### The Ising model and phase transitions in magnetic systems

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#### Abstract

#### 1 Introduction

### 2 Methods

#### 2.1 The Ising model

The Ising model consists of variables s that can exist in two states, typically +1 or -1. These variables represent magnetic dipole moments of atomic spin, and are ordered in a two dimensional lattice consisting of  $L \times L$  spins. Given that there is no magnetic field, the energy of the system is modelled as

$$E = -J \sum_{\langle ij \rangle} s_i s_j, \tag{1}$$

where J is a coupling constant expressing the strength of the interaction between neighbouring spins. The symbol  $\langle ij \rangle$  means that the sum is over neighbouring spins. The magnetisation of the system is simply the sum of all the spins

$$M = \sum_{i} s_{i}.$$
 (2)

#### 2.2 Monte Carlo simulations

#### 2.3 Metropolis' algorithm

The Metropolis' algorithm is an algorithm that can be used to solve the Ising model.

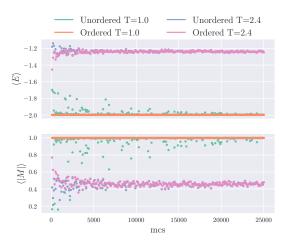


Figure 1:

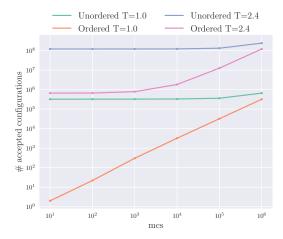


Figure 2:

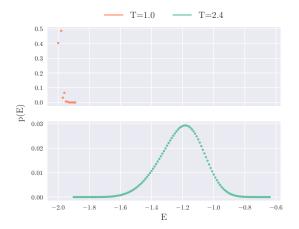


Figure 3:

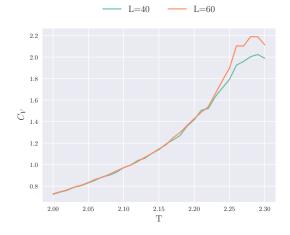


Figure 5:

$$\begin{array}{c|cc} T & \langle E \rangle & \sigma^2 \\ \hline 1.0 & -1.997 & 0.025 \\ 2.4 & -1.237 & 8.116 \\ \end{array}$$

Table 1:

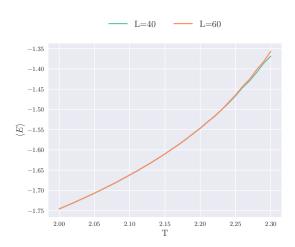


Figure 4:

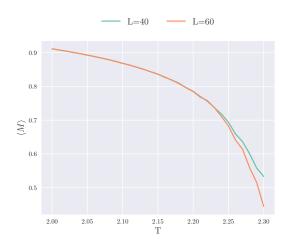


Figure 6:

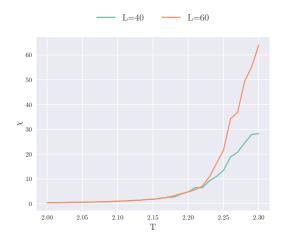


Figure 7:

## 3 Results

## 4 Discussion

# Appendix