prices and can then be expressed in an implied volatility matrix.

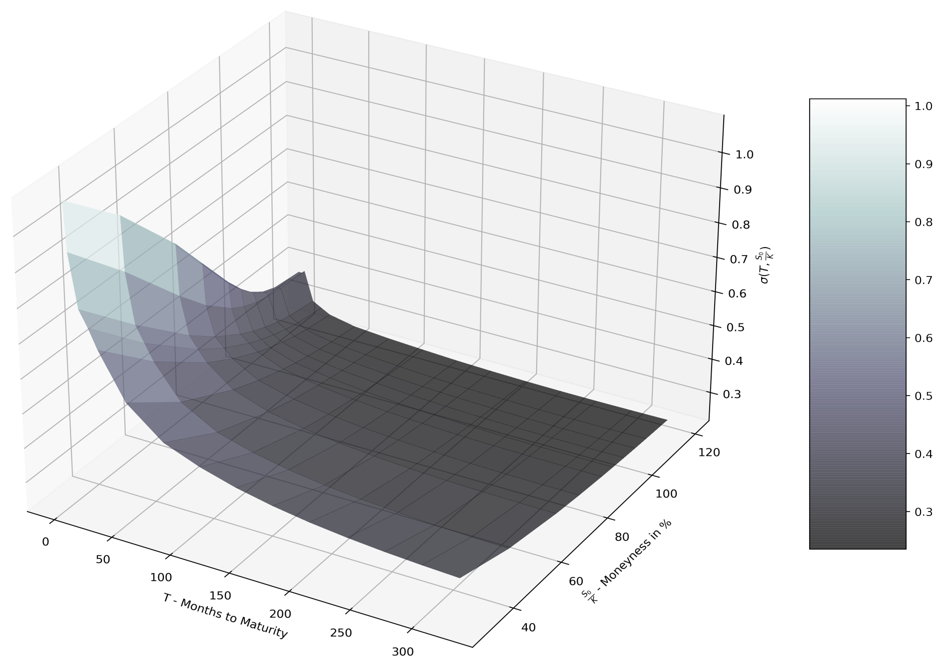

Figure 2.6: Volatility Surface Credit Agricole, Source: Internal CS

Exotic equity derivatives usually require a more sophisticated model than the Black-Scholes model. The most used model in practice is a local volatility model (LocVol) called Dupire's Local Volatility Model. It is often considered as a complete model as it allows hedging based on the underlying and consistent as it does not contain any contradiction. In LocVol models, the volatility is a deterministic function of the asset's level. Contrary to the assumption of the BS Model, the fact that we have a skew is the market telling us that the asset's log-returns have an implied distribution that is not Normal. In this model, given a set of vanilla option prices we can try to find a distribution which corresponds exactly to all vanilla prices taken from the skew. In fact, LocVol extends beyond skew and can also capture term structure. It can therefore theoretically supply us with a model that gives the exact same prices for vanillas taken from a whole implied volatility surface. On the other hand, stochastic volatility models are useful because they explain in a self-consistent way why it is that options with different strikes and expirations have different Black-Scholes implied volatilities.

Steven Heston introduced one of the most widely used model in this 1993 paper *"A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options"*, allowing us to include a random distribution to the volatility. This is a useful model for simulating stochastic volatility and its effect on the potential paths an asset can take over the life of an option. Its popularity mainly comes from the fact that it is an easy closed-form solution for European Option pricing, which allows for more effective modelling than the Black-Scholes formula. The Heston Model provides a dynamic for the underlying asset and the volatility with a "reversion effect around a target" which allows us to set long term targets with macroeconomic dynamics in our pricing. With this model we define the diffusion of the stock and volatility as follows:

$$\boxed{dS_t = \mu S_t dt \ + \ \sqrt{v_t} S_t dW_t^S}$$

$$\boxed{dv_t = k\left(\theta - v_t\right) d_t + \sigma \sqrt{v_t} dW_t^V}$$

The first equation is similar to the BS but includes v_t which is modelled according to the second equation with the following:

- v_t is the stochastic volatility at t
- σ is the "constant volatility" of the "stochastic volatility" also called "Vol on Vol"
- The two Brownian Motions are correlated: $d\left\langle W_t^S, W_t^V\right\rangle = \rho dt$ which allows our volatility to revert to its long term mean: θ at a certain speed k
- We also add a necessary condition proven in 1951 by Feller, which requires the volatility process to be positive: $2k\theta > \sigma^2$

It is interesting to notice that the correlation between the two Brownian Motions, which reflects the dependence between the asset price and volatility, is obseved in real life. In fact in periods of high volatility there is a tendency for stocks to drop in value. The set of parameters for the Heston model have to be determined using market observed prices of European options for various strike prices and maturities, it uses therefore implied volatilities. When this set of parameters have been determined, the model can be used to price European, American or more exotic options, which includes auto callable products. Because of the additional effort necessary to precisely calibrate a pricing engine using the Heston model to have realistic parameters and the resulting increase in computation time, the simpler Black-Scholes model was chosen for this paper.

2.3 Optimizing our product using Greeks

The autocallable structure described in the first part accumulates sensitivities to a wide range of variables which can be hard to understand from the point of view of the investor.

2.3.1 Greeks Analysis

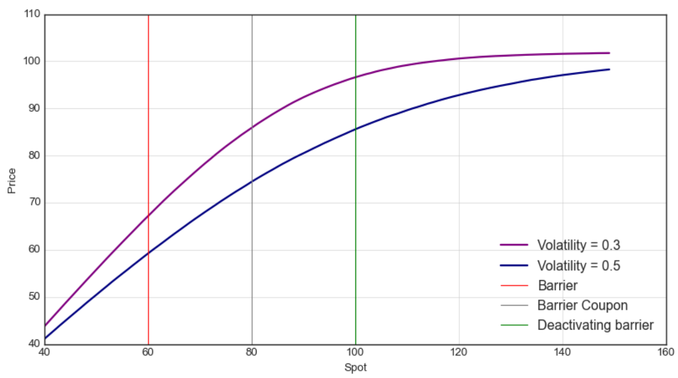

Figure 2.7: Price of Phoenix Autocall based on spot

◇ **First and Second-Order Exposition to the Underlying**

The investor is globally bullish on the underlying as the investor bets on the underlying being above specific coupons and recall levels. Therefore we have positive delta exposure which is decreasing with the spot level, it roughly goes from approximately 1 when the short PDI is deep In-the-Money, to 0 when the worst performing stock crosses the autocall barrier. In fact as our product's performance is capped at a certain point as the maximum maturity outcome is already known. We therefore obtain the following delta on our product, positive but decreasing as we are capped at a certain level. As we can see, volatility increase lower the impact of this first order exposure on our pricing.

Also we observe that the second order sensitivity to the underlying is globally negative. This Short Gamma exposure is due to to the fact that the investor is selling a Put-Down-And-In option at maturity. The negative gamma is also easily explained by the concave shape of our product payoff.

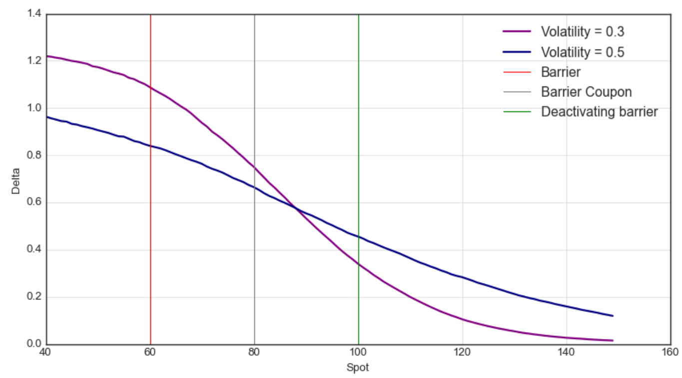

Figure 2.8: Delta Exposure

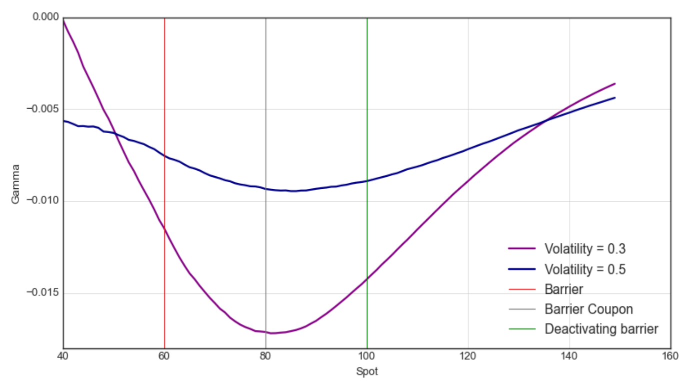

Figure 2.9: Gamma Exposure

◇ **Exposition to Dividends**

The forward value is one of the most important factor to take into consideration when choosing our underlying. It represents the estimated future value of an underlying given the spot value S_t, the forward rates curve r (as a proxy for the risk-free rate), and the dividend yield q and is verified by non-aritrage condition with the following formula :

$$\boxed{S_t = S_0 e^{(r-q)t}}$$

This value is extremely important for Structured products as it sets at the present value the expected probability of the underlying to cross its different barriers and directly affect its future cash flows. In order to lower the cost of our product we understand that we have to lower our probability of breaching the coupon and autocall barriers which can be achieved by using underlyings with low forward values. Low forwards also make the EKIP deeper in-the-money and therfore increase its value. The best scenariis would have to be an opposite view on the market, as a low forward value would imply a great opportunity. By betting on underlyings with low forward values the investor bet on the non-occurence of this scenario. By using the Forward Value formula defined in the previous part we understand that dividends play a great role in the structured product pricing.

Dividends are therefore directly impacting our product's value as they impacts the underlying future levels. Dividends are also very difficult to hedge for the bank and represent a huge uncertainty for market-makers due to their difficulty to hedge. Generally dividends model are a function of future underlying level and time as they are considered to be increasing with stock level and if stocks drops on a long period below a certain threshold it is likely that the company will cut or reduce its dividends. We are therefore short dividends as a rise in dividends will lower the forward value and therefore decrease the return of our product.

We need to consider that dividends represent an important risk exposure for the bank. If the dividends do not go as low as expected, the future value of the underlying will not match the Forward value used when pricing our product. This happened during the Covid-19 crisis where banks had issues due to dividend cut and trading desks were not prepared enough for this change in dividend on a global perspective. We will further develop in the next part how this dividend risk is resulting in the developement of new underlying strategies.

◇ **Exposition to Volatility**

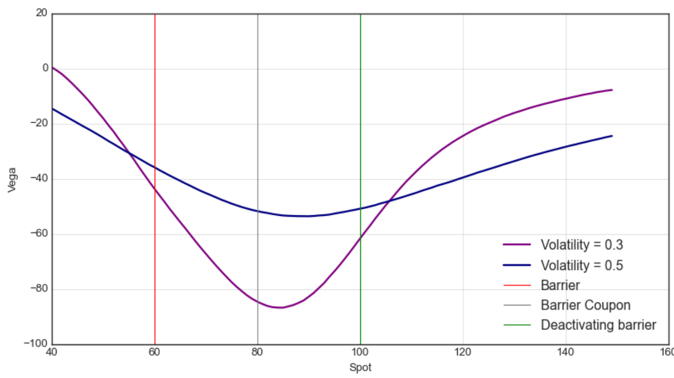

Figure 2.10: Price of Phoenix Autocall based on spot value

As we can see in Figure 2.10 we have a negative volatility exposition, short vega, as the investor is selling volatility with the European Knock In Put. However this is not always the case if we change our products parameters. In fact depending on our product's features, conditional coupons during our product's life time and autocallability will also impact the vega of our product as they imply discontinuities in our product's payoff. It would therefore be more interesting to visualise vega sensitivities accross time. Also the product is sensitive to the evolution of the implied volatility smile. When selling the EKIP the investor is also exposed to the skew of the product due to the activating barrier. The investor is therefore short of the "implied volatility skew (smile)"

◇ **Exposition to Interest Rate**

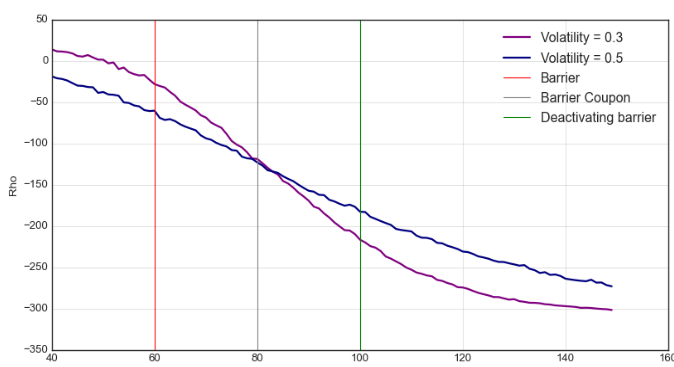

Figure 2.11: Rho Exposure

As we can see, this structure has a negative exposition to interest rates moves due to the ZC bond embedded in it. In fact a rise in interest rates directly impact the present value of future cash flows and therefore the product's price. Just as with the Heston Model, we could implement stochastic rates in our pricing with for instance the Vasicek Model. However, in practice and due to time efficiency, most banks tends to compute an additional cost, also called a provision, which is also the case for many different parameters. This implies that depending on the bank and on the trading desk risk appetite and book fillings we can have very different pricing.

We also note that the investor is long correlation between stock returns and interest rates. In fact if a rise in rates impacts equity value positively then this directly shorten the expected autocall date, which implies receiving back nominal plus coupons, as well as the fact that the investor will be able to reinvest his nominal and coupons at more advantageous levels compared to his initial investment.

2.3.2 Basket Optimization

Underlying selection is the most important performance factor when it comes to structured products. Depending on the underlying the client will need to adapt both his short and long legs as it directly impacts the cost of the different options. However it can be a quite tideous and time consuming process to run our pricer on every desired underlying in order to find the cheapest product, i.e. the one with highest possible coupon payments, all other parameters being equal. In fact, more than often, clients will come with a pre-selected underlyings universe, and a fixed structure and ask us to optimize the product's coupons. This can take a very long time as in the CAC40 alone we can create 780 combinations ob basket of 2 or 9 880 baskets of 3. We will for this part to select the possible underlyings based on the previously described phoenix structure.

We know that the main drivers of our product's price are the forward value and volatility of our product. Therefore we can try to find underlyings with low forward value and high volatility levels. The lowest forward in the market tends to be stocks with very high implied dividends yield. This can be simply viewed in the Forward Formula. The dividend yield, expressed as a percentage, is a financial ratio (dividend/price) that shows how much a company pays out in dividends each year relative to its stock price. We can note that mature companies in the utility and consumer industries are the most likely to pay higher dividend yields. We can also note that a high dividend yield can also result from a declining stock price. This can be especially usefull for an investor looking to invest in structured product as a lower underlying price both indicate a lower level for entry as well as a higher dividend yield. The dividend level used are available in the appendix of this Thesis, please see Figure 17 in order to view the dividend levels for each CAC40 stocks. This simple reasonning lead us to the results in appendix, figure 18. Regarding dividend levels, it is important to note that we are using the implied dividends of the market. The market valuation of dividends is found from the relationship between stock prices and futures. For instance, on SX5E we hae the following implied dividend levels :

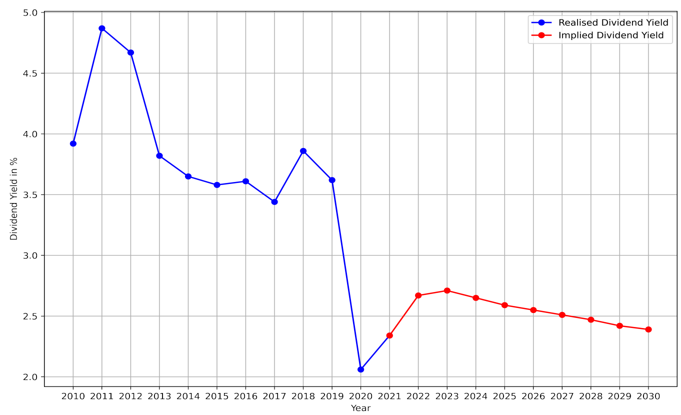

Figure 2.12: SX5E Dividend Yield, Source : Internal CS

It is frequent for the investor to select multiple underlyings instead of one. We now have to take one more important factor into consideration, correlation. It is a statistic that measures the degree to which two securities move in relation to each other. Correlation shows the strength of a relationship between two variables and is expressed numerically by the correlation coefficient. The correlation coefficient's values range between -1.0 and 1.0. To have the best possible results we should also ty to match our historical results with the duration of our product, i.e. expected time to maturity based on our simulations. The correlation on this project will be based on historical correlation between underling's over one year between the selected CAC40 stock constituents. We can Visualise correlation with a Heat Map in figure 2.13 between each constituents of the CAC40.

- The Equally weighted basket (EWB), composed of the weighted average performance of each underlying.

$$\boxed{\text{Basket Call}_{\text{payoff}} = max\Big[0, \sum_{i=1}^{n} w_i R_i - K\Big]}$$

- The Worst of or Second Worst of Basket (WOB), where the worst or second worst performance of the basket.

$$\boxed{\text{WO Put}_{\text{Payoff}} = max[0, K - min(S_1(T), S_2(T), ..., S_n(T))]}$$

- And similarly the Best of Basket where we retain the best underlying's performance.

We need to incorporate in our pricer a multidimensional process. To price multi-asset products it is necessary to simulate correlated assets. This can be done using a multivariate Geometric Brownian Motion, with each price process observing:

$$\boxed{dS_t^i = \mu S_t^i + \sigma S_t^i dW_t^i}$$

And which require a Wiener process of the following form $E\left(dW_t^i dW_t^j\right) = \rho_{i,j} dt$ with $\rho_{i,j}$ the correlation between two assets i and j.

$$\boxed{A = LL^* = \begin{bmatrix} 1 & 0 & 0 \\ a_{2,1} & a_{2,2} & 0 \\ a_{3,1} & a_{3,2} & a_{3,2} \end{bmatrix} \begin{bmatrix} 1 & a_{1,2} & a_{1,3} \\ 0 & a_{2,2} & a_{2,3} \\ 0 & 0 & a_{3,3} \end{bmatrix}}$$

In the case of coupon maximisation the investor will generally decide to invest in a WoF basket, which is one of the most commonly used type of basket due to the fact that the forward used in this type of basket will always be lower (or equal to) each of his components. This strategy is however much riskier as the probability of reaching the autocall barrier becomes much lower. Another factor has to be taken into account when choosing a basket of stocks is the correlation, which represents the evolution of a stock relatively to another. It is measured by the correlation coefficient computed by using the standard deviation σ_i of S_i and the covariance of each assets.

$$\boxed{\rho_{i,j} = \frac{cov(S_i, S_j)}{\sigma_i \sigma_j}}$$

On a WoF Basket the best possible situation for maximizing coupons is when correlation is equal to -1. Since we are only looking at the lowest value of the underlying basket it guarantees that no matter which way the underlying moves it will affect our basket. Using our pricer can now compute the Delta, Vega and Correlation Risk for our product.

Then we can simply generate combinations of n stocks within the CAC40 universe. And compute the difference in forward, volatility and correlation with our benchmark basket. We then apply the first order sensitivities to this difference to have an idea of the ranking of each baskets. We can then select the first n ranked basket and recompute the final price using our pricer.

For instance, in order to find the best possible underlyings on the structure defined in the first part of this structure, but this type on a Worst of Basket of two underlyings I start by pricing my product on Bouygues and Orange, which allow for a coupon of 12.5% p.a. using Credit Suisse Pricer. On this basket we have a Vega Risk of -67.63%, a Delta Risk of 48.20% and a Correlation Risk 13.36%.

Name	Vol	Forward
CAGR.PA	29.32%	75.01%
STLA.PA	35.34%	84.36%

Risk	Move	Contribution
delta	-2.93%	-1.41%
vega	7.43%	-5.03%
Correlation	2.54%	0.34%

Mktdata	Basket Avg
Vol	32.33%
Forward	79.69%
Correlation	70.29%

Then, by using the difference in Forward, Volatility and Correlation of each basket and by applying the different sensi to these differences we can obtain the following ranking :

1	2	3	4	5
STLA.PA URW.AS	CAGR.PA URW.AS	SOGN.PA STLA.PA	BNPP.PA STLA.PA	CAGR.PA STLA.PA

The First Basket on Stellantis and Unibail-Rodamco-Westfield once priced, allow us to show a coupon of 25,50% p.a., a differential of more than 13% in coupon with our initial basket.

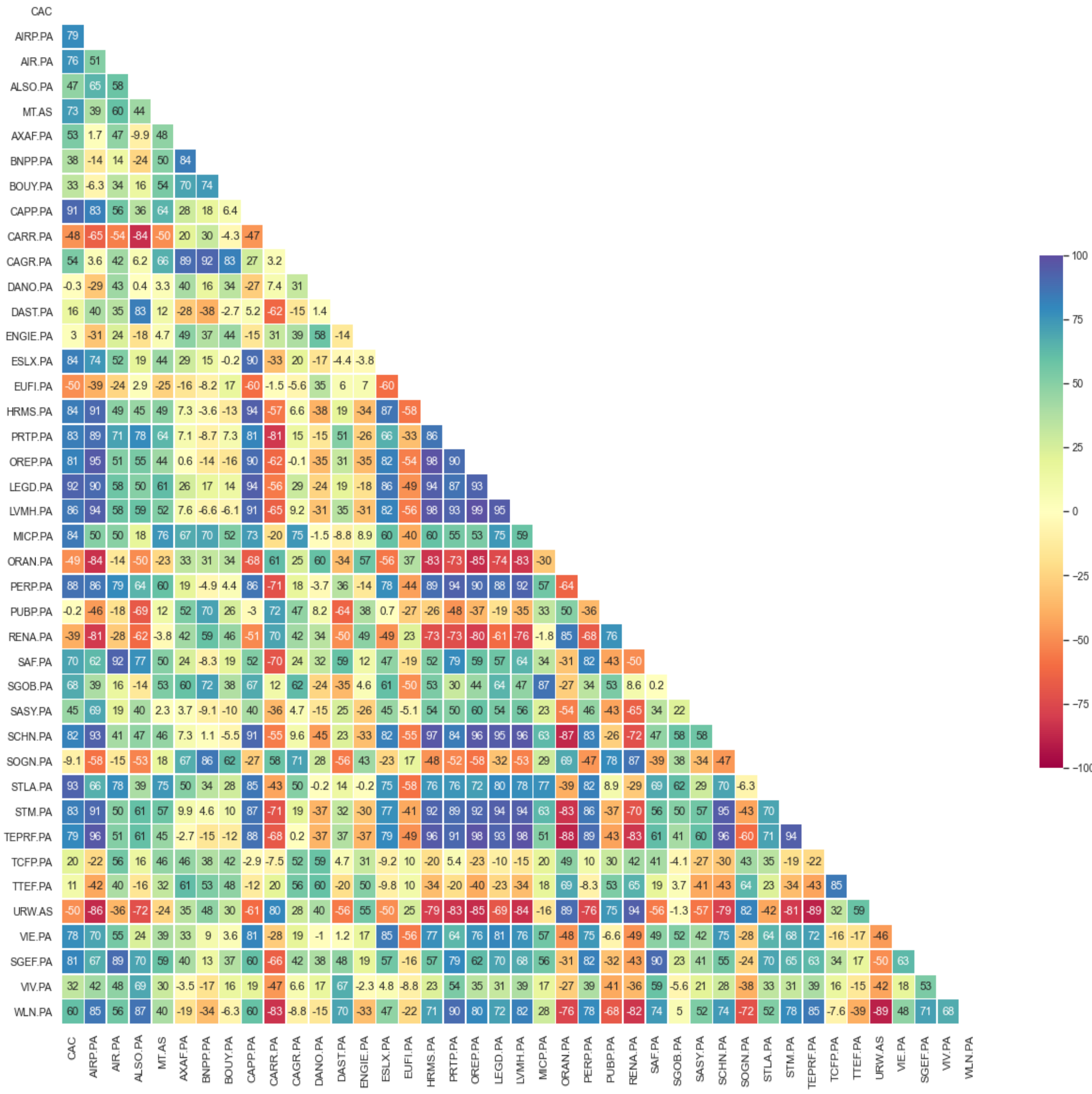

Figure 2.13: Historical Correlation of the CAC 40 constituents

2.4 Reducing Hedging Costs : Fix Dividends Underlyings

Due to their popularity, billions of euros of autocallables are issued each year and autocallables on indices represents the vast majority of them. As a result in Europe banks tend to accumulate the same risk in their books. In fact they are long dividends and volatility on the same indices, which is especially the case for the Eurostoxx 50. This has led to the developpement of proprietary indices which allow to transfer some of these risks to the investor, in exchange for higher coupon payments. In this part we review their functioning, their risk and if this extra complexity has been paying out for the investor so far.

As explained in the previous part, future dividend expectation will both impact the client and the bank. In fact, when a product is priced we use the market average estimate in order to compute the fair price of our product. In this situation the bank is the one collecting the dividend payments during the life time of the product, which also mean that the issuer is the one bearing the dividend risk. Having correct assumptions on future dividends is a very tricky process especially when we have to account for every stocks listed in the market. This explains why issuers will need to hedge dynamically their dividend expositions by selling dividend swaps, meaning that the issuer pays the dividends distributed in exchange for a fixed amount.

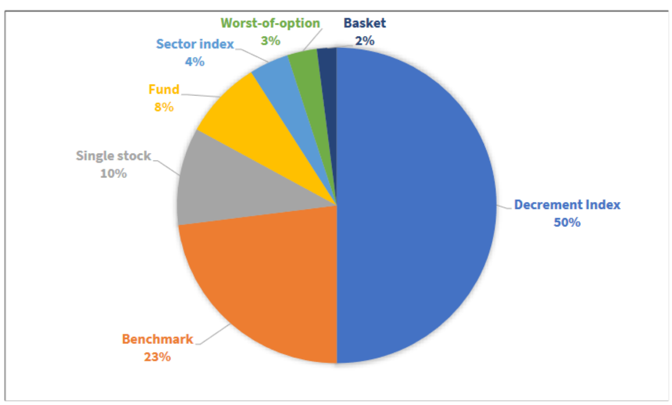

Figure 2.14: Underlying assets on the French market of structured products in Q1 2021, Source : Structured Products Intelligence[2]

However, since most issuers have long dividend expositions on the same underlyings due to the increasing popularity of autocalls, it is resulting an unbalanced market and as a result creates higher hedging costs for the bank, which decreases the possible coupon payments for the investor in the end. It is important to note that this phenomenon also accelerates when markets decreases, as issuers need to sell even more swaps, and which result to higher losses for trading desks. This both impacts the issuer and the investors as banks will tend to add higher buffers to compensate with the added risk in the market, which leads to lower returns for the investor. It is also important to note that on figure 2.12 we can see a downward slopping term structure for dividends on SX5E as long term dividends have implied levels even lower than short term one. This is partly due to the fact that structured product's desk are constantly selling long term dividends to hedge autocallables products. This accentuate the fact that the Eurostoxx doesn't provide very attractive coupons on its own.

2.4.1 Decrement Index

The first decrement index weas created in 2014 with the objective to significantly reduce the hedging costs for the issuers, and thus to give them the possibility to offer structured products with better features. A decrement index, takes as a base a total return index and detaches dividends, but here those dividends are set in advance. This synthetic dividend is called decrement. It is usually paid continuously (daily at close in practice) and is either defined as a percentage of the index when launched or as an absolute number of points. As a result the decrement index will tend to diverge with the total return index. It is important to note that in most cases this decrement index does not have any economic meaning as the decrement is set to a level unrelated to real dividends. This allows for banks to free themselves from their dividend risk as this amount of dividend is set in advance. Only the volatility risk therefore remains.

Figure 2.15: Computation of a Decrement Index

One might question the interest for the investor of such underlyings. But this allows for more competitive pricing from the bank as they do not need to add any buffers for dividends in order to price autocallables. In addition decrement index tend, by construction to have lower performance than a real index. The difference in performance will be exactly the difference between the real dividend yield and the decrement index. For instance, if an underlying has a real dividend yield of 2.5%, a 5% decrement index will underperform the real index of 2.5%. In addition to the choice of decrement type and their decrement frequency, indexes can differ based on their weighting policy (equally weighted or value weighted for instance). This might lead to overperformance of these index compared to the initial benchmark index due to their different methodology. We will therefore focus on indexes with similar features. The ideal situation would be if the investor would anticipate higher dividend yield compared to the market anticipation. As a result, by choosing an index matching the investor's view but with a higher decrement than the implied one, the structured product should feature a higher coupon with no significative underperformance. While choosing a higher decrement would lead to high coupons but at the almost cetain cost of underperformance.

The investor must also keep in mind when choosing for the right index that point decrement features will generally lead to a higher risk of underperformance. Imagine an index starting at 1000 points. Let us assume that index has a natural dividend yield of 3%, and we create two decrement indices one with 5% decrement and one with 50 points decrement. With an initial index level at 1000, both decrement seem equivalent as they have the same yield of 5% (50/1000 = 5%). If the total return index goes quickly to 700 and stays there for one year, the price return index will be roughly 97% *700 = 679. The five percent decrement index will be at 95%*700 = 665, and the 50 points decrement will be 650 (700-50). When computing the forward and assuming 0 interest rate and no market moves, we can quickly view that on a 10 year period point decrement have a much lower forward value compared to the percentage one's. Starting at 1 000 points on each and with a 50 point decrement vs. 5% over 10 years and assuming no interest rates we have:

$$Forward\,Point\,Decrement\,Index\;: S_0 * (-q * T) = 1000 * (-50 * 10) = 500.00$$

$$Forward\,\%age\,Decrement\,Index\;: S_0 * (e^{(-q*T)} = 1000 * e^{(-5\%)*10}) = 606.53$$

Using the same logic, one could however point out the fact that in rising market point decrement index could lead to an overperformance compared to their percentage version. However we need to keep in mind that these indexes are mostly used on autocalls and as a result the investor does not benefit from the upside of an index as the product should either get autocalled before getting to a large momentum or see its performance getting capped by the coupon future values.

2.4.2 Performance review

Let's review in this sub-section how this additionnal risk has resulted for the investor by backtesting a Phoenix Autocall using the same features, but comparing how they have payed out on, Classic Eurostoxx 50 vs. Decrement 5% verson of this index. We will be using a 5Y Maturity product, with a EKIP of 60%, a coupon barrier level of 80% and a callable level of 100% with annual observation dates. Those parameters selected for this product showcase one of the most common and basic product regarding autocallables structured products.

In the context of my internship I will be using the coupon levels from Credit Suisse and not from my pricer as it does not refelect the different spreads and bumbers used by trading desks on the different underlying. On Athena 1, on Eurostoxx (SX5E) we obtain a coupon of 6.07% p.a. compared to 7.95% on the Euro iStoxx 50 decrement 5%.

For each product, we perform these backtests on every working day from 01/01/2001 to 12/04/2017, since we are backtesting 5Y maturity products. We can then compute the return of each product by using the internal rate of return (IRR) of our product, which represent the discount rate that makes the Net Present Value of the Cash Flows equal to zero. This method is commonly used to compute the profitability of an investment. We obtain the following results:

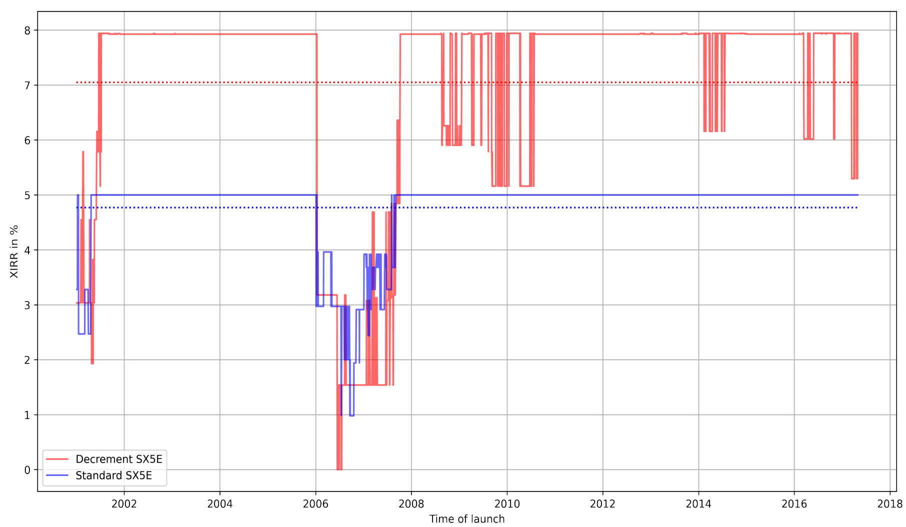

Figure 2.16: Rate of Return

On these structure it would seem that the decrement version of this index has paid out for the investor, however it only represents one version of a product on one underlying. Using a study from StructuredRetailProducts.com, provides us with a bigger picture :

	Number of Products	Expected Coupon if paid / average strike level	Average EKIP Level	Callable Frequency	Average Annualised Rate of Return
Benchmark Indices	354	6.05% / 100.36%	60.60%	28.25%	5.53%
Decrement Indices	283	7.32% / 99.84%	59.77%	23.32%	7.06%

Their study of autocallable products sold in France that maturing or knocking out early between April 2018 and March 2019 confirms our result by looking at a larger sample of the autocallable market. They also point out the following:

"As with any investment however, there is a trade-off that investors accept when opting for products linked to decrement indices. Despite their higher returns, the likelihood of these products being called decreased from 28% to 23% during the 12 months considered. On the other hand, and taking into account the bear market, decrement products have provided significantly better returns in a recovering market, compared with strategy and thematic indices, which featured low volatility and risk control features."

We might conclude that decrement indexes propose a very interesting solution both for banks and investors as they ease the dividend risk for trading desks and help the investor generate on average a higher return on their investments. However the complexity of these product and the additional risk for the investor might not always be clearly understood by the final investor. Since the creation of the first decrement indexes in 2014, the portion of decrement indexes among the underlying of the structured products sold to French retail investors has kept growing, so it will be interesting to see how products using a decrement index pay out in the future.

Complexity in structured finance is a great concern for regulators. In France, the AMF (Autorité des Marchés Financiers) and the ACPR (Autorité de Contrôle Prudentiel et de Résolution) control the issuance of structured products sold to retail investors. They impose that promotional communications relating to a public offer have to be communicated to the AMF prior to their publication, and that the number of mechanisms included in the product payoff cannot exceed 3 in the case of a public offer. Breaking down a payoff into different complexity mechanisms allows regulators to control the overall complexity level of a product by imposing a limit on a product's number of mechanisms. Therefore, issuers must ensure that investors are well informed of the risks associated with the product. This is where the previously described KID and SRRI comes in.

A study from Baule and Münchhalfen in 2020 [2] show that investors consider costs and product structure to be more important than information on risk, when choosing a product. If they admit that unexperienced investors tend to take more into considerations risk levels and documentations it can still lead to difficulties for small investor to fully understand the products they are invested in. In fact, during my internship it was common for clients not understanding the secondary valuations of the products thinking that they should fluctuate with the same path of the underlying. This situation is even more true with the rise of fix dividends stocks, as the risk of a dividend cut is greater on a single underlying compared to on a diversified index. Therefore it seems that this added complexity justifies the improved protection of individuals due to the additional risks that are not always taken into account by the final investor.

Conclusion

The purpose of this Master Thesis was to understand how we can price and optimize Autocallable Structured products in order to maximize the return and to assess if they represent a good investment opportunity.

In the first Chapter, I introduced the reader to the world of Equity structured products by describing the main types and categories of products sold on the market. I have also developped the different products that compose the short and long leg of our product and explained how by combining them we can propose attractive investment solutions for the investor. I also reviewed the pros and cons of investing in structured products as they allow to shape their return distribution to meet the client needs.

In the second chapter of this Thesis I introduced the reader to autocallables products and detailled the payoff of a Phoenix Note, one of the most commonly sold product on the French Market. I then explained the pricing methodology for our structure and created a path dependent Monte Carlo Pricer for a Phoenix Note in Python. I also analysed the Greeks in a third part and explained how we can efficienly use them in order to select the best underlyings for our needs. Finally I introduced the notions of decrement indexes and explained how they can be both beneficial for the bank and for the investor by reviewing their pros and cons and by performing backtests on these products.

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Appendix

Company	Sector	GICS Sub-Industry	Ticker	Spot	Forward 10Y	Forward in %	AtmVol	AtmSkew	Expected DivYield
Air Liquide	Basic Materials	Industrial Gases	AIRP.PA	161,86	168,803	104,29%	20,26%	0,88%	1,37%
Airbus	Industrials	Aerospace & Defense	AIR.PA	108	116,697	108,05%	31,63%	0,78%	1,02%
Alstom	Industrials	Rail Transport	ALSO.PA	21,79	23,3726	107,26%	32,87%	1,04%	1,10%
ArcelorMittal	Basic Materials	Steel	MT.AS	28,845	30,5959	106,07%	39,50%	0,85%	1,21%
AXA	Financial Services	Life & Health Insurance	AXAF.PA	26	17,5783	67,61%	21,09%	1,24%	5,73%
BNP Paribas	Financial Services	Diversified Banks	BNPP.PA	50,19	33,2936	66,34%	26,12%	0,91%	5,93%
Bouygues	Industrials	Construction & Engineering	BOUY.PA	30,93	23,03	74,46%	23,08%	0,62%	4,75%
Capgemini	Technology	IT Consulting & Other Services	CAPP.PA	205,4	222,588	108,37%	28,16%	1,01%	1,00%
Carrefour	Consumer Defensive	Hypermarkets & Super Centers	CARR.PA	19,015	17,6835	93,00%	26,22%	0,20%	2,51%
Crédit Agricole	Financial Services	Regional Banks	CAGR.PA	10,614	6,7792	63,87%	25,30%	0,94%	6,32%
Danone	Consumer Defensive	Packaged Foods & Meats	DANO.PA	50,26	41,2075	81,99%	19,92%	1,01%	3,78%
Dassault Systèmes	Technology	Application Software	DAST.PA	46,31	54,5422	117,78%	27,52%	0,90%	0,19%
Engie	Utilities	Gas Utilities	ENGIE.PA	11,928	8,2155	68,88%	21,52%	1,23%	5,55%
EssilorLuxottica	Healthcare	Apparel, Accessories & Luxury Goods	ESLX.PA	170,6	181,489	106,38%	24,58%	1,07%	1,18%
Eurofins Scientific	Healthcare	Biotechnologies	EUFI.PA	91,32	100,255	109,78%	24,67%	1,34%	0,87%
Hermès	Consumer Cyclical	Apparel, Accessories & Luxury Goods	HRMS.PA	1310,5	1531,82	116,89%	24,85%	0,82%	0,26%
Kering	Consumer Cyclical	Apparel, Accessories & Luxury Goods	PRTP.PA	584,3	605,098	103,56%	28,38%	1,19%	1,45%
L'Oréal	Consumer Defensive	Personal Products	OREP.PA	375,5	403,695	107,51%	23,03%	1,12%	1,08%
Legrand	Industrials	Electrical Components & Equipment	LEGD.PA	88,28	87,9741	99,65%	23,96%	1,09%	1,83%
LVMH	Consumer Cyclical	Apparel, Accessories & Luxury Goods	LVMH.PA	662,8	718,393	108,39%	26,55%	1,28%	1,00%
Michelin	Industrials	Tires & Rubber	MICP.PA	124,65	116,108	93,15%	26,65%	0,85%	2,49%
Orange	Communication Services	Integrated Telecommunication Services	ORAN.PA	10,674	7,4381	69,68%	17,36%	0,76%	5,42%
Pernod Ricard	Consumer Defensive	Distillers & Vintners	PERP.PA	201,1	208,368	103,61%	21,51%	1,26%	1,44%
Publicis	Communication Services	Advertising	PUBP.PA	55,5	46,6774	84,10%	27,15%	0,92%	3,51%
Renault	Consumer Cyclical	Automobile Manufacturers	RENA.PA	24,185	24,9699	103,25%	33,96%	1,01%	1,47%
Safran	Industrials	Aerospace & Defense	SAF.PA	105	121,38	115,60%	32,14%	0,98%	0,36%
Saint-Gobain	Industrials	Building Products	SGOB.PA	54,2	50,4276	93,04%	27,70%	0,87%	2,50%
Sanofi	Healthcare	Pharmaceuticals	SASY.PA	95,27	82,3624	86,45%	19,81%	0,90%	3,24%
Schneider Electric	Industrials	Electrical Components & Equipment	SCHN.PA	156,36	161,471	103,27%	25,45%	1,18%	1,47%
Société Générale	Financial Services	Diversified Banks	SOGN.PA	22,955	15,056	65,59%	30,14%	1,10%	6,05%
Stellantis	Consumer Cyclical	Automobile Manufacturers	STLA.PA	14,722	10,754	73,05%	29,01%	0,87%	4,94%
STMicroelectronics	Technology	Semiconductors	STM.PA	38,125	44,5683	116,90%	35,14%	0,94%	0,25%
Teleperformance	Communication Services	Outsourcing	TEPRF.PA	351,8	385,292	109,52%	25,57%	0,23%	0,89%
Thales	Industrials	Aerospace & Defense	TCFP.PA	117,85	117,516	99,72%	25,42%	0,46%	1,82%
TotalEnergies	Energy	Integrated Oil & Gas	TTEF.PA	46,24	34,3648	74,32%	22,47%	1,14%	4,77%
Unibail-Rodamco-Westfield	Real Estate	Retail REITs	URW.AS	68	65,7148	96,64%	37,27%	0,79%	2,13%
Veolia	Industrials	Multi-Utilities	VIE.PA	28,82	24,8878	86,36%	25,93%	1,16%	3,25%
Vinci	Industrials	Construction & Engineering	SGEF.PA	88,57	76,136	85,96%	24,84%	1,06%	3,29%
Vivendi	Communication Services	Movies & Entertainment	VIV.PA	12,01	12,0249	100,12%	25,23%	0,68%	1,77%
Worldline	Technology	Data Processing & Outsourced Services	WLN.PA	39,38	47,2541	120,00%	35,32%	1,10%	0,00%

Figure 17: CAC40 data, Source : Internal CS & Bloomberg, as of April 2022

	Underlying	Delta Risk	Vega Risk	Correl Risk				Best ticker
Sensis Obtained with Pricer	STLA.PA	68,77%	-36,89%	0,00%			1	SOGN.PA
							2	CAGR.PA
	Forward in %	AtmVol					3	BNPP.PA
Data for STLA.PA	73,05%	29,01%						

Ticker	Forward in %	AtmVol	Delta Forward	Delta AtmVol	Delta Forward Impact on price	Delta AtmVol Impact on price	Forward + AtmVol Impact on price	Rank
AIRP.PA	104,29%	20,26%	31,24%	-8,75%	21,49%	3,23%	24,71%	31
AIR.PA	108,05%	31,63%	35,01%	2,62%	24,07%	-0,97%	23,11%	27
ALSO.PA	107,26%	32,87%	34,22%	3,86%	23,53%	-1,42%	22,11%	26
MT.AS	106,07%	39,50%	33,02%	10,49%	22,71%	-3,87%	18,84%	19
AXAF.PA	67,61%	21,09%	-5,44%	-7,92%	-3,74%	2,92%	-0,82%	4
BNPP.PA	66,34%	26,12%	-6,71%	-2,89%	-4,62%	1,07%	-3,55%	3
BOUY.PA	74,46%	23,08%	1,41%	-5,93%	0,97%	2,19%	3,16%	8
CAPP.PA	108,37%	28,16%	35,32%	-0,85%	24,29%	0,31%	24,60%	30
CARR.PA	93,00%	26,22%	19,95%	-2,79%	13,72%	1,03%	14,75%	18
CAGR.PA	63,87%	25,30%	-9,18%	-3,71%	-6,31%	1,37%	-4,94%	2
DANO.PA	81,99%	19,92%	8,94%	-9,09%	6,15%	3,35%	9,50%	11
DAST.PA	117,78%	27,52%	44,73%	-1,49%	30,76%	0,55%	31,31%	39
ENGIE.PA	68,88%	21,52%	-4,17%	-7,49%	-2,87%	2,76%	-0,11%	5
ESLX.PA	106,38%	24,58%	33,34%	-4,43%	22,92%	1,63%	24,56%	29
EUFI.PA	109,78%	24,67%	36,74%	-4,34%	25,26%	1,60%	26,87%	35
HRMS.PA	116,89%	24,85%	43,84%	-4,16%	30,15%	1,53%	31,68%	40
PRTP.PA	103,56%	28,38%	30,51%	-0,63%	20,98%	0,23%	21,22%	24
OREP.PA	107,51%	23,03%	34,46%	-5,98%	23,70%	2,21%	25,91%	33
LEGD.PA	99,65%	23,96%	26,61%	-5,05%	18,30%	1,86%	20,16%	23
LVMH.PA	108,39%	26,55%	35,34%	-2,46%	24,30%	0,91%	25,21%	32
MICP.PA	93,15%	26,65%	20,10%	-2,36%	13,82%	0,87%	14,69%	17
ORAN.PA	69,68%	17,36%	-3,36%	-11,65%	-2,31%	4,30%	1,99%	7
PERP.PA	103,61%	21,51%	30,57%	-7,50%	21,02%	2,77%	23,79%	28
PUBP.PA	84,10%	27,15%	11,06%	-1,86%	7,60%	0,69%	8,29%	10
RENA.PA	103,25%	33,96%	30,20%	4,95%	20,77%	-1,83%	18,94%	20
SAF.PA	115,60%	32,14%	42,55%	3,13%	29,26%	-1,15%	28,11%	37
SGOB.PA	93,04%	27,70%	19,99%	-1,31%	13,75%	0,48%	14,23%	16
SASY.PA	86,45%	19,81%	13,40%	-9,20%	9,22%	3,39%	12,61%	14
SCHN.PA	103,27%	25,45%	30,22%	-3,56%	20,78%	1,31%	22,10%	25
SOGN.PA	65,59%	30,14%	-7,46%	1,13%	-5,13%	-0,42%	-5,55%	1
STLA.PA	73,05%	29,01%	0,00%	0,00%	0,00%	0,00%	0,00%	6
STM.PA	116,90%	35,14%	43,85%	6,13%	30,16%	-2,26%	27,90%	36
TEPRF.PA	109,52%	25,57%	36,47%	-3,44%	25,08%	1,27%	26,35%	34
TCFP.PA	99,72%	25,42%	26,67%	-3,59%	18,34%	1,32%	19,66%	21
TTEF.PA	74,32%	22,47%	1,27%	-6,54%	0,87%	2,41%	3,29%	9
URW.AS	96,64%	37,27%	23,59%	8,26%	16,22%	-3,05%	13,18%	15
VIE.PA	86,36%	25,93%	13,31%	-3,08%	9,15%	1,14%	10,29%	12
SGEF.PA	85,96%	24,84%	12,91%	-4,17%	8,88%	1,54%	10,42%	13
VIV.PA	100,12%	25,23%	27,08%	-3,78%	18,62%	1,39%	20,02%	22
WLN.PA	120,00%	35,32%	46,95%	6,31%	32,29%	-2,33%	29,96%	38

Figure 18: CAC40 Stock Selection (single underlying), Source : Using CS sensi