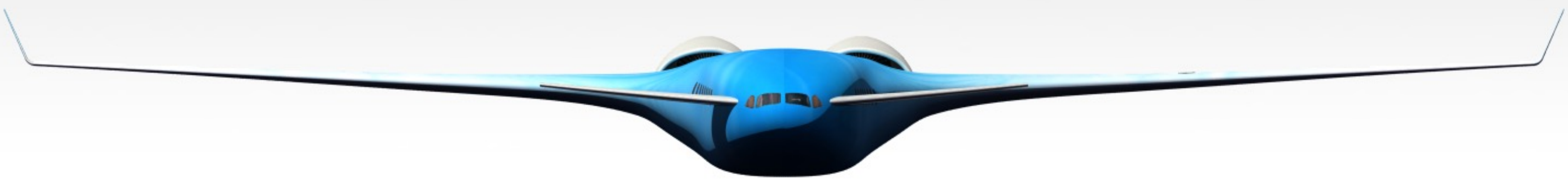

AE4238: Aero Engine Technology Tutorial: cycle calculation

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Outline of the tutorial

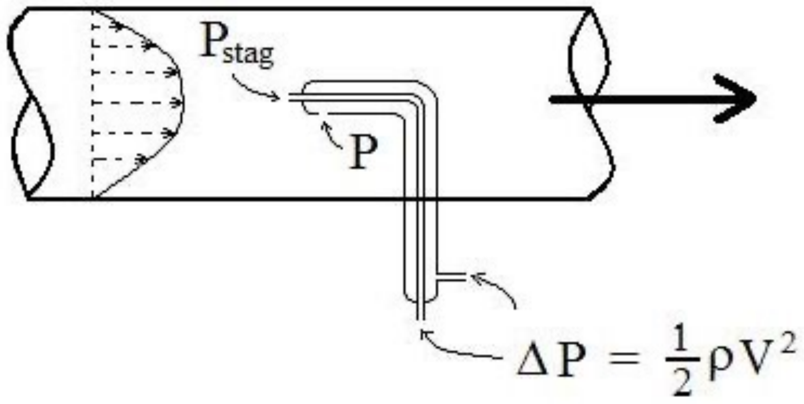
- Single-shaft turboprop
- Twin-spool turbofan (in-class exercise)

General Procedure

- Basic idea is to mathematically model all the processes of the engine's thermodynamic cycle using the known data for the engine considering real-Brayton case.
- Necessary simplification may be made as and when they are required
- The following are the main steps involved:
  - Calculate the parameters at the inlet of the engine
  - Model the compression process assuming adiabatic compression
  - The heat addition process can be analysed from the turbine inlet conditions
  - Like compression, expansion in a turbine can also be modelled as adiabatic expansion
  - Afterburner (if present) can be treated similarly to the main heat addition process
  - Lastly evaluating the expansion in the exit nozzles will yield the exhaust parameters
  - Thrust and other performance indicators can be evaluated once the exhaust parameters are known

Note

- Total properties *(T_t, P_t)* are used to account for the fluid's motion during the whole cycle calculation. Static properties are only used at stations inlet of the engine (station 1) and exit of the nozzle (station 8) to compute the thrust



- Nomenclature note: total properties can be noted with the sub-index *t* or *0*

Useful formula

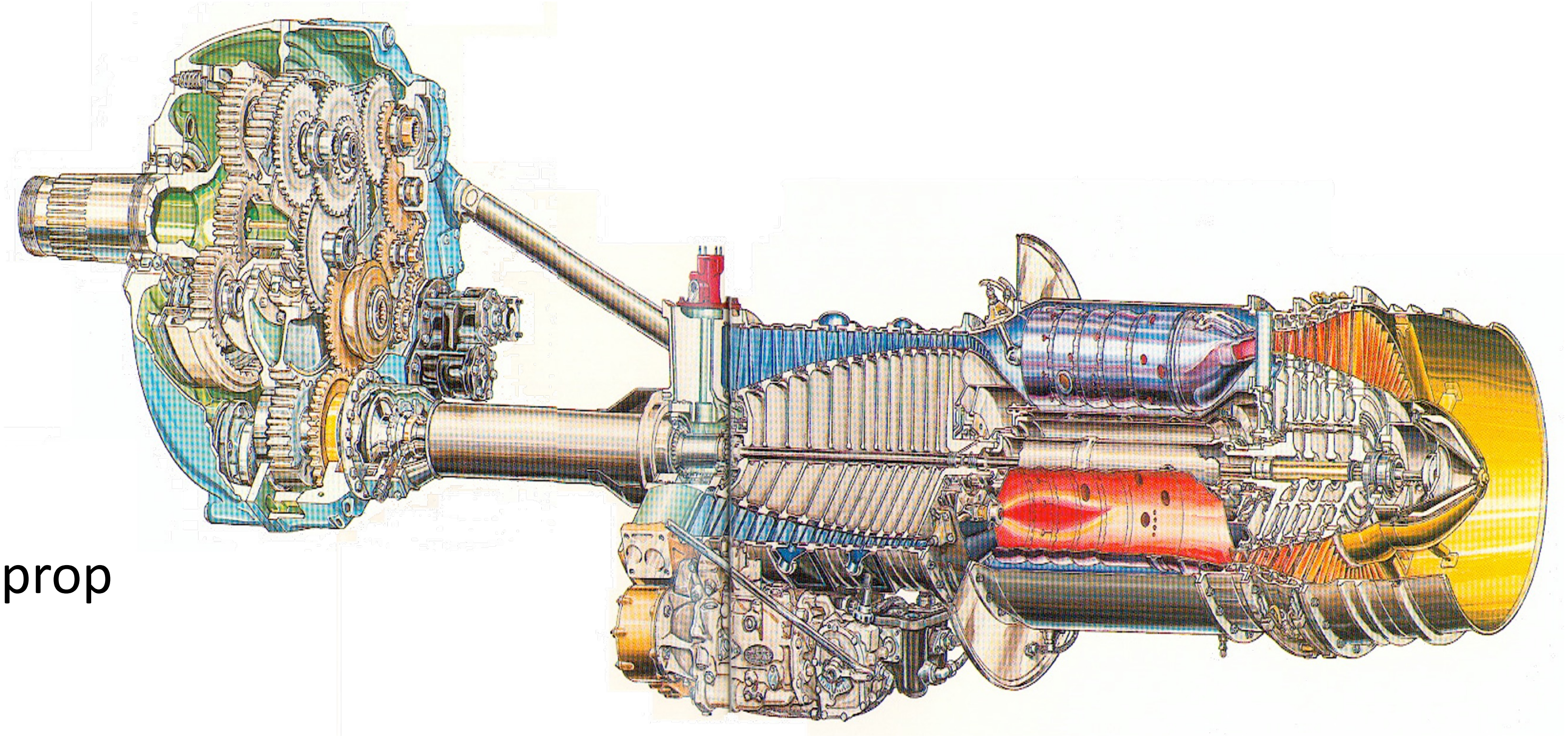
- Thrust: $F = \dot{m}(v_j - v_0)$
- Specific fuel consumption: $\mathrm{SFC} = \frac{\dot{m}_f}{F}$
- Nozzle critical pressure ratio

$$p_{t,7} / p_{critical} = \left[\frac{1}{\left(1 - \left(\frac{1}{\eta_j}\right) \cdot \left(\frac{\kappa_g - 1}{\kappa_g + 1}\right)\right)^{\frac{\kappa_g}{\kappa_g - 1}}} \right]$$

Example of a single shaft turboprop engine

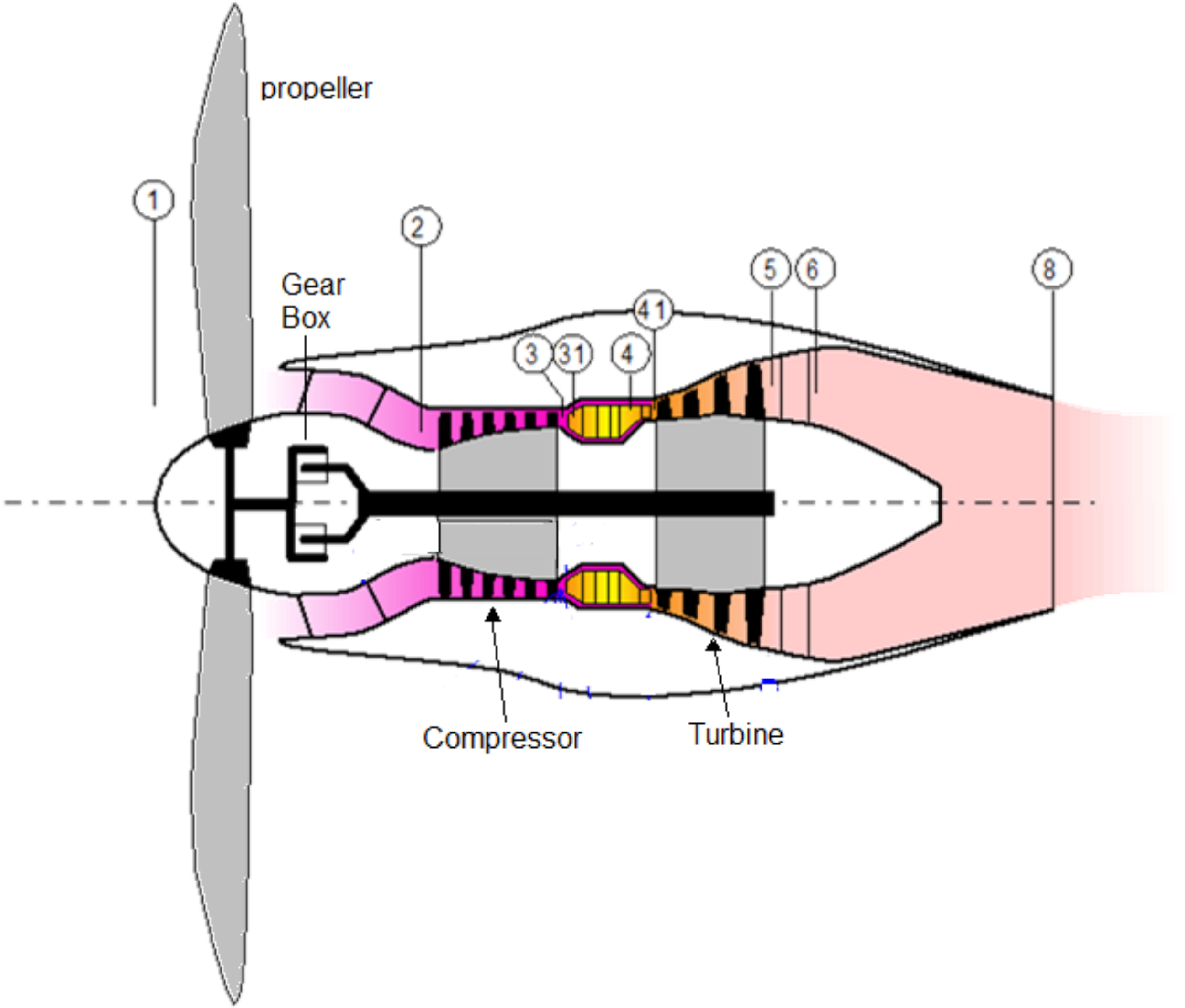
E-2C Hawkeye

Rolls Royce T56-A Series IV



Single shaft turboprop
Length: 3.71 m
Diameter: 0.69 m
Basic weight: 1940 lb
Power-to-weight ratio: 2.75:1

Engine Nomenclature



T56-A Series IV Specifications

General characteristics at SL, ISA take off conditions, Mach = 0.5

- Compressor= 14 stages axial flow
- Combustors= 6-cylindrical flow-through
- Turbine= 4 stages axial flow
- Nozzle= Convergent
- Shaft power = 5250 shp (3.91MW)
- Compressor Pressure Ratio = 11.5
- Combustor Exit Temperature (T4) = 1130K
- Engine mass flow rate= 35 kg/s
- Intake Pressure ratio= 1.0

Calculate pressure, temperature and mass flow at every station. Calculate the total thrust

- Propeller efficiency = 0.9
- Compressor isentropic efficiency = 0.85
- Turbine isentropic efficiency = 0.89
- Mechanical efficiency = 0.99
- Combustion efficiency = 0.995
- Combustion chamber Pressure Ratio = 0.96
- Nozzle efficiency = 0.95
- C_{Pair} = 1000 J/kg-K; kappa air = 1.4
- C_{Pgas} = 1150 J/kg-K; kappa gas = 1.33
- Gas constant= 287 J/kg K
- Fuel calorific value = 42.8 MJ/kg
- Ambient Pressure = 101325 Pa
- Ambient Temperature = 288.15K

T56-A Series IV Engine

T_a = 288,15 K, p_a = 101325 Pa

Mach = 0.5

$$T_{0a} = T_a \cdot \left(1 + \frac{\kappa_a - 1}{2} \cdot M_a^2\right) = 302.56K$$

$$p_{0a} = p_a \cdot \left(1 + \frac{\kappa_a - 1}{2} \cdot M_a^2\right)^{\frac{k_a}{k_a - 1}} = 120193.00Pa$$

$$V_0 = M_a \cdot \sqrt{\kappa_a \cdot R \cdot T_a} = 170.13m/s$$

Inlet pressure ratio is 1.0

Thus $p_{02} = p_{0a}$ = 120193 Pa ; $T_{02} = T_{0a}$ = 302.56K

T56-A Series IV Engine

Compressor

$p_{03} = p_{02}$*11.5 = 1382219.45Pa;

T_{03} = 661.84K; $m_{dot,3}$= 35 kg/s

$$\frac{T_{0,3}}{T_{0,2}} = 1 + \frac{1}{\eta_{is}}\left[\left(\frac{p_{0,3}}{p_{0,2}}\right)^{\left(\frac{\kappa_a - 1}{\kappa_a}\right)} - 1\right]$$

Combustion Chamber

T_{04} = 1130 K (given);

$m_{dot,fuel}$ = 0.4425 kg/s

p_{04} = 0.96*p_{03} = 1326930.67 Pa = 1326.93 kPa

$m_{dot,4} = m_{dot,3} + m_{dot,fuel}$ = 35.4425 kg/s

$$\dot{m}_{fuel} = \frac{\dot{m}_3 \cdot C_{Pgas} \cdot \Delta T}{LHV \cdot \eta_{cc}}$$

T56-A Series IV Engine

Work done on Compressor

$$W_c = \dot{m}_3 \cdot Cp_{air} \cdot (T_{03} - T_{02}) = 35 * 1000 * (661.84 - 302.56) = 12574800W$$

Turbine

$$T_{04} - T_{05} = (W_C + Wprop) / (\dot{m}_4 \cdot C_{p\,gas} \cdot \eta_{mech}) = 408.65K$$

$$T_{05} = 1130 - 408.65 = 721.35K$$

$$p_{05} = 158238.59Pa = 158.239kPa$$

$$\boxed{\frac{T_{05}}{T_{04}} = 1 - \eta_{is,T}\left[1 - \left(\frac{p_{05}}{p_{04}}\right)^{\left(\frac{\kappa_g - 1}{\kappa_g}\right)}\right]}$$

T56-A Series IV Engine

Nozzle

$T_{t8} = T_{t5}$=721.35 K

$P_{t8} = P_{t5}$=158239 Pa

$$E_{cr} = p_{05} / p_{critical} = \left[\frac{1}{\left(1 - \left(\frac{1}{\eta_{noz}} \right) \cdot \left(\frac{\kappa_g - 1}{\kappa_g + 1} \right) \right)^{\frac{\kappa_g}{\kappa_g - 1}}} \right] = 1.916$$

Check if the nozzle is choked?

$$p_{05} / p_a = 158238.59 / 101325 = 1.56 < E_{cr}$$

The nozzle is **NOT CHOKED**

<u>Note</u>: Since the nozzle is not chocked, the static values at station 8 depend on ambient conditions. Static temperature is calculated using the isentropic efficiency of an expansion (η_{nozzle}), being:

<u>Static pressure</u>:

$$p_{s8} = p_a = 101325 Pa$$

$$\frac{T_{s8}}{T_{t8}} = 1 + \eta_{nozzle} \cdot \left[\left(\frac{P_{s8}}{P_{t8}} \right)^{\frac{\kappa_g - 1}{\kappa_g}} - 1 \right]$$

<u>Static temperature</u>:

$$T_{s8} = 649.5\ K$$

Velocity:

$$v_8 = \sqrt{2 \cdot c_{p_g} \cdot (T_{t_8} - T_{s8})} = 406.51\, m/s \qquad (T_{t8} = T_{t_7})$$

Thrust core: $F_N = \dot{m}_8 \cdot (v_8 - v_0) = 8377.9N$

Propeller Thrust:

$$F_{prop} = \frac{\eta_p \cdot W_{prop}}{V_0} = 20710.21N$$

Total Thrust:

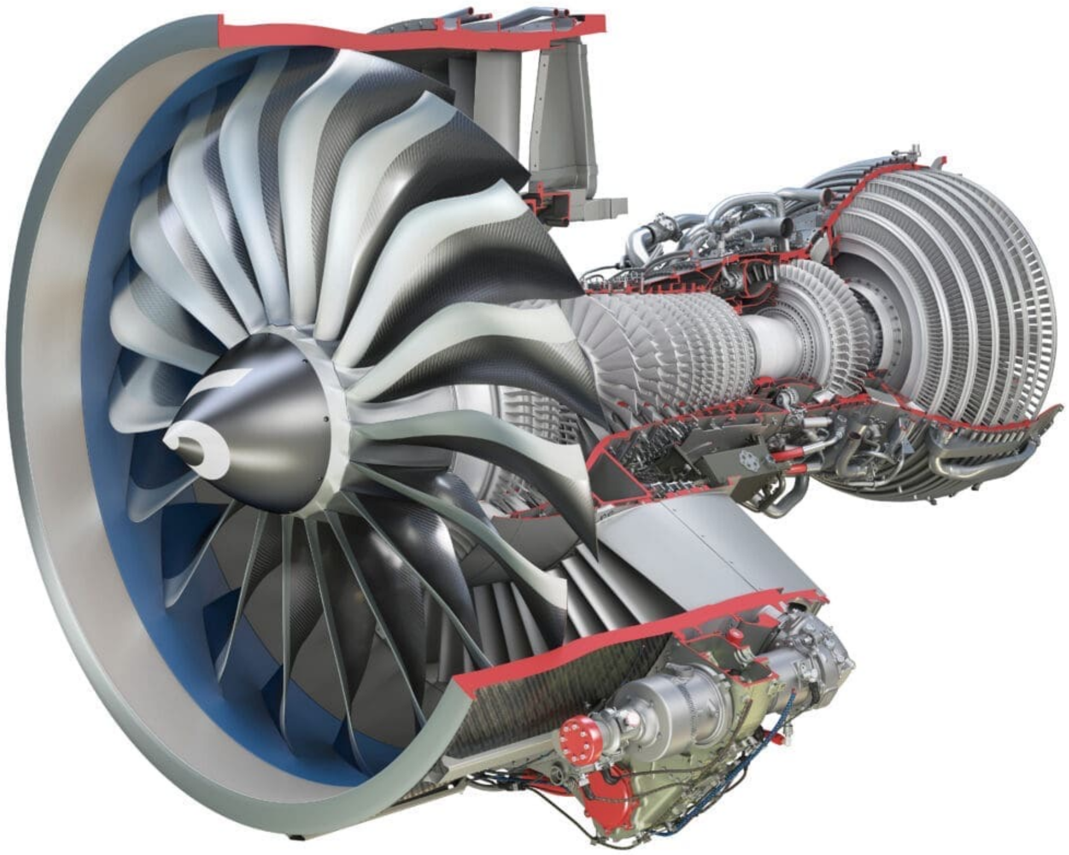
$$F_{total} = F_N + \mathrm{F_{prop}} = 8301.3 + 20710.2 = 29011.51\ \mathrm{N} = 29.012\ \mathrm{kN}$$

Example of a twin-spool unmixed turbofan engine

A320 neo

Figure source: [Airbus]

LEAP-1A turbofan engine

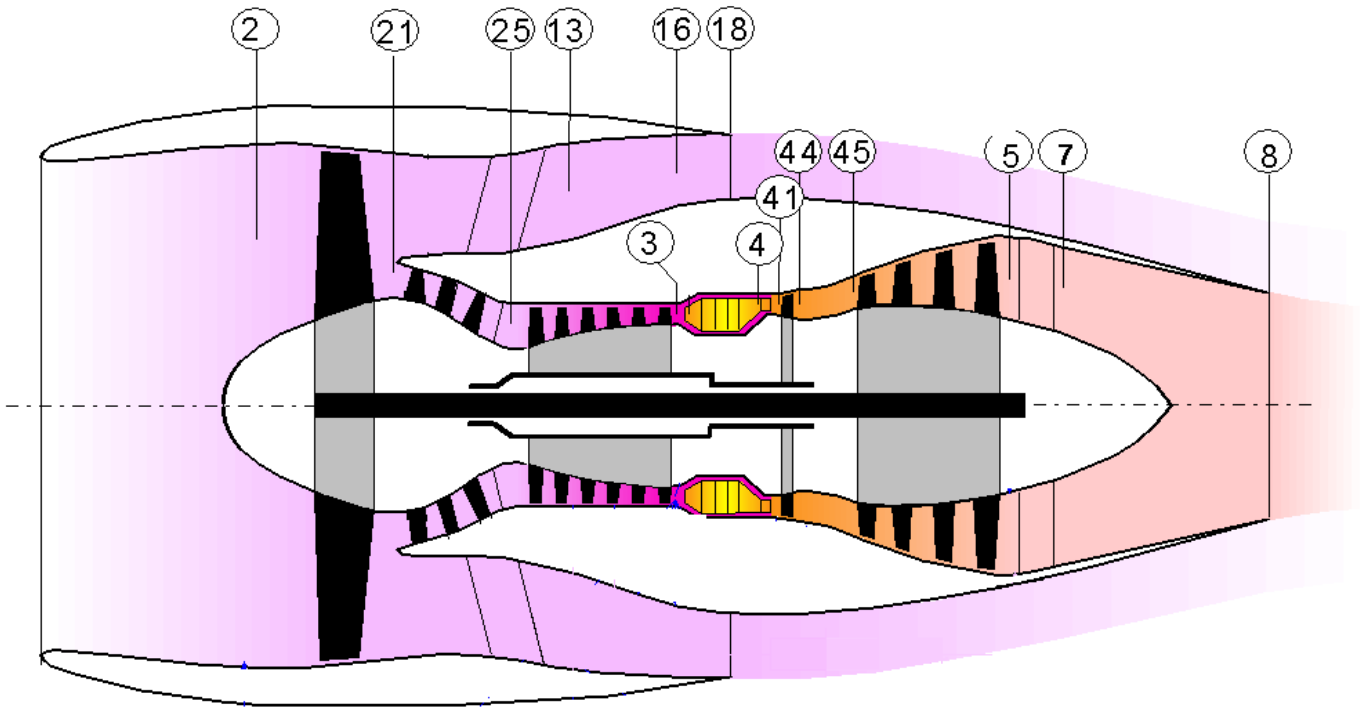


Twin spool turbofan:

- Compressor: 1 fan, 3-stage low pressure compressor, 10-stage high pressure compressor
- Turbine stages: 2-stage high pressure turbine , 7-stage Low pressure turbine
- Take off thrust: ~120 kN

Figure taken from: https://ses.aero/engines/

Turbofan Nomenclature



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LEAP-1A Turbofan

LEAP-1A at cruise condition(Mach = 0.78 Altitude = 10668 meter):

- Inlet pressure ratio = 0.98
- Engine air mass flow rate = 173 kg/s
- Bypass ratio = 12
- Fan Pressure Ratio = 1.4
- LPC Pressure Ratio = 1.7
- HPC Pressure Ratio = 12.5
- Combustor Exit Temperature (T4) = 1400 K
- Fan isentropic efficiency = 0.90
- LPC & HPC isentropic efficiency = 0.92
- LPT & HPT isentropic efficiency = 0.90
- Mechanical efficiency = 0.99
- combustor efficiency = 0.995
- combustor pressure ratio = 0.96
- Nozzle= Convergent
- Nozzle efficiency = 0.98
- Ambient Temp. = 218.8 K
- Ambient Press. = 23842 Pa
- Gas constant= 287 J/kg K
- Fuel calorific value (LHV) = 43MJ/kg
- $c_{P,air}$ = 1000 J/kg.K; k_{air} = 1.4
- $c_{P,gas}$ = 1150 J/kg.K; k_{gas} = 1.33

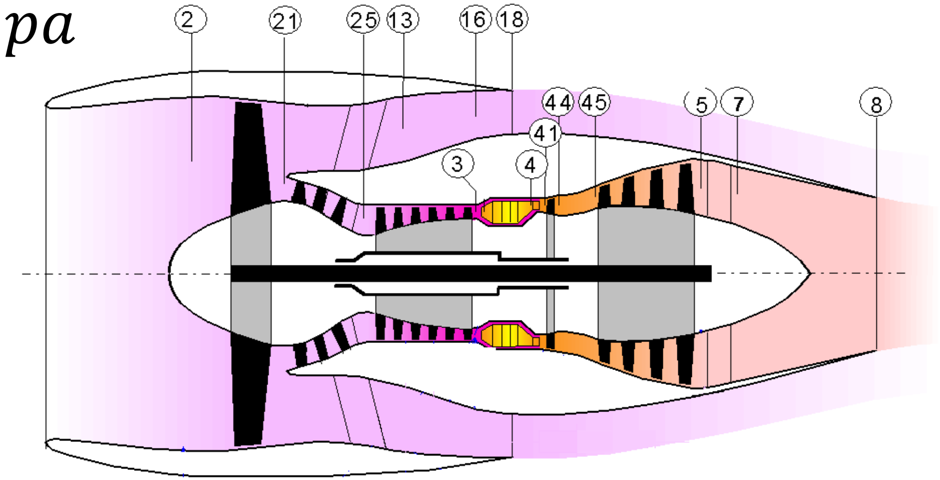
Calculate the thrust and specific fuel consumption!

$$\frac{T_t}{T} = 1 + \frac{\kappa_a - 1}{2} M^2$$

$$\frac{p_t}{p} = \left(1 + \frac{\kappa_a - 1}{2} M^2\right)^{\frac{\kappa}{\kappa - 1}}$$

LEAP-1A Turbofan: Inlet Conditions

- M = 0.78, p_a = 23842 Pa, T_a = 218.8 K
- Total conditions in the ambient: $T_{t,a} = 245.42K, p_{t,a} = 35635.6\, pa$
- Total temperature across the inlet remains constant, but for the pressure the 2% pressure loss needs to be accounted for.
- Thus, $T_{t,2} = 245.42\, K, p_{t,2} = 34922.89\, pa$



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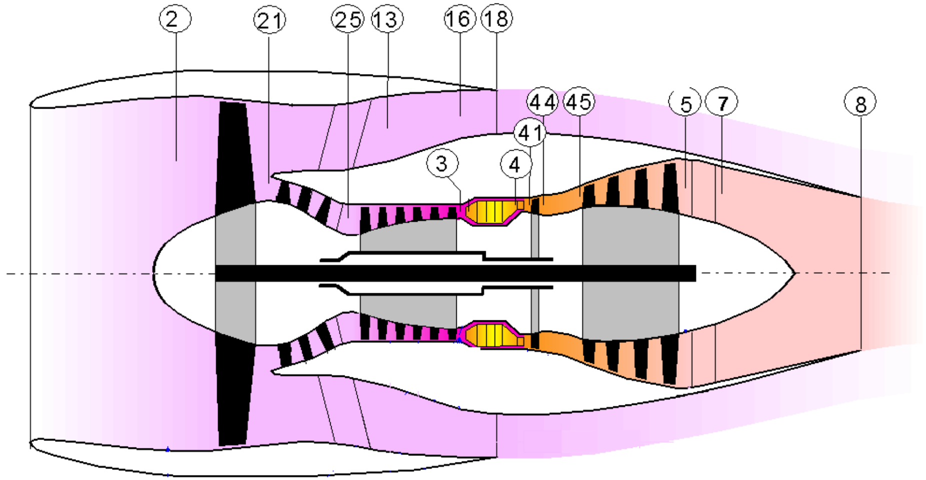
$$BPR = m_{bypass}/mcore$$

LEAP-1A Turbofan: Inlet Conditions

- Inlet mass flow rate: $\dot{m} = 173\ kg/s$
- Bypass ratio BPR = 12 (given)
- $\dot{m}_{core} = \dot{m}_{21} = 13.31\ kg/s$
- $\dot{m}_{bypass} = \dot{m}_{13} = 159.69\ kg/s$



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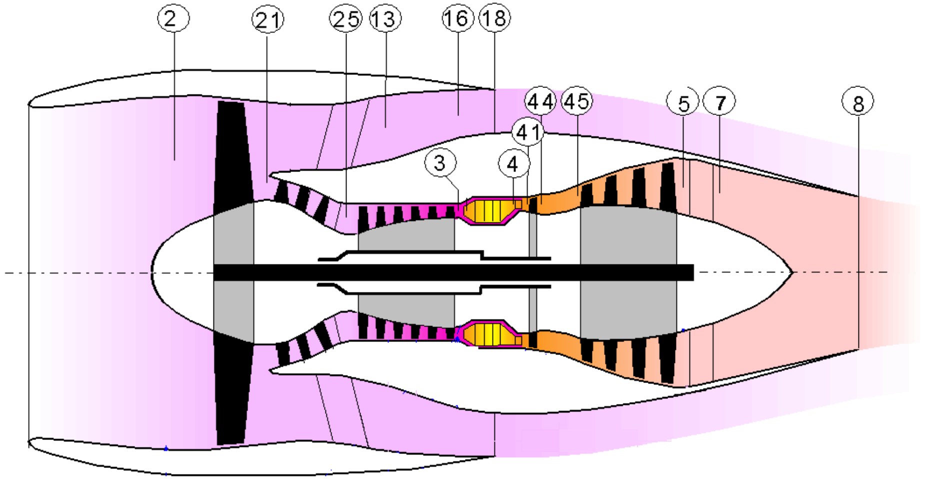
LEAP-1A Turbofan: Fan Calculations

$$\frac{T_{t,21}}{T_{t,2}} = 1 + \frac{1}{\eta_{is}}\left[\left(\frac{p_{t,21}}{p_{t,2}}\right)^{\left(\frac{\kappa_a - 1}{\kappa_a}\right)} - 1\right]$$

- Pressure ratio = 1.4, $\eta_{is,f} = 0.90$
- $p_{t,13} = p_{t,21} = 1.4 * p_{t,2} = 48892.05\ pa$
- $T_{t,13} = T_{t,21} = 272.94\ K$
- $\dot{m}_{21} = 13.31\ kg/s$
- $\dot{m}_{13} = 159.69\ kg/s$
- Work done on fan

$$W_{fan} = \dot{m}_2 C_{p,a}\left(T_{t,21} - T_{t,2}\right)$$
$$= 4.76\ MW$$



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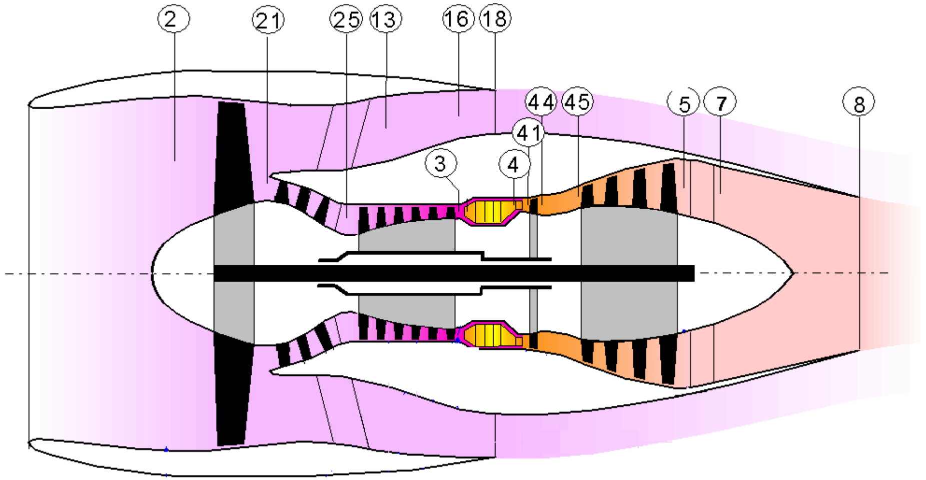
LEAP-1A Turbofan: LPC Calculations

$$\frac{T_{t,25}}{T_{t,21}} = 1 + \frac{1}{\eta_{is}}\left[\left(\frac{p_{t,25}}{p_{t,21}}\right)^{\left(\frac{\kappa_a - 1}{\kappa_a}\right)} - 1\right]$$

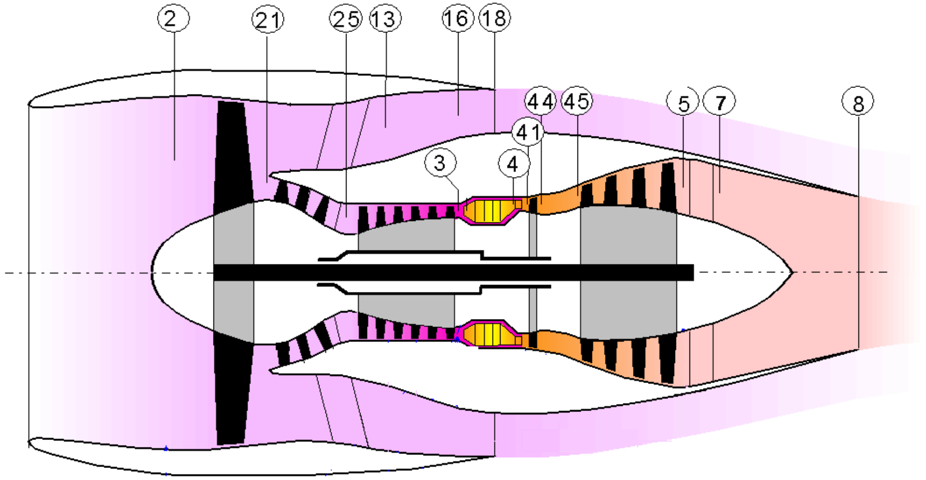
- Pressure ratio = 1.7, $\eta_{is,LPC} = 0.92$
- $p_{t,25} = 1.7 * p_{t,21} = 83116.48\, pa$
- $T_{t,25} = 321.51K$
- $\dot{m}_{25} = 13.31\, kg/s$
- Work done on LPC:
  - $W_{LPC} = \dot{m}_{25} \cdot C_{p,a} \cdot (T_{t,25} - T_{t,21})$
  - $W_{LPC} = 0.65\, MW$



LEAP-1A Turbofan: HPC Calculations

$$\frac{T_{t,3}}{T_{t,25}} = 1 + \frac{1}{\eta_{is}}\left[\left(\frac{p_{t,3}}{p_{t,25}}\right)^{\left(\frac{\kappa_a - 1}{\kappa_a}\right)} - 1\right]$$

- Pressure ratio = 12.5, $\eta_{is,HPC} = 0.92$
- $p_{t,3} = 12.5 * p_{t,25} = 1038955.98\ pa$
- $T_{T,3} = 691.17\ K$
- $\dot{m}_3 = 13.31\ kg/s$
- Work done on HPC:
  - $W_{HPC} = \dot{m}_3 \cdot C_{p,a} \cdot \left(T_{t,3} - T_{t,25}\right)$
  - $W_{HPC} = 4.92\ MW$



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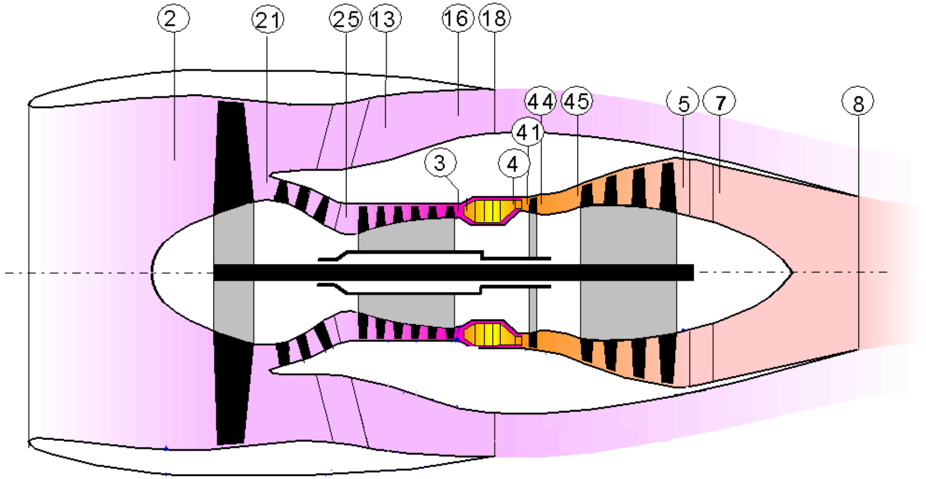
$$Heat\ added = \dot{m} * (h_{out} - h_{in}) = \dot{m} * c_p\,(T_{out} - T_{in}) = \dot{m}_f * LHV * \eta_{cc}$$

$$\rightarrow \dot{m}_f = \frac{\dot{m}_3 * c_P * (T_{t,4} - T_{t,3})}{\eta_{cc} * LHV}$$

LEAP-1A Turbofan: Combustion

- Turbine inlet temperature: $T_{t,4} = 1400\ K$
- Combustion efficiency = 0.995
- Combustion Chamber Pressure ratio = 0.96
- Fuel flow required:
  - $\dot{m}_f = \frac{\dot{m}_3 * C_{p,g} * (T_{t,4} - T_{t,3})}{\eta_{cc} * LHV} = 0.25\ kg/s$
- $\dot{m}_4 = \dot{m}_3 + \dot{m}_f = 13.56\ kg/s$
- $p_{t,4} = 0.96 * p_{t,3} = 997397.74\ pa$



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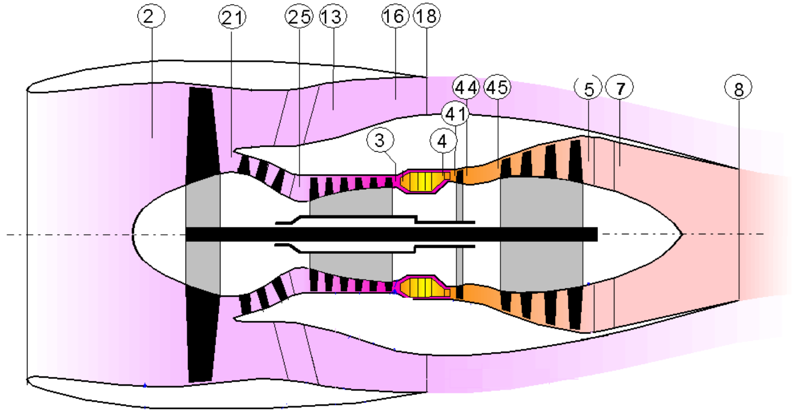
LEAP-1A Turbofan: HPT Calculations

$$\text{Work done} = \dot{m} * \Delta h$$
$$= \dot{m} * c_p\,(\Delta T)$$

$$\frac{T_{t,45}}{T_{t,41}} = 1 - \eta_{is}\left[1 - \left(\frac{p_{t,45}}{p_{t,41}}\right)^{\left(\frac{\kappa_g - 1}{\kappa_g}\right)}\right]$$

- <u>**HP turbine drives the HP compressor**</u>
- Work done on HPC, $W_{HPC} = 4.92\ MW$
- Delivered by the HPT with $\eta_{m,HPS} = 0.99$
- $\dot{m}_4 \cdot C_{p,g} \cdot \left(T_{t,4} - T_{t,45}\right) = W_{HPT} = \frac{W_{HPC}}{\eta_m}$
- $T_{t,4} - T_{t,45} = \frac{W_{HPC}}{\eta_m \dot{m}_4 C_{p,g}} = 318.69K \rightarrow T_{t,45} = 1081.31K$
- With $\eta_{is,HPT} = 0.9$
- $p_{t,45} = 307996.88\ pa \quad \dot{m}_{45} = 13.56\ kg/s$



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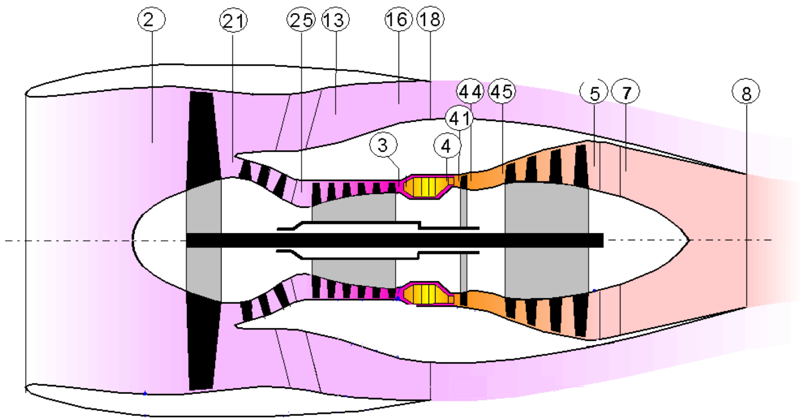
LEAP-1A Turbofan: LPT Calculations

$$Work\ done = \dot{m} * \Delta h$$
$$= \dot{m} * c_p (\Delta T)$$

$$\frac{T_{t,5}}{T_{t,45}} = 1 - \eta_{is}\left[1 - \left(\frac{p_{t,5}}{p_{t,45}}\right)^{\left(\frac{\kappa_g - 1}{\kappa_g}\right)}\right]$$



- <u>**LP turbine drives the LP compressor and fan**</u>
- Work required done on LPC, $W_{LPC} = 0.65\ MW$
- Work required by fan, $W_{fan} = 4.76\ MW$
- Delivered by LPT with $\eta_{m,LPS} = 0.99$
- $\dot{m}_{45} \cdot C_{p,g} \cdot \left(T_{t,45} - T_{t,5}\right) = W_{LPT} = \dfrac{W_{LPC} + W_{fan}}{\eta_m}$
- $T_{t,45} - T_{t,5} = \dfrac{W_{LPC} + W_{fan}}{\eta_m \cdot \dot{m}_{45} \cdot C_{p,g}} = 350.43\ K \rightarrow T_{t,5} = 730.88K$
- With $\eta_{is,LPT} = 0.90$; $p_{t,5} = 50947.09\ pa$; $\dot{m}_5 = 13.56\ kg/s$

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LEAP-1A Turbofan: Core Nozzle

- No change between station 5 and 7,

  $p_{t,7} = p_{t,5}$ and $T_{t,7} = T_{t,5}$

- Nozzle efficiency 0.98

- $p_{t,7}/p_a = \frac{50947.09}{23842} = 2.137 > 1.876$; Therefore ***the nozzle is choked***

- Static temperature and pressure at the nozzle exit:

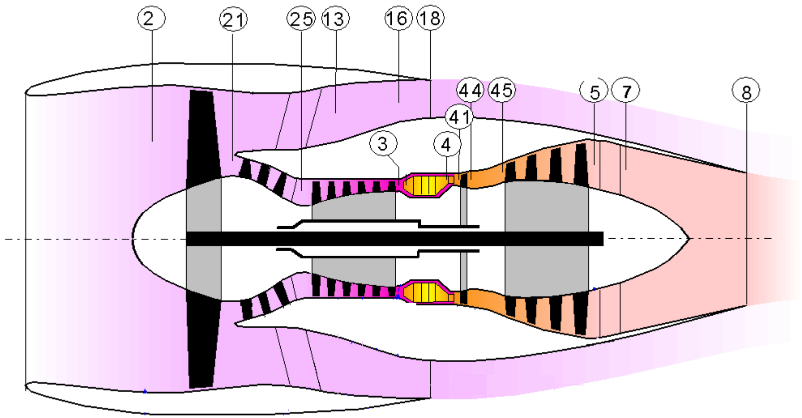
- $T_8 = T_{t,7}\left(\frac{2}{\kappa_g+1}\right) = 627.36\ K$

- $p_8 = \frac{p_{t,8}}{1.876} = 27157.30\ Pa$

Critical Pressure for Core Nozzle

$$\frac{p_{t,7}}{p_{critcal}} = \left(1 - \frac{1}{\eta_n}\left(\frac{\kappa_g - 1}{\kappa_g + 1}\right)\right)^{\left(\frac{-\kappa_g}{\kappa_g - 1}\right)}$$

$$= 1.876$$



LEAP-1A Turbofan: Core Nozzle

- The nozzle is choked, therefore, $v_8 = \sqrt{\kappa_g R T_8} = 489.36\, m/s$
- $\rho_8 = \frac{p_8}{R * T_8} = 0.151 kg/m^3$
- $A_8 = \frac{\dot{m}_8}{\rho_8 v_8} = 0.184\, m^2$
- With the flight speed,$v_{\infty} = M * \sqrt{\kappa_a R T_a} = 231.27 m/s$
- Hence, the thrust generated in the core is given as;
- $F_{core} = \dot{m}_8 (v_8 - v_{\infty}) + A_8 (p_8 - p_a)$
- $F_{core} = 3499.70 + 610.02 = 4109.72\, N$

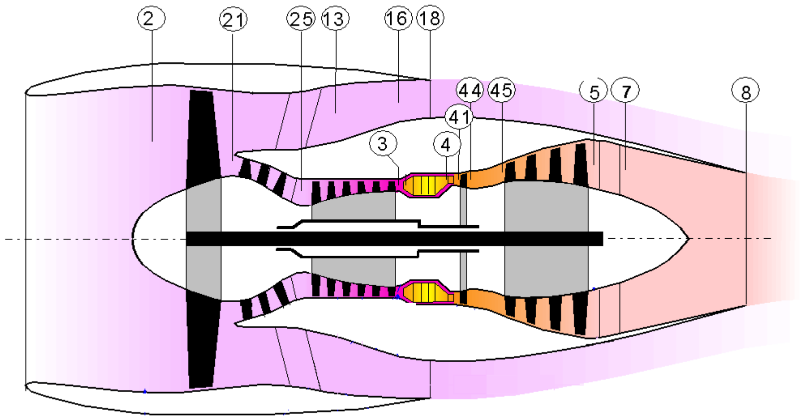
LEAP-1A Turbofan: Bypass Nozzle

- No change between station 13 and 16
  - $p_{t,16} = p_{t,13}$ and $T_{t,16} = T_{t,13}$
- Nozzle efficiency 0.98.
- $p_{t,16}/p_a = \frac{48892.05}{23842} = 2.05 > 1.92$, Therefore ***the nozzle is choked***
- Static temperature and pressure at the nozzle exit:
- $T_{18} = T_{t,18}\left(\frac{2}{\kappa_a+1}\right) = 227.45K$
- $p_{18} = \frac{p_{t,16}}{1.92} = 25464.61\ Pa$

Critical Pressure for Bypass Nozzle

$$\frac{p_{t,16}}{p_{critcal}} = \left(1 - \frac{1}{\eta_n}\left(\frac{\kappa_a - 1}{\kappa_a + 1}\right)\right)^{\left(\frac{-\kappa_a}{\kappa_a - 1}\right)}$$

$$= 1.920$$



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LEAP-1A Turbofan: Bypass Nozzle

- The nozzle is choked, therefore, $v_{18} = \sqrt{\kappa_a R T_{18}} = 296.5\ m/s$
- $\rho_{18} = \frac{p_{18}}{R * T_{18}} = 0.390\ kg/m^3$
- $A_{18} = \frac{\dot{m}_{18}}{\rho_{18} v_{18}} = 1.38\ m^2$
- With the flight speed, $v_\infty = M * \sqrt{\kappa_a R T_a} = 231.27\ m/s$
- Hence, the thrust generated in the bypass is given as;
- $F_{bypass} = \dot{m}_{18}(v_{18} - v_\infty) + A_{18}(p_{18} - p_a)$
- $F_{bypass} = 10416.58 + 2239.20 = 12655.78\ N$

LEAP-1A Turbofan: Overall Performance

- Total Thrust generated by the engine:
  - $F_N = F_{core} + F_{bypass} = 4.11 + 12.66 = 16.77\ kN$
- Thrust Specific Fuel Consumption (TSFC):
  - $TSFC = \frac{\dot{m}_f}{F_N} = \frac{0.25\ kg/s}{17.24\ kN} = 14.91\ {}^{g}/_{kN.s}$

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