Analysis 3

Week 1: Differential operators

Gradient: For Ω open, $f : \Omega \to \mathbb{R}$ is defined as:

$$\nabla f(x) = (\frac{\partial f}{\partial x_1}, ...)$$

Divergence: For Ω open, $f : \Omega \to \mathbb{R}^n$ is defined as:

$$divF(x) = (\nabla \cdot F)(x) = \frac{\partial F_1}{\partial x_1} + ... + \frac{\partial F_n}{\partial x_n}$$

Rotational: Let $F : \Omega \to \mathbb{R}^n$

if $n = 2$ then:

$$rotF(x, y) = \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}$$

if $n = 3$ then:

$$rotF(x, y) = (\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y})$$

The best way to remember the formula for the case $n = 3$ is by using the determinant formula for the following matrix:

$$\begin{bmatrix} e_1 & e_2 & e_3 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{bmatrix}$$

Laplacian: Let $F : \Omega \to \mathbb{R}$, then:

$$lap(f) = \Delta f = div(grad(f)) = \frac{\partial F_1^2}{\partial {x_1}^2} + ... + \frac{\partial F_n^2}{\partial {x_n}^2}$$

If $\Delta f = 0$ then f is harmonic.

Important result: Let $\Omega \subset \mathbb{R}^n$ and f a scalar map with $f \in C^2$ and $F : \Omega \to \mathbb{R}^n$ with $F \in C^2$ then:

1. $div\ gradf = \Delta f$
2. for $n = 2$, $rot\ gradf = 0$
3. for $n = 3$, $rot\ gradf = \vec{0}$

Week 2: Line integrals, Greens theorem

$R \subset \mathbb{R}^n$ is a simple regular curve if there exists an interval $[a, b] \subset \mathbb{R}$ and a function $f : [a, b] \to \mathbb{R}^n$ such that:

1. $R = f([a, b])$
2. r is injective on $[a, b[$
3. $r \in C^1$
4. $||r'(t)|| \neq 0$

If the first two properties hold, the curve is said to be *simple*. If the two start and endpoints meet, the curve is said to be *closed*.

Some visuals below: