CmpE 548 Monte Carlo Methods Term Project

A Closer Look at Ant Colony Optimization

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Problem Statement

Ant colony optimization algorithm solves different NP-hard problems such as:

- Routing problems (Traveling Salesman, Vehicle Routing)
- Assignment problems (Graph Coloring, Quadratic Assignment)
- Scheduling problems (Project Scheduling, Open Shop)
- Subset problems (Multiple Knapsack, Set Covering)
- Other (Bayesian Networks, Protein Folding)

Introduction

First *Ant Colony Optimization* (ACO) algorithm which is known as *Ant System* (AS) was first proposed by Dorigo et al in 1991 to solve *Traveling Salesman Problem* (TSP) under the umbrella of swarm intelligence methods that are inspired from the social behaviors of insects and animals, in this case ants.[1]

Each ant in the ant colony tends to follow the best path between the nest and the target place. The most favorable paths to food sources are the ones that are mostly marked by *pheromones* deposited by ants. After AS, some other ACO algorithms such as *MAX- MIN Ant System*, *Ant Colony System* etc. were introduced but AS is better for TSP. Also, ACO was extended to other combinatorial problems.[1]

We applied AS to TSP due to the stochastic decision making mechanism of ants while choosing the next city to visit and stochastic city traversal which makes ACO a Monte Carlo Method. Then, we solved TSP with *Simulated Annealing* and compared the results with AS. On the other hand, each ant symbolizes a particle. Therefore, ACO was applied to *Particle Filter* (PF) to solve impoverishment problem.[2]

ANT SYSTEM

- Ants build solutions by traversing the fully connected *construction graph*.
- At each construction step, the partial solution s^p is extended by adding a feasible solution component(edge) from the set $\mathbb{N}(s^p) \subseteq C$.
- Ants born in random cities in the beginning of each tour. Therefore, ants explore the graph randomly in each tour.
- The probability of going city j from city i:

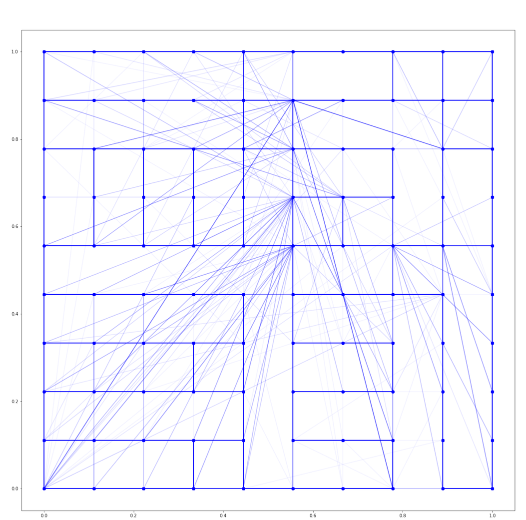
$$p_{ij}^k = \begin{cases} \frac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\Sigma_{c_{il} \in \mathbb{N}(s^p)} \tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}}, & \text{if } c_{ij} \in \mathbb{N}(s^p), \\ 0, & \text{otherwise.} \end{cases} \qquad (1)$$

$$\eta_{ij} = 1/d_{ij} \quad d_{ij}\text{:the distance between cities i-j} \qquad (2)$$

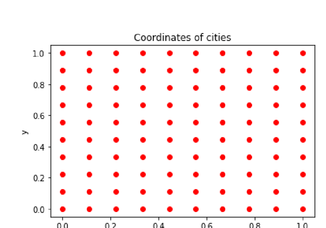
- the pheromone τ update at the end of each tour

$$\tau_{ij} \leftarrow (1-\rho) \cdot \tau_{ij} + \Sigma_{k=1}^{m} \Delta\tau_{ij}^{k} \qquad (3)$$

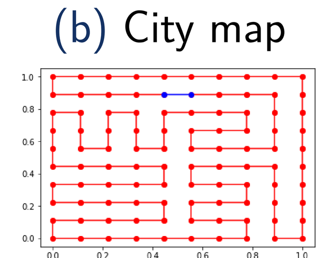
$$\Delta\tau_{ij}^{k} = \begin{cases} Q/L_k, & \text{if ant k used edge (i,j) in its tour,.} \\ 0, & \text{otherwise.} \end{cases} \qquad (4)$$



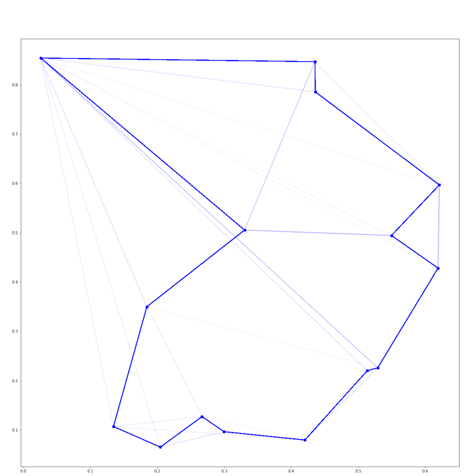
(a) Utilization of roads



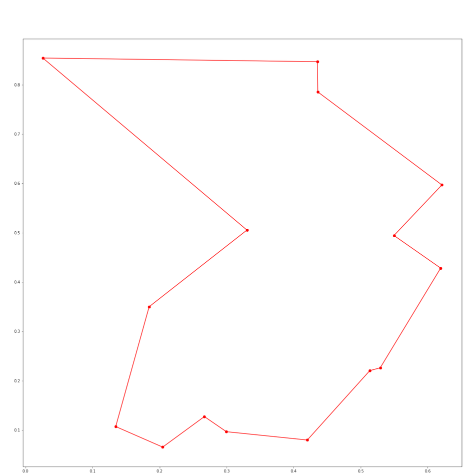
(b) City map



(c) Shortest tour



(a) Utilization of roads



(b) Shortest tour

Road selection becomes more stable as evaporation rate decreases. All ants head towards the shortest path as α/β increases.

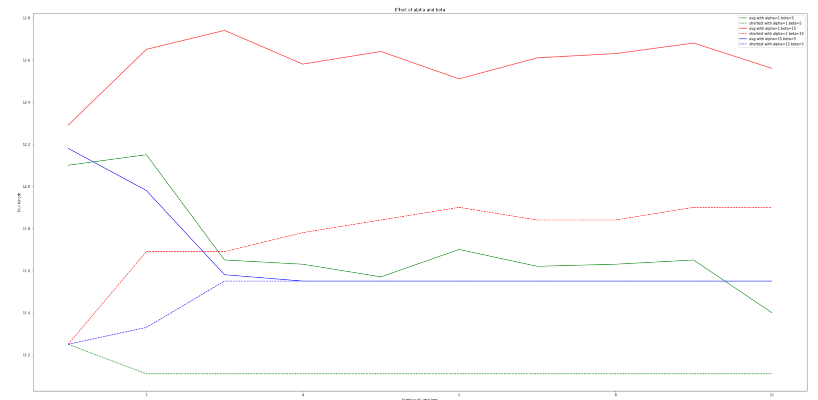


Figure: Effect of alpha and beta

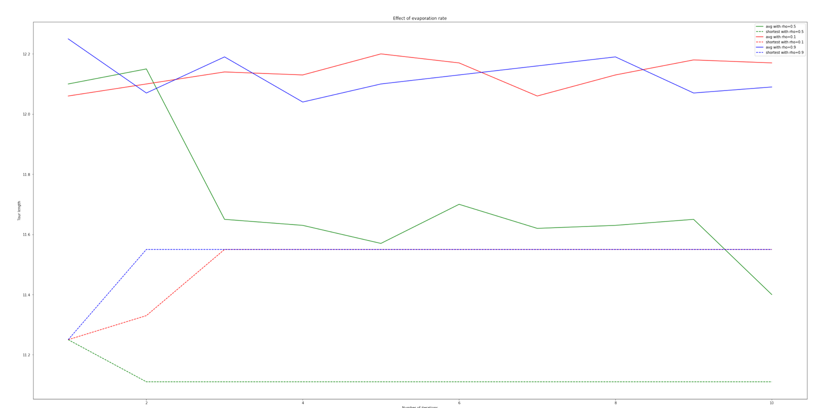


Figure: Effect of evaporation rate

SIMULATED ANNEALING

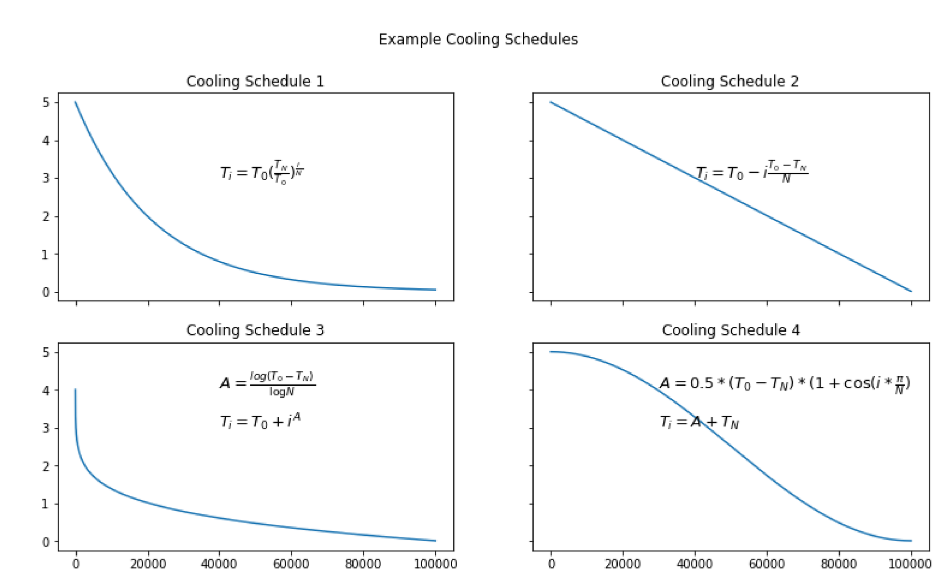


Figure: Different Cooling Schemes

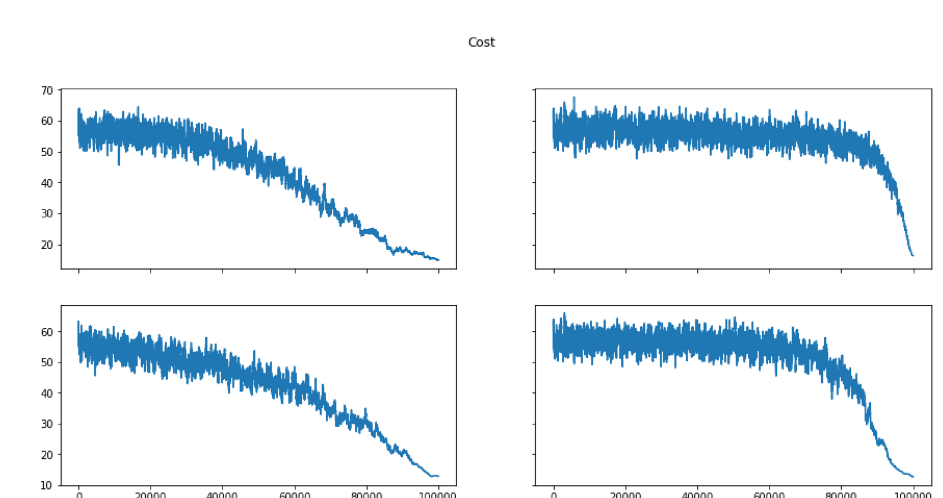


Figure: Corresponding Tour Cost Graphs

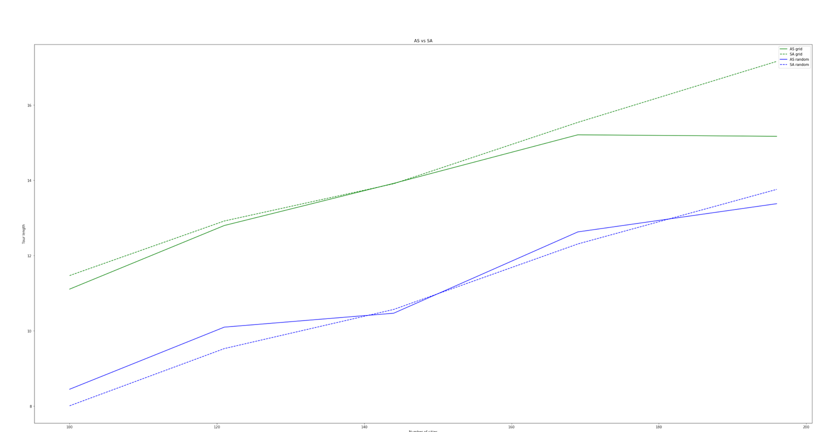


Figure: ACO vs Simulated Annealing

PARTICLE FILTER WITH ACO

- Particle Impoverishment
- Diversity Loss

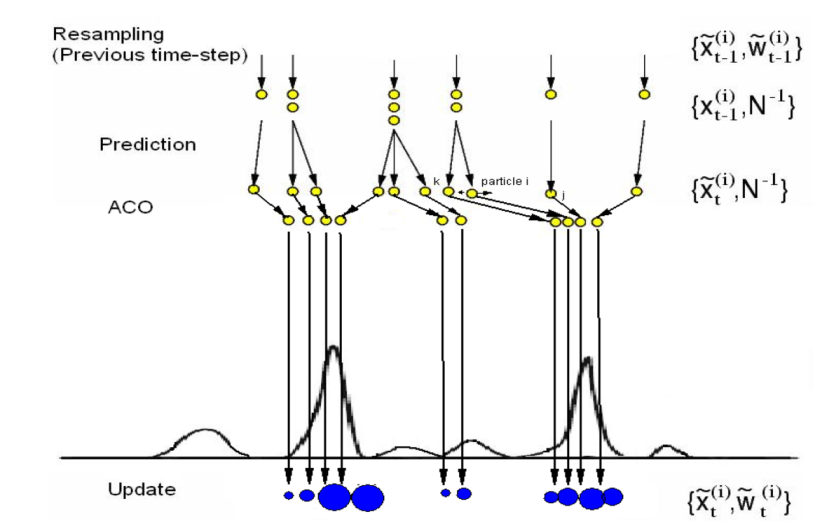


Figure: Mechanism of PF with ACO [2]

$$\tau_{ij}(t+1) = \begin{cases} (1-\rho)\tau_{ij}(t) + \Delta v, & \text{if } j \text{ is target particle.} \\ (1-\rho)\tau_{ij}(t), & \text{otherwise.} \end{cases}$$

$$x(t+1) = 0.5 * x(t) + \frac{25 * x(t)}{1 + x(t)^2} + \cos(1.2 * t) + w$$

$$z(t) = x(t)^2 * 0.05 + r$$

$$w \sim \mathcal{N}(0, 0.01) \quad r \sim \mathcal{N}(0, 0.1)$$

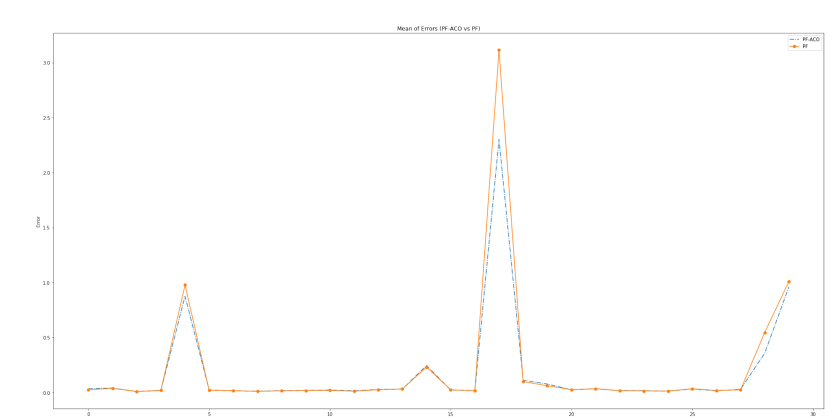


Figure: PF vs PF with ACO MSE

github.com/claudiusalp/MonteCarloProject-ACO

References

[1] Brittari M. Dorigo M. and Stutzle T.
Ant colony optimization.
IEEE Computational Intelligence Magazine, 2006.

[2] Junpei Z. and Yu-Fai F.
Case study and proofs of ant colony optimisation improved particle filter algorithm.
IET Control Theory and Applications, 2012.