AICC 2 - Bixio Rimoldi

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1 Week 2

We begin by defining **entropy** as Shannon put it:

Definition 1. ***Entropy***
Note that this definition assumes base 2 aka. binary.

$$H(s) = -\sum_{s \in A} p(s) \log_2 p(s) \text{ } \textit{A being our alpahabet aka. sample space}$$

and thus an equivalent definition is:

$$H(s) = E[-\log_2 p(s)]$$

And similarly, Shannon defines information as:

$$-\log_2 p(s)$$

For a random distribution we get:

Example 1.1.

$$\forall x \in A \; p(x) = \frac{1}{|A|}, \; -\log_2 p(s) = \log_2 |A|$$

Hence the entropy function $H(s) = E[\log_2 |A|] = \underbrace{\log_2 |A|}_{\textit{do the algebra}}$

And now we present the **information theory inequality**

Definition 2. ***IT inequality***

$$\log_b r \leq (r-1) \log_b(e)$$

Proof. Given that

$$\ln(r) \leq (r-1)$$

and that

$$ln(r) = \frac{\log_b(r)}{\log_b(e)}$$

we are done. □

And now we present the Entropy bound theorem:

Theorem 1.1.

$$S \in A \; 0 \leq H(S) \leq \log |A|$$

Proof. We only show the RHS as the LHS is more or less trivial. Our goal is to show:

$$\begin{aligned} \text{need to reach} H(s) - \log |A| \leq 0 \\ E[-\log p(s)] - \log |A| \\ = E[\log \frac{1}{p(s)|A|}] \\ = \sum_{s \in A} p(s) (\log \frac{1}{p(s)|A|}) \\ \underbrace{\leq log(e) \sum [\frac{1}{|A|} - p(s)]}_{\text{using IT ineq.}} = 0 \end{aligned}$$

□