Two methods of solving linear second-order differential equations with Dirichlet boundary conditions

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Abstract

1 Introduction

We encounter wave phenomena everywhere in the natural sciences. From quantum mechanics to oceanography, we find that be it the motion of a particle or the ocean, we require knowledge of wave-like behaviour to solve the problem. In quantum mechanics, a particle's wave function is described by a complex-valued diffusion equation, the Schrödinger equation, while in oceanography, we can describe ocean waves using the wave equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \tag{1}$$

where x and t denote the spatial and temporal coordinates, respectively. The wave equation will be the topic of this paper, in particular, we will model Rossby waves, first identified by Rossby (1939). These are inertial, planetary waves in the Earth's atmosphere and ocean which motions contribute to extreme weather Mann et al. (2017),

$\mathbf{n}$	$\mathbf{t_g/t_s}$	$\mathbf{t_{LU}/t_s}$
10	2.08	3.70
10^2	1.89	$1.00 \cdot 10^2$
10^3	1.48	$1.05 \cdot 10^4$
10^4	1.43	$1.18 \cdot 10^6$
10^5	1.39	-
10^6	1.41	-
10^7	1.39	-

Table 1: Ratio between CPU time for the general algorithm ($\mathbf{t_g}$), the special algorithm ($\mathbf{t_g}$) and the LU decomposition algorithm ($\mathbf{t_{LU}}$) for different matrix sizes ($\mathbf{n}$). The LU decomposition crashed for $\mathbf{n}$ greater than 10^4.

2 Methods

3 Results

4 Discussion

References

Michael E Mann, Stefan Rahmstorf, Kai Kornhuber, Byron A Steinman, Sonya K Miller, and Dim Coumou. Influence of anthropogenic climate change on planetary wave resonance and extreme weather events. *Scientific Reports*, 7:45242, 2017.

Carl-Gustaf Rossby. Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semi-permanent centers of action. *J. Mar. Res.*, 2:38–55, 1939.

A

$$A = \begin{bmatrix} b_1 & c_1 & 0 & \dots & \dots & 0 \\ a_1 & b_2 & c_2 & 0 & \dots & 0 \\ 0 & a_2 & b_3 & c_3 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \ddots & a_{n-2} & b_{n-1} & c_{n-1} \\ 0 & \dots & \dots & 0 & a_{n-1} & b_n \end{bmatrix},$$

all_matrix_compare.pdf

Figure 1: The numeric solution using different solving algorithms. The graphs for n=100 and n=1000 are so similar that they are not distinguishable.