Student Information

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Answer 1

a)

Since we know that X is selected, and there is 2 green balls and 6 balls in total, the probability that we pick a green ball from box X is $\frac{2}{6} = \frac{1}{3}$.

b)

The probability that a red ball is selected from box X is equal to the probability that selected box is X times number of red balls divided by total number of balls. $0.4 \times \frac{1}{3} = \frac{2}{15}$. Same formula applies for box Y so, the probability that we pick red ball from box Y is $0.6 \times \frac{1}{5} = \frac{3}{25}$. Since we computed the probability of red ball selected from boxes X and Y, we can add these probabilities. $\frac{2}{15} + \frac{3}{25} = \frac{19}{75}$.

c)

Since both boxes contains blue colored balls, first we need to compute the probability of getting blue ball. We can simply add probabilities of picking blue ball from X and Y. That is
$\frac{1}{3} \times 0.4 + \frac{2}{5} \times 0.6 = \frac{28}{75}$.

Given that blue ball is picked, the probability that we had chose box Y is equal to the probability of choosing blue ball from box Y divided by the probability of choosing blue ball (by conditional probability formula 2.7). The probability of getting blue ball from box Y is equal to $\frac{2}{5} \times 0.6 = \frac{6}{25}$. The probability of getting a blue ball is equal to $\frac{28}{75}$.

Let's say picking a blue ball is event B and choosing box Y is event Y. The solution is $\frac{P\{B \cap Y\}}{P\{B\}}$. $P\{B \cap Y\} = \frac{6}{25}$ and $P\{B\} = \frac{28}{75}$. After dividing these fractions we get $\frac{9}{14}$ that is the probability of we had chosen the box Y given that picked ball is blue.

Answer 2

a)

If $\overline{A}$ and $\overline{B}$ are exhaustive that means $\overline{A} \cup \overline{B} = \Omega$ (from the definition 2.8). From the equation at page 13, $\overline{A} \cup \overline{B} = \overline{A \cap B}$ and since this equals to whole space, complement of it $(A \cap B = \emptyset)$ equals to empty space. Thus, from the definition 2.7, A and B are mutually exclusive.

b)

If $\overline{A}$, $\overline{B}$ *and* $\overline{C}$ are exhaustive, (from the definition 2.8) $\overline{A} \cup \overline{B} \cup \overline{C} = \Omega$. From the equation at page 13, $\overline{A} \cup \overline{B} \cup \overline{C} = \overline{A \cap B \cap C} = \Omega$, Which also implies that the intersection of A, B and C is empty set. However these events still can have intersections pairwise, so this disproves that A, B and C are mutually exclusive if and only if $\overline{A}$, $\overline{B}$ *and* $\overline{C}$ are exhaustive.

Answer 3

a)

In order to have exactly 2 successful dice, out of 5 dice rolls we need to get 3 unsuccessful dice. With one dice roll we have $\frac{1}{3}$ probability of getting successful dice and $\frac{2}{3}$ probability of getting unsuccessful dice. We can simply compute $\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ which is equal to $\frac{8}{243}$. After that we need to consider all the orders of successful dice, so we chose 2 successful dice in 5 dice rolls that is being $\binom{5}{2}$ which is equal to 10. Finally we can conclude the answer as $\frac{8}{243} \times 10 = \frac{80}{243}$.

b)

The probability that we get at least 2 successful dice is
1 - (probability of getting 1 successful dice + probability of getting 0 successful dice).
The probability of getting 1 successful dice is $\binom{5}{1} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{80}{243}$. And the probability of getting all of them unsuccessful is $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{32}{243}$.

After plugging the values $1 - (\frac{80}{243} + \frac{32}{243}) = \frac{131}{243}$. The probability that we get at least 2 successful dice is $\frac{131}{243}$.

Answer 4

a)

On the fifth and seventh lines of the table, our random variables A and C are 1 and 0 respectively. We can calculate P(A=1, B=b, C=0) as addition of fifth and seventh lines of table which is $P_{A,B,C}(1,1,0) + P_{A,B,C}(1,0,0) = 0.06 + 0.09 = 0.15$

b)

We need to add all the probabilities that have B variable as 1 so, P(A=a, B=1, C=c) = $P_{A,B,C}(0,1,0) + P_{A,B,C}(0,1,1) + P_{A,B,C}(1,1,0) + P_{A,B,C}(1,1,1) = 0.21 + 0.02 + 0.09 + 0.08 = 0.40$

c)

Definition of independence of random variables (3.4) says that $P_{(X,Y)}(x,y) = P_X(x)P_Y(y)$ if and only if X and Y are independent. We start checking the equality by computing for X=0 and Y=0.
$P_A(0) = 0.14 + 0.08 + 0.02 + 0.21 = 0.55$
$P_B(0) = 0.14 + 0.08 + 0.06 + 0.32 = 0.6$
$P_A(0)P_B(0) = 0.55 \times 0.6 = 0.33$
$P_{(A,B)}(0,0) = 0.14 + 0.08 = 0.32$
Since $P_{(A,B)}(0,0) = 0.32 \neq 0.33 = P_A(0)P_B(0)$ we do not need to check other pairs, we can say that A and B are not independent random variables.

d)

Since it is given that C=1. We need to look at the conditional probabilities of these events while considering the independence and we also know the formula for conditional probability (2.7). Now we can start checking with the values A=0 and B=0.
$P\{C=1\}=0.5$

$P_A(A=0 \mid C=1)=(0.08+0.02)/0.5=0.2$
$P_B(B=0 \mid C=1)=(0.08+0.32)/0.5=0.8$
$P_A(A=0 \mid C=1)P_B(B=0 \mid C=1)=0.16$
$P_{A,B}(A=0,B=0 \mid C=1)=0.08/0.5=0.16$
Since $P_{A,B}(A=0,B=0 \mid C=1)=P_A(A=0 \mid C=1)P_B(B=0 \mid C=1)$ we need to check for other values.

$P_A(A=0 \mid C=1)=(0.08+0.02)/0.5=0.2$
$P_B(B=1 \mid C=1)=(0.02+0.08)/0.5=0.2$
$P_A(A=0 \mid C=1)P_B(B=1 \mid C=1)=0.04$
$P_{A,B}(A=0,B=1 \mid C=1)=0.02/0.5=0.04$
Since $P_{A,B}(A=0,B=1 \mid C=1)=P_A(A=0 \mid C=1)P_B(B=1 \mid C=1)$ we continue checking.

$P_A(A=1 \mid C=1)=(0.32+0.08)/0.5=0.8$
$P_B(B=0 \mid C=1)=(0.08+0.32)/0.5=0.8$
$P_A(A=1 \mid C=1)P_B(B=0 \mid C=1)=0.64$
$P_{A,B}(A=1,B=0 \mid C=1)=0.32/0.5=0.64$
Since $P_{A,B}(A=1,B=0 \mid C=1)=P_A(A=1 \mid C=1)P_B(B=0 \mid C=1)$ we continue checking.

$P_A(A=1 \mid C=1)=(0.32+0.08)/0.5=0.8$
$P_B(B=1 \mid C=1)=(0.02+0.08)/0.5=0.2$
$P_A(A=1 \mid C=1)P_B(B=1 \mid C=1)=0.16$
$P_{A,B}(A=1,B=1 \mid C=1)=0.08/0.5=0.16$
With these values $P_{A,B}(A=1,B=1 \mid C=1)=P_A(A=1 \mid C=1)P_B(B=1 \mid C=1)$ and there is no more values to check for A and B so we say that, A and B are conditionally independent given that C=1.