Planetary waves: a numerical study of the wave equation

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Abstract

In this article, we study Rossby waves using different both implicit and explicit schemes for solving the wave equation numerically. We consider both the advantages and disadvantages of both methods, as well as their efficiency and accuracy. From this analysis, we find that

1 Introduction

We encounter wave phenomena everywhere in the natural sciences. From quantum mechanics to oceanography, we find that be it the motion of a particle or the ocean, we require knowledge of wave-like behaviour to solve the problem. In quantum mechanics, a particle's wave function is described by a complex-valued diffusion equation, the Schrödinger equation, while in oceanography, we can describe ocean waves using the wave equation,

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \tag{1}$$

where x and t denote the spatial and temporal coordinates, respectively. The wave equation will be the topic of this paper, in particular, we will model Rossby waves, first identified by Rossby (1939). These are inertial, planetary waves in the Earth's atmosphere and ocean which motions contribute to extreme weather (Mann et al., 2017), might drive the El-Ninõ southern oscillation (ENSO) (Bosc and Delcroix, 2008), and is further produced by ENSO, see Battisti (1989). While we hope the reader appreciate the wide range of phenomena related to these waves, our article is a numerical study of the waves isolated from other processes. We therefore begin by describing fundamental theory of waves and partial differential equations in the Theory section, present our algorithm and the technicalities relating to its implementation. In the Results section, we present our data as figures, before discussing their implications in the Discussion section. Concluding our paper, we look at the learning outcomes, and present our final thoughts on the topic of simulating Rossby waves.

2 Theory

2.1 Wave theory

2.2 Partial differential equations

2.3 Implementation

3 Results

4 Discussion

5 Conclusion

References

David S Battisti. On the role of off-equatorial oceanic rossby waves during enso. *Journal of physical Oceanography*, 19(4):551–560, 1989.

$\mathbf{n}$	$\mathbf{t_g/t_s}$	$\mathbf{t_{LU}/t_s}$
10	2.08	3.70
10^2	1.89	$1.00 \cdot 10^2$
10^3	1.48	$1.05 \cdot 10^4$
10^4	1.43	$1.18 \cdot 10^6$
10^5	1.39	-
10^6	1.41	-
10^7	1.39	-

Table 1: Ratio between CPU time for the general algorithm ($\mathbf{t_g}$), the special algorithm ($\mathbf{t_g}$) and the LU decomposition algorithm ($\mathbf{t_{LU}}$) for different matrix sizes ($\mathbf{n}$). The LU decomposition crashed for $\mathbf{n}$ greater than 10^4.

Christelle Bosc and Thierry Delcroix. Observed equatorial rossby waves and enso-related warm water volume changes in the equatorial pacific ocean. *Journal of Geophysical Research: Oceans*, 113(C6), 2008.

Michael E Mann, Stefan Rahmstorf, Kai Kornhuber, Byron A Steinman, Sonya K Miller, and Dim Coumou. Influence of anthropogenic climate change on planetary wave resonance and extreme weather events. *Scientific Reports*, 7:45242, 2017.

Carl-Gustaf Rossby. Relation between variations in the intensity of the zonal circulation of the atmosphere and the displacements of the semi-permanent centers of action. *J. Mar. Res.*, 2:38–55, 1939.

A

$$A = \begin{bmatrix} b_1 & c_1 & 0 & \dots & \dots & 0 \\ a_1 & b_2 & c_2 & 0 & \dots & 0 \\ 0 & a_2 & b_3 & c_3 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \ddots & a_{n-2} & b_{n-1} & c_{n-1} \\ 0 & \dots & \dots & 0 & a_{n-1} & b_n \end{bmatrix},$$

all_matrix_compare.pdf

Figure 1: The numeric solution using different solving algorithms. The graphs for n=100 and n=1000 are so similar that they are not distinguishable.