GENERAL PHYSICS 2

1. Week 3

1.1. Electric field distribution on surfaces

Granted that an electric field is simply

$$E = \frac{F}{Q_t}$$

A single point charge will induce an electric field of size

$$\vec{E} = \frac{Q}{4\pi\epsilon r^2}$$

Similarly, for a collection of point charges, the electric field becomes the vector sum of the single E_i

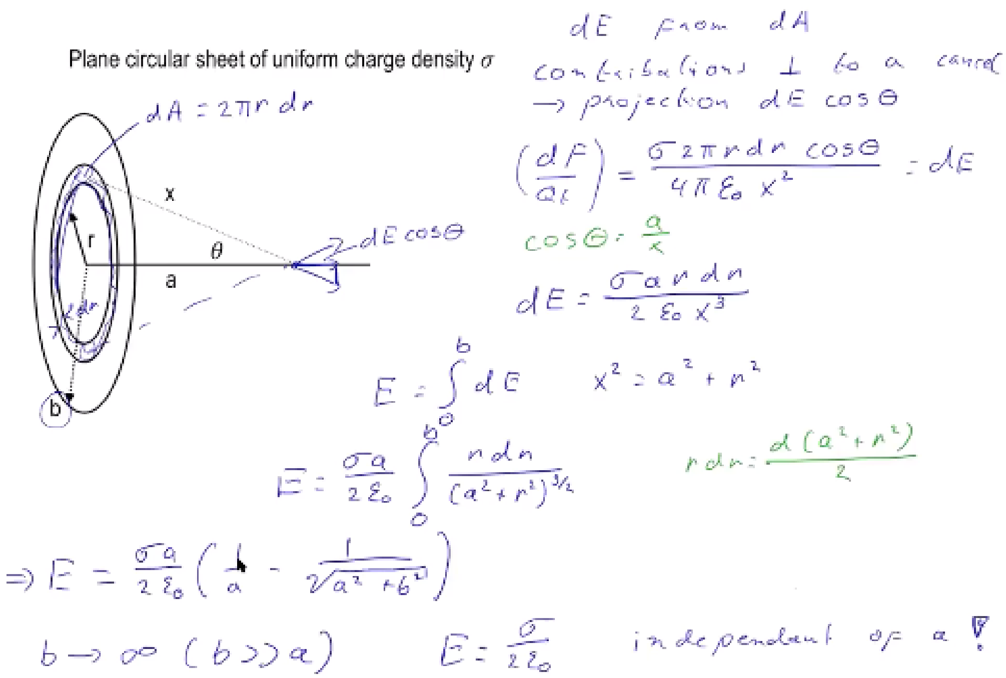
The rod
An recurent problem is the electric field from continous charge distribution on a rod of length L. This is given by:

$$F = \frac{Q\lambda L}{4\pi\epsilon_0 x\sqrt{x^2 + (\frac{L}{2}^2)}}$$

Plane circular sheet

$$E = \frac{\sigma}{2\epsilon_0}$$

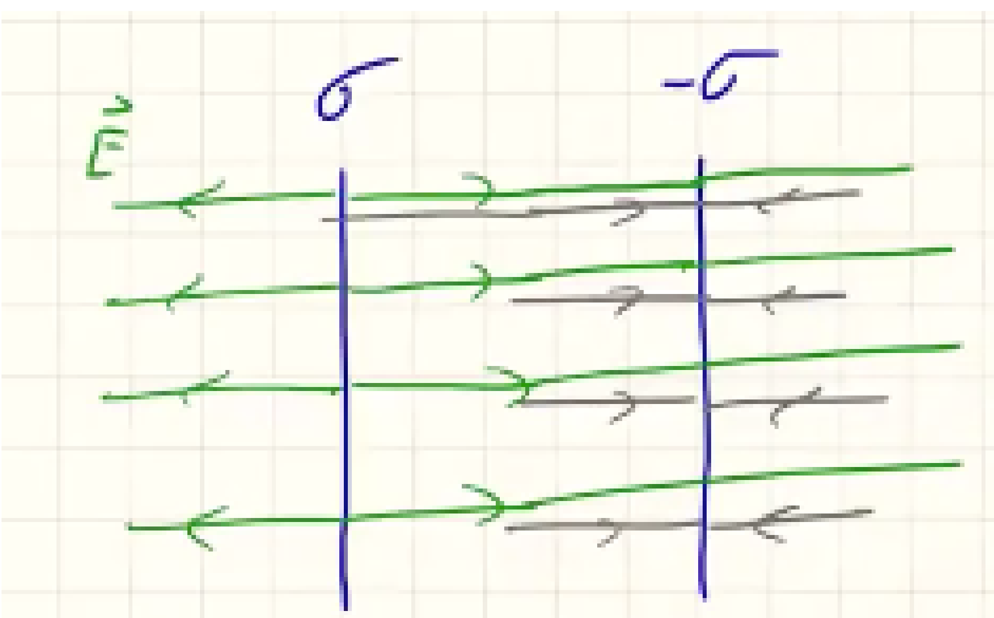
Here is the derivation of the above:



Note that the given formula holds only when $b >> a$, that is we are significantly close to the plate which means that the most RHS term in the derivation will go to 0.

Homogenous field around two plates

Imagine we put two plates, of the same charge but opposite signs. Ie. something like this:



Now in between the fields, the total field is simply the sum of the two plates which is

$$E_{\text{between}} = \frac{\sigma}{\epsilon_0}$$

And outside of the plates, if we observe the vector lines, the fields cancel each other out, resulting in

$$E_{\text{outside}} = 0$$

Electric potential

Let's recall some definitions:

E-field: Force per test charge, vector field
E-potential: Potential energy per test charge, scalar field

More formally, potential from point A to B is

$$V_{AB} = \frac{W_{AB}}{Q_t}$$

Where the W_{AB} in the above is defined as:

$$W_{AB} = \int_A^B -\vec{F} \cdot d\vec{L} = \int_A^B -\vec{Q}_t \vec{E} \cdot d\vec{L} \rightarrow V_B = \int_A^B -\vec{E} \cdot d\vec{L}$$

An **important** point is that the potential is **independent** of path taken.

Using the integral definition of V_p we have that for a point charge

$$V_P = \int_\infty^P -\frac{Q\vec{r} \cdot d\vec{L}}{4\pi\epsilon_0 r^2}$$

Similarly, the potential of a collection of point charges is

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{Q_i}{r_i}$$

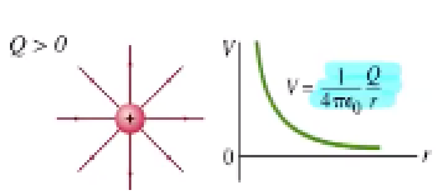
Electric potential around uniform charged disk

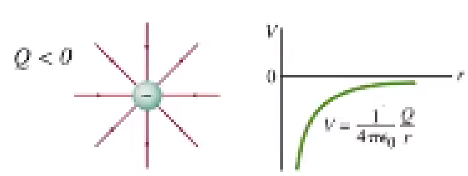
Returning to the disk from earlier on, the expression for potential becomes

$$V = \frac{\sigma}{2\pi\epsilon_0}(\sqrt{a^2 + b^2 - a})$$

Now we let b go to infinity, well shouldnt the potential then go to infinity? The problem with this approach is that when b goes to infinity, it will almost start touching our object which is a problem.

And here is how potential varies for a positive and negative charge:





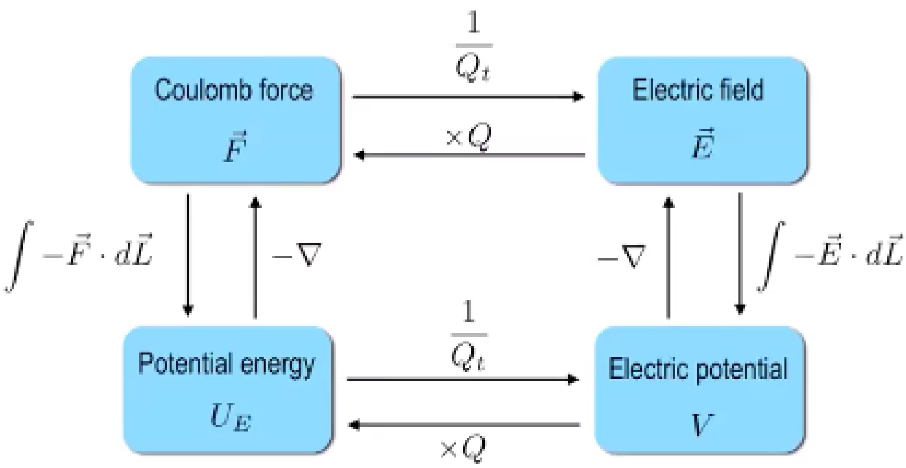
Obtaining E-field from V

$$\vec{E}(\vec{r}) = -\nabla V(x, y, z)$$

Finding force from E-field:

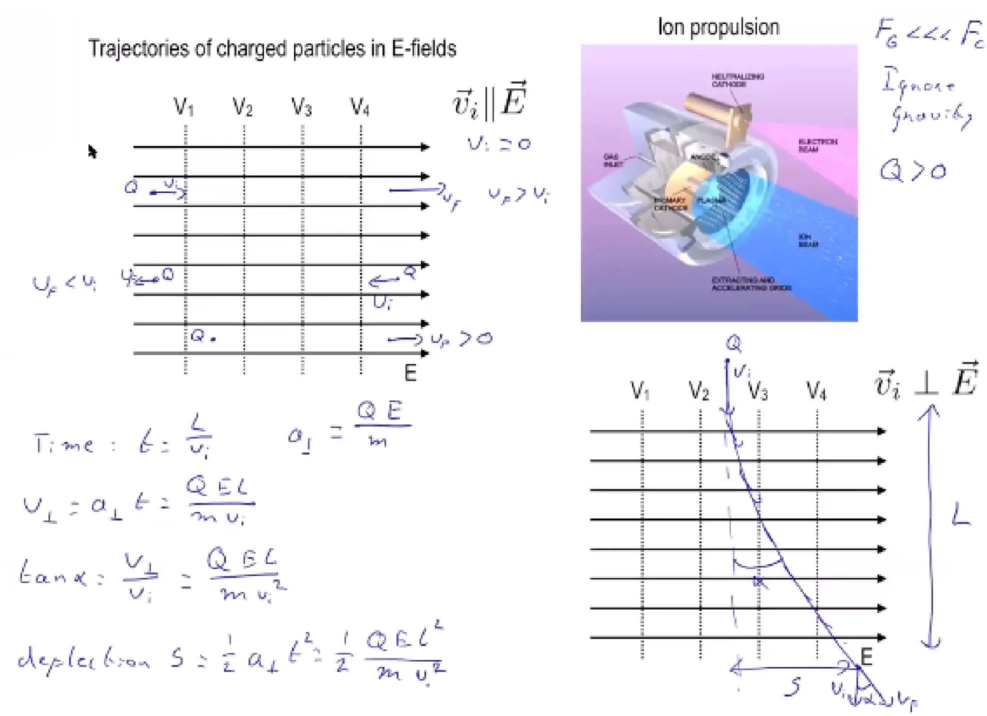
$$\vec{F} = Q\vec{E} = Q(V_B - V_A)$$

Week 4



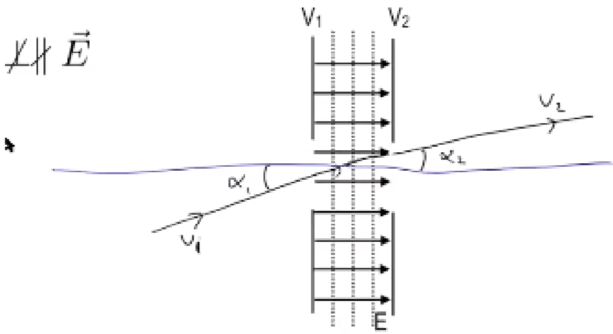
We introduce the **Electronvolt** which is energy of e^{-1} passing $1V$ of potential difference.

Now let's analyse the kinematics of a charged particle in an electric field:



A more interesting case is a charged particle entering an electric field at an angle for which we have the following result

$\vec{v}_i \not\parallel \vec{E}$

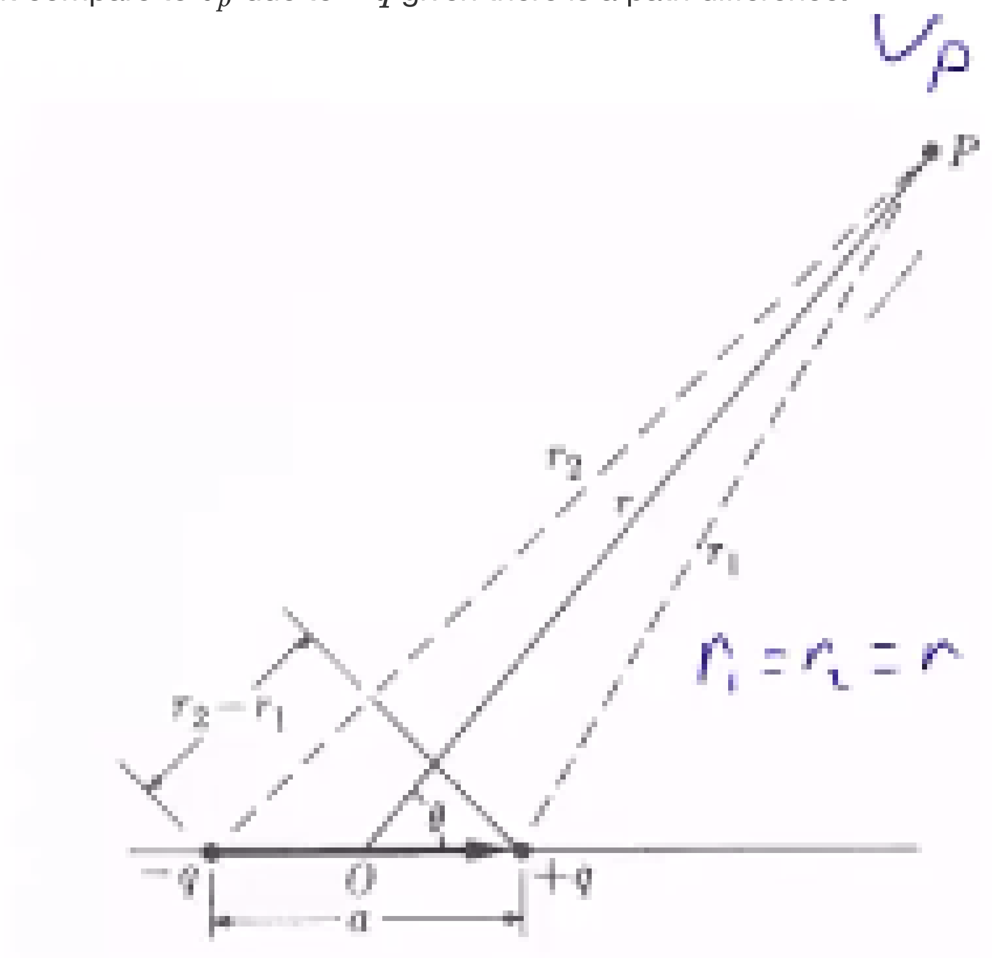


$$\frac{\sin\alpha_1}{\sin\alpha_2} = \frac{v_2}{v_1} = \sqrt{\frac{V_2}{V_1}}$$

Electric dipole

An electric dipole is a two-point charge with opposite charges such that $\sum Q = 0$ We have that $+q$ and $-q$ are at a distance a. An $ideal$ dipole is one where a is small compared to other distances.

The *dipole moment* is $\vec{p} = q\vec{a}$. Now a natural question to ask is, what is v_p due to $+q$ and how does it compare to v_p due to $-q$ given there is a path difference.



We have:

$$v_q = \frac{q}{4\pi\epsilon_0 r}$$

and also

$$v_{-q} = -v_q - dv_q$$

Thus the net v_p at the point p becomes $v_q + v_{-q} = -dv_q$ where $-dv_q$ is the distance over which potential has changed multiplied by the rate of change which is

$$-dv_q = -a\frac{\partial \frac{q}{4\pi\epsilon_0 r}}{\partial x}$$

Taking everything not depending on x outside we obtain

$$v_p = -\frac{q}{4\pi\epsilon_0}\frac{\partial \frac{1}{r}}{\partial x} = \frac{q}{4\pi\epsilon_0}\frac{\partial r}{\partial x}$$

which simplifies to

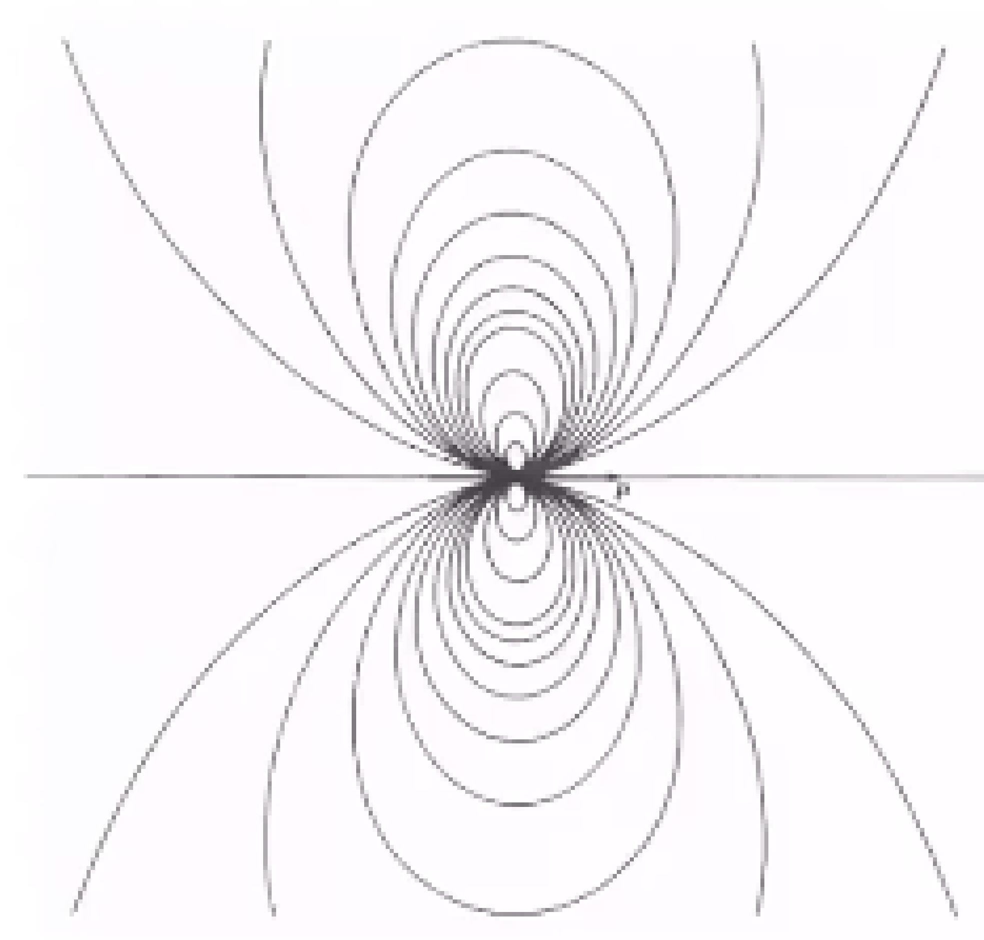
$$\frac{qa\cos\theta}{4\pi\epsilon_0 r^2}$$

Which in terms of the dipole moment becomes

$$\frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^2}$$

The last formula gives the *potential of a dipole*

The electric field of a dipole then looks like



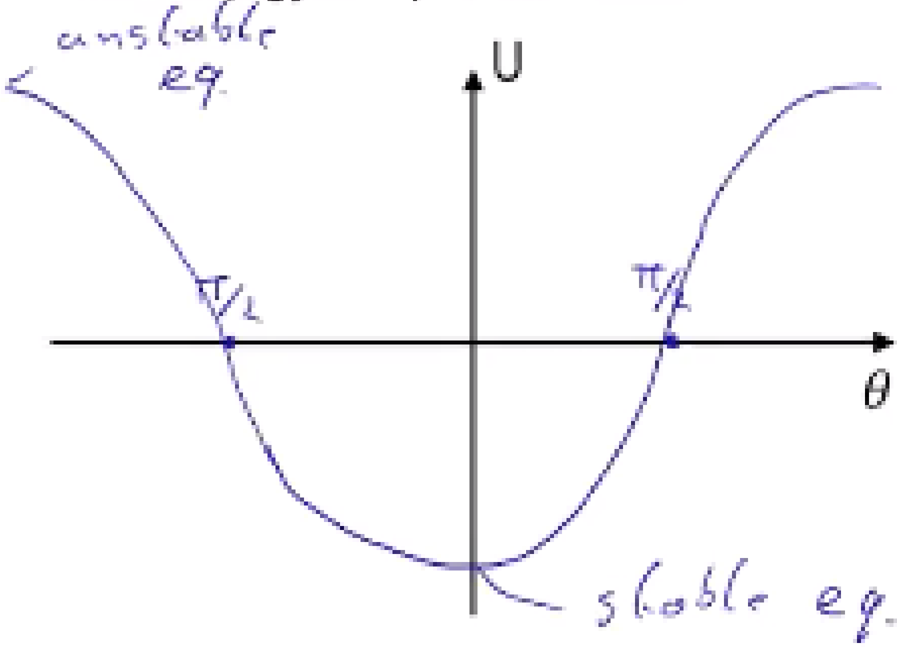
where

$$|E_\theta| = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$$

$$|E_r| = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}$$

Ofcourse the total electric field is the vector sum of the above two expressions.

Potential energy of dipole in E-field



Multipoles

	Potential V ∝	E Field ∝
Monopole	$\frac{Q}{r}$	$\frac{Q}{r^2}$
Dipole	$\frac{p}{r^2}$	$\frac{p}{r^3}$
Quadrupole	$\frac{q}{r^3}$	$\frac{q}{r^4}$