Student Information

Name :Alperen Oğuz Çakmak
ID :2237162

Answer 1

a)

Firstly, in order to find P(X) for all values of X we need to add rows in this table because the table shows joint probabilities of variables X and Y but we want marginal pmf of X in this question. Since we don't have anything to do with Y in this part of question, we can add 2 rows of these table to find P(X) for values x = 1, 2, 3. Computing for these x values we have the following:

$P_x(0) = \frac{3}{12}$, $P_x(1) = \frac{6}{12}$, $P_x(2) = \frac{3}{12}$

E(X) can be calculated by using expectation for discrete case formula (definition 3.3) from textbook. This formula implies that we have to calculate x.P(x) for all discrete X values and add them.
$E(X) = (0).\frac{3}{12} + (1).\frac{6}{12} + (2).\frac{3}{12} = 1$

For the variance of X , from variance formula (definition 3.6) on textbook, we can calculate all $(x-\mu)^2.P(x)$ for X values and add them. μ stands for mean value in other words expectation so it is equal to 1 from the previous part.

$Var(X) = (0-1)^2.P(0) + (0-0)^2.P(1) + (1-0)^2.P(2)$

$Var(X) = (0-1)^2.\frac{3}{12} + (0-0)^2.\frac{6}{12} + (1-0)^2.\frac{3}{12} = 1/2$

b)

X+Y can take values of minimum 0 and maximum of 4 which we need to consider while calculating the pmf. So we start calculating for values of X+Y.

$P_{X+Y}(0) = P\{X+Y=0\} = P\{X=0 \cap Y=0\} = P(0,0) = 1/12$
$P_{X+Y}(1) = P\{X+Y=1\} = P\{X=1 \cap Y=0\} = \frac{4}{12} = 1/3$
$P_{X+Y}(2) = P\{X=0 \cap Y=2\} = P\{X=2 \cap Y=0\} = \frac{2}{12} + \frac{1}{12} = 1/4$
$P_{X+Y}(3) = P\{X=1 \cap Y=2\} = \frac{2}{12} = 1/6$
$P_{X+Y}(4) = P\{X=2 \cap Y=2\} = \frac{2}{12} = 1/6$

c)

From the definition 3.8 on page 51 in textbook Cov(XY) = E(XY) - E(X)E(Y), we can compute E(XY) as following:
$E(XY) = (2).(1).\frac{2}{12} + (2).(2).\frac{2}{12} = 1$
From part a we know that E(X)=1 and we can compute E(Y) using marginal pmf of Y.
$E(Y) = (0).\frac{3}{12} + (1).\frac{6}{12} + (2).\frac{3}{12} = 1$

Now we can plug in the values: Cov(XY) = 1 - 1.1 = 0.

d)

Considering 2 random variables A and B if A and B are independent, from the property 3.5 on page 49 of textbook, E(AB) = E(A)E(B).
Knowing that, we can move on to the covariance formula, from definition 3.8 on page 51 of textbook, Cov(A,B) = E(AB) - E(A)E(B). Using previous part, we can write E(AB) to wherever we see E(A)E(B). So our equation is Cov(A,B) = E(AB) - E(AB) = 0.

e)

To show independency we need to compare joint probabilities with the multiplication of corresponding marginal probabilities. Let's start with X=0 and Y=0:
$P_{(X,Y)}(0, 0) = \frac{1}{12}$
$P_X(0) \times P_Y(0) = \frac{3}{12} \times \frac{6}{12} = \frac{18}{12} = \frac{3}{2}$
We can clearly see that $P_{(X,Y)}(0, 0) \neq P_X(0) \times P_Y(0)$. From the definition 3.4 on page 45 in textbook we can say that X and Y are dependent.

Answer 2

a)

We can compute the probability of getting excatly 0, 1, and 2 broken pens and substract from 1 to get the probability that getting at least 3 broken pens among 12 pens. Let's start with getting 0 broken pen using formula 3.9 on page 58 in textbook:

$P(0) = \binom{12}{0}.(0.2)^0.(1 - 0.2)^{12} = 0.069$ (For this question p is probability of getting broken pen)

$P(1) = \binom{12}{1}.(0.2)^1.(1 - 0.2)^{11} = 0.206$

$P(2) = \binom{12}{2}.(0.2)^2.(1 - 0.2)^{10} = 0.283$

$1 - (P(0) + P(1) + P(2)) = 1 - 0.558 = 0.442$ is the probability that among 12 pens at least 3 of them are broken.

b)

The probability that the fifth pen is the second broken pen we get is equal to getting exactly one broken pen among 4 pens times the probability that getting a broken pen. Firstly we must compute getting 1 broken pen out of 4. We can again use definition 3.9 as we did in last part.

$P(1) = \binom{4}{1}.(0.2)^1.(0.8)^3 = 0.409$
The probability that the fifth pen we test will be the second broken pen is $0.409 \times 0.2 = 0.0818$

c)

Definition 3.13 says that in order to find required number of trials to get k numbers of successes is negative binomial distribution. We need 4 broken pens so k=4 and p=0.2 since probability of getting broken pen is given. Question asks average number of trials to get 4 broken pens so we need to find mean value of this distribution in other words expected value. From the properties of negative binomial distribution (3.12) we see that E(X) $= \frac{k}{p}$ which is equal to $\frac{4}{0.2} = 20$. On the average in order to get 4 broken pens we need to pick 20 pens.

Answer 3

a)

Question 3 has phone calls as time events which are rare and number of calls within a time period has poisson distribution. The probability that bob does not get a phone call at least 2 hours is equal to the probability that bob does not get any phone call within 2 hours. Since Bob gets a phone call every 4 hours on average, within 2 hours expected number of calls is 0.5 which is equal to λ.
Finding that $\lambda = 0.5$ and the number of phone calls (x) $= 0$, we can look at the poisson distribution table (table A3) in the textbook. By the table P{X=0}=0.607 which is the probability that Bob doesn't get a phone call for at least 2 hours.

b)

For this part, we can again use the poisson distribution table since phone calls are rare time events. Given that Bob gets a phone call every 4 hours on average, in 10 hours, expected number of phone calls is 2.5 which is our new λ(lambda). We can simply redefine the question as poisson distribution $P\{X \leq 3\}$, $\lambda = 2.5$

From the poisson distribution table (table A3) in the textbook, $P\{X \leq 3\} = 0.758$ which is the probability of getting at most 3 calls for the 10 hours.

c)

Let's say the probability of not getting more than 3 phone calls for the first 10 hours is event A and the probability of not getting more than 3 phone calls for the first 16 hours is event B. Bayes rule (2.9) states that the probability of B given that A is equal to:

$P\{B|A\} = \frac{P\{A|B\}P\{B\}}{P\{A\}}$

From part b we know the probability of A which is 0.758 . The probability of B can be evaluated using same steps as in part b which is to take λ=4 (in 16 hours average number of successes is 4) and x=3 . Looking at the table A3, the probability of B is equal to 0.433 . Now all we need is to calculate $P\{A|B\}$ which is a very simple step because given that in 16 hours Bob did not get more than 3 phone calls, it is impossible for him to get more than 3 phone calls within 10 hours. So $P\{A|B\} = 1$, now all we need to do is substitute the values to the bayes formula.

$P\{B|A\} = \frac{1 \times 0.433}{0.758} = 0.571$ which is the probability of Bob did not get more than 3 phone calls for the first 16 hours given that he did not get more than 3 phone calls for the first 10 hours.