



# Maratona **Cln**

**SELETIVA 2020**

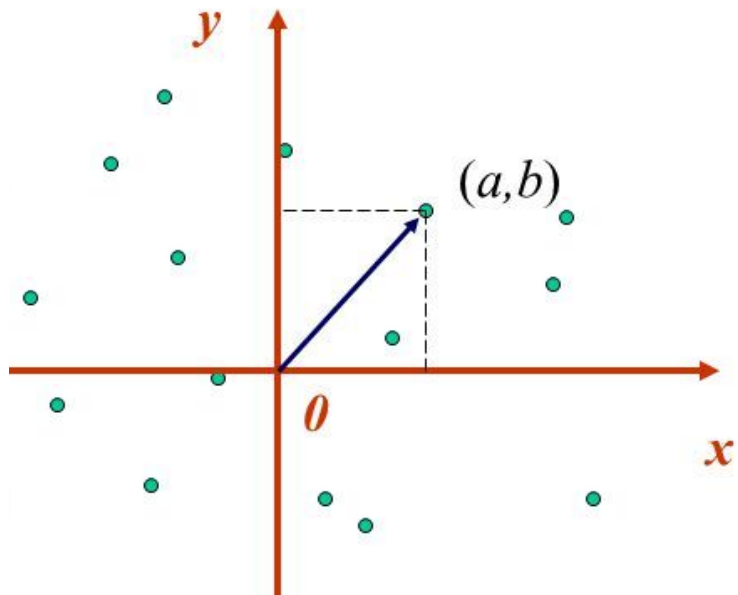
# **GEOMETRIA**

## **Aula 10**

# GEOMETRIA

## Vetor

- ❖ Coordenadas
  - $(a, b)$
  - $(x, y, z)$
  - `vector<tipo> X`



# GEOMETRIA

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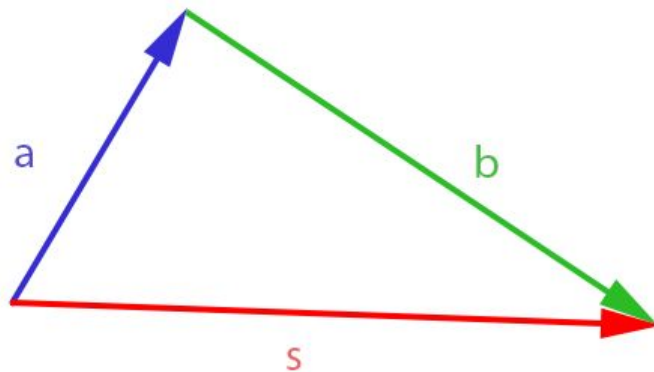
## Vetor

- ❖ Soma de vetores

$$a + b = (a.x + b.x, a.y + b.y)$$

- ❖ Subtração de vetores

$$a - b = (a.x - b.x, a.y - b.y)$$



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## Vetor

❖ Multiplicação por escalar

❖ Divisão por escalar

$$a * k = (a.x * k, a.y * k)$$

$$a / k = (a.x / k, a.y / k)$$



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## Vetor

### ❖ Magnitude

$$\sqrt{x^2 + y^2}$$

// Vetores 2D

$$\sqrt{x^2 + y^2 + z^2}$$

// Vetores 3D

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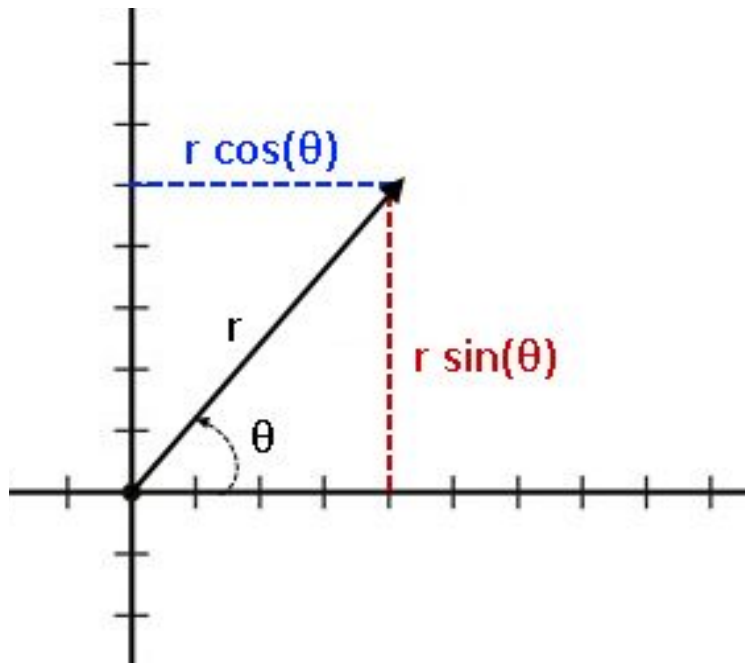
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## Vetor

❖ Ângulo

$$\Theta = \text{atan2}(y, x)$$

// Retorna de  $-\pi$  a  $\pi$



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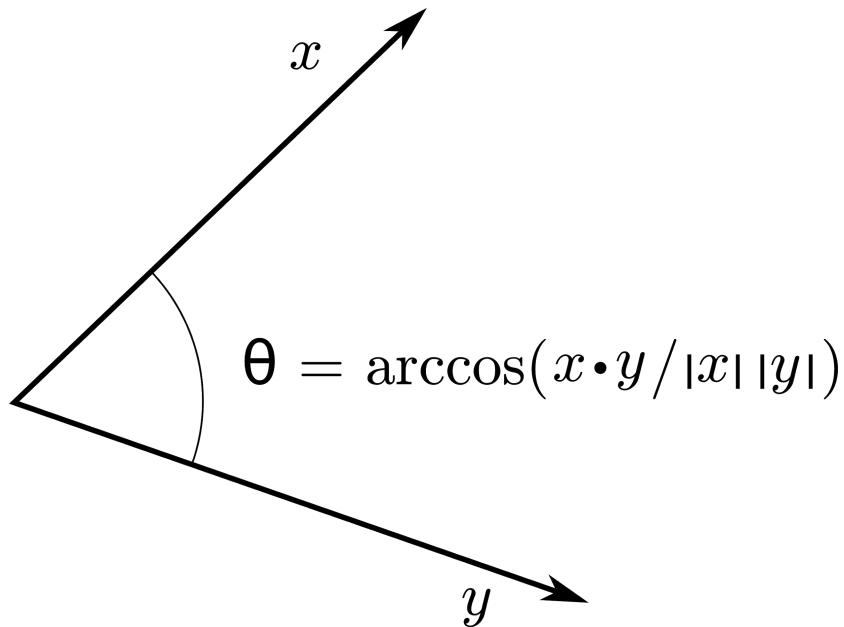


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## Vetor

### ❖ Produto Escalar

$$a \cdot b = (a.x * b.x) + (a.y * b.y)$$





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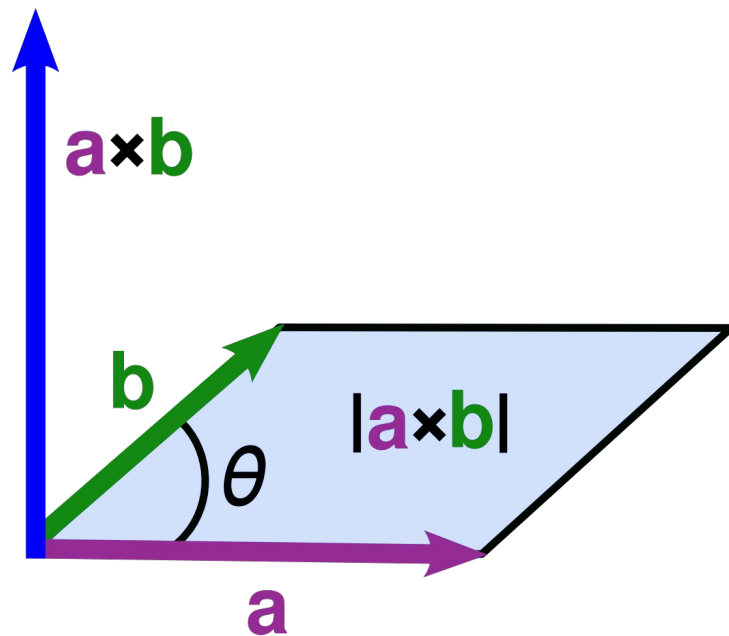


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## Vetor

### ❖ Produto Vetorial

$$a \times b = (a.x * b.y) - (a.y * b.x)$$



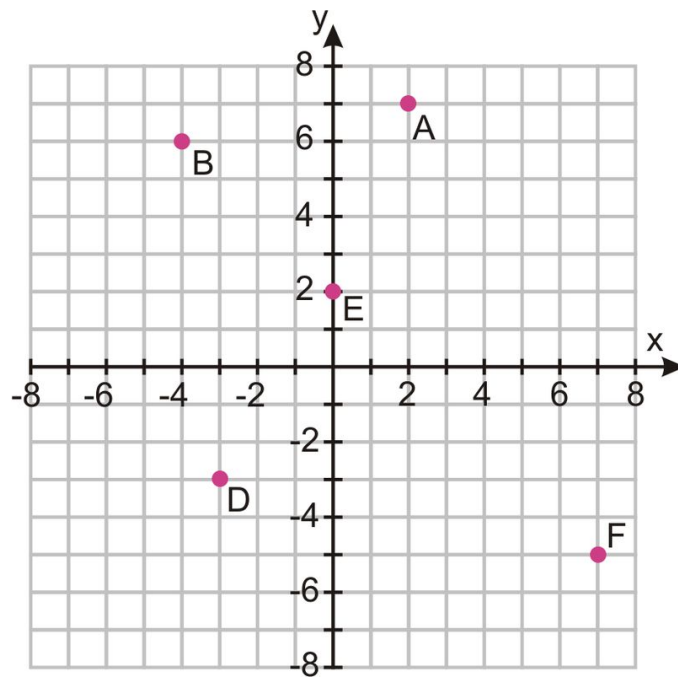
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## Ponto



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## Ponto

```
ld x, y;
```

```
PT(ld x = 0, ld y = 0): x(x), y(y) {}
```

```
PT operator + (const PT P){  
    return PT(x + P.x, y + P.y);}
```

```
PT operator - (const PT P){  
    return PT(x-P.x, y-P.y);}
```

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## Ponto

```
PT operator * (const ld d){  
    return PT(x*d, y*d);}
```

```
PT operator / (const ld d){  
    return PT(x/d, y/d);}
```

```
ld operator * (const PT P){  
    return x*P.x + y*P.y;}
```

```
ld operator ^ (const PT P){  
    return x*P.y - y*P.x;}
```

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## Ponto

```
ld dist(){  
    return sqrt(x*x+y*y);}  
  
ld angle(){  
    ld a = atan2l(y, x);  
    if(a<0) a += 2*PI;  
    return a;}
```

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## Ponto

- ❖ Distância euclidiana entre P e Q

$(P-Q).dist()$

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## Ponto

- ❖ Distância de Manhattan entre P e Q

$$\text{abs}(P.x - Q.x) + \text{abs}(P.y - Q.y)$$

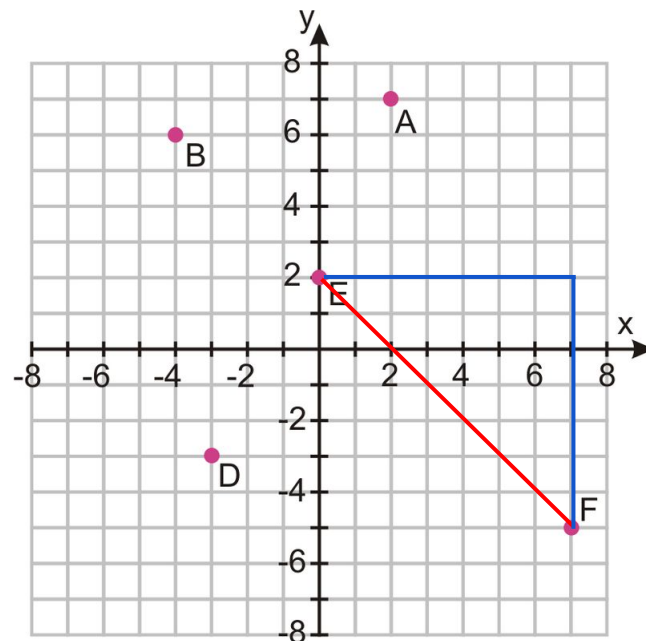
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## Ponto

- ❖ Distância **euclidiana**
- ❖ Distância de **Manhattan**





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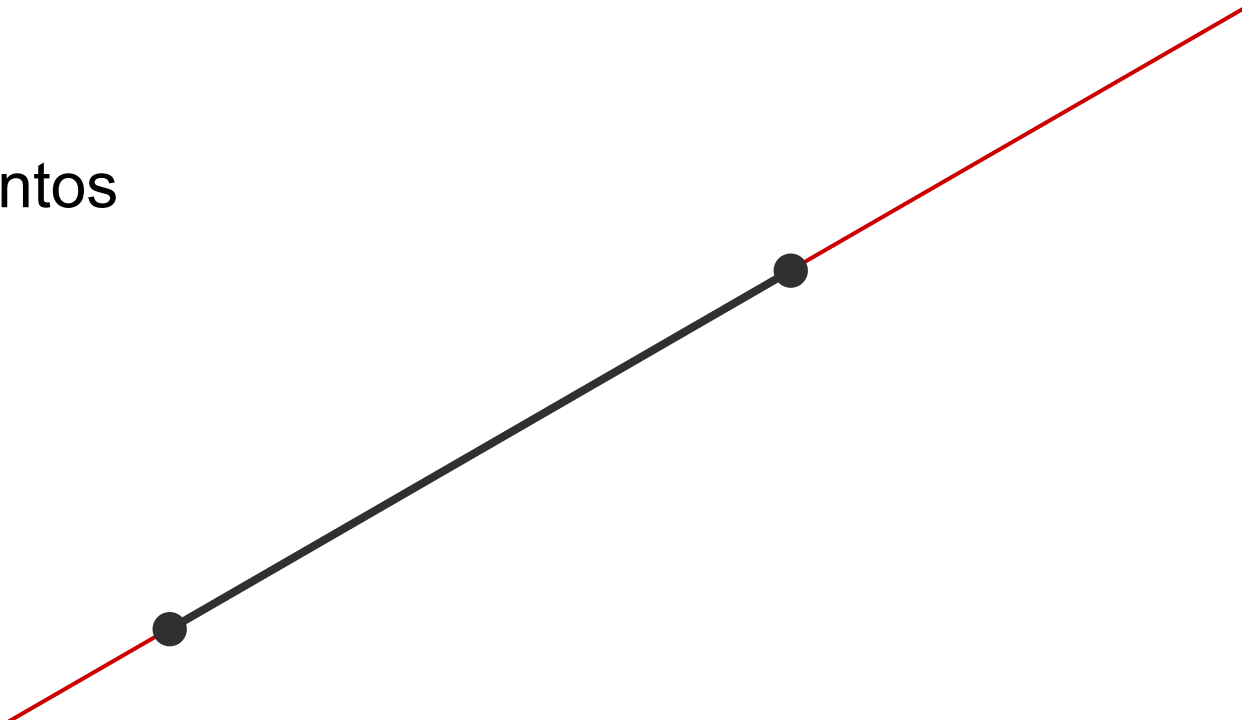


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## Reta

❖ Dois pontos distintos

- A
- B



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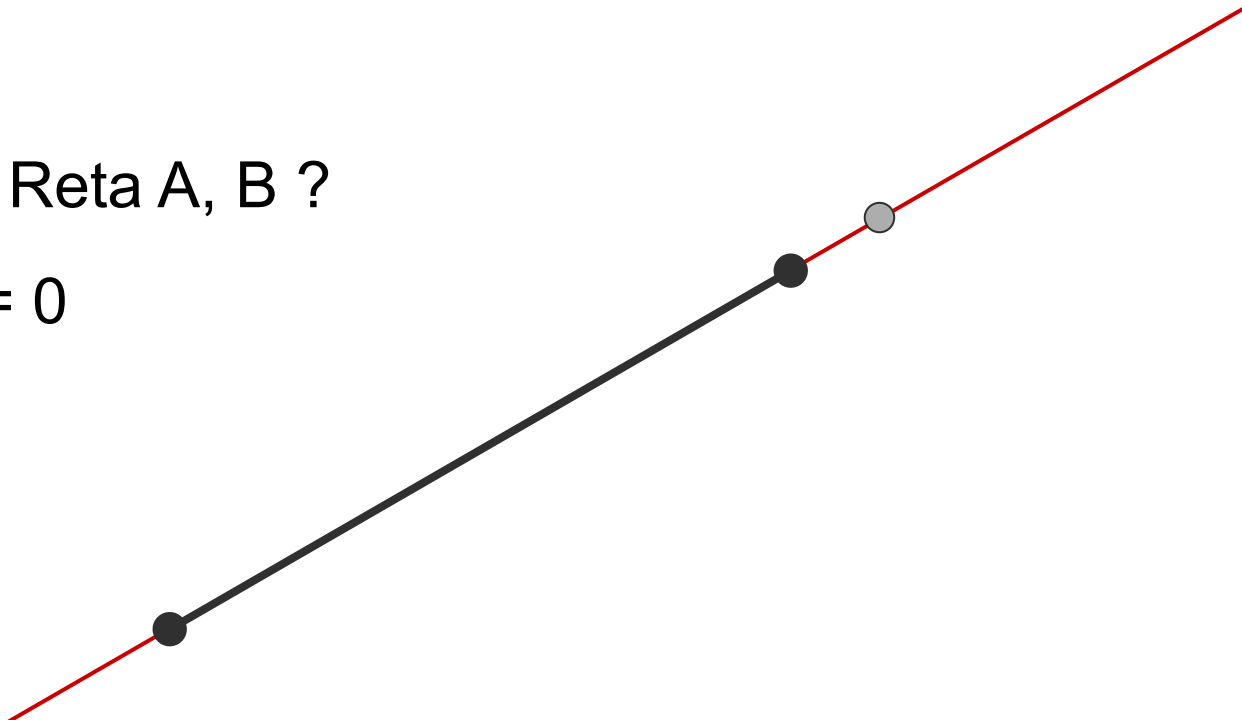


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## Reta

❖ Ponto C sobre a Reta A, B ?

$$(C - A) \wedge (B - A) = 0$$



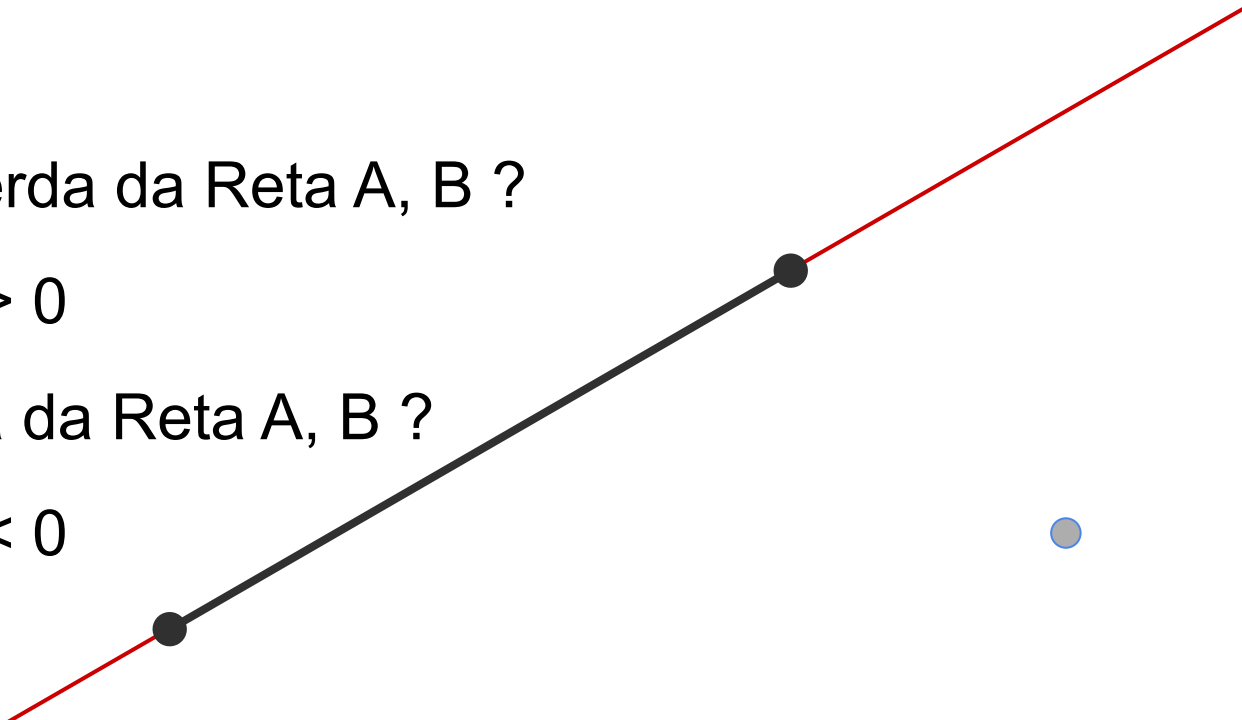
## Reta

- ❖ Ponto C à esquerda da Reta A, B ?

$$(C - A) \wedge (B - A) > 0$$

- ❖ Ponto C à direita da Reta A, B ?

$$(C - A) \wedge (B - A) < 0$$



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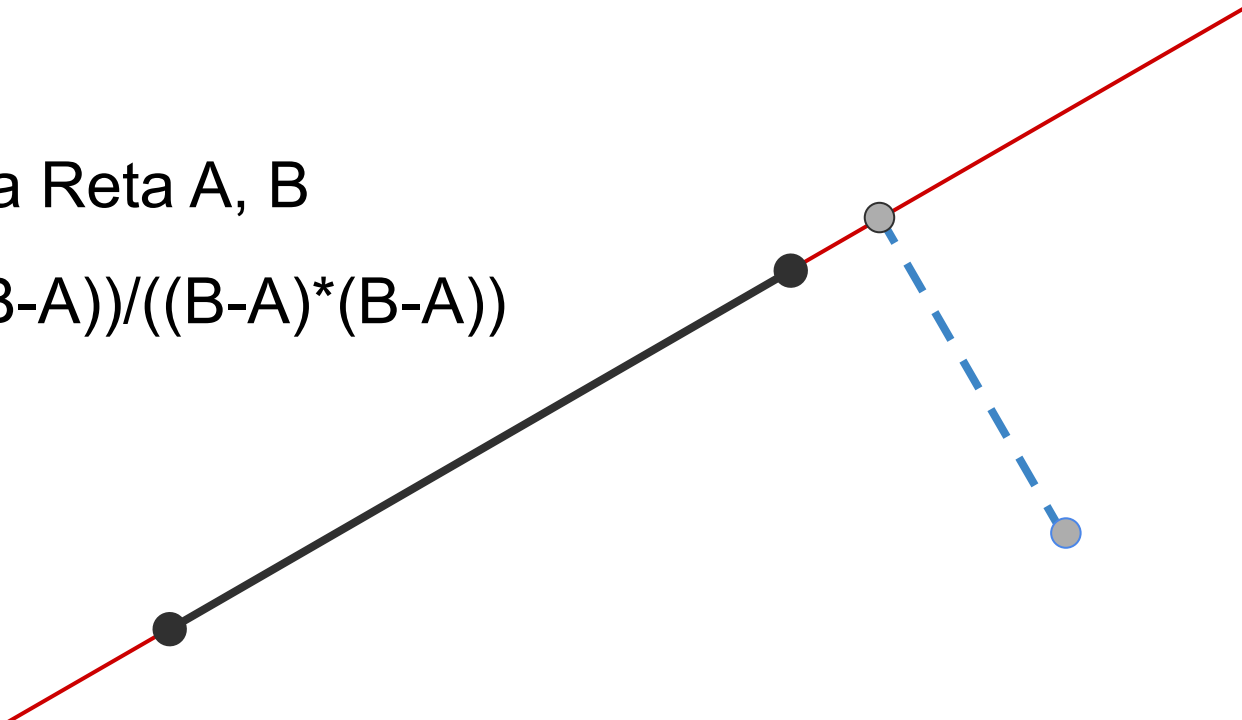


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## Reta

❖ Projeção de C na Reta A, B

$$A + (B-A) * ((C-A)*(B-A))/((B-A)*(B-A))$$



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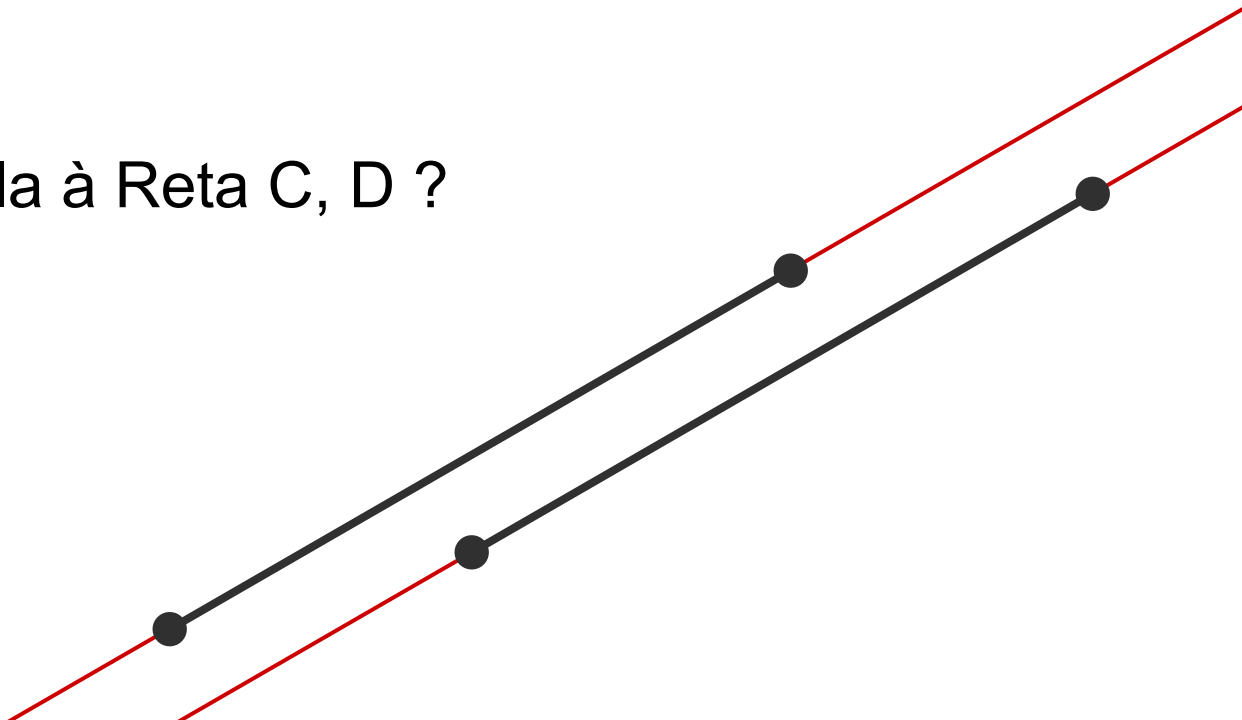


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## Reta

- ❖ Reta A, B paralela à Reta C, D ?

$$(B-A)^{(C-D)} = 0$$



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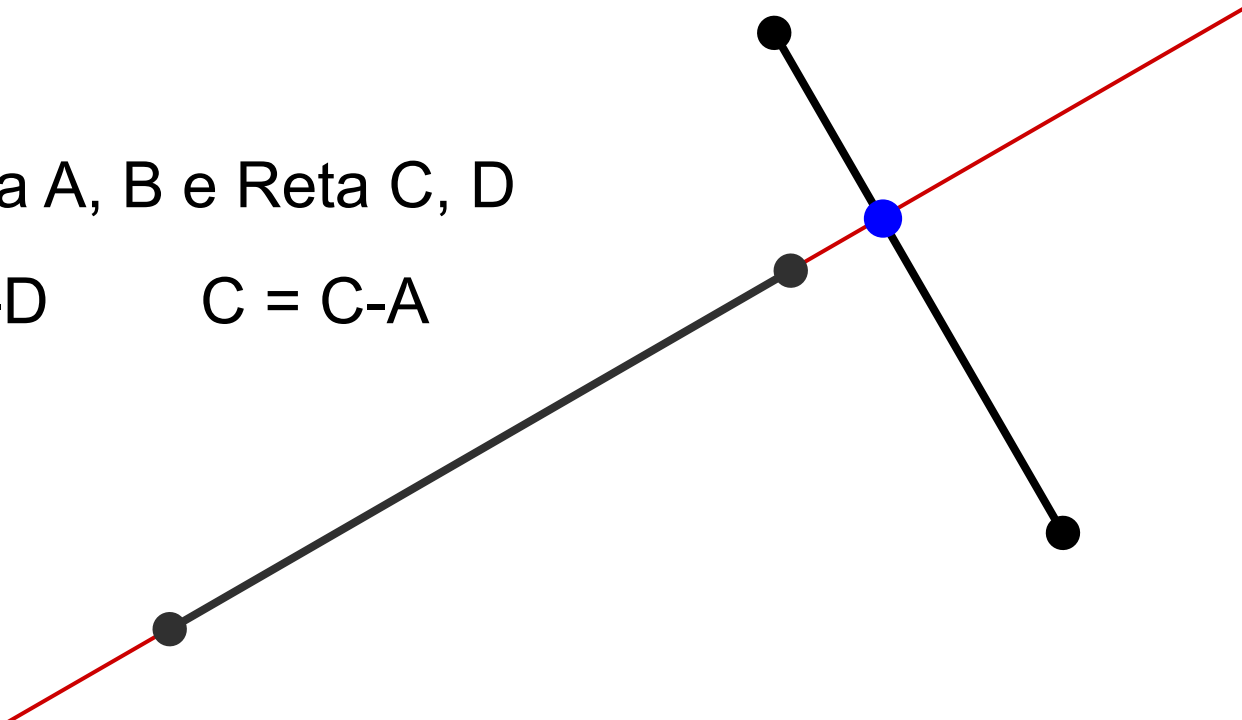
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## Reta

❖ Cruzamento Reta A, B e Reta C, D

$$B = B - A \quad D = C - D \quad C = C - A$$

$$A + B * (C^D) / (B^D)$$



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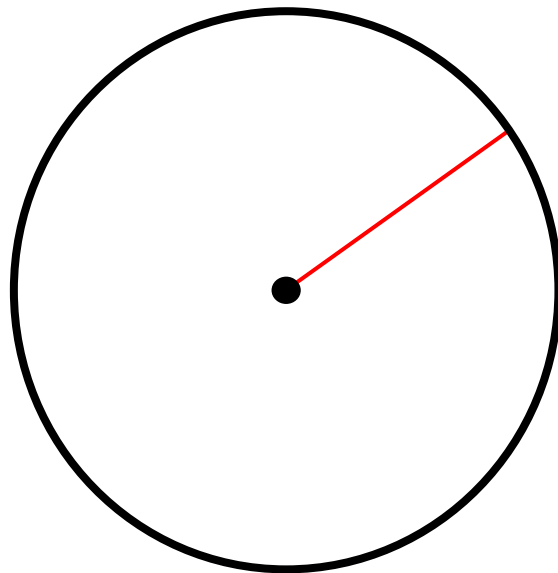
## Círculo

❖ Centro

- P

❖ Raio

- R



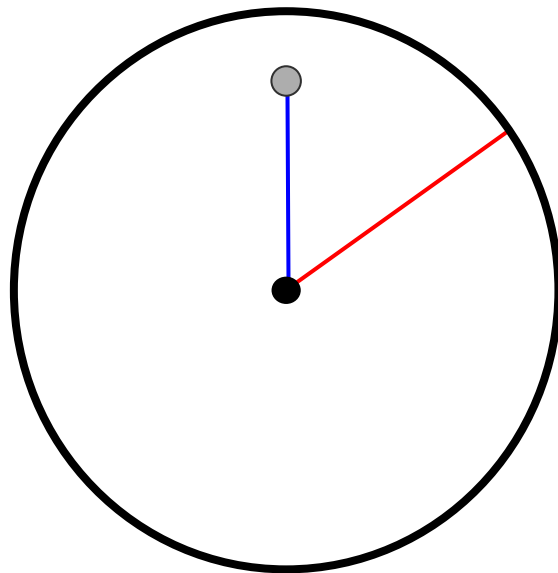
# GEOMETRIA

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## Círculo

❖ Ponto Q dentro do círculo C ?

$$D < R$$





# GEOMETRIA

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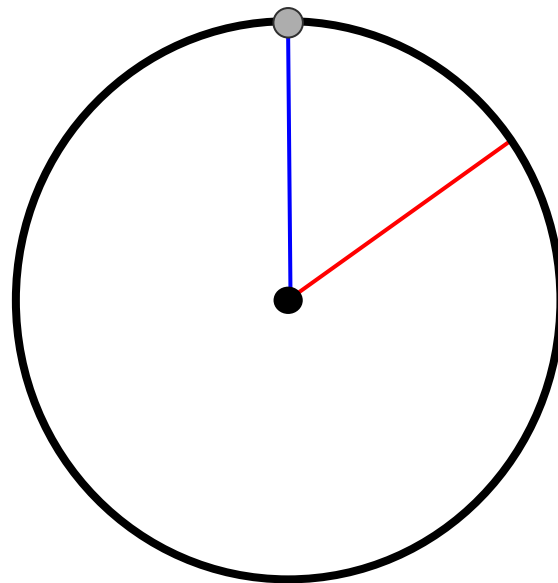


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## Círculo

❖ Ponto Q dentro do círculo C ?

$$D = R$$



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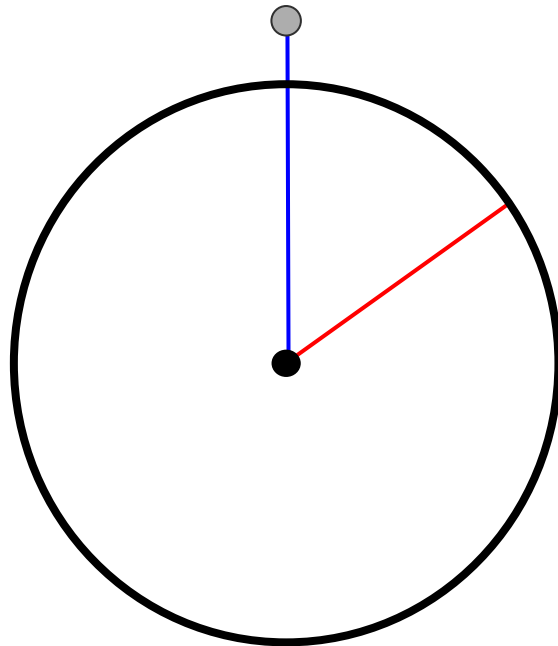


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## Círculo

❖ Ponto Q dentro do círculo C ?

$$D > R$$



# GEOMETRIA

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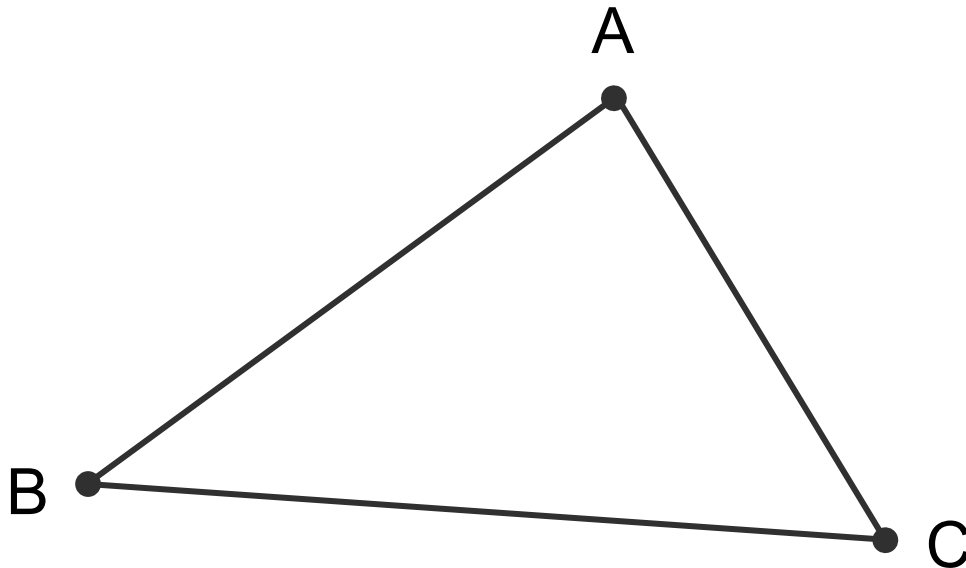


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## Triângulo

❖ 3 pontos

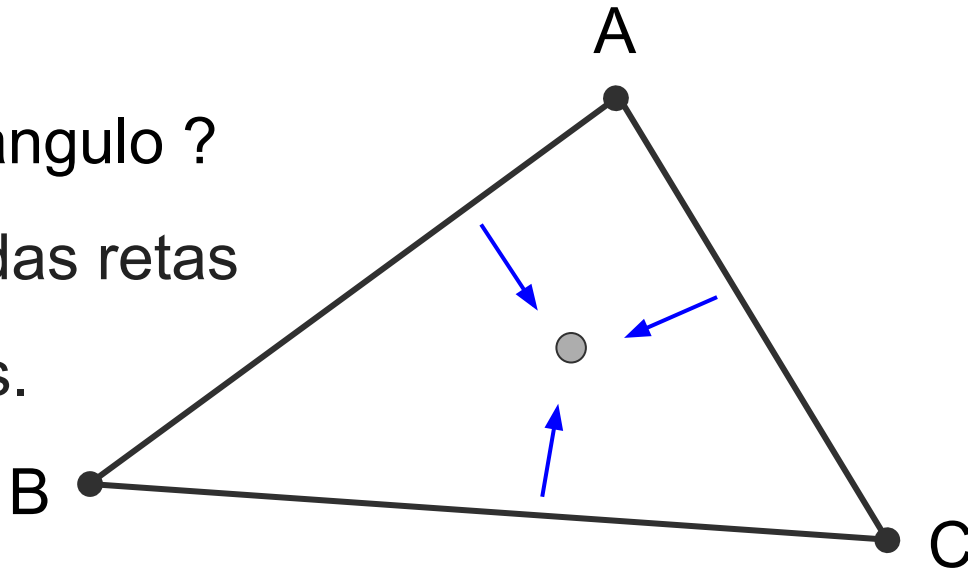
- A
- B
- C



## Triângulo

❖ Ponto dentro do triângulo ?

À direita ou esquerda das retas  
que compõem os lados.



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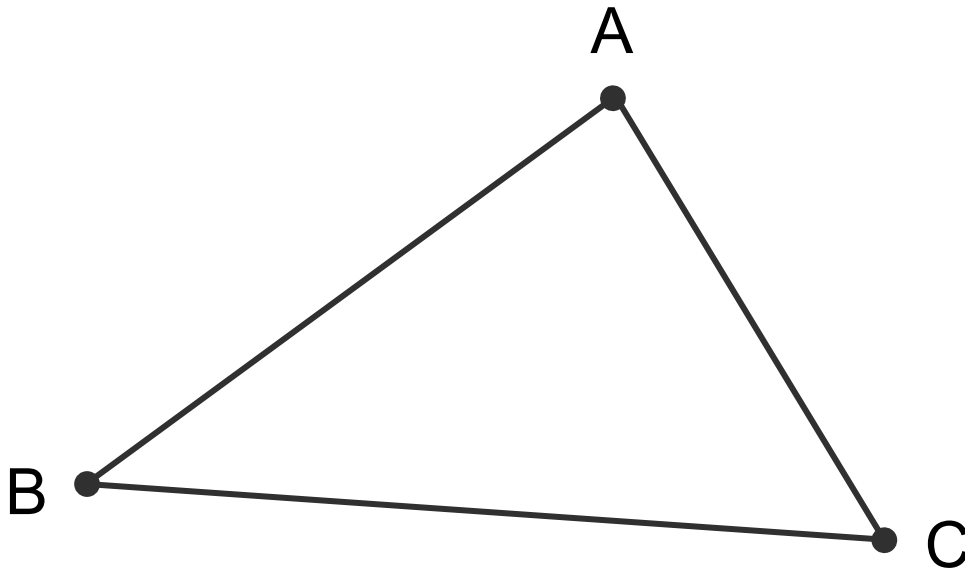


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## Triângulo

❖ Área

$$- |(B-A)^{\wedge}(C-A)| / 2$$

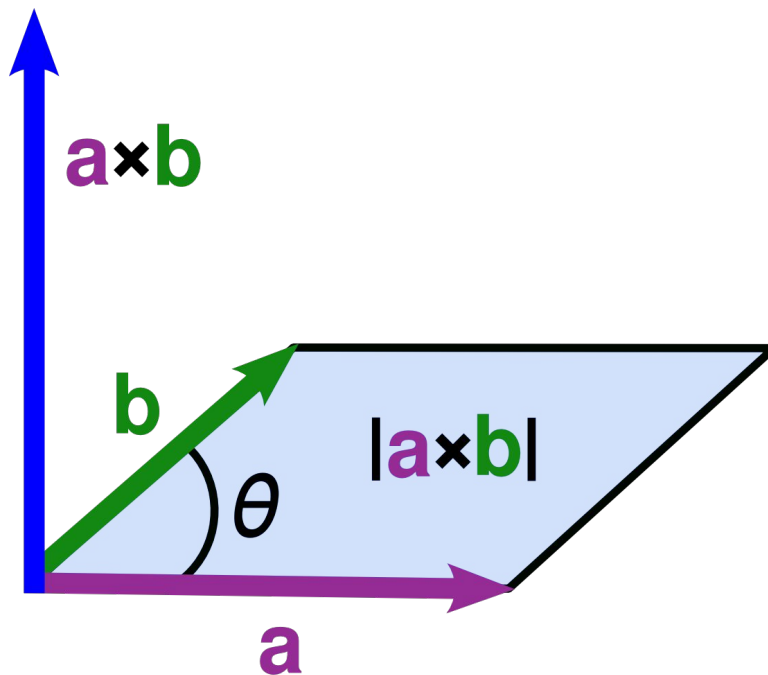


# GEOMETRIA

## Triângulo

❖ Área

$$- |(B-A) \wedge (C-A)| / 2$$



# GEOMETRIA

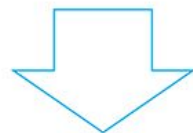
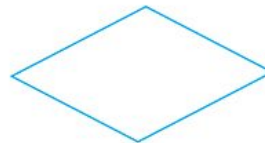
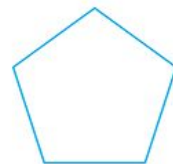
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## Polígonos

- ❖ Pontos que formam o polígono
  - Vetor de pontos  $P$



# GEOMETRIA

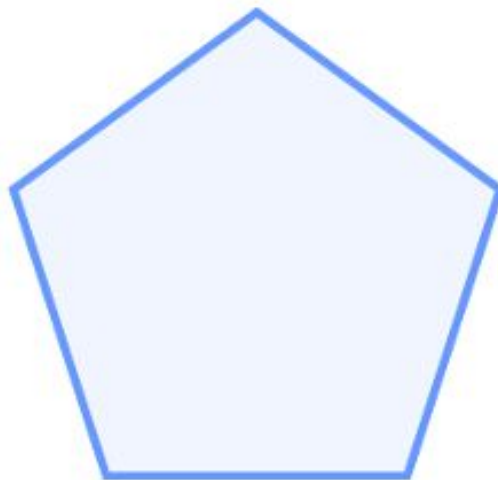
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## Polígonos

❖ Convexos x Côncavos





# GEOMETRIA

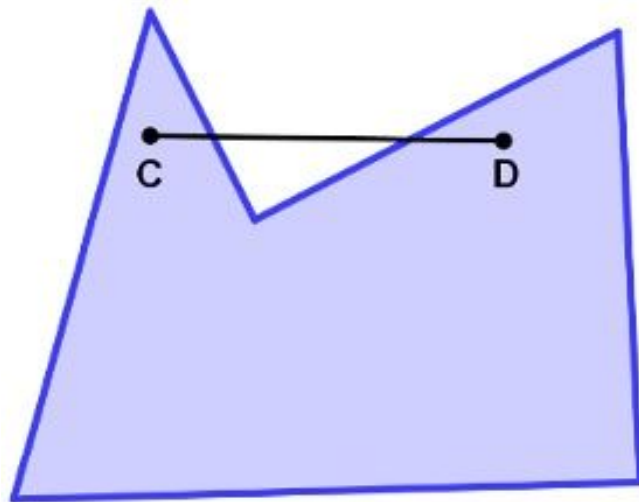
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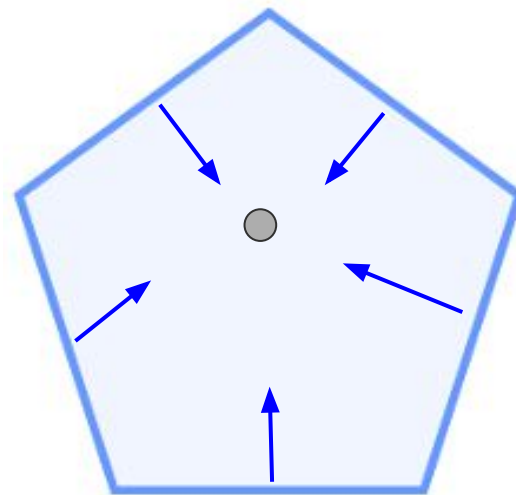
## Polígonos

❖ Convexos x Côncavos



## Polígonos

- ❖ Ponto dentro do polígono convexo ?
  - Mesma estratégia do triângulo

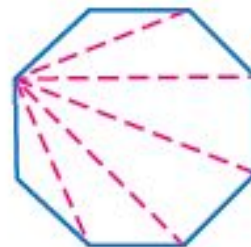
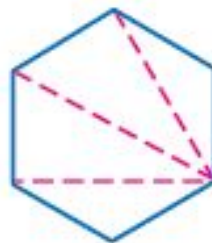
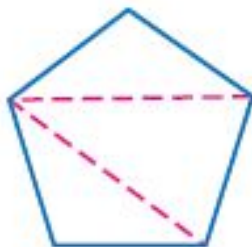
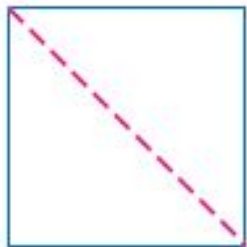


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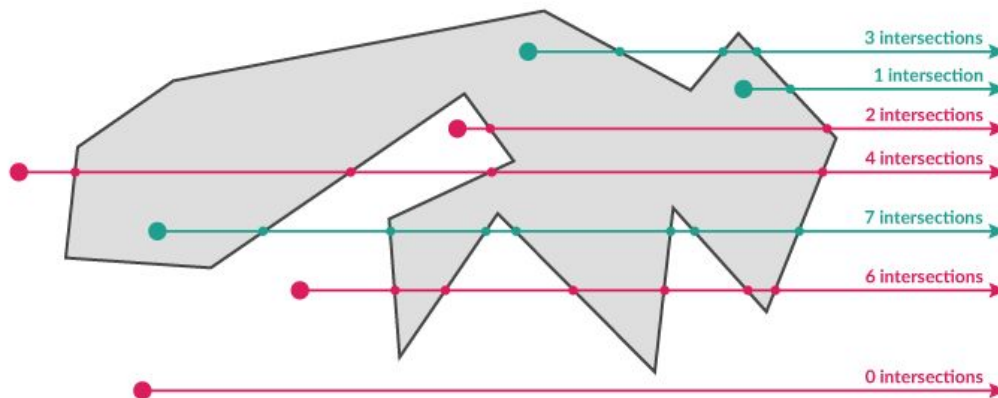
## Polígonos

❖ Ponto dentro do polígono convexo ?



## Polígonos

❖ Ponto dentro do polígono ?

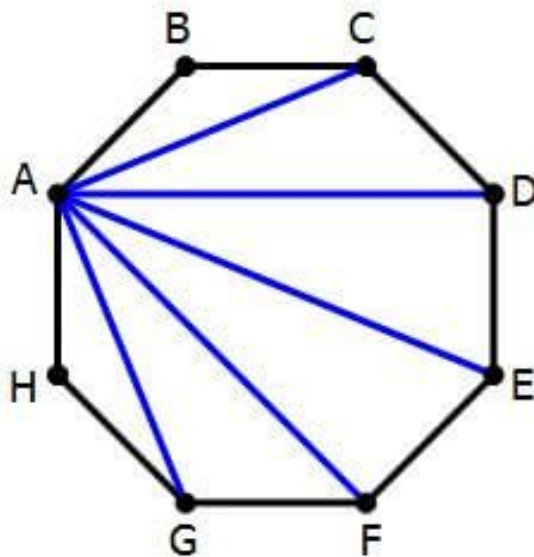


# GEOMETRIA

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## Polígonos

❖ Área do polígono



# GEOMETRIA

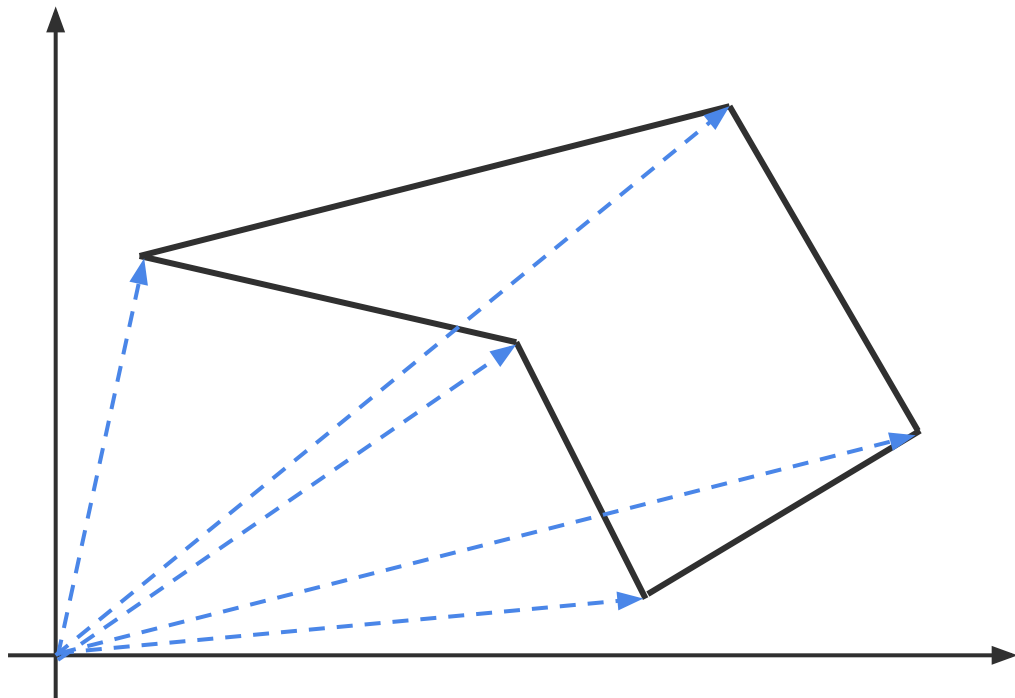
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## Polígonos

❖ Área do polígono



# GEOMETRIA

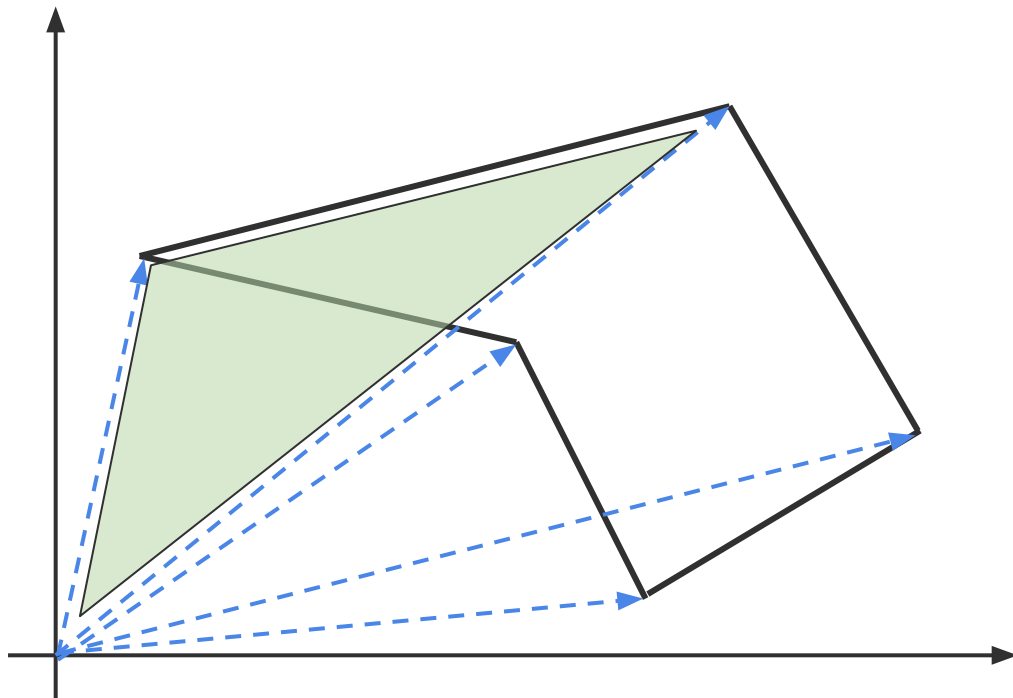
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## Polígonos

❖ Área do polígono



# GEOMETRIA

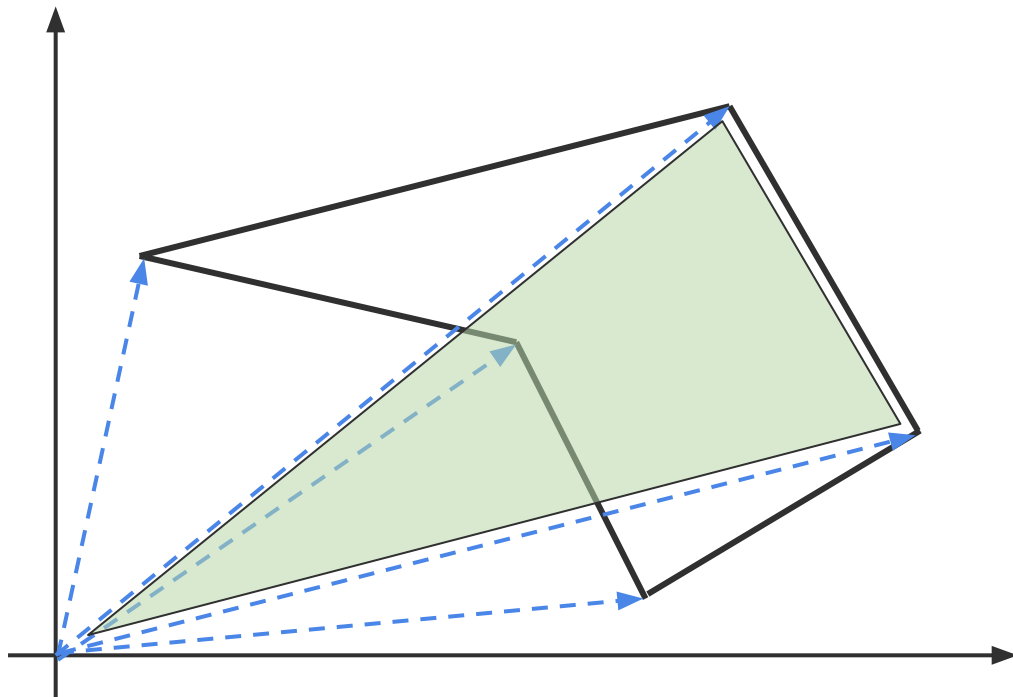
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## Polígonos

❖ Área do polígono





# GEOMETRIA

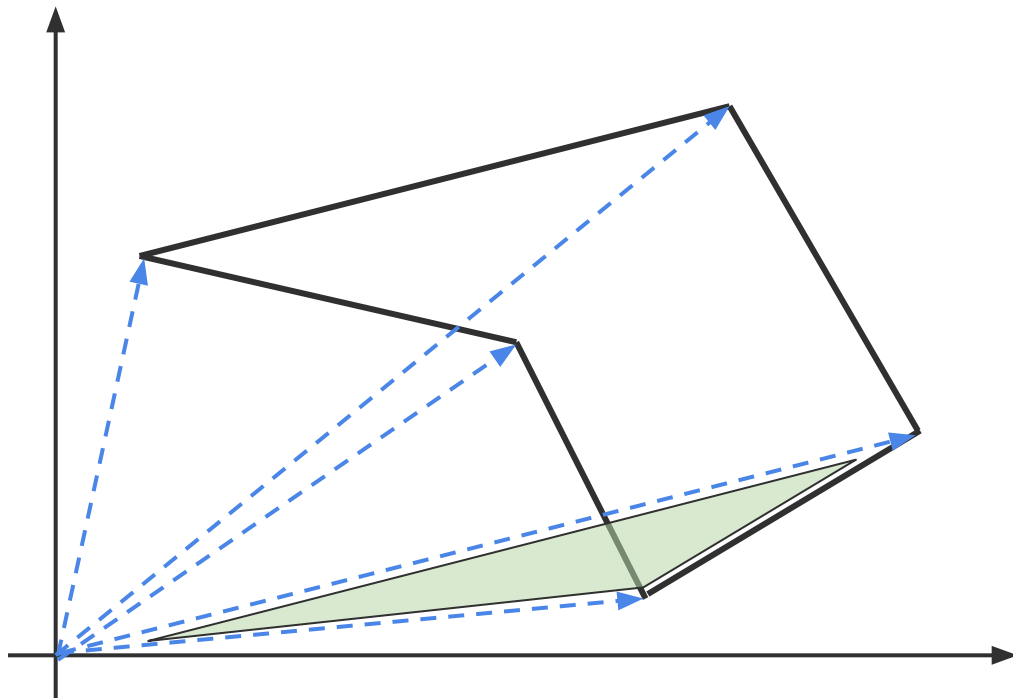
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## Polígonos

❖ Área do polígono



# GEOMETRIA

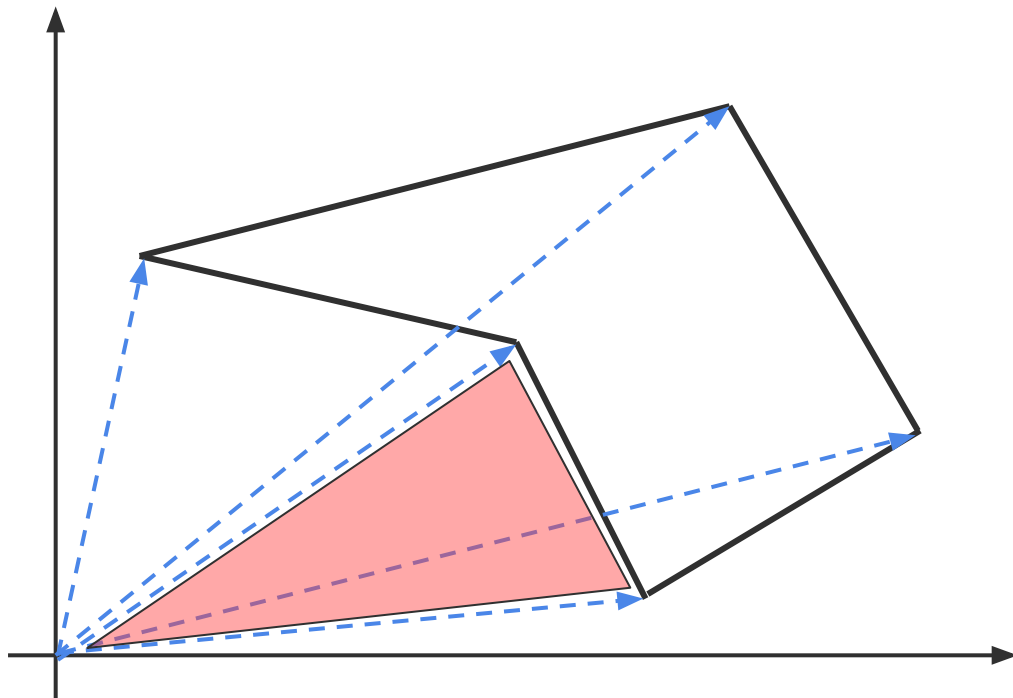
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## Polígonos

❖ Área do polígono



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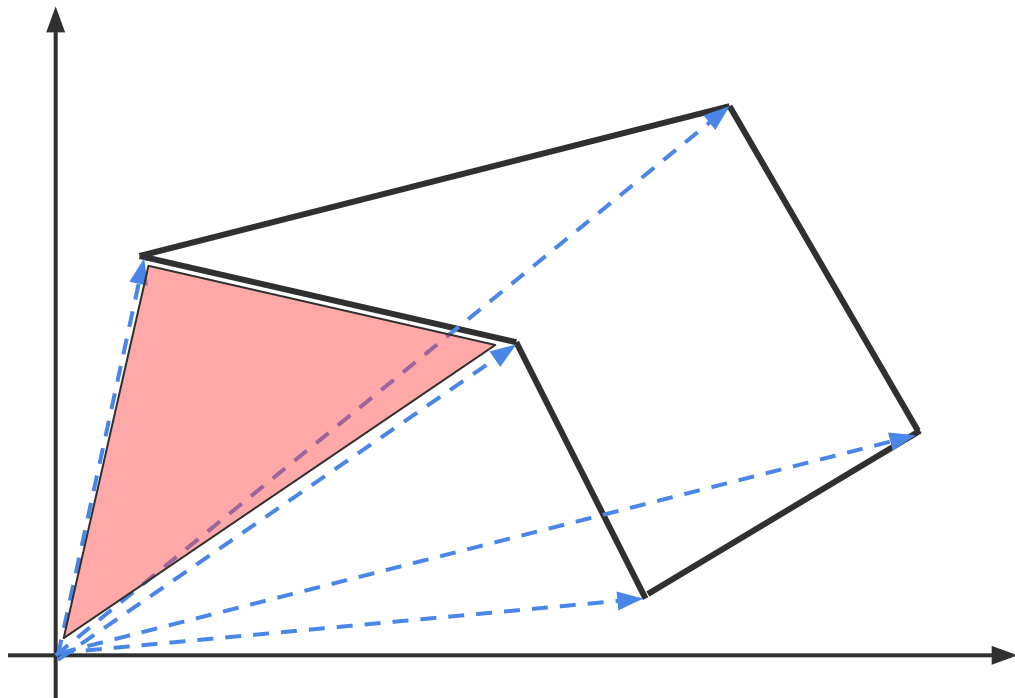
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## Polígonos

❖ Área do polígono



# GEOMETRIA

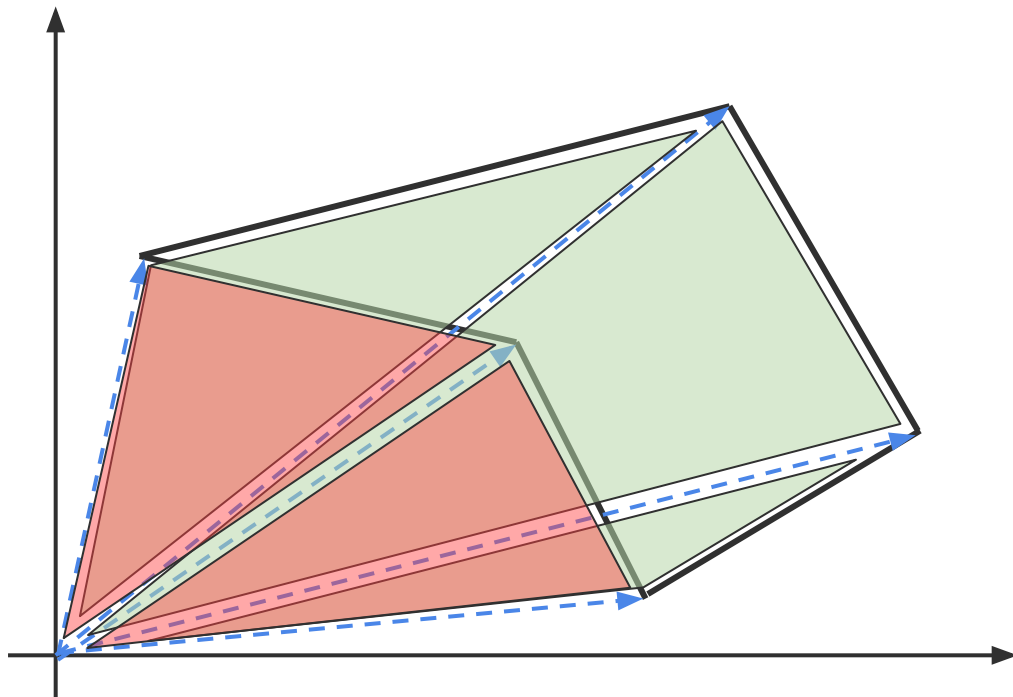
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## Polígonos

❖ Área do polígono



## Considerações finais

### ❖ Erros de precisão

- Sempre que possível, use int ou long long
- Caso precise usar double, adicione um l nas funções

Ex: `senl(x)`, `cosl(x)`, `atan2l(y,x)`



*"That's all Folks!"*