**Lab 06**

**Rayleigh and Rician Fading Channel Models**

Objectives:

1. Recall Multipath and Fading Channels
2. Rayleigh Fading Channel Modelling
3. Rician Fading Channel Modelling
4. Fading Channel Impact on Signal Constellation

**A) Recall Basic concepts:**

Multipath Fading: a propagation phenomenon that results in signals reaching the receiver by two or more paths, which we experience in real-world wireless systems.

**Multipath:**

All realistic wireless channels include many “reflectors”, given that RF signals bounce. Any object between or near the transmitter (Tx) or receiver (Rx) can cause additional paths the signal travels along. Each path experiences a different phase shift (delay) and attenuation (amplitude scaling).

At the receiver, all the paths add up. They can add up constructively, destructively, or a mix of both. We call this concept of multiple signal paths “multipath”. There is the Line-of-Sight (LOS) path, and then all other paths.

In the example below, we show the LOS path and a single non-LOS path:

A diagram of a path

Description automatically generated

Destructive interference can happen if you get unlucky with how the paths sum together. Consider the example above with just two paths. Depending on the frequency and the exact distance of the paths, the two paths can be received 180 degrees out of phase at roughly the same amplitude, causing them to null out each other (depicted below). You may have learned about constructive and destructive interference in physics. In wireless systems when the paths destructively combine, we call this interference “deep fade” because our signal briefly disappears.

A diagram of a mathematical equation

Description automatically generated

Paths can also add up constructively, causing a strong signal to be received. Each path has a different phase shift and amplitude, which we can visualize on a plot in the time domain called a “power delay profile”:

A black and white image of a building

Description automatically generated

The first path, the one closest to the y-axis, will always be the LOS path (assuming there is one) because there’s no way for any other path to reach the receiver faster than the LOS path. Typically, the magnitude will decrease as the delay increases, since a path that took longer to show up at the receiver will have travelled further.

**Fading**

What tend to happen is we get a mix of constructive and destructive interference, and it changes over time as the Rx, Tx, or environment is moving/changing. We use the term “fading” when referring to the effects of a multipath channel changing over time. That’s why we often refer to it as “multipath fading”; it’s really the combination of constructive/destructive interference and a changing environment. What we end up with, is a SNR that varies over time; changes are usually on the order of milliseconds to microseconds, depending on how fast the Tx/Rx is moving. Beneath is a plot of SNR over time in milliseconds that demonstrates multipath fading.

A graph of a graph

Description automatically generated

There are two types of fading from a **time domain** perspective:

* **Slow Fading**: also known as log-normal shadowing, is primarily caused by obstacles such as buildings, trees, and terrain features. As the name suggests, slow fading occurs over relatively long-time scales (the channel doesn’t change within one packet’s worth of data), resulting in gradual changes in signal strength. This type of fading is particularly prevalent in outdoor environments where the signal path encounters varying obstructions. The characteristics of slow fading necessitate robust error correction and detection mechanisms to ensure reliable communication, especially in scenario where the signal power fluctuates slowly over time.
* **Fast Fading**: in contrast to slow fading, fast fading manifests as rapid fluctuations in signal strength, typically occurring over short time intervals (the channel changes very quickly compared to the length of one packet). This phenomenon is often attributed to multipath propagation, where the transmitted signal takes multiple paths to reach the receiver, causing constructive and destructive interference. Fast fading poses significant challenges for wireless communication systems, as the rapid variations in signal strength can lead to signal distortion and errors. Mitigating the impact of fast fading requires adaptive modulation, coding techniques, and dynamic channel equalization to compensate for the rapid signal variations.

A graph showing the distance between the two different levels of the distance

Description automatically generated with medium confidence

There are also two types of fading from a **frequency** **domain** perspective:

**Frequency Selective Fading**: The constructive/destructive interference changes within the frequency range of the signal. When we have a wideband signal, we span a large range of frequencies. Recall that wavelength determines whether it’s constructive or destructive. Well, if our signal spans a wide frequency range, it also spans a wide wavelength range (since wavelength is the inverse of frequency). Consequently, we can get different channel qualities in different portions of our signal (in the frequency domain). Hence the name frequency selective fading.

**Flat Fading:** Occurs when the signal’s bandwidth is narrow enough that all frequencies experience roughly the same channel. If there is a deep fade, then the whole signal will disappear (for the duration of the deep fade).

In the figure below, the red shape shows our signal in the frequency domain, and the black curvy line shows the current channel condition over frequency. Because the narrower signal is experiencing the same channel conditions throughout the whole signal, it’s experiencing flat fading. The wider signal is very much experiencing frequency selective fading.

A diagram of a graph

Description automatically generated with medium confidence

Here is an example of 16 MHz wide signal that is continuously transmitting. There are several moments in the middle where there’s a period of time a piece of signal is missing. This example depicts frequency selective fading, which causes holes in the signal that wipe out some frequencies but not others.

A yellow and blue screen

Description automatically generated with medium confidence

In this Lab activity we are going to focus on the Frequency Flat Fading. In these scenarios we typically find that the received amplitude to be statistically distributed either as Rayleigh or Rician distributions.

**B) Simulating Rayleigh Fading and Rayleigh Distribution**

**Theory of Rayleigh Fading**

Let us denote the complex impulse response h(t) of the flat fading channel as follows:

where and are zero mean Gaussian distributed. Therefore, the fading envelope is Rayleigh distributed and is given by:

The probability density function (Rayleigh distribution) of the above-mentioned amplitude response is given by:

, where

1. **Simple Scenario with two multipaths**

The delays associated with different signal paths in a multipath fading channel change in an unpredictable manner and can only be characterized statistically. When there is a large number of paths, the central limit theorem can be applied to model the time-variant impulse response of the channel as a complex-valued Gaussian random process. When the impulse response is modelled as a zero-mean complex valued process, the channel is said to be a Rayleigh fading channel.

Rayleigh Fading is used to model fading over time, when there is no significant LOS path. On the other hand, Rician fading which we will investigate later is similar to that of Rayleigh fading, except that in Rician fading a strong dominant component is present. This dominant component can for instance be the LOS wave.

In this section, the Rayleigh Fading model is assumed to have only two multipath components and . Rayleigh Fading can be obtained from zero-mean complex Gaussian processes ( and ). Simply adding the two Gaussian Random variables and taking the square root (envelope) gives a single tap Rayleigh distributed process. The phase of such random variable follows uniform distribution.

Consider two Gaussian random variables with zero mean and same variance and .

Let’s define a complex Gaussian random variable (to mimic IQ channel) as:

Now, the envelope of the complex random variable is given by:

And the phase is given by:

The envelope follows Rayleigh distribution, and the phase will be uniformly distributed.

The probability density function (Rayleigh distribution) of the above-mentioned amplitude response () is given by:

The following MATLAB script explains how to generate a Rayleigh probability density function (PDF):

**Rayleigh\_PDF**

clc

close all

clear all

% Input Section

N = 1000000; %Number of samples to generate

variance = 0.5; % Variance of underlying Gaussian random variables

% Independent Gaussian random variables with zero mean and unit variance

x = randn(1, N);

y = randn(1, N);

% Rayleigh fading envelope with the desired variance

r = sqrt(variance\*(x.^2 + y.^2));

%Define bin steps and range for histogram plotting

step = 0.1; range = 0:step:3;

%Get histogram values and approximate it to get the pdf curve

h = hist(r, range);

approxPDF = h/(step\*sum(h)); %Simulated PDF from the x and y samples

%Theoritical PDF from the Rayleigh Fading equation

theoretical = (range/variance).\*exp(-range.^2/(2\*variance));

plot(range, approxPDF,'b\*', range, theoretical,'r');

title('Simulated and Theoretical Rayleigh PDF for variance = 0.5')

legend('Simulated PDF','Theoretical PDF')

xlabel('r --->');

ylabel('P(r)---> ');

grid;

% PDF of phase of the Rayleigh envelope

theta = atan(y./x);

figure(2)

hist(theta); %Plot histogram of the phase part

% Approximate the histogram of the phase part to a nice PDF curve

[counts,range] = hist(theta,100);

step=range(2)-range(1);

% Normalizing the PDF to match theoretical curve

approxPDF = counts/(step\*sum(counts)); %Simulated PDF from the x and y samples

bar(range, approxPDF,'b');

hold on

plotHandle=plot(range, approxPDF,'r');

set(plotHandle,'LineWidth',3.5);

axis([-2 2 0 max(approxPDF)+0.2])

hold off

title('Simulated PDF of Phase of Rayleigh Distribution ');

xlabel('\theta --->');

ylabel('P(\theta) --->');

grid;

In the above script, two independent identically distributed (i.i.d) Gaussian random arrays are generated using “randn” function in MATLAB and the envelope of their sum is computed to give the Rayleigh Fading process. Histogram is used to plot the PDF of the generated process and its phase plot is also drawn. From the plots it is evident that the envelope of the random variable (created by adding two i.i.d Gaussian random variables) follows Rayleigh distribution and the phase follows uniform distribution as we can see the following figures:





1. **Rayleigh Fading simulation: Clarke’s Model – Sum of Sinusoids**

A multipath fading channel can be modelled as an FIR (Finite Impulse Response) filter with the following impulse response:

where is the time varying impulse response of the multipath fading channel having L multi-paths and and denote the time varying complex gain and excess delay of the -th path. The above-mentioned impulse response can be implemented as an FIR filter as shown below:

A diagram of a diagram

Description automatically generated

The channel under consideration can be modelled as a multipath fading channel in which the impulse response follows the Rayliegh distribution.

Different method of simulation techniques was proposed to simulate/model multipath channels. Some of the models include Clarke’s reference model, Jake’s model, Young’s model, filtered Gaussian noise model etc.

A Rayleigh fading channel (flat fading channel) is considered is this section. For simplicity we fix the excess delays in the above equation and we generate that follows Rayleigh distribution. In this simulation Clarke’s Rayleigh fading model is used. This model is also called mathematical reference model and is commonly considered as a computationally inefficient model compared to Jake’s Rayleigh Fading simulator.

**Clarke’s Rayleigh Fading model:**

The random process of flat Rayleigh fading with M multi-paths can be simulated with the sum-of-sinusoid method described as

A math equations and formulas

Description automatically generated

Simulation:

1. The Rayleigh fading model is implemented as a function in MATLAB with following parameters:

M = number of multipaths in the fading channel,

N = number of samples to generate,

f\_d = maximum Doppler spread in Hz,

T\_s = sampling period.

We will use the Clarke’s Rayleigh Fading model (given below) and check the statistical properties of the random process generated by the model against the statistical properties of Rayleigh distribution.

function to generate Rayleigh Fading samples based on Clarke's model

M = number of multi-paths in the channel N = number of samples to generate fd = maximum Doppler frequency Ts = sampling period

A white text with green and black text

Description automatically generated

1. A test program is coded which uses the above-mentioned function is used to generate Rayleigh Fading samples with the following values for the function arguments. M=15, N=10^5, fd=100 Hz, Ts=0.0001 second.

**Clarke's Rayleigh Fading model**

**A screenshot of a computer program

Description automatically generated**

**A screenshot of a computer program

Description automatically generated**

1. Let’s now investigate the statistical properties of samples generated using Clarke’s model. From the above test program, the mean and variance of the real and imaginary parts of generated samples are:

Mean of real part =0

Mean of imaginary part =0

Variance of real part =0.4989 =0.5

Variance of imaginary part =0.4989 =0.5

The results imply that the mean of the real and imaginary parts is same and are equal to zero. The variances of the real and imaginary parts are approximately equal to 0.5.

1. Nex the PDF of the real part of the simulated samples are plotted and compared against the PDF of a Gaussian distribution (with mean=0 and variance=0.5).



1. The PDF of the generated Rayleigh fading samples are plotted and compared against that of Rayleigh distribution (with variance=1)



1. From 4) and 5) we confirm that the samples generated by Clarke’s model follows Rayleigh distribution (with variance=1) and the real and imaginary part of the samples follow Gaussian distribution (with mean=0 and variance=0.5).
2. The Magnitude and Phase response of the generated Rayleigh Fading samples are plotted here:



**C) Rician Fading and Rician Distribution**

The model behind Rician fading is similar to that of Rayleigh fading, except that in Rician fading a strong dominant component is present. This dominant component can for instance be the line-of-sight wave. This is modelled by using two Gaussian random variables – one with zero mean and another with non-zero mean.

Consider two Gaussian random variables and . Here and are the means of the distributions and is the variance.

Let us define a complex Gaussian random variable (to mimic IQ channel) as:

Now, the envelope of the complex random variable is given by:

And the phase is given by:

Since, the two variables X and Y have different “means”, a non-centrality parameter (indicating the non-central mean) is defined:

The non-centrality parameter (the imbalance in the means) is caused by the presence of dominant path in a Rician Fading environment. Due to this, the Rician K factor – representing the ratio of power of Line-of-sight (LOS) (or dominant multipath component) and the power of Non-Line-of-Sight (NLOS) (or the remaining multipath components) is defined in such scenario as:

The envelope () follows Rician distribution, whose PDF is given by:

Where, is the modified zero-th order Bessel function of the first kind.

**Simulating Rician PDF:**

In the following simulation, Rician fading envelope is simulated for various non-centrality parameters and for the underlying Gaussian random variables X and Y. The random variables X and Y used to generate Rician distribution differ only in their means. Their variances remain the same. For convenience, one of the Gaussian random variables (say X) is generated with mean=s (the non-centrality parameter) and variance=.

Note that (, or ) and are not the means and variance of the Rician distribution.

The following MATLAB code uses this concept to simulate a Rician PDF for various non-centrality parameters and for the underlying Gaussian random variable. Theoretical PDF curves are superimposed on the simulated curves for verification.

N = 100000; % Number of Samples

sigma = 1; % Variance of underlying Gaussian Variables

s = [0 1 2 4]; % Non-Centrality Parameter

plotStyle = {'b-','r-','k-','g-'};

% Simulating the PDF from two Gaussian Random Variables

for i = 1: length(s)

X = s(i) + sigma.\*randn(1,N); % Gaussian RV with mean=s and given sigma

Y = 0 + sigma.\*randn(1,N); % Gaussian RV with mean=0 and same sigma as Y

Z = X+1i\*Y; % channel coefficient

[val,bin] = hist(abs(Z),1000); % pdf of generated Raleigh Fading samples

plot(bin,val/trapz(bin,val),plotStyle{i}); % Normalizing the PDF to match theoretical result

% Trapz function gives the total area under the PDF curve. It is used as the normalization factor

legendInfo{i} = ['Simulated Rician PDF s = ' num2str(s(i))];

hold on;

end

%Theoretical PDF computation

for i = 1:length(s)

x = s(i);

m1 = sqrt(x);

m2 = sqrt(x\*(x-1));

r = 0:0.01:9;

ss = sqrt(m1^2+m2^2);

x = r.\*ss/(sigma^2);

f = r./(sigma^2).\*exp(-((r.^2+ss^2)./(2\*sigma^2))).\*besseli(0,x);

plot(r,f,plotStyle{i},'LineWidth',2.5);

legendInfo{i+length(s)} = ['Theoretical Rician PDF s = ' num2str(s(i))];

hold on;

end

legend(legendInfo);

title('Rician PDF for various non centrality parameter and underlying sigma=1')

The simulation result is the following:



**D) Fading Channel Impact on Signal Constellation**

In this part we are going to create a random sequence of bits and modulate it using BPSK (seen in previous Lab), where the modulated signal is depicted as “” representing the transmitted signal. We will investigate a flat fading channel that also adds AWGN noise to the modulated signal.

The received signal “” can be represented as:

Where, n is the noise contributed by AWGN which is Gaussian distributed with zero mean and unit variance and h is the complex channel amplitude scaling factor that follows Rayleigh or Rician distribution. It is to note that for a simple AWGN channel without Fading the received signal can be represented as y=x+n.

These are the following steps that you should follow to generate the signals:

1. Create a BPSK signal to be transmitted over the Fading channel.

You can use the following.

M\_mod = 2; % modulation order N = 10^5; % number of samples

bitSequence = randi([0, 1], N, 1);

x = pskmod(bitSequence, M\_mod);

1. Plot the signal constellation of x using

refC = [-1, 1];

constDiag = comm.ConstellationDiagram('ReferenceConstellation', refC, 'XLimits',[-7 7],'YLimits',[-7 7]);

constDiag1(x)

1. Generate the noise from a Gaussian distribution with zero mean and unit variance.

nois\_var = 0.1;

noise = (nois\_var)\*(randn(1,N) + 1i\*randn(1,N)); % AWGN noise with mean=0 var=0.1

1. Generate two channel coefficients h\_ray and h\_ric for both the Rayleigh and Rician distribution. For Rician use s1=0, s2=1, s3=2, s4=3.

You can use the functions that we investigated in the previous section.

M = 15; N = 10^5; fd = 100; Ts = 0.0001; sigma=1;

h\_ray = rayleighFading(M,N,fd,Ts);

h\_ric\_s1 = ricianFading(s1, sigma, N);

h\_ric\_s2 = ricianFading(s2, sigma, N);

h\_ric\_s3 = ricianFading(s3, sigma, N);

h\_ric\_s4 = ricianFading(s4, sigma, N);

1. Generate a signal y\_ray representing the received signal through Rayleigh Channel based on .
2. Generate multiple signals y\_ric\_s1, y\_ric\_s2, y\_ric\_s3, y\_ric\_s4 representing the received signal through Rician channel with different Rician K factor.
3. For each received signal plot the signal constellation using:

update(constDiag, y');

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