GENERAL PHYSICS 2

Week 3

Electric field distribution on surfaces

Granted that an electric field is simply

$$E = \frac{F}{Q_t}$$

A single point charge will induce an electric field of size

$$\vec{E} = \frac{Q}{4\pi\epsilon r^2}$$

Similarly, for a collection of point charges, the electric field becomes the vector sum of the single E_i

The rod

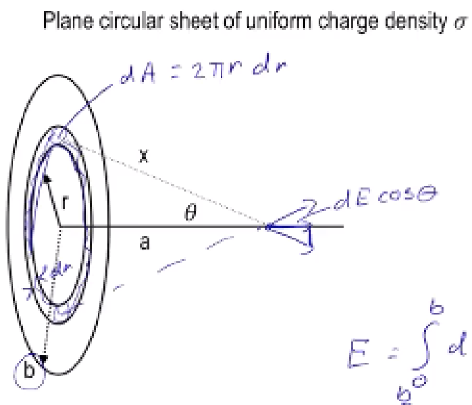
An recurent problem is the electric field from continous charge distribution on a rod of length L. This is given by:

$$F = \frac{Q\lambda L}{4\pi\epsilon_0 x\sqrt{x^2 + (\frac{L}{2}^2)}}$$

Plane circular sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

Here is the derivation of the above:



dE from dA

contributions ⊥ to a cancel

→ projection $dE\cos\theta$

$$\left(\frac{dF}{dq}\right) = \frac{\sigma\, 2\pi r\, dr \cos\theta}{4\pi\varepsilon_0 x^2} = dE$$

$$\cos\theta = \frac{a}{x}$$

$$dE = \frac{\sigma a r\, dr}{2\varepsilon_0 x^3}$$

$$E = \int_0^b dE \qquad x^2 = a^2 + r^2$$

$$E = \frac{\sigma a}{2\varepsilon_0}\int_0^b \frac{r\, dr}{(a^2+r^2)^{3/2}} \qquad r\, dr = \frac{d(a^2+r^2)}{2}$$

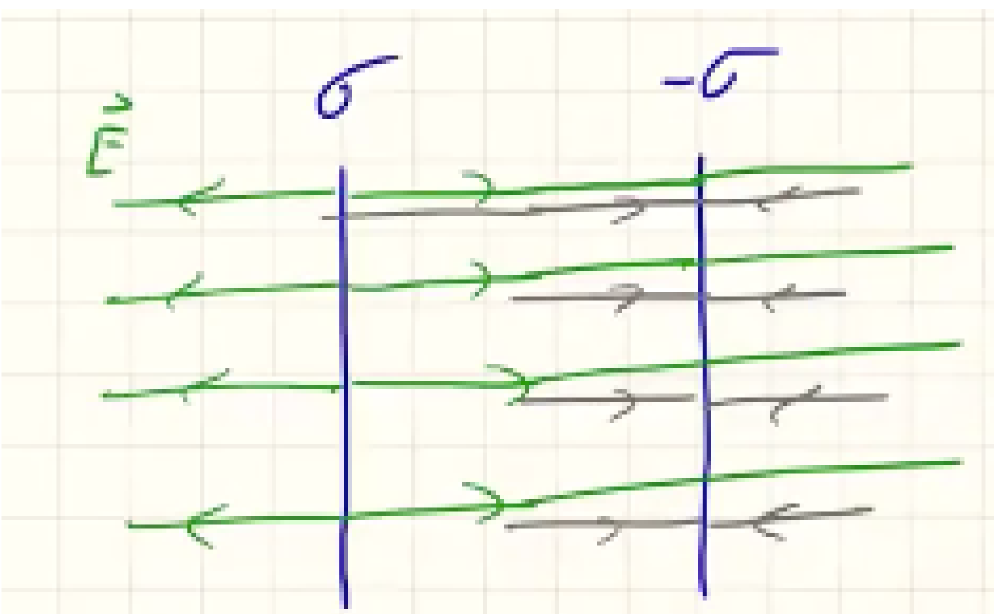
$$\Rightarrow E = \frac{\sigma a}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\sqrt{a^2+b^2}}\right)$$

$b \to \infty$ $(b >> a)$ $\quad E = \frac{\sigma}{2\varepsilon_0}$ independent of a !

Note that the given formula holds only when $b >> a$, that is we are significantly close to the plate which means that the most RHS term in the derivation will go to 0.

Homogenous field around two plates

Imagine we put two plates, of the same charge but opposite signs. Ie. something like this:



Now in between the fields, the total field is simply the sum of the two plates which is

$$E_{\text{between}} = \frac{\sigma}{\epsilon_0}$$

And outside of the plates, if we observe the vector lines, the fields cancel each other out, resulting in

$$E_{\text{outside}} = 0$$

Electric potential

Let's recall some definitions:

E-field: Force per test charge, vector field
E-potential: Potential energy per test charge, scalar field

More formally, potential from point A to B is

$$V_{AB} = \frac{W_{AB}}{Q_t}$$

Where the W_{AB} in the above is defined as:

$$W_{AB} = \int_A^B -\vec{F} \cdot d\vec{L} = \int_A^B -\vec{Q}_t \vec{E} \cdot d\vec{L} \rightarrow V_B = \int_A^B -\vec{E} \cdot d\vec{L}$$

An **important** point is that the potential is **independent** of path taken.

Using the integral definition of V_p we have that for a point charge

$$V_P = \int_\infty^P -\frac{Q\vec{r} \cdot d\vec{L}}{4\pi\epsilon_0 r^2}$$